

# MathLinks

# 7-8

STUDENT PACKET

## MATHLINKS: GRADE 7 STUDENT PACKET 8 EXPLORING EXPRESSIONS AND EQUATIONS

<b>8.1</b>	<b>Equivalent Expressions</b>	1
	<ul style="list-style-type: none"><li>• Write, evaluate, and simplify expressions.</li><li>• Describe geometric patterns numerically, pictorially, symbolically, and verbally.</li><li>• Interpret expressions in terms of their geometric context.</li><li>• View algebra as a useful mathematical tool.</li></ul>	
<b>8.2</b>	<b>Hundreds Chart Patterns</b>	10
	<ul style="list-style-type: none"><li>• Make conjectures about number patterns.</li><li>• Use algebraic expressions to prove conjectures.</li><li>• Write and simplify algebraic expressions.</li><li>• View algebra as a useful mathematical tool.</li></ul>	
<b>8.3</b>	<b>Polygon Area Puzzle</b>	18
	<ul style="list-style-type: none"><li>• Write algebraic expressions and equations.</li><li>• Evaluate algebraic expressions.</li><li>• Solve equations.</li><li>• Use algebra to solve problems.</li></ul>	
<b>8.4</b>	<b>Skill Builders, Vocabulary, and Review</b>	25

Commentary on the packet will be in red in text boxes along the way.

Welcome to a *MathLinks* Student Packet (SP). This packet is from *MathLinks: Grade 7* and is SP8, meaning it is the 8<sup>th</sup> packet out of 16.

On the cover sheet you will find the titles, goals, and page numbers of the three concept lessons as well the location of the fourth section which is always the Skill Builders, Vocabulary, and Review.

## WORD BANK

Word or Phrase	Definition or Description	Example or Picture
deductive reasoning		<div style="border: 1px solid black; padding: 5px; color: red;"> <p>All major vocabulary for the SP is found in the Word Bank, though some words are introduced and defined within the lessons. All words are defined or explained in the Resource Guide.</p> <p>The Resource Guide also includes explanations and examples. It replaces the examples and glossary of a traditional textbook.</p> <p>Students will receive the resource guide in two parts, roughly corresponding to the two semesters in the school year.</p> </div>
distributive property		
equation		
equivalent expressions		
evaluate an expression		
expression		
greatest common factor		
inductive reasoning		
solve an equation		
variable		

# EQUIVALENT EXPRESSIONS

<b>Summary</b>	<b>Goals</b>
<p>We will write numerical expressions to represent geometric patterns, describe patterns in words, and generalize using variable expressions. We will apply properties of arithmetic to generate equivalent expressions that include integers.</p>	<ul style="list-style-type: none"> <li>Write, evaluate, and simplify expressions.</li> <li>Describe geometric patterns numerically, pictorially, symbolically, and verbally.</li> <li>Interpret expressions in terms of their geometric context.</li> <li>View algebra as a useful mathematical tool.</li> </ul>

### Warmup

Rewrite each arithmetic problem below as an expression, horizontally on one line. Do not compute.

<p>1.        23           46       + 54  _____</p>	<p>2.        132           - 67  _____</p>	<p>3.        19 <math>\overline{)451}</math>  _____</p>
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4. Rewrite problem 3 using fraction notation.

Demonstrate whether each of the following pairs of expressions is equivalent or not.

Expressions		Yes / No
5.	a. $-8 - 3$	b. $-8 + (-3)$
6.	a. $-6 - 5$	b. $5 - (-6)$
7.	a. $4 - 9$	b. $-9 + 4$
8.	a. $-2 - 7$	b. $7 - (-2)$

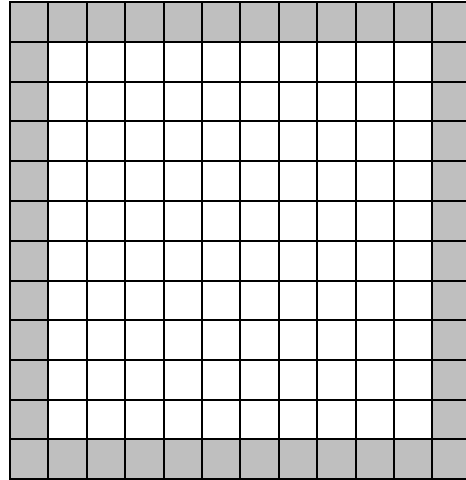
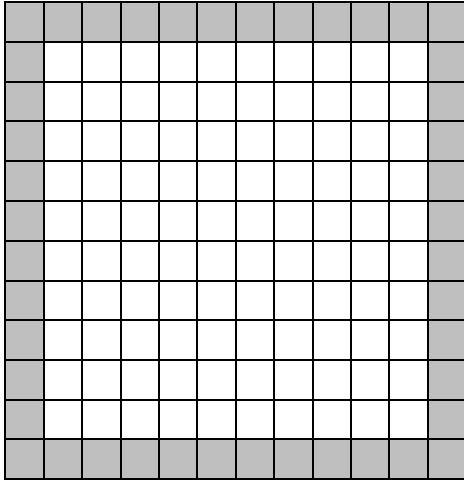
The black strip along the top of this page, along with the Summary and Goals of this lesson, signifies the beginning of a new lesson.

All lessons begin with a Warmup that reviews or previews knowledge for the new lesson.

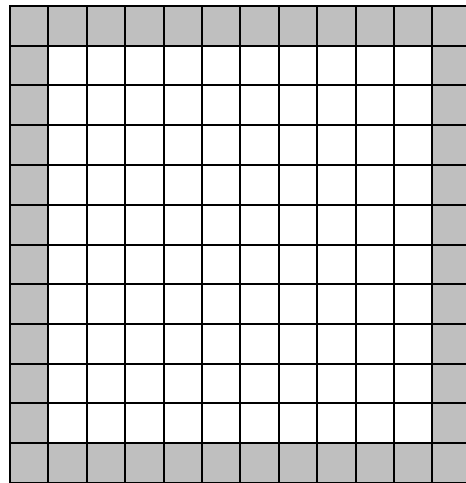
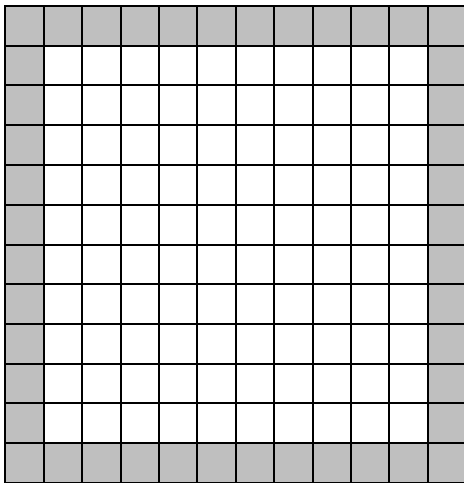
9. Find the greatest common factor (GCF) of 12 and 28.

# THE BORDER PROBLEM

1. The  $12 \times 12$  grids below have  $1 \times 1$  squares along the borders. Your teacher will give you directions for writing numerical expressions about the areas of the borders.

