LESSON 14: TRANSFER FUNCTIONS OF DC MOTORS

ET 438a Automatic Control Systems Technology

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LEARNING OBJECTIVES

After this presentation you will be able to:

- Write the transfer function for an armature controlled dc motor.
- Write a transfer function for a dc motor that relates input voltage to shaft position.
- Represent a mechanical load using a mathematical model.
- Explain how negative feedback affects dc motor performance.

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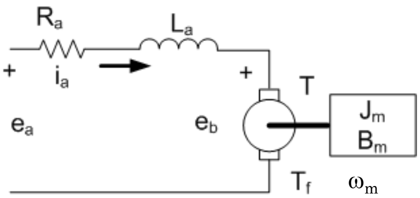
Steady-State Operation of Separately Excited Dc Motors

Consider steady-state model



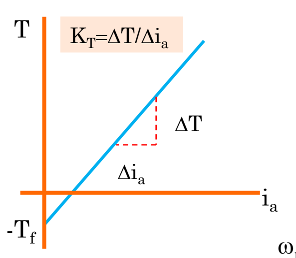
i_a = armature current
e_b= back emf
e_a= armature terminal voltage
ω_m = motor speed (rad/sec)
T = motor torque
T_f = static friction torque
R_a = armature resistance
L_a = armature inductance
J_m = rotational inertia
B_m = viscous friction

Review the steady-state relationships
Of machine

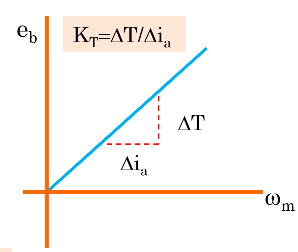
Steady-State Operation of Separately Excited Dc Motors

Relationships of Separately Excited Dc Motor

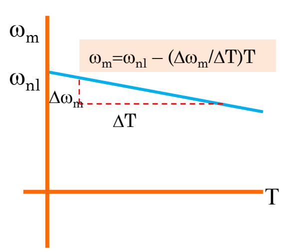
Torque-Current Curve



Back EMF Curve



Speed-Torque Curve



Steady-State Motor Equations

Developed Torque

$$T = K_T \cdot i_a - T_f \quad \text{N-m}$$

T = motor torque
K_T = torque constant
T_f = motor friction torque
i_a = armature current

KVL in Armature Circuit

$$e_a = i_a \cdot R_a + e_b \quad \text{V}$$

e_a = armature voltage
e_b = back emf
R_a = armature resistance

Back EMF

$$e_b = K_e \cdot \omega_m \quad \text{V}$$

ω_m = shaft speed (rad/s)
e_b = back emf
K_e = back emf constant

Developed Power

$$P = \omega_m \cdot T \quad \text{W}$$

P = shaft power

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Steady-State Motor Equations

Combining the previous equations gives:

$$\omega_m = \frac{K_T \cdot e_a - (T - T_f) \cdot R_a}{K_T \cdot K_e} \quad (1)$$

$$\omega_m = \frac{e_a - i_a \cdot R_a}{K_e} \quad (2)$$

If the load torque is zero (T=0) then the above equation (1) gives the no-load speed

$$\omega_{nl} = \frac{K_T \cdot e_a - (T_f) \cdot R_a}{K_T \cdot K_e}$$

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Steady-State Motor Operation

Example 14-1: An armature-controlled dc motor has the following ratings: T_f=0.012 N-m, R_a=1.2 ohms, K_T=0.06 N-m/A, K_e=0.06 V-s/rad. It has a maximum speed of 500 rad/s with a maximum current of 2 A. Find: a) maximum output torque, b) maximum mechanical output power, c) maximum armature voltage, d) no-load speed at maximum armature voltage.

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Example 14-1 Solution (1)

Define given variables

$T_f = 0.012\ \text{N-m}$ $\quad K_T = 0.06\ \text{N-m/A}$ $\quad \omega_{max} = 500\ \text{rad/sec}$

$R_a = 1.2\ \Omega$ $\quad K_e = 0.06\ \text{V-s/rad}$ $\quad i_{a\,max} = 2.0\ \text{A}$

a) T_{max} occurs at I_{max} so….

$$T_{max} = K_T i_{a\,max} - T_f$$

$$T_{max} = (0.06\ \text{N-m/A})(2.0\ \text{A}) - 0.012\ \text{N-m}$$

$$\boxed{T_{max} = 0.108\ \text{N-m}}$$

Answer

b) Find P_{max}

$$P_{max} = \omega_{max} T_{max}$$

$$P_{max} = (500\ \text{rad/s})(0.108\ \text{N-m})$$

$$P_{max} = \boxed{54\ \text{W}}$$

Answer

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Example 14-1 Solution (2)

c) Find maximum back emf

$$e_a = i_a R_a + e_b$$
$$e_a = i_a R_a + K_e \omega_{max}$$
$$e_a = (1.2\Omega)(2.0) + (0.06\ \text{V-s/rad})(500\ \text{rad/s})$$
$$\boxed{e_a = 32.4\ \text{V}}$$

Answer

d) Find no-load motor speed

At no-load, T=0. Load torque is zero.

$$\frac{K_T e_a - (T+T_f)R_a}{K_e K_T} = \omega_m \qquad \frac{K_T e_a + T_f(R_a)}{K_e K_T} = \omega_{nl}$$

T=0

$$\omega_{nl} = \frac{(0.06)(32.4) + 0.212(1.2)}{(0.06)(0.06)} = \boxed{536\ \text{rad/s}}$$

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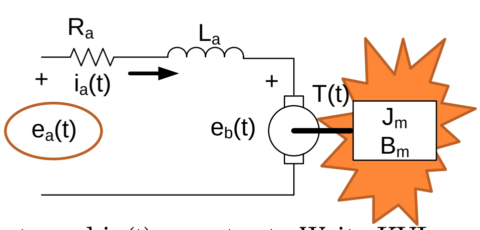
Transfer Function of Armature-Controlled DC Motor

Write all variables as time functions

Write electrical equations and mechanical equations. Use the electromechanical relationships to couple the two equations.



