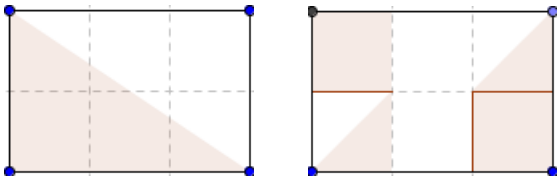
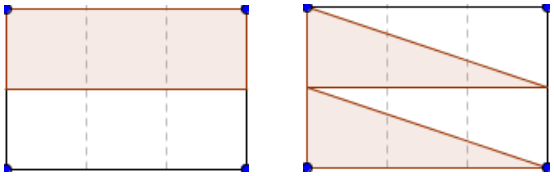
Shaded Fractional Areas and Equivalence

Student Probe

Which of the following figures are equivalent to ½?





Answers and Misconceptions

All four figures are equivalent to ½.

Figure A

- Visualize (looks like ½): most students should recognize that dividing a rectangle diagonally results in ½ the region.
- Numeric: students might recognize the pieces as 3 pieces out of 6 (3/6=1/2); however this would require them to rearrange pieces to form three whole square units.

Figure B

- Visualize (looks like ½): most students will not immediately recognize this arrangement of shaded squares as ½. Students could rearrange the parts to make the shaded region represent three whole continuous square units horizontally dividing the rectangle.
- Numeric: students might recognize the pieces as 3 pieces out of 6 (3/6=1/2)

Figure C

- Visualize: most students should be able to recognize the shaded portion as ½ without any manipulation or numeric calculation.
- Numeric: students might recognize the pieces as 3 pieces out of 6 (3/6=1/2)

Figure D

- Visualize: students can manipulate the pieces around to create a shape similar to figure C. Students might also notice that the 2 shaded portions represent ½ of ½ (¼) and when put together make ½ .
- Numeric: students might recognize the pieces as 3 pieces out of 6 (3/6=1/2)

At a Glance

What: Understand fraction equivalence by manipulating various shaded fractional areas.

Common Core Standards: CC.4.NF.1. Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

Mathematical Practices:
Reason abstractly and quantitatively.
Attend to precision.
Look for and make use of structure.

Who: Students who do not understand that equivalent fractions may look different.

Grade Level: 4

Prerequisite Vocabulary: rectangle, dimensions

Prerequisite Skills: naming fractional parts of a whole

Delivery Format: individual, small group

Lesson Length: 15-30 minutes

Materials, Resources, Technology: scissors, attached worksheets

Student Worksheets: Painter's Problem, Fraction Equivalence Worksheets for ¼ and ½.

Students who do not see B and D as equivalent are struggling with the concept that fractional parts of an area must have the same size but not necessarily the same shape. This lesson could be used as a companion lesson to Equivalent Fractions.

Lesson Description

The lesson is intended to help students develop an understanding of fraction equivalence using visual models. Students will cut out shaded portions of various rectangles which look different but are equivalent. Students will work with ½ and ¼ to develop the idea of equivalence. The scaffolding of the fractional pieces allows students to use the idea of "doubling" to help make sense of the various fractional parts. At the end of the lesson students will respond to questions that address the idea of equivalence and create their own set of equivalent shaded models.

Rationale

Students often fail to understand that fractional amounts can be equivalent without being the exact same shape. This misconception often keeps students from generating accurate models of equivalent fractions. This spatial visualization is a critical part of students' ability to conceptualize equivalence. Students need to be given opportunities to manipulate fractional parts to create shapes that represent fractional pieces that they are familiar with (traditional models of ½, ¼, etc.) These opportunities help students build a foundational understanding of equivalence that allows them the flexibility with their thinking about equivalence to see that "equivalence doesn't depend upon the congruence of shapes".

Preparation

Provide copies of grid paper and colored pencils for each student to sketch and shade their rectangles. Have color tiles available for students who may need them. Make student copies and provide students with a copy of the formative assessment after instruction has been given.