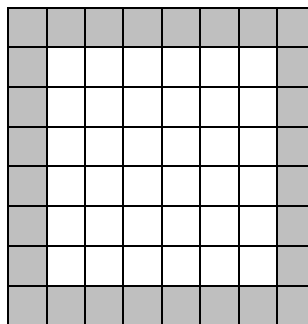
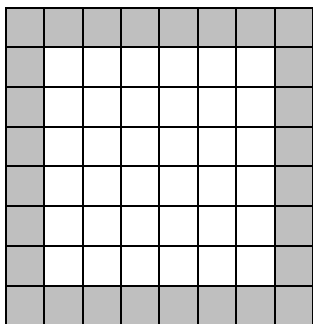
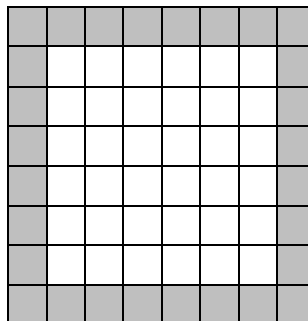
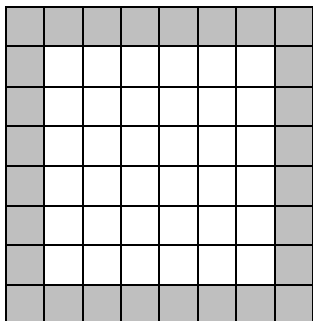


*MathLinks packets are not "workbooks." This is one of many pages in this packet where students follow the teacher's lead before exploring mathematical concepts. Students report their findings to the class, and the teacher facilitates a rich discussion.*



## THE BORDER PROBLEM (Continued)

2. Your teacher will give you directions for writing numerical expressions for the areas of the borders of these  $8 \times 8$  grids below.



3. Your teacher will give you directions for writing numerical expressions for the borders of a  $5 \times 5$  grid.

## EXPRESSIONS AND THE DISTRIBUTIVE PROPERTY

Two numerical expressions are equivalent if they represent the same value.

Two mathematical expressions are equivalent, if for any possible substitution of values for the variables, the two resulting values are equal.

1. Refer to the page 3. Write three equivalent numerical expressions that you generated for the  $8 \times 8$  grid. Simplify the expressions to show they are equivalent.

2. Consider the algebraic expression  $x + 3x + y + 2y + 5x$ .

a. Simplify the expression as much as possible.

b. Rewrite the expression using the distributive property of 3 and the simplified expression from part (a).

$$3(\text{_____} + \text{_____})$$

c. Substitute the values  $x = -1$  and  $y = 4$  into the expressions in parts (a) and (b) above. Do you think that the expressions are equivalent? Explain.

This follow up page highlights how we move from an exploration of a concept to addressing necessary, more traditional skills.

Some lesson pages look like "workbook" pages, but this is generally not the intent. In this case, structured workspace gives teachers references for discussing definitions and students a place to record and apply what they are learning.

In the Teacher Packet (TP), which is in the Teacher Guide, you will find more information to help you deliver lessons.

The distributive property states that  $a(b + c) = ab + ac$  and  $(b + c)a = ba + ca$  for any three numbers  $a$ ,  $b$ , and  $c$ .

d. Illustrate the distributive property with the expressions in parts (a) and (b) above.

$$\text{_____} = \text{_____}$$

e. How do you know the expressions on both sides of the equal sign are equivalent?

3. Refer to your expressions for the  $12 \times 12$  grid on page 2. Choose one expression that can be rewritten using the distributive property and rewrite it. Then show both expressions are equivalent by simplifying the numerical expressions.

## PRACTICE 1

Evaluate each expression for $m = 5$ .			If the expressions are equivalent, write the algebraic expression in the simplest form
1.	a. $-m + (-m)$	b. $-m - m$	
2.	a. $-m + m$	b. $-m - (-m)$	Targeted practice is included in the lesson. More practice is located in Skill Builders in this packet and in future packets.
3.	a. $m - m$	b. $m - (-m)$	
4.	a. $m(m - 1)$	b. $m \cdot m - 1$	

5. In problems 1-4 above, if we let  $m = -5$  instead, would any of the “equivalent” expressions change to “not equivalent?” Explain.

6. Tere looked at the expressions  $2n$  and  $n^2$ . She substituted the value of 2 for  $n$  in both expressions, and then said, “They both are equal to 4, so they must be equivalent expressions.” Critique Tere’s reasoning.

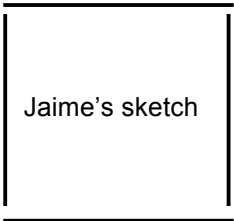
Use the distributive property to rewrite each expression so that it is a sum (or difference) of terms. Do not simplify. Then draw arrows to match pairs of equivalent expressions.

7. $-3(9 + 5)$	8. $-3(9 - 5)$	9. $3(-9 + 5)$	10. $3(-5 - 9)$
11. $-3(x + 5)$	12. $-3(x - 5)$	13. $3(-x + 5)$	14. $3(-5 - x)$

## REVISITING THE BORDER PROBLEM

Jaime recorded the sketch to the right and then wrote expressions to describe his thinking.

- Complete the table below for Jaime using numerical expressions that match his drawing on the right. Evaluate each expression.



	Expression 1	Expression 2	Expression 3
$12 \times 12$	$12 + 12 + 10 + 10$	$2(\underline{\quad}) + 2(\underline{\quad})$	$2(\underline{\quad} + \underline{\quad})$
$8 \times 8$			
$5 \times 5$			

- How do you know that the numerical expressions for the  $8 \times 8$  grid are equivalent?

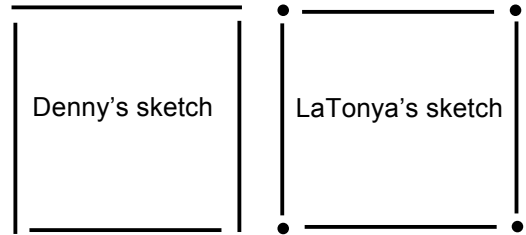
- Use Jaime's sketch and patterns to write three expressions for an  $n \times n$  grid. Simplify each expression to show they are equivalent.

	Expression 1	Expression 2	Expression 3
$n \times n$	<div style="border: 1px solid black; padding: 5px; margin: 0 auto; width: 80%;"> <p style="color: red; font-size: small;">Students' experiences with the concrete border visuals in the beginning of the lesson help them to arrive at the important algebraic skill of generalizing using symbolic notation. Students can more easily make sense of abstract ideas because they connect to something meaningful.</p> </div>		



## REVISITING THE BORDER PROBLEM (Continued)

Write two expressions below that could describe Denny's and LaTonya's thinking for the  $12 \times 12$  grid to the right. Then follow the same pattern in each column for the  $8 \times 8$ ,  $5 \times 5$  and  $n \times n$  grids. Simplify the algebraic expressions to show they are equivalent.



### 4. Denny's method

	Expression 1	Expression 2
$12 \times 12$		
$8 \times 8$		
$5 \times 5$		
$n \times n$		

### 5. LaTonya's method

	Expression 1	Expression 2
$12 \times 12$		
$8 \times 8$		
$5 \times 5$		
$n \times n$		

**PRACTICE 2**

Use the distributive property to rewrite each expression so that it is a sum or difference of terms.

1. $-9(f + 4)$	2. $9(-f + k)$	3. $(-3f - 6)(-2)$
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Use the distributive property and the GCF of the terms to rewrite each expression so that it is a product of factors.

