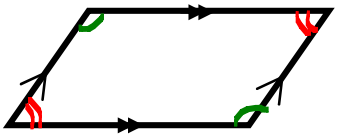
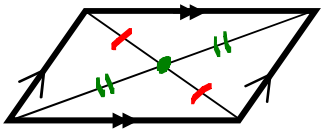
Sec 2.10 Geometry – Quadrilateral Properties Name:

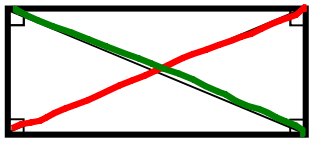
Name all of the properties of a parallelogram and its diagonals.





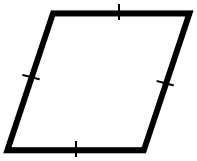
1. Opposite Sides are parallel
2. Opposite Sides are congruent
3. Opposite Angles are congruent
4. Consecutive Angles are supplementary
5. Diagonals bisect each other

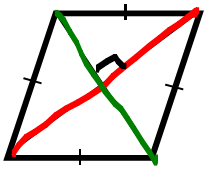
The properties of a rectangle and its diagonals:



1. All angles are right
2. Opposite Sides are parallel
3. Opposite Sides are congruent
4. Diagonals bisect each other
5. Diagonals are congruent

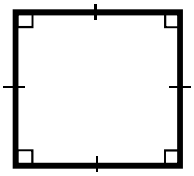
The properties of a rhombus and its diagonals:

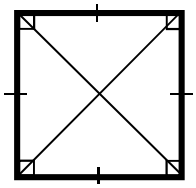




1. Opposite angles are congruent
2. Opposite Sides are parallel
3. All Sides are congruent
4. Consecutive angles are supplementary
5. Diagonals are perpendicular
6. Diagonals bisect interior angles.

The properties of a square and its diagonals:

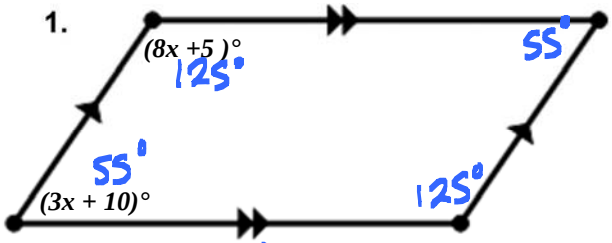




1. All angles are right
2. Opposite Sides are parallel
3. All Sides are congruent
4. Diagonals bisect each other
5. Diagonals are perpendicular
6. Diagonals bisect angles
7. Diagonals are congruent

Find the value of *x* in each diagram below using properties of quadrilaterals.

1.



$(3x+10)+(8x+5)=180$

$11x+15=180$

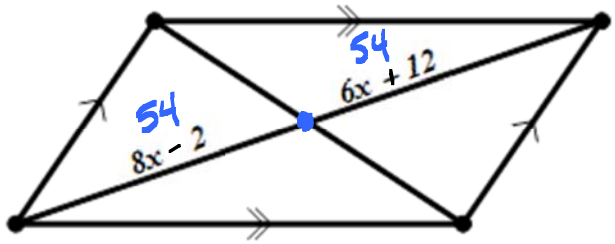
$-15 \quad -15$

$\frac{11x}{11} = \frac{165}{11}$

$x=15$

1. $x = 15$

2.



$8x-2=6x+12$

$-6x \quad -6x$

$2x-2=12$

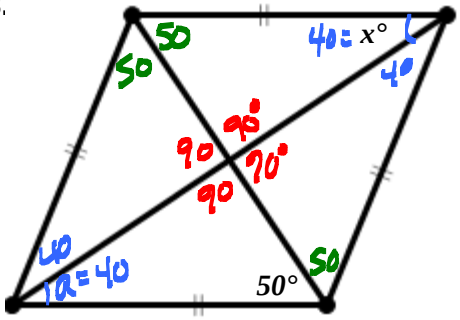
$+2 \quad +2$

$\frac{2x}{2} = \frac{14}{2}$

$x=7$

2. $x = 7$

3.