Lesson

The teacher says or does…	Expect students to say or do…	If students do not, then the teacher says or does…
1. Refer to The Painter's Problem. What fraction of Figure A is shaded? What fraction of Figure B is shaded?	Students should respond correctly with "½ of the area is shaded". This is a very visual and easy model to see.	If students are having trouble stating that each is ½ of the area have them cut out the shaded portion of the figures to prove that each piece fits exactly on top of the other.

The teacher says or does…	Expect students to say or do…	If students do not, then the teacher says or does…
2. A painter wants to paint the various patterns (shown in figures 1-4) in four different classrooms. He only has a 6 feet by 4 feet piece of plywood to paint the figure on for each class. He wants to make sure that each classroom is getting the same amount of space painted. However, after looking at the different patterns he is not sure that all the classes are getting the same amount painted. He is asking for your help to determine if each class is getting the same amount painted. Use the shaded figures 1-4 to help the painter answer his question. What do you think--are they the same or not?	Students might say that the different patterns will have different areas because they look different. This is the misconception the lesson addresses.	Do not tell students at this time whether they are the same or not. They will discover this when they cut them out and rearrange them.
3. Have students cut out the different shaded portions for figure 1-4 and rearrange them to see if they cover the same amount of space. You will need to use **Figures A and B** to help determine if the areas for the different classrooms are the same or different.	Students will spend a few minutes cutting and arranging shaded portions to lay on top of Figures A and B to determine if the shaded portions for Figures 1-4 are the same or different.	Prompt the students to move or place certain shaded pieces in a particular place to get them started identifying similar areas. Focus the conversation on the size of the shaded pieces and not the orientation or shape of them. This needs to be constantly revisited with students while working and rearranging pieces.

The teacher says or does…	Expect students to say or do…	If students do not, then the teacher says or does…
4. When you arranged the shaded pieces, what fractional amount(s) did you find being occupied for each classroom? How many squares (after you rearranged the pieces) where shaded for each figure?	The areas are the same because I can place them directly on top of each other as an exact fit or match. Students should identify the shaded portions for each classroom as ½ of the piece of plywood. Students should realize that all the figures have 12 out of 24 squares shaded.	Provide students with explicit instruction on how to move the various pieces to cover Figure A or B. How many pieces do you see that are shaded and how many squares of that same size it would take to fill the entire "whole"?
5. Now that you have helped the painter determine how to find figures with different shapes that have the same area, you need to create patterns of your own that have the same area. Using the grid paper provided (Fraction Equivalence Worksheet ½) make five different patterns that have ½ the rectangle shaded but look entirely different.	Students will generally start out with horizontal, vertical, and diagonal divisions of the rectangle to show different parts. Students should be pushed to come up with more complex ways. Simple divisions do not promote the understanding necessary to correct the misconception.	