4. $4t + 12$	5. $-8w - 12$	6. $-49 - 7u$
7. $4a + 28b$	8. $3a + 3b - 3$	9. $6 + 12x$

Simplify each expression. Then use the distributive property and the GCF of the terms to rewrite each expression so that it is a product of factors.

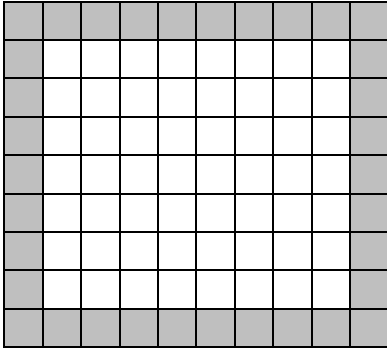
10. $n + 1 + n + 1$	11. $4(n + 2) + 2n + 6$	12. $-12(n + 1) + 2n - 8$
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13. Trista looked at the expression in problem 10 and said, "That's just two groups of  $n + 1$ , so I'll write  $2(n + 1)$ ." Explain whether Trista is correct or not.

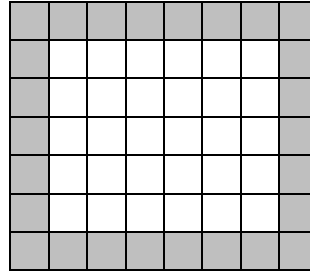
## A DIFFERENT BORDER PROBLEM

1. Write at least two numerical expressions for the number of shaded border squares in these grids. Two of them have been drawn for you.

$9 \times 10$



$7 \times 8$



$4 \times 5$

Students extend and apply their knowledge to different situations.

2. Consider the pattern that seems to be established in the grids above.

If the shorter side has length  $n$ , then the longer side has length ( \_\_\_\_\_ + \_\_\_\_\_ ).

Write at least two variable expressions for the number of shaded border squares and simplify them. Circle the simplified expressions to show that they are equivalent.

# HUNDREDS CHART PATTERNS

## Summary

We will investigate patterns on the hundreds chart. We will write algebraic expressions and use them to prove conjectures based on the patterns. We will find equivalent expressions that involve variables and rational numbers.

## Goals

- Make conjectures about number patterns.
- Use algebraic expressions to prove conjectures.
- Write and simplify algebraic expressions.
- View algebra as a useful mathematical tool.

## Warmup

List at least three patterns that you observe on the hundreds chart below.

The black strip tells us we are starting the second lesson.

Summary, Goals, and Warmup always kick off the start of a new lesson.

This hundreds chart is also provided in the Teacher Guide in the Reproducible tab so it can be posted.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

# HUNDREDS CHART

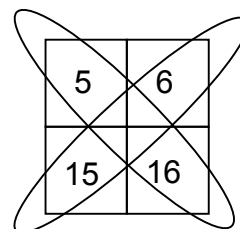
Let the variable  $n$  represent some number on the hundreds chart. Write variable expressions for the following, in the cases that the space described is in the hundreds chart.

1. The number that is one space to the right of  $n$  is \_\_\_\_\_.
2. The number that is one space to the left of  $n$  is \_\_\_\_\_.
3. The number that is one space directly below  $n$  is \_\_\_\_\_.
4. The number that is one space directly above  $n$  is \_\_\_\_\_.
5. The number that is two spaces below and three spaces to the right of  $n$  is \_\_\_\_\_.
6. The number that is three spaces below  $n$  is \_\_\_\_\_.
7. The number that is three times the value of  $n$  is \_\_\_\_\_.
8. Explain using the hundreds chart why  $3 + n$  and  $3n$  are not equivalent expressions.

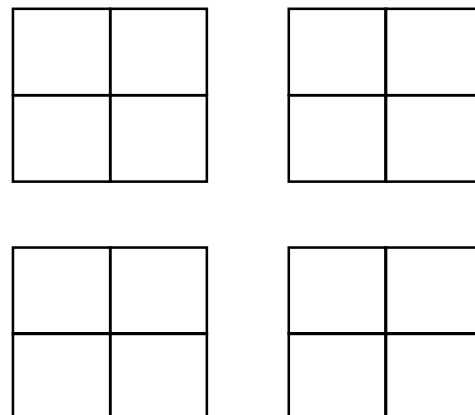
A new context is introduced for exploring more algebra topics.

Experiment with four numbers on the hundreds chart that form a  $2 \times 2$  square. One such group of numbers is to the right and the diagonals are circled.

9. List anything you notice about the numbers.



10. Consider at least four more  $2 \times 2$  squares. Make a conjecture about these diagonals.



## 2 BY 2 SQUARE CONJECTURES

Inductive reasoning is a form of reasoning in which the conclusion is supported by the evidence, but it is not proved.

1. Based on the evidence, write a conjecture that the class will attempt to prove about the diagonals of any  $2 \times 2$  squares in the hundred chart.

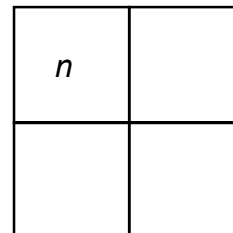
2. For how many  $2 \times 2$  squares would you have to find sums in order to prove that your conjecture is true?

The "friendly" context and ample white space keep students from being overwhelmed, yet we address the relatively sophisticated ideas of inductive vs. deductive reasoning, conjecture, and proof.

Deductive reasoning is a form of reasoning in which the conclusion is justified by an argument based on definitions, known facts, and accepted rules of logic.

3. Prove the conjecture by using algebra to label a generalized square and then by taking the sum of the values in each diagonal.

Define your variable by letting  $n =$  \_\_\_\_\_.

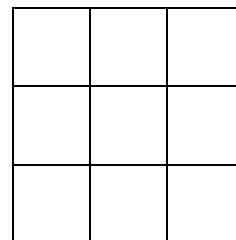
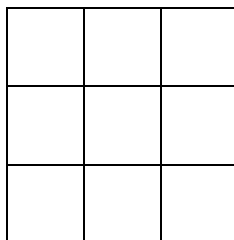
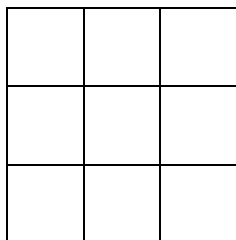
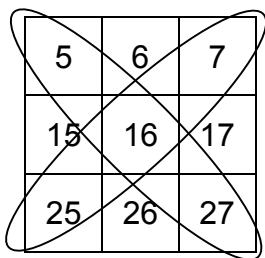


Sum of Values along Diagonal 1	Sum of Values along Diagonal 2
$( \quad ) + ( \quad )$	$( \quad + \quad ) + ( \quad + \quad )$

4. Why does your work prove the conjecture?

### 3 BY 3 SQUARE CONJECTURES

- Do you think a conjecture like the one we proved for  $2 \times 2$  squares also holds for  $3 \times 3$  squares? Try the same experiment with the given  $3 \times 3$  square. Find the sum of the numbers in each diagonal and compare the sums. Then do this for a few more  $3 \times 3$  squares.

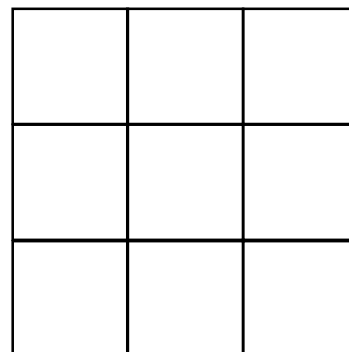


- Write your conjecture in words.