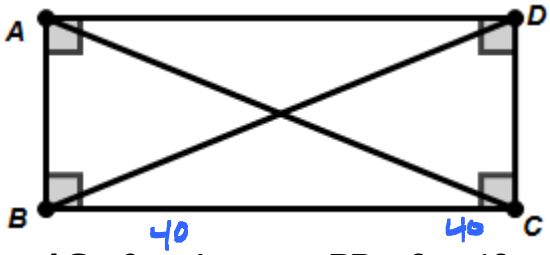
$a+90+50=180$

$a+140=180$

$a=40$

3. $x = 40°$

4.



AC = 9x + 4 **BD = 6x + 16**

$AC = BD$

$9x+4=6x+16$

$-6x \quad -6x$

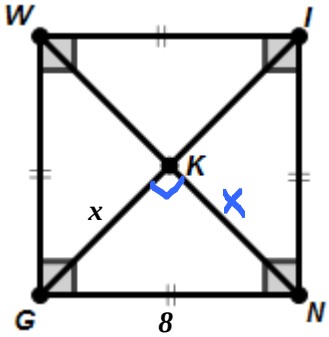
$3x+4=16$

$-4 \quad -4$

$\frac{3x}{3} = \frac{12}{3}$

4. $x = 4$

5.



$a^2+b^2=c^2$

$\sqrt{32}$

8 4

4 4 2 2

2 2 2 2

$x^2+x^2=8^2$

$\frac{2x^2}{2} = \frac{64}{2}$

$\sqrt{x^2} = \sqrt{32}$

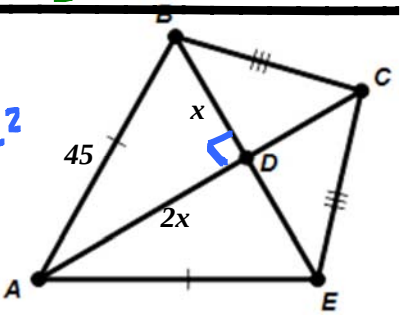
$x=\sqrt{32} = \sqrt{4}\cdot\sqrt{4}\cdot\sqrt{2}$

$= 2\cdot 2\sqrt{2} = 4\sqrt{2}$

5. $x = \sqrt{32} = 4\sqrt{2}$

$x \approx 5.66$

6.



$a^2+b^2=c^2$

$(2x)^2+(x)^2=45^2$

$4x^2+x^2=2025$

$\frac{5x^2}{5} = \frac{2025}{5}$

$\sqrt{x^2} = \sqrt{405}$

$x=\sqrt{405} = \sqrt{9}\cdot\sqrt{9}\cdot\sqrt{5}$

$= 3\cdot 3\cdot\sqrt{5} = 9\sqrt{5}$

$\sqrt{405}$

5 81

5 9 9

5 3 3 3 3

6. $x = \sqrt{405} = 9\sqrt{5}$

$x \approx 20.12$

Plot points A(-3, -1), B(-1, 2), C(4, 2), and D(2, -1).