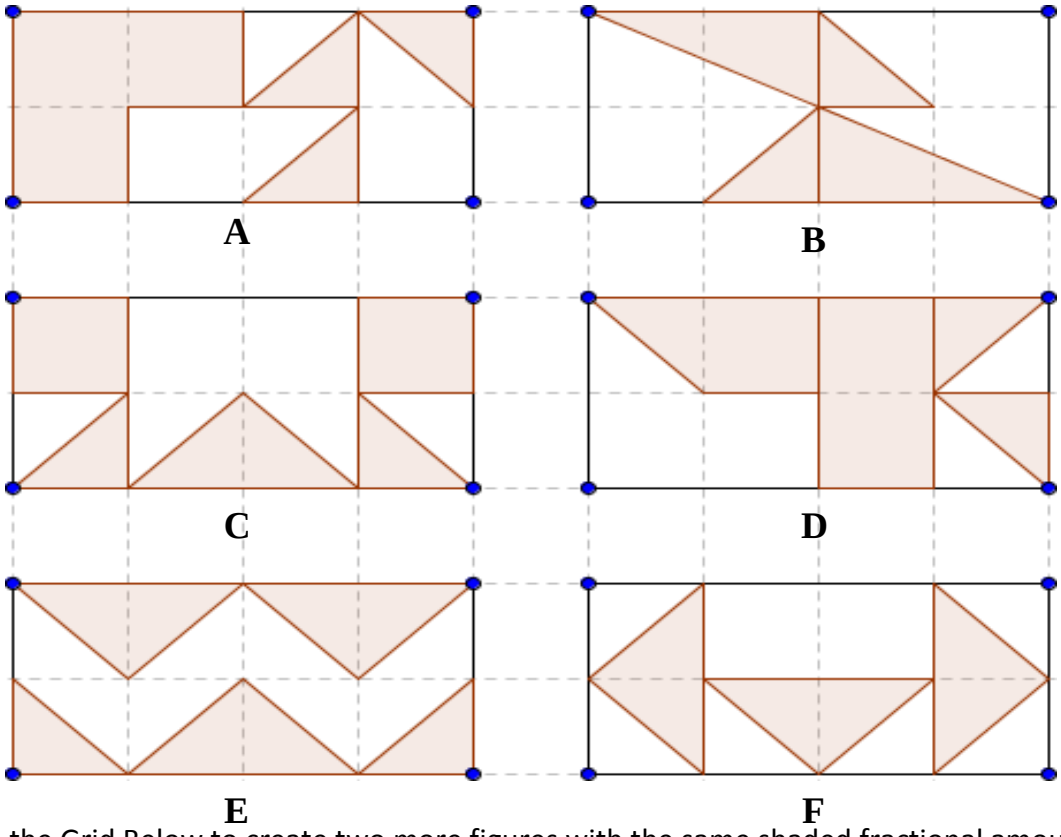
The teacher says or does…	Expect students to say or do…	If students do not, then the teacher says or does…
6. How did you know that your different patterns all had the same area?	Student responses should be focused around physical and visual manipulation and rearranging of the figures to new places on the grid to create new patterns. Students need to be reminded that nothing is being taken away or added when pieces are being moved around. Conversation may focus on the number of squares shaded versus the total number of squares.	Have students create a simple pattern then cut apart the pattern into smaller pieces that can be moved around and reformed. Have the students compare the original figure with the new. Ask the students if both the original and the new contained the same amount of shaded squares. The fractional name for both parts will be the same.
7. Repeat Steps 5-6 with Fraction Equivalence Worksheet ¼ for patterns focusing on ¼ the area of a rectangle.		
8. Reflect and write down what mathematical ideas you have learned or discovered during the lesson.	Topics might include: Fraction pieces must be the same size but not necessarily the same shape. Understand that fractional parts can be subdivided (broken up) and rearranged to make a new figure but retain the same fractional value.	

Teacher Notes

1. A discussion about the numerator and denominator in terms of squares shaded and total squares on the piece of plywood is a secondary point within this lesson “it is not the primary focus of the lesson”.
2. By discussing how to rearrange their pieces and then cover Figures A and B, students should understand that fractional amounts can look differently but still have the same amount.

Formative Assessment

1. Identify which figures below have the same fractional area shaded and explain how you know the figures contain the same fractional amount.



2. Use the Grid Below to create two more figures with the same shaded fractional amount as one of the figures pictured in question 1. What is the fractional amount of each?

References

Russell Gersten, P. (n.d.). *RTI and Mathematics IES Practice Guide - Response to Intervention in Mathematics*. Retrieved 2 25, 2011, from rti4sucess

Marjorie Montague, Ph.D. (2004, 12 7). *Math Problem Solving for Middle School Students With Disabilities*. Retrieved 4 25, 2011, from The Iris Center