These next two pages ask students to extend and apply their knowledge to different situations.

- Prove your conjecture algebraically. Define your variable.

Let  $n =$  \_\_\_\_\_ .

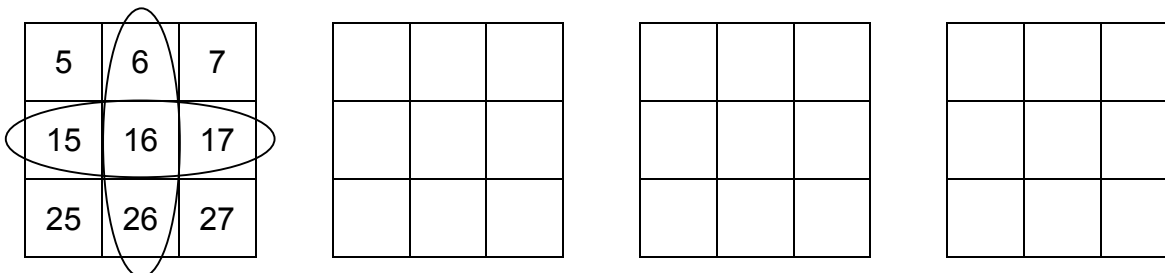


Sum of Values in Diagonal 1	Sum of Values in Diagonal 2

- Does the conjecture hold for all  $3 \times 3$  squares? Explain.

## PLUS PATTERN CONJECTURES

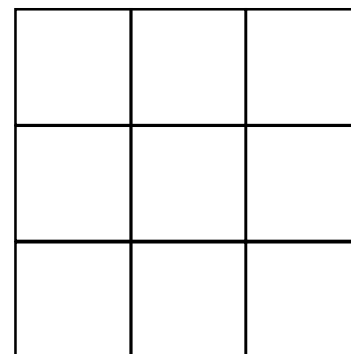
1. Try an experiment with the given “plus” pattern below from the hundreds chart. Add the numbers in the column and add the numbers in the row. Compare the sums. Repeat this process for plus patterns for several  $3 \times 3$  squares.



2. Write your conjecture in words.

3. Prove your conjecture algebraically. Define your variable.

Let  $n =$  \_\_\_\_\_ .



Sum of Values in Column	Sum of Values in Row

4. Does your conjecture hold? Explain.



**REWRITING EXPRESSIONS**

Rewrite the following expressions using the distributive property.

1. $\frac{3}{4}(u - v)$	2. $-\frac{3}{4}(u + v)$	3. $-\frac{2}{3}(u - \frac{1}{2})$	4. $-\frac{2}{3}(u - \frac{1}{2}v)$
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Simplify.

5. $-6p - 5m - p - 2m$	6. $-2(p - m) + 3(p + m)$
<div style="border: 1px solid black; padding: 5px; color: red;">Procedural skills that students need for success in any algebra unit are mixed into the lesson.</div>	
7. $4(-p - m) - 4(p + 2m)$	8. $2.4x + 3.5y - 1.8x$
9. $-2.4x - 3.5y - 1.8x$	10. $-2.4x - (3.5y - 1.8x)$

**REWRITING EXPRESSIONS (Continued)**

Simplify.

11. $\frac{1}{3}d + \frac{2}{3}h - \frac{3}{4}h$	12. $\frac{1}{5}(d - h) + \frac{3}{10}(h - d)$
13. $\frac{1}{3}(a + 6b) + \frac{2}{3}a$	14. $-\frac{3}{8}(h - d) - \frac{3}{4}h - d$
15. $(-2\frac{7}{10}b) - (\frac{9}{10} - 0.5b)$	16. $-\frac{5}{2}(2a + 2b) + \frac{5}{4}b$
17. $-2 + 1.5(b - \frac{1}{4}a)$	18. $2(-a + 1.5b) - \frac{1}{4}a$

## PRACTICE WITH EQUIVALENT EXPRESSIONS

Fill in the blanks so that the expressions on both sides of the equal sign are equivalent.

1. $\frac{1}{2}b - \frac{1}{6}a - \frac{4}{5}b + \frac{2}{3}a = \begin{array}{ c } \hline \phantom{x} \\ \hline \phantom{x} \\ \hline \end{array} a - \begin{array}{ c } \hline \phantom{x} \\ \hline \phantom{x} \\ \hline \end{array} b$	<div style="color: red; text-align: center; padding: 5px;"> <p>Remember that skills practice is located within lessons and also in Skill Builders in this packet and in future packets.</p> </div>
2. $\frac{1}{4}x + \frac{1}{8}y = \frac{1}{2} \left( \begin{array}{ c } \hline \phantom{x} \\ \hline \phantom{x} \\ \hline \end{array} x + \begin{array}{ c } \hline \phantom{x} \\ \hline \phantom{x} \\ \hline \end{array} y \right)$	3. $2.8f - 1.4k = 0.07(\text{ \_\_\_\_\_\_ } f + \text{ \_\_\_\_\_\_ } k)$
4. $\frac{5x}{3} + \frac{2}{3} = \frac{\begin{array}{ c } \hline \phantom{x} \\ \hline \phantom{x} \\ \hline \end{array}}{3}$	5. $\frac{10x + 2}{4} = \frac{\begin{array}{ c } \hline \phantom{x} \\ \hline \phantom{x} \\ \hline \end{array}}{4} + \frac{\begin{array}{ c } \hline \phantom{x} \\ \hline \phantom{x} \\ \hline \end{array}}{4}$

6. Circle all the expressions that are equivalent to  $-4(3x - 2)$ .

$-12x - 2$

$-12x - 8$

$-12x + 8$

$-2(6x - 4)$

7. Circle all the expressions that are equivalent to  $5 + \frac{1}{2}(x + 8)$ .

$5\frac{1}{2}(x + 8)$

$\frac{1}{2}x + 9$

$\frac{1}{2}x + 13$

$5 + \frac{1}{2}x + 4$

8. Circle all the expressions that are equivalent to  $\frac{-4(x + 1)}{6}$ .

$\frac{-4x - 4}{6}$

$\frac{-4x}{6} - \frac{4}{6}$

$\frac{-2(x + 1)}{3}$

$-\frac{2x}{3} - \frac{2}{3}$

# POLYGON AREA PUZZLE

## Summary

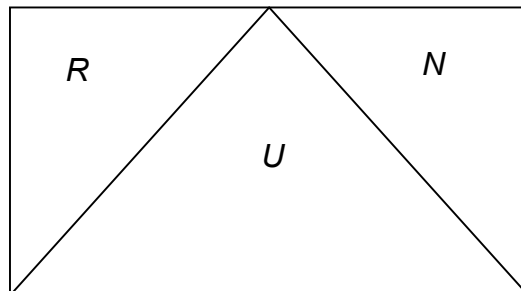
We will create expressions and equations based on the areas of polygon puzzle pieces. We will evaluate expressions, solve equations, and solve problems.

## Goals

- Write algebraic expressions and equations.
- Evaluate algebraic expressions.
- Solve equations.
- Use algebra to solve problems.

## Warmup

The rectangle to the right contains three triangles. Let  $R$ ,  $U$ , and  $N$  represent the areas of each triangle. The triangle with area  $U$  is an isosceles triangle.