1. What specialized geometric figure is quadrilateral ABCD? Support your answer mathematically.

ABCD IS A PARALLELOGRAM

SLOPE AB = 3/2 = SLOPE DC ✓ PARALLEL

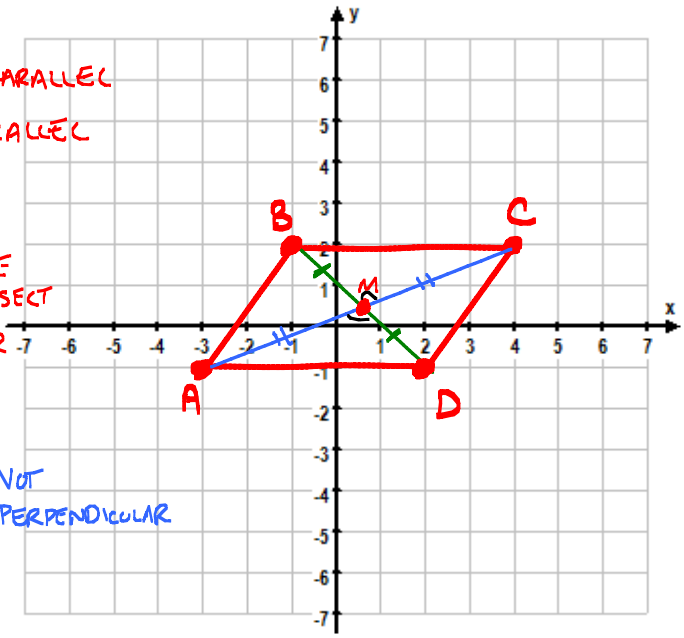
SLOPE AD = 0 = SLOPE BC ✓ PARALLEL

2. Draw the diagonals of ABCD. Find the coordinates of the midpoint of each diagonal. What do you notice?

MIDPOINT BD $\left(\frac{-1+2}{2}, \frac{2+-1}{2}\right) = (0.5, 0.5)$

MIDPOINT AC $\left(\frac{-3+4}{2}, \frac{-1+2}{2}\right)$

THEY ARE THE SAME & THUS BISECT EACH OTHER



3. Find the slopes of the diagonals of ABCD. What do you notice?

SLOPE BD $m = \frac{(y_2-y_1)}{(x_2-x_1)} = \frac{(-1-2)}{(2--1)} = \frac{-3}{3} = -1$

SLOPE AC $m = \frac{(y_2-y_1)}{(x_2-x_1)} = \frac{(2--1)}{(4--3)} = \frac{3}{7}$

NOT PERPENDICULAR

4. The diagonals of ABCD create four small triangles. Are any of these triangles congruent to any of the others? Why or why not?

$\triangle AMD \cong \triangle CMB$ BY SAS

$\triangle AMB \cong \triangle CMD$ BY SAS

Plot points E(1, 2), F(2, 5), G(4, 3) and H(5, 6).

$1^2 + 3^2 = c^2$

$1 + 9 = c^2$

$\sqrt{10} = \sqrt{c^2}$

5. What specialized geometric figure is quadrilateral EFHG? Support your answer mathematically using two different methods.

EFHG IS A RHOMBUS

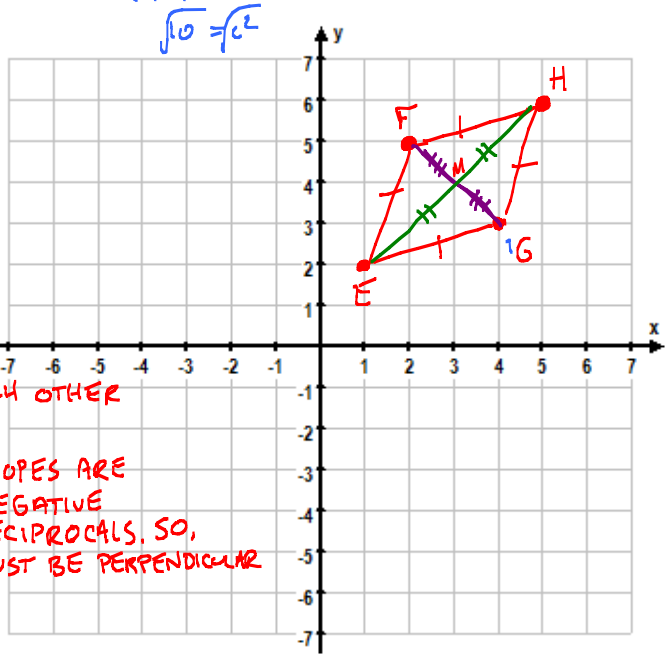
$FE = FH = HG = EG = \sqrt{1^2+3^2} = \sqrt{10}$

6. Draw the diagonals of EFHG. Find the coordinates of the midpoint of each diagonal. What do you notice?

MIDPOINT FG $\left(\frac{2+4}{2}, \frac{5+3}{2}\right) = (3, 4)$

MIDPOINT EH $\left(\frac{1+5}{2}, \frac{2+6}{2}\right) = (3, 4)$

SAME, SO DIAGONALS BISECT EACH OTHER



7. Find the slopes of the diagonals of EFHG. What do you notice?

SLOPE FG $m = \frac{(y_2-y_1)}{(x_2-x_1)} = \frac{(3-5)}{(4-2)} = \frac{-2}{2} = -1$

SLOPE EH $m = \frac{y_2-y_1}{x_2-x_1} = \frac{(6-2)}{(5-1)} = \frac{4}{4} = 1$

SLOPES ARE NEGATIVE RECIPROCALS, SO, MUST BE PERPENDICULAR

8. The diagonals of EFHG create four small triangles. Are any of these triangles congruent to any of the others? Why or why not?