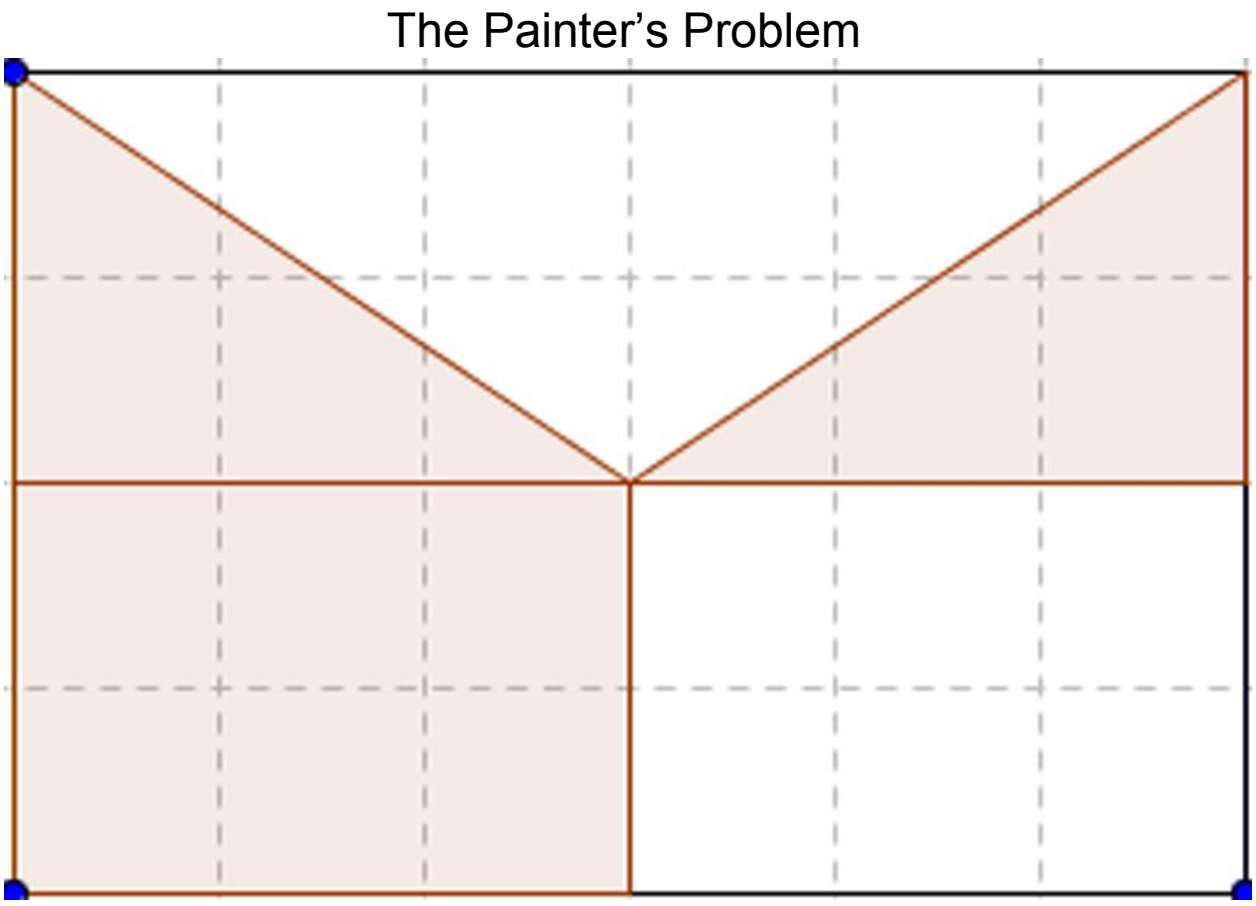


Figure 1