1. Explain the relationship between the triangle with area  $R$  and the triangle with area  $N$ .

2. Explain the relationship between the triangle with area  $R$  and the triangle with area  $U$ .

3. If  $R = \frac{3}{4}$ , what is the value of  $U$ ?

This lesson introduces another context to motivate student exploration of expressions and equations.

The next packet is all about solving equations in traditional and non-traditional ways.

4. If  $U = 4.3$ , what is the value of  $N$ ?

## WRITING EQUATIONS

Your teacher will give you some polygon puzzle pieces. The area of each shape is written inside the shape. For example, the rectangle with a “ $B$ ” inside has an area of  $B$  square units.

1. Write equations based upon the areas of the shapes.

Example:  $A = 2B$ .

See the intact puzzle pieces on page 37 (also in the Reproducible section for making extra copies for students to cut out if desired). Teachers also like to make sturdy sets of the puzzle pieces to use over and over.

2. Refer to the equations you wrote above. Also consider equations that classmates shared. Organize these equations into categories. Explain how you sorted them.

## RELATIONSHIPS AMONG POLYGON AREAS

Use your knowledge of the areas of the polygon pieces and the given values to find the missing values. Work space is provided. Each problem is independent of the others.

<p>1. Given: <math>D = 4</math> square units</p> <p><math>C =</math> _____</p> <p><math>\frac{1}{2}B =</math> _____</p> <p><math>A + C =</math> _____</p>	<p>2. Given: <math>N = 3</math> square units</p> <p><math>P =</math> _____</p> <p><math>C =</math> _____</p> <p><math>E + 2F =</math> _____</p>	
<p>3. Given: <math>A = 40</math> square units</p> <p><math>F =</math> _____</p> <p><math>3B =</math> _____</p> <p><math>2G + H =</math> _____</p>	<p>4. Given: <math>H = 36</math> square units</p> <p><math>G =</math> _____</p> <p><math>\frac{1}{3}J =</math> _____</p> <p><math>A + B =</math> _____</p>	
<p>5. Given: <math>D = 30</math> sq. units</p> <p><math>M =</math> _____</p> <p><math>L =</math> _____</p>	<p>6. Given: <math>N = \frac{1}{3}</math> sq. unit</p> <p><math>Q + D + F =</math> _____</p>	<p>7. Given: <math>C = \frac{4}{5}</math> sq. unit</p> <p><math>Q + D + F =</math> _____</p>

## SOLVING POLYGON AREA EQUATIONS

Use symbols, words, or diagrams to explain how you know that the following equations are true. Then solve for the unknown using any method. All values are given in square units.

1. Explain why  $D = Q$ .

Then find  $D$  if  $Q = 97.43$ .

2. Explain why  $E = 2F$ .

Then find  $F$  if  $E = 4.8$ .

Students may use equation solving strategies learned in 6<sup>th</sup> grade or any other methods they know. This is a nice way to informally assess what your students already know about substitution and solving equations. Equation solving strategies, both traditional and not, are the focus of the next packet.

3. Explain why  $2K = L + D$ .

Then find  $K$  if  $L = 6$  and  $D = 2$ .

4. Explain why  $2G + H = P + 2(N + D)$ .

Then find  $D$  if  $G = 3$ ,  $H = 18$ ,  
 $P = 12$ , and  $N = 2$ .

**SOLVING POLYGON AREA EQUATIONS (Continued)**

Use symbols, words, or diagrams to explain how you know that the following equations are true. Then solve for the unknown using any method. All values are given in square units.

5. Explain why  $L = C + D$ .

Then find  $D$  if  $L = \frac{3}{8}$  and  $C = \frac{1}{4}$ .

6. Explain why  $D = \frac{1}{2} C$ .

Then find  $C$  if  $D = \frac{3}{8}$ .

7. Explain why  $\frac{1}{3} J = \frac{1}{2} B$ .

Then find  $B$  if  $J = 18$ .

8. Explain why  $2N = \frac{M}{2}$ .

Then find  $N$  if  $M = 1.6$ .



## WRITING EQUATIONS REVISITED

Write each given expression in terms of the specified variable.

	Given expression	Written in terms of this variable	Equation
Example 1	$A$	$B$	$A = 2B$
Example 2	$B$	$A$	$B = \frac{1}{2}A$
1.	$C$	$F$	$C =$
2.	$F$	$C$	<div style="border: 1px solid black; padding: 5px; color: red;">           Writing an expression in terms of a specific variable is probably a new challenge for students.         </div>
3.	$G$	$H$	
4.	$H$	$G$	
5.	$B + C$	$D$	
6.	$J$	$A$	
7.	$2(B + C)$	$J$	
8.	$2N$	$P$	
9.	$M - D$	$M$	
10.	$L + D$	$K$	
11.	$L$	$K$	
12.	$D$	$L$	

## PAINTINGS ON THE WALL

Meghan has a room with a wall that is  $12\frac{1}{4}$  feet wide.

- She wants to paint four square canvases that are all the same size to hang side-by-side across the wall from left to right.
- She wants  $\frac{3}{4}$  feet between each of the four canvases.
- She wants to leave  $1\frac{1}{4}$  feet between the left edge of the wall and the first canvas and  $1\frac{1}{4}$  between the right edge of the wall and the last canvas.

1. Sketch and label the wall below with the four canvases on it. Then find the side length of each square canvas.

This problem is like a  
"performance task."

2. Rahim likes the way Meghan's wall looks with the canvases on it and wants to do the same on his wall, but he doesn't know its total width. Write an expression Meghan could give Rahim for the side length he should use for his square canvases given that his wall's width is  $w$  feet.

# SKILL BUILDERS, VOCABULARY, AND REVIEW

## SKILL BUILDER 1

Find the GCF and LCM of the following pairs of numbers.

1. 9, 12	2. 16, 24
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Write  $<$ ,  $=$ , or  $>$  to make a true statement.

3. $10 - 2(3) \underline{\hspace{1cm}} 3(-4) - 6$	4. $13(-1) - 2 \underline{\hspace{1cm}} (-3) + (-12)$
5. $\frac{4 + (-8)}{2(-2)} \underline{\hspace{1cm}} \frac{5(-6)}{-3 - 2}$	6. $\frac{-2(5 - 3)}{6 - 8} \underline{\hspace{1cm}} \frac{3(-9 + 2)}{6 - 9}$

7. Jack and Jill began to walk up a hill, starting at an elevation of 5 feet below sea level. They walked up 20 feet to fetch a pail of water. Jack slipped and fell 2 feet per second for 12 seconds. He finally stopped falling when he broke his crown. At what elevation did Jack break his crown?

*Skill Builders usually have 3-4 pages of review at the start, strategically built in, both to include review for fluency's sake, and to support learning in the current or an upcoming packet.*

Compute.

8. $7.8 - 29.3$	9. $(-4.2)(6.1)$
10. $-2\frac{3}{4} + 5\frac{3}{8}$	11. $\left(-\frac{3}{10}\right)\left(-\frac{2}{9}\right)$

## SKILL BUILDER 2

1. Consider the probability of picking a jack of diamonds from a standard deck of cards. Circle all of the words below that appropriately describe this situation.

certain    impossible    possible    likely    unlikely    probable    improbable

Each of the letters of the word *CALIFORNIA* are written on separate pieces of paper that are then folded, put in a hat, and mixed thoroughly. One piece of paper is chosen (without looking) from the hat.