$\triangle FME \cong \triangle FMH \cong \triangle GMH \cong \triangle GME$

BY SSS.

$m = \frac{1}{1}$ $\perp m = -\frac{1}{1}$

NEGATIVE RECIPROCALS SUGGESTS PERPENDICULAR LINES

Plot points P(4, 1), W(-2, 3), M(2,-5), and K(-6, -4).

□PWKM IS A KITE

9. What specialized geometric figure is quadrilateral PWKM? Support your answer mathematically.

$PW = \sqrt{6^2+2^2} = \sqrt{40} = \sqrt{6^2+2^2} = PM$ ✓

$KW = \sqrt{4^2+7^2} = \sqrt{65} = \sqrt{1^2+8^2} = KM$ ✓

10. Draw the diagonals of PWKM. Find the coordinates of the midpoint of each diagonal. What do you notice?

MIDPOINT KP: $\left(\frac{-6+4}{2}, \frac{1+-4}{2}\right) = (-1, -1.5)$

MIDPOINT WM: $\left(\frac{-2+2}{2}, \frac{3+-5}{2}\right) = (0, -1)$

KP BISECTS WM

11. Find the lengths of the diagonals of PWKM. What do you notice?

$KP = \sqrt{10^2+5^2} = \sqrt{125} = 5\sqrt{5} \approx 11.18$

$WM = \sqrt{4^2+8^2} = \sqrt{80} = 4\sqrt{5} \approx 8.94$

12. Find the slopes of the diagonals of PWKM. What do you notice?

SLOPE KP: $M = \frac{(y_2-y_1)}{(x_2-x_1)} = \frac{(1--4)}{(4--6)} = \frac{5}{10} = \frac{1}{2}$

SLOPE WM: $M = \frac{(y_2-y_1)}{(x_2-x_1)} = \frac{(-5-3)}{(2--2)} = \frac{-8}{4} = \frac{-2}{1}$

SLOPES OF DIAGONALS ARE NEGATIVE RECIPROCALS. SO, DIAGONALS ARE PERPENDICULAR

13. The diagonals of ABCD create four small triangles. Are any of these triangles congruent to any of the others? Why or why not?

$\triangle KOM \cong \triangle KOW$ BY SSS

$\triangle POW \cong \triangle POM$ BY SSS

Plot points A(1, 0), B(-1, 2), and C(2, 5).

14. Find the coordinates of a fourth point D that would make ABCD a rectangle. Justify that ABCD is a rectangle.

D(4,3) ✓

SLOPE $AD = \frac{y_2-y_1}{x_2-x_1} = \frac{3-0}{4-1} = \frac{3}{3} = 1$

SLOPE $AB = \frac{y_2-y_1}{x_2-x_1} = \frac{2-0}{-1-1} = \frac{2}{-2} = -1$

SLOPE $BC = \frac{y_2-y_1}{x_2-x_1} = \frac{5-2}{2-1} = \frac{3}{3} = 1$

SLOPE $CD = \frac{y_2-y_1}{x_2-x_1} = \frac{3-5}{4-2} = \frac{-2}{2} = -1$

CONSECUTIVE SIDES HAVE NEG. RECIPROCAL SLOPES WHICH MEANS PERPENDICULAR.

15. Find the coordinates of a fourth point D that would make ABCD a parallelogram that is not also a rectangle. Justify that ABCD is a parallelogram but is not a rectangle.

D(0,7)

SLOPE $AB = \frac{y_2-y_1}{x_2-x_1} = \frac{2-0}{-1-1} = \frac{2}{-2} = -1$

SLOPE $AC = \frac{y_2-y_1}{x_2-x_1} = \frac{5-0}{2-1} = \frac{5}{1} = 5$

SLOPE $CD = \frac{y_2-y_1}{x_2-x_1} = \frac{7-5}{0-2} = \frac{2}{-2} = -1$

SLOPE $BD = \frac{y_2-y_1}{x_2-x_1} = \frac{7-2}{0-1} = \frac{5}{1} = 5$

OPPOSITE SIDES HAVE THE SAME SLOPE AND THEREFORE PARALLEL