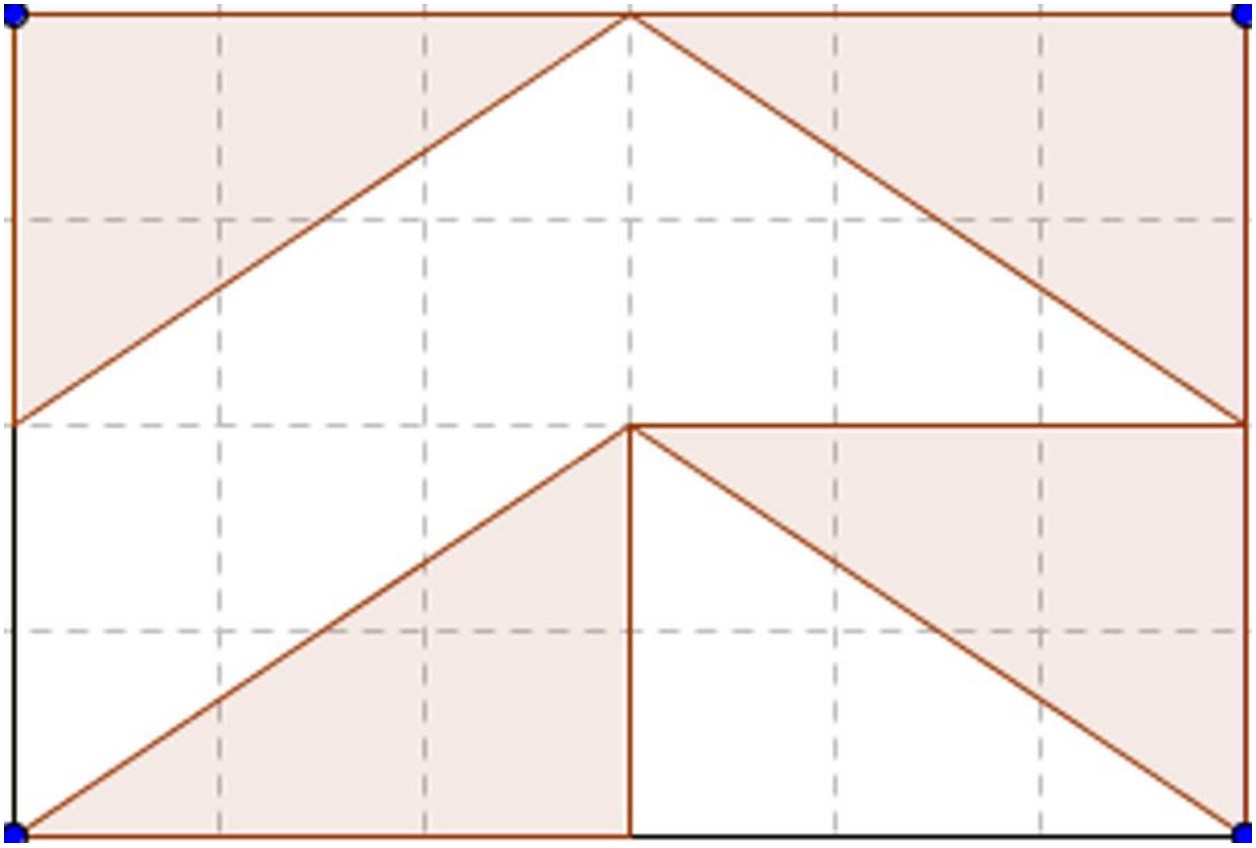


Figure 2