<p>a. What is the probability that an <i>A</i> will be chosen?</p>	<p>b. If you picked a letter, replaced it in the hat, and repeated the experiment 20 times, about how many times would you expect to pick an <i>A</i>?</p>
<p>c. Write the letters <i>CALIFORNIA</i> on pieces of paper and perform the experiment described in part (b) above 20 times. Record your results with tally marks.</p>	<p>d. Is your result from part (c) equal to your prediction in part (b)? _____ Explain why there could be discrepancies.</p>

2. Create a display to the right that shows all the possible outcomes from rolling a number cube and spinning a spinner with four equal-sized sectors with colors green, red, orange, and green on it.

Using the display above, find the following probabilities.

<p>3. <math>P(6 \text{ and green})</math></p>	<p>4. <math>P(\text{odd and orange})</math></p>	<p>5. <math>P(\text{less than 6 and black})</math></p>
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### SKILL BUILDER 3

Below are the prices and weights of different-sized pretzel products.

<b>Product</b>	<b>Price</b>	<b>Total Ounces</b>
A. Small bag of Pretzels	\$3	20 oz
B. Box of 10 mini-packs of pretzels, each 1 oz	\$4	10 oz
C. Big bag of Pretzels	\$6	40 oz

1. Demonstrate that two pretzel products have the same unit, in terms of dollars per ounce, while the “other” does not. Explain your reasoning.
  
  
  
  
  
  
  
  
  
  
  
2. What would be the price of the “other” pretzel product in order for it to have the same units price as the other two?

For problems 3 and 4, assume that pretzels are bagged and sold for a price of \$30 for each 20 ounces.

3. How much would a 30 oz bag of pretzels cost?
  
  
  
  
  
  
  
  
  
  
  
4. What would be the weight of a bag of pretzels that costs \$12?

**SKILL BUILDER 4**

Solve.

1. $3.5x = 35$	2. $0.9y = 0.36$	3. $0.60 = 1.5n$
4. $\frac{a}{5} = 2.2$	5. $4 = \frac{b}{1.5}$	6. $\frac{d}{\frac{1}{3}} = 2$
7. $\frac{1}{2}x = 20$	8. $\frac{1}{4}m = \frac{5}{8}$	9. $\frac{4}{9} = \frac{2}{3}p$

10. Lena spent  $\frac{7}{10}$  of the money  $m$  she had in her pocket at the grocery store.

a. Write an expression for the amount of money Lena spent.

b. Lena has \$29.40 left in her pocket. Write an equation to determine how much money she had to start, and then solve the equation.

### SKILL BUILDER 5

Demonstrate whether each of the following pairs of expressions is equivalent or not.

Expressions		Yes / No
1.	a. $-9 - 4$	b. $-9 + (-4)$
2.	a. $-10 - 2$	b. $2 - (-10)$

First page of practice of current packet's work appears here.

Evaluate each expression for $m = 3$		If equivalent, write the algebraic expression in its simplest form
3.	a. $-m + (m)$	b. $m - m$
4.	a. $m(m + 1)$	b. $m \cdot m + m$
5.	a. $-m(m - 1)$	b. $-m \cdot m - m$

6. Consider the expression  $w + 3x + 4w + x + 6x$ .

a. Simplify the expression.

b. Rewrite the expression as a product of 5 and an expression with two terms.

$$5(\underline{\hspace{1cm}} + \underline{\hspace{1cm}})$$

c. Substitute the values  $w = -3$  and  $x = 6$  into the expressions. Based on this evidence, do you think the expressions appear to be equivalent?

Part (a):

Part (b):

d. Illustrate the distributive property with two of the variable (algebraic) expressions above to show that the expressions are equivalent.

$$\underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

## SKILL BUILDER 6

- What are two factors of the expression  $5(8 + 7)$ ? \_\_\_\_\_ and \_\_\_\_\_ .
- Rewrite the expression above as a sum of terms using the distributive property. Do not simplify.

Use the distributive property to rewrite each expression so that it is a sum of two terms.

3. $-4(g + 2)$	4. $8(-x + y)$	5. $(5w - 8)(-3)$
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Use the distributive property and the GCF of the terms to rewrite each expression so that it is a product of two factors.

6. $3t + 15$	7. $-4w - 20$	8. $-56 - 8u$
9. $12a + 24b$	10. $6a + 6b - 12$	11. $-3 + 21x$

Simplify each expression. Then use the distributive property and the GCF of the terms to rewrite each expression so that it is a product of factors.

12. $2n + 2 + 2n + 2$	13. $3(n + 4) + 3n + 6$	14. $-10(n + 1) - 2n + 6$
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## SKILL BUILDER 7

1. Explain the difference between inductive and deductive reasoning.

Simplify.

<p>2. <math>\frac{1}{2}(-4x + 2) + \frac{1}{6}</math></p>	<p>3. <math>-\frac{7}{4}(4y - x) + \frac{3}{4}x</math></p>
<p>4. <math>-0.5x + 2.5(x + 2y)</math></p>	<p>5. <math>\frac{1}{3}y + 6(2y - 0.5x)</math></p>

Fill in the blanks so that the expressions on each side of the equal sign are equivalent.

6.  $\frac{1}{9}x + \frac{1}{6}y = \frac{1}{3} \left( \begin{array}{|c|} \hline \phantom{x} \\ \hline \phantom{x} \\ \hline \end{array} x + \begin{array}{|c|} \hline \phantom{y} \\ \hline \phantom{y} \\ \hline \end{array} y \right)$

7.  $3.5f - 1.5k = 0.05(\text{ \_\_\_\_\_\_ } f + \text{ \_\_\_\_\_\_ } k)$

8. Circle all the expressions that are equivalent to  $-4x + 6$ .

$-6 + 4x$

$6 - 4x$

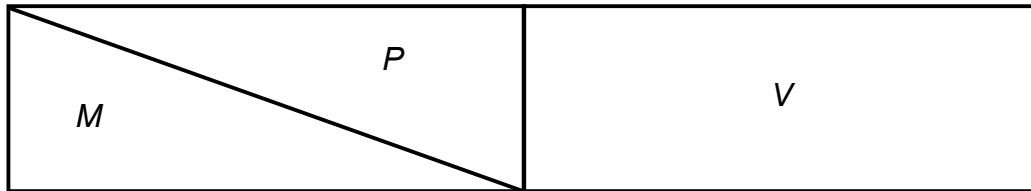
$-2(2x + 6)$

$-2(-2x + 3)$

$\frac{-12x + 18}{3}$

## SKILL BUILDER 8

1. The shape with area  $M$  exactly covers the shape with area  $P$ . The shape with area  $V$  exactly covers the shapes with areas  $M$  and  $P$  combined. Write at least four true equations based upon the areas of the figures. Use fractions in at least one of the equations.



Evaluate each expression.