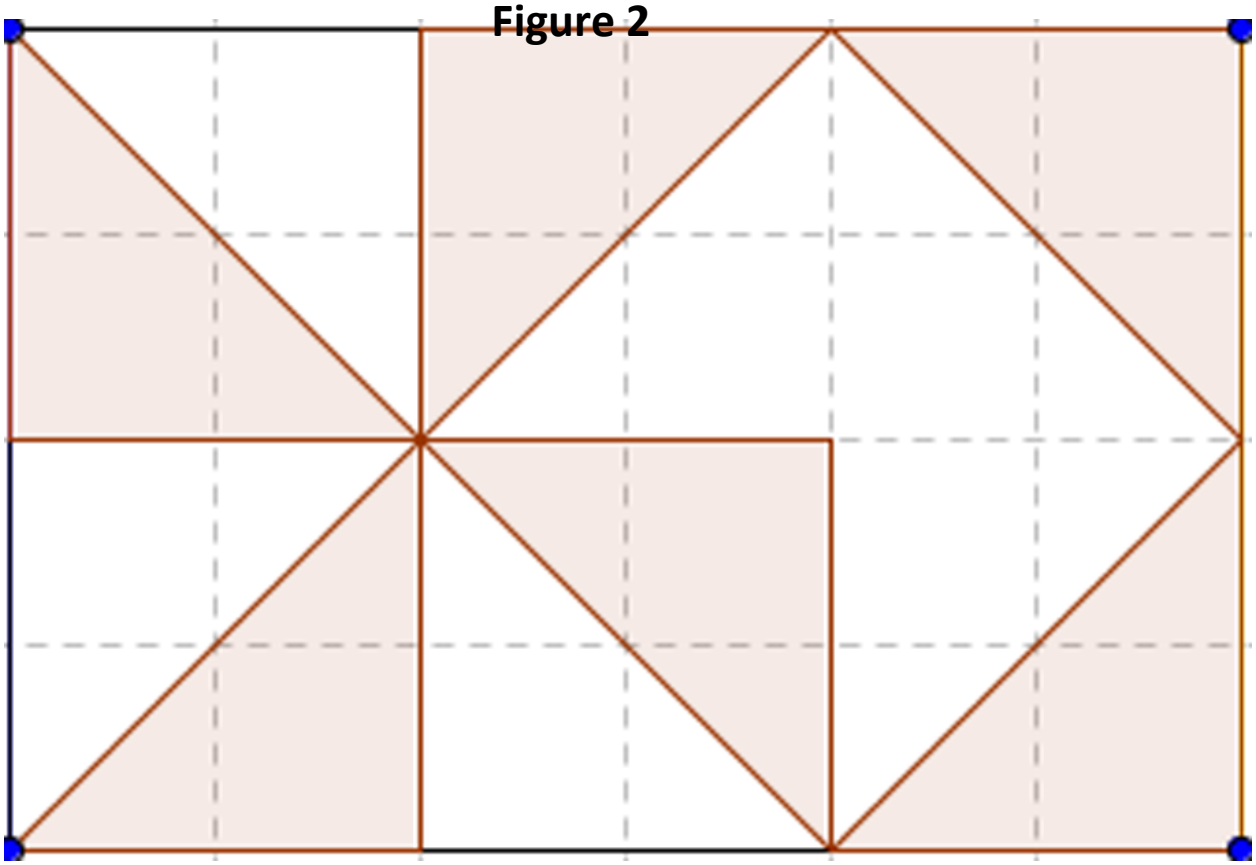


Figure 3

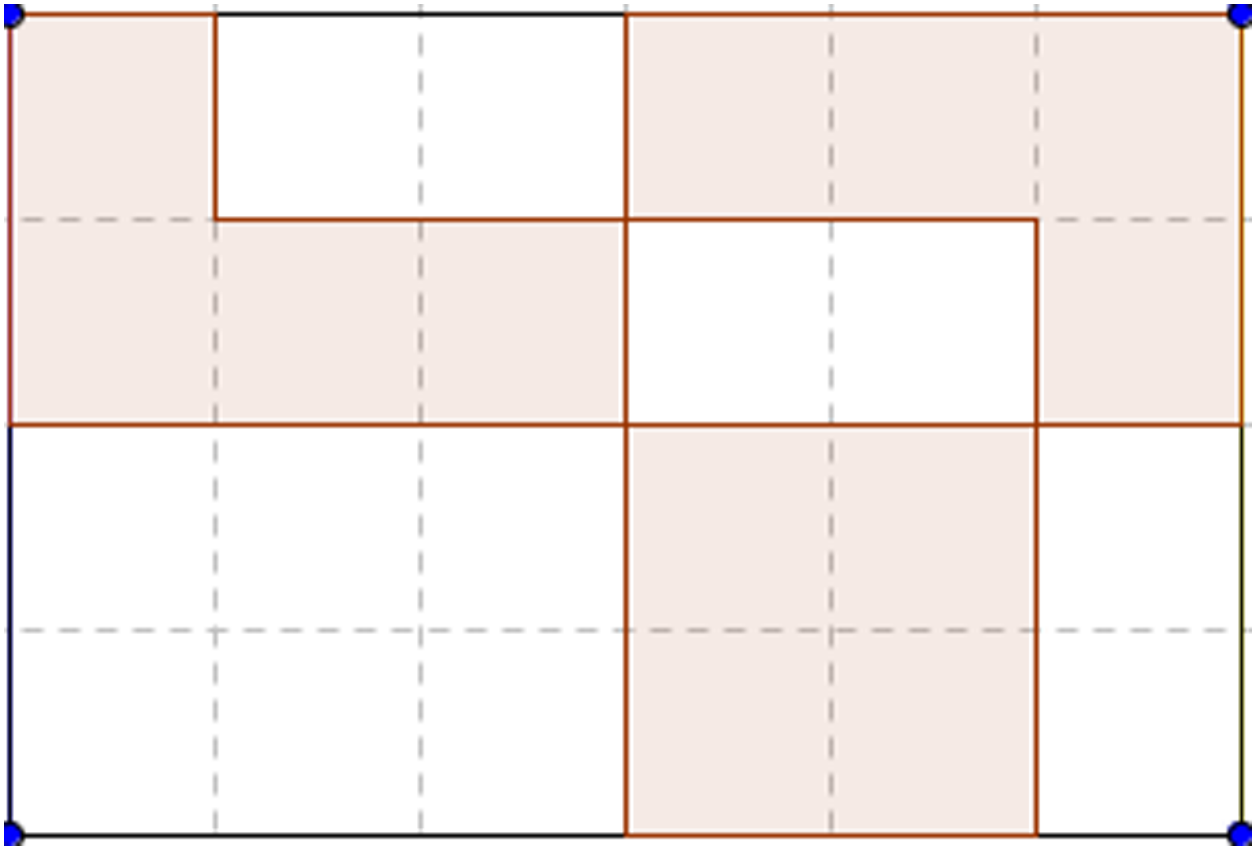
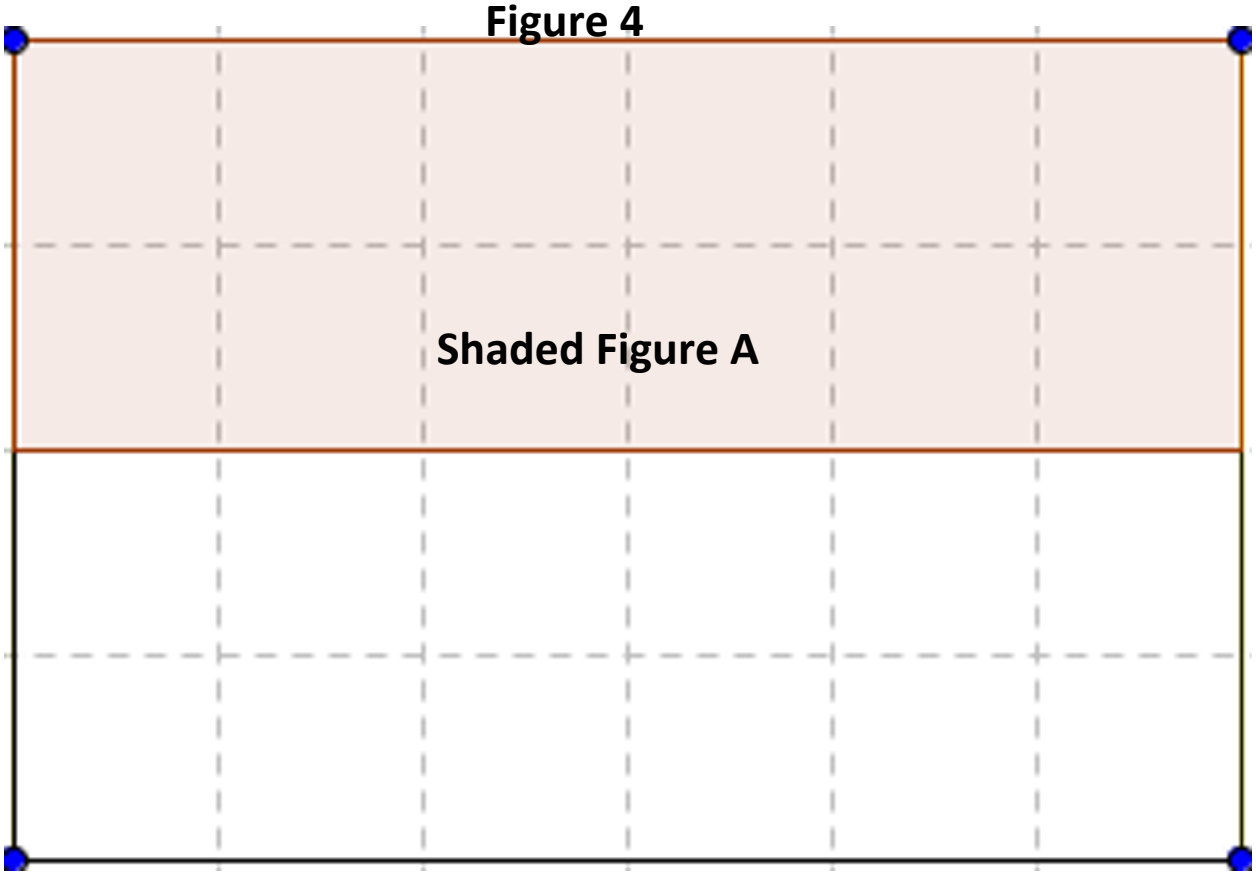