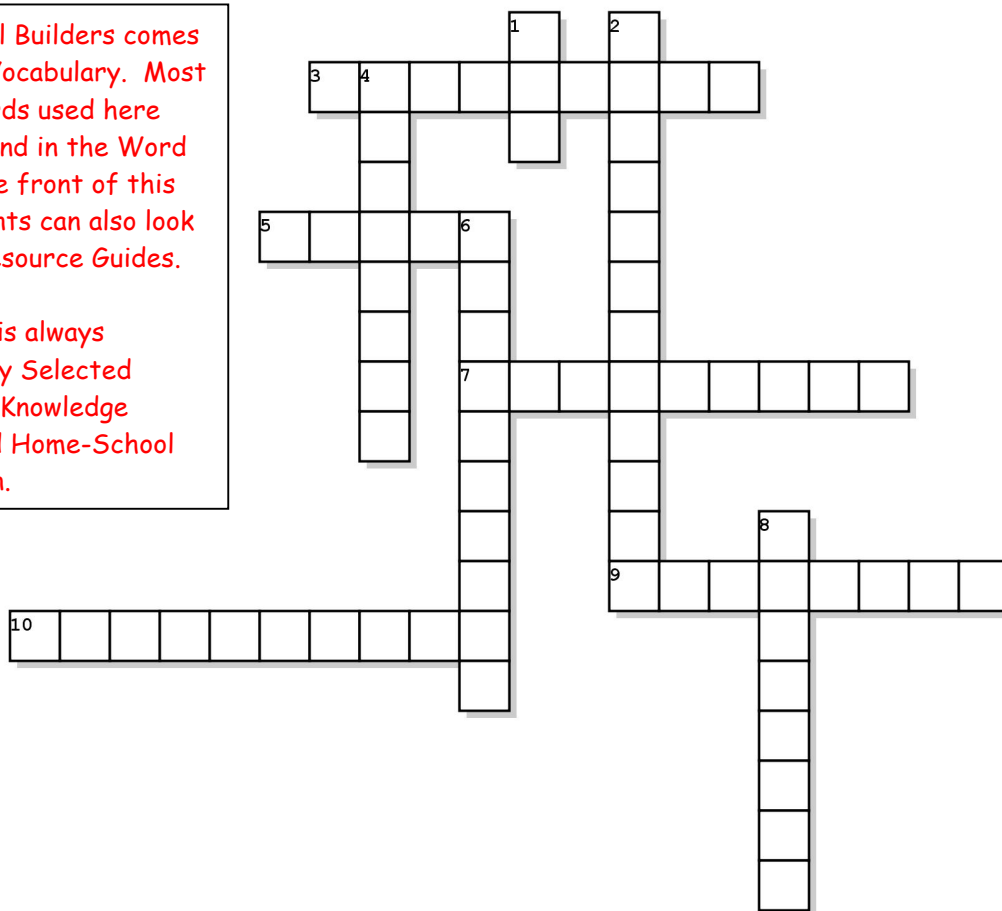
<p>2. Given <math>P = 3</math>, find:</p> <p>a. <math>V</math></p> <p>b. <math>\frac{1}{2}M</math></p> <p>c. <math>\frac{1}{4}(M + P)</math></p>	<p>3. Given <math>M = \frac{1}{3}</math>, find:</p> <p>a. <math>V</math></p> <p>b. <math>2P</math></p> <p>c. <math>\frac{1}{2}V + 2M</math></p>
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4. The curator of an art gallery wants to display 4 surfboards on a wall that is 9.5 feet wide. The surfboards are 1.5 feet wide at their widest point. They should be 0.75 feet away from the wall on both sides, and there should be an equal distance between neighboring boards.
- a. Sketch a picture of this situation.
- b. Translate the information into an equation. Let  $x$  = the distance between each pair of boards. Clearly indicate how far apart each board should be.

## FOCUS ON VOCABULARY

After Skill Builders comes Focus on Vocabulary. Most of the words used here can be found in the Word Bank in the front of this SP. Students can also look in their Resource Guides.

This page is always followed by Selected Response, Knowledge Check, and Home-School Connection.



**Across**

- 3 Reasoning based on rules of logic
- 5 Find a solution that makes an equation true.
- 7 Reasoning based on examples
- 9 Statement that two numerical expressions have the same value
- 10 A sum of terms, for example

**Down**

- 1 Greatest factor that two numbers share in common, in short
- 2 The \_\_\_\_\_ property links addition and multiplication.
- 4 Substitute values for the variables and calculate the result.
- 6 Two numerical expressions with the same value.
- 8 A quantity that may vary

(For word hints, see the word bank and other vocabulary used in this packet.)

**SELECTED RESPONSE**

Show your work on a separate sheet of paper and choose the best answer(s).

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1. Choose ALL expressions that are equivalent to  $x - 3 + (5x)$ .

A.  $4x - 3$

B.  $6x + 3$

C.  $3(2x - 1)$

These are exercises where more than one answer may be correct help to prepare students for these types of items on SBAC or PARCC.

2. Choose ALL of the true equations for all values of the variable.

A.  $4g - 16 = 4(4 - g)$

B.  $8k - 4 = 4(2k - 1)$

C.  $5y + 25 = 5(5 + y)$

D.  $12 + 20m = (3 + 5m)(4)$

3. Which expression is NOT equivalent to  $m + 2$ ?

A.  $1\frac{3}{4}\left(m + \frac{4}{7}\right) + \frac{1}{4}m$

B.  $1 - \frac{1}{2}(-2m - 2)$

C.  $0.8 + 2m - (-1.2) - m$

D.  $-4(0.5 - 0.25m) + 4$

4. Simplify.  $-2\frac{1}{2}(w + 4) - 1\frac{1}{2}w$

A.  $-1\frac{1}{2}w + 10$

B.  $-4w + 10$

C.  $-4w - 10$

D.  $-1\frac{1}{2}w - 10$

5. Use the equation  $A = B + Y$  to find the value of  $B$ , if  $A = -1.87$  and  $Y = 5.9$ .

A.  $-4.03$

B.  $4.03$

C.  $7.77$

D.  $-7.77$

6. Which of the following are solutions to the equation  $2B = \frac{M}{2}$ .

A.  $B = 4, M = 16$

B.  $B = 6.5, M = 13.5$

C.  $B = \frac{1}{2}, M = 1$

D.  $B = 1\frac{1}{4}, M = 5$

## KNOWLEDGE CHECK

Show your work on a separate sheet of paper and write your answers on this page.

### 8.1 Equivalent Expressions

1. Simplify.  $4(m - 6)$
2. Use the distributive property and the GCF of the terms to rewrite  $9y + 36$  so that it is the product of two factors.

### 8.2 Hundreds Chart Patterns

Simplify each expression.

3.  $-8(w + 5) + 2(6 - w)$
4.  $6.8x + 2.5y - 1.9x$
5.  $-2\frac{1}{8}n + 1\frac{3}{4}m - 4\frac{1}{2}n$

These problems are somewhat representative of those in each lesson, and may be used for review or formative assessment.

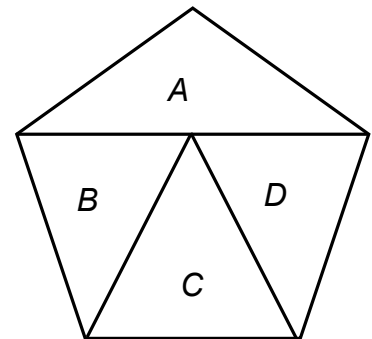
### 8.3 Polygon Area Puzzle

Use the figure to the right. The area of each triangle is indicated.

$$A + B + C + D = E; \quad A = \frac{2}{7}E$$

6. If  $E = 14$ ,  $A =$  \_\_\_\_\_

7. If  $A = 20$ ,  $E =$  \_\_\_\_\_



8.  $B + C + D =$    $\cdot E$

## HOME-SCHOOL CONNECTION

Here are some problems to review with your young mathematician.

1. Consider the expression  $y + 4y + 6x + x + y + 5x$ .

a. Simplify the expression.

b. Rewrite the expression as a product of 3 and an expression  $v$ .

c. Is the expression you wrote in part (b) equivalent to the original expression? \_\_\_\_\_  
Explain.

d. Verify that the expressions in parts (a) and (b) above lead to equivalent numerical expressions if  $x = -2$  and  $y = 4$ .

Part (a):

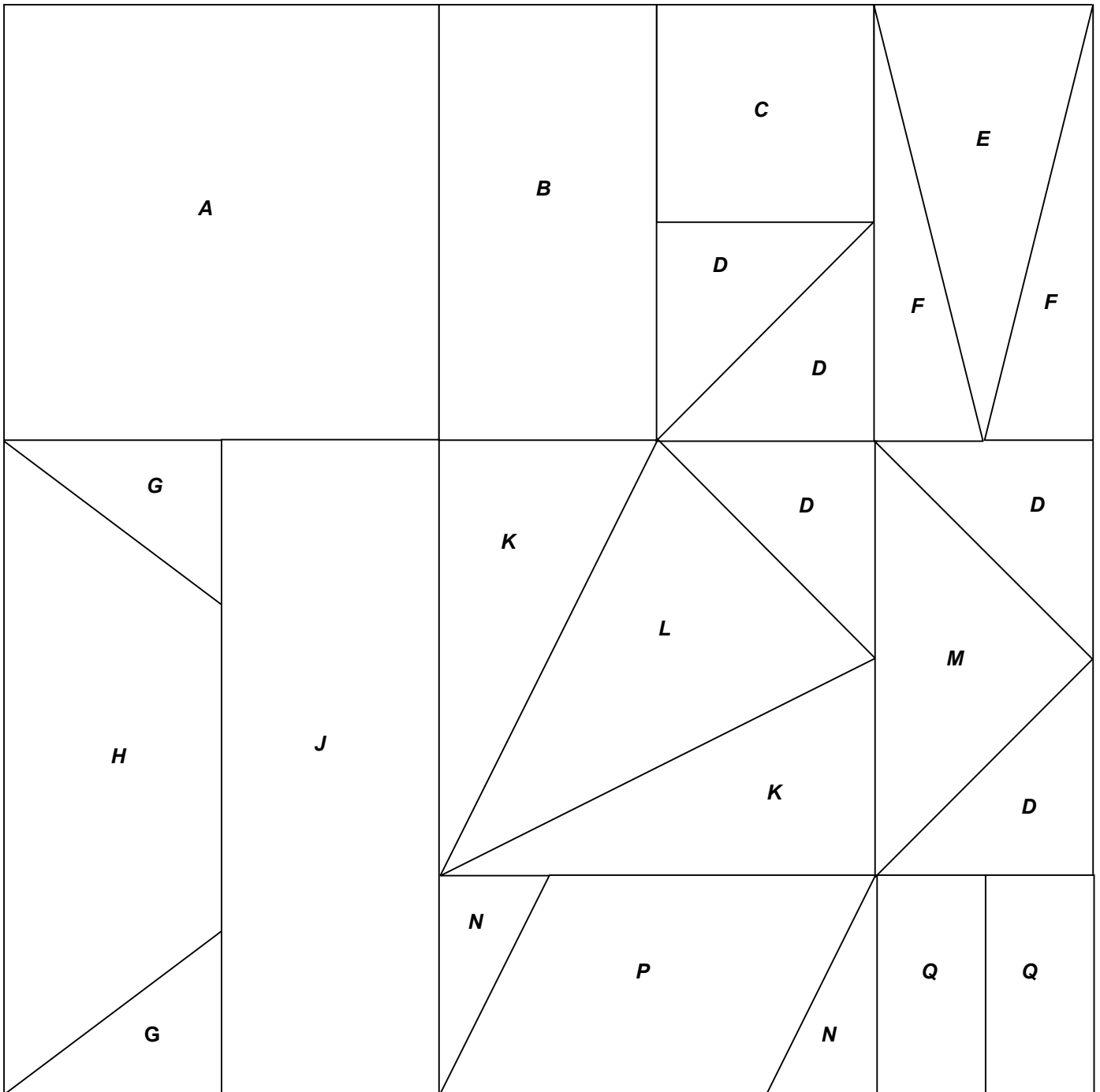
Part (b):

2. Simplify.  $\frac{3}{4}(w + 8d) + 2\frac{3}{4}w$

The intent of this page is to provide an opportunity for students to explain to parents or guardians what they are learning. At this time parents or guardians might check to see if students are completing their work in the packet.

Parent (or Guardian) Signature \_\_\_\_\_

# POLYGON PUZZLE PIECES



A Reproducible page like this in the back is unique to this packet. We think it will be helpful for students to have access to this page midway through lesson 3.

# COMMON CORE STATE STANDARDS – MATHEMATICS

STANDARDS FOR MATHEMATICAL CONTENT	
<b>6.NS.B*</b>	<b>Compute fluently with multi-digit numbers and find common factors and multiples.</b>
6.EE 3*	Apply the properties of operations to generate equivalent expressions. <i>For example, apply the distributive property to the expression <math>3(2 + y)</math>; apply properties of operations to write the equivalent expression <math>6 + 3y</math>;</i>
6.EE 4*	Identify when two expressions are equivalent. <i>For example, <math>3x + 3y</math> and <math>3(x + y)</math> are equivalent because they define the same function for every value of <math>x</math> and <math>y</math>.</i>
<b>6.EE.B*</b>	<b>Reason about and solve one-variable equations and inequalities.</b>
6.EE 6*	Use variables to represent numbers in a real-world or mathematical problem; understand solving an equation or inequality as a process of reasoning; explain each step in solving a one-variable equation or inequality (e.g., adding or subtracting the same number to both sides of an equation); justify the solution.
<b>7.NS.A</b>	<b>Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.</b>
7.NS.3	Solve real-world and mathematical problems involving the four operations with rational numbers.
<b>7.EE.A</b>	<b>Use properties of operations to generate equivalent expressions.</b>
7.EE 1	Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
7.EE 2	Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. <i>For example, <math>a + 0.05a = 1.05a</math> means that “increase by 5%” is the same as “multiply by 1.05.”</i>
<b>7.EE.B</b>	<b>Solve real-life and mathematical problems using numerical and algebraic expressions and equations.</b>
7.EE.3	Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. <i>For example: If a woman making \$25 an hour gets a 10% raise, she will make an additional <math>\frac{1}{10}</math> of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar <math>9\frac{3}{4}</math> inches long in the center of a door that is <math>27\frac{1}{2}</math> inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.</i>

These are the major content standards that are addressed in this packet. It is common for a standard to fully play out over multiple lessons and multiple packets. Note that the "cross-out" above means that the phrase was not included in this packet. It is addressed elsewhere in the program.

The practice standards above are not an exhaustive list, but are good examples of MPs in these lessons (See Teacher Note 1 in TP8). All of the MPs are revisited frequently throughout each course.

\*Content essential for success in 7<sup>th</sup> grade

STANDARDS FOR MATHEMATICAL PRACTICE	
MP1	Make sense of problems and persevere in solving them.
MP2	Reason abstractly and quantitatively.
MP3	Construct viable arguments and critique the reasoning of others.
MP4	Model with mathematics.
MP7	Look for and make use of structure.