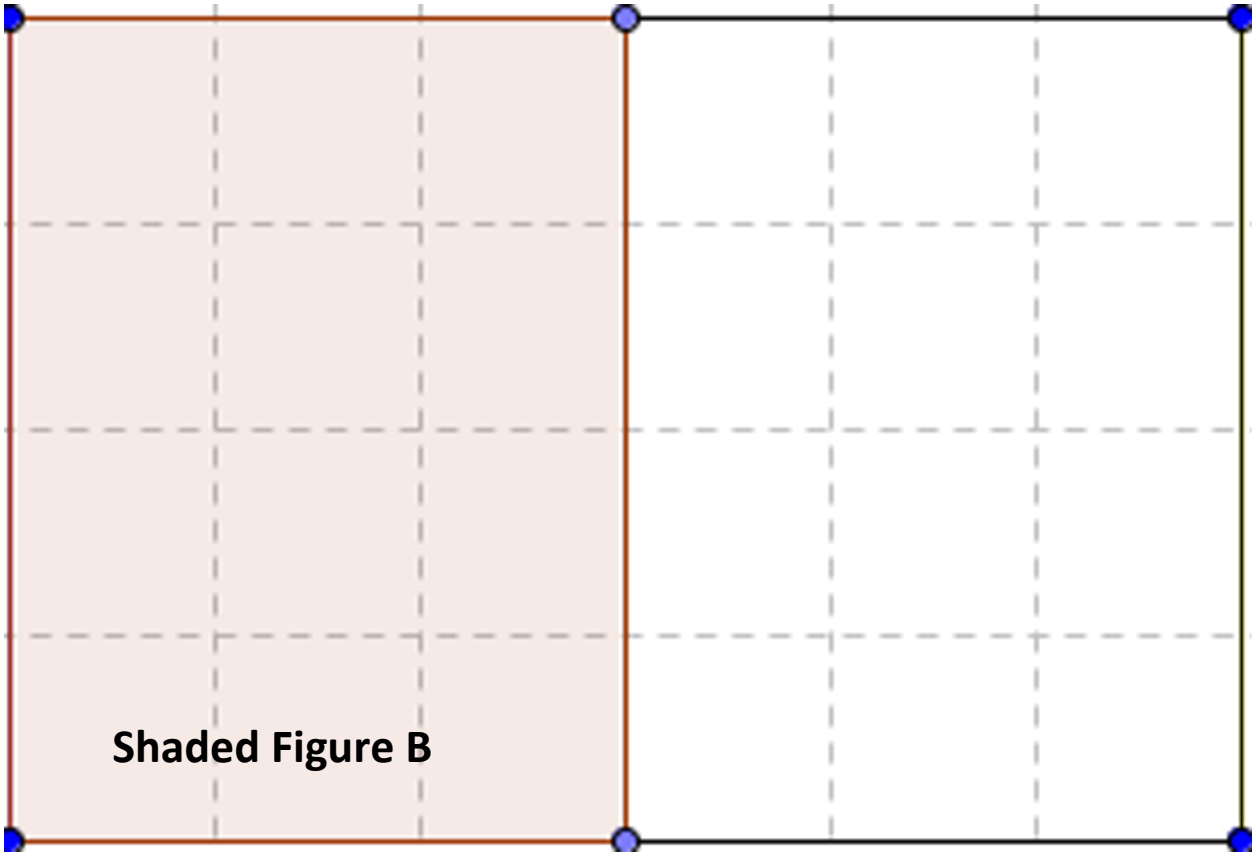


Figure 4




Shaded Figure B