Consider $e_a(t)$ and $e_b(t)$ as inputs and ia(t) as output. Write KVL around armature

$$e_a(t) = R_a \cdot i_a(t) + L \cdot \frac{di_a(t)}{dt} + e_b(t)$$

Mechanical Dynamics $T(t) = J_m \cdot \frac{d\omega_m(t)}{dt} + B_m \cdot \omega_m(t)$

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TRANSFER FUNCTION OF ARMATURE-CONTROLLED DC MOTOR

Electromechanical equations

$$e_b(t) = K_E \cdot \omega_m(t)$$
$$T(t) = K_T \cdot i_a(t)$$

Find the transfer function between armature voltage and motor speed

$$\frac{\Omega_m(s)}{E_a(s)} = ?$$

Take Laplace transform of equations and write in I/O form

$$E_a(s) = L \cdot s \cdot I_a(s) + R_a \cdot I_a(s) + E_b(s)$$
$$E_a(s) = (L \cdot s + R_a) \cdot I_a(s) + E_b(s)$$
$$E_a(s) - E_b(s) = (L \cdot s + R_a) \cdot I_a(s)$$
$$I_a(s) = \left[\frac{1}{L \cdot s + R_a}\right]\left[E_a(s) - E_b(s)\right]$$

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TRANSFER FUNCTION OF ARMATURE-CONTROLLED DC MOTOR

Laplace Transform of Electromechanical Equations

$$E_b(s) = K_E \cdot \Omega_m(s)$$
$$T(s) = K_E \cdot I_a(s)$$

Laplace Transform of Mechanical System Dynamics

$$T(t) = J_m \cdot \frac{d\omega_m(t)}{dt} + B_m \cdot \omega_m(t)$$

Rewrite mechanical equation as I/O equation

$$T(s) = \left[J_m \cdot s + B_m\right] \cdot \Omega_m(s) \quad \Rightarrow \quad \Omega_m(s) = \left[\frac{1}{J_m \cdot s + B_m}\right] \cdot T(s)$$

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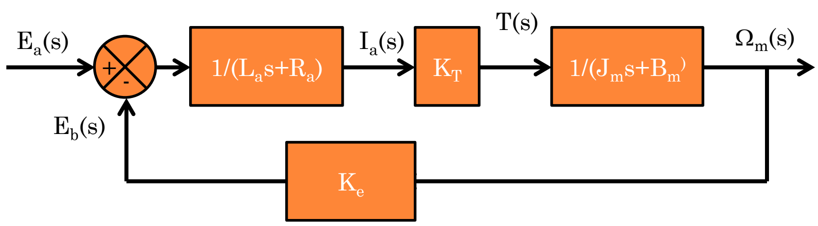
Block Diagram of Armature-Controlled Dc Motor

Draw block diagram from the following equations

$$I_a(s)=\left[\frac{1}{L\cdot s+R_a}\right]\left[E_a(s)-E_b(s)\right] \qquad T(s)=K_T\cdot I_a(s) \qquad \Omega_m(s)=\left[\frac{1}{J_m\cdot s+B_m}\right]\cdot T(s)$$



$$E_b(s)=K_E\cdot\Omega_m(s)$$

Note: The dc motor has an inherent feedback from the CEMF. This can improve system stability by adding a electromechanical damping

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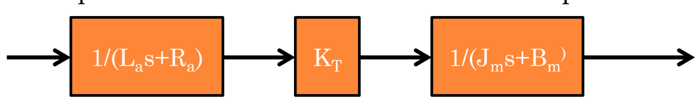
Transfer Function of Armature-Controlled Dc Motor

Use the feedback formula to reduce the block diagram

$$\frac{\Omega_m(s)}{E_a(s)}=\frac{G(s)}{1+G(s)\cdot H(s)}$$

$$H(s)=K_E$$

G(s) is the product of all the blocks in the forward path



$$G(s)=K_T\cdot\left[\frac{1}{L_a\cdot s+R_a}\right]\cdot\left[\frac{1}{J_m\cdot s+B_m}\right]$$

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Simplification of Transfer Function

Substitute G(s) and H(s) into the feedback formula

$$\frac{\Omega_m(s)}{E_a(s)} = \frac{\dfrac{K_T}{(L_a \cdot s + R_a)\cdot(J_m \cdot s + B_m)}}{1+\left[\dfrac{K_T}{(L_a \cdot s + R_a)\cdot(J_m \cdot s + B_m)}\right]\cdot K_E}$$

G(s)

H(s)

G(s)

Simplify by multiplying numerator and denominator by factors $(L_a s+R_a)(J_m s+B_m)$

$$\frac{\Omega_m(s)}{E_a(s)} = \frac{K_T}{(L_a \cdot s + R_a)\cdot(J_m \cdot s + B_m) + K_T \cdot K_E}$$

Expand factors and collect like terms of s

Final Formula

$$\frac{\Omega_m(s)}{E_a(s)} = \frac{K_T}{L_a \cdot J_m \cdot s^2 + (R_a \cdot J_m + B_m \cdot L_a)\cdot s + (K_T \cdot K_E + R_a \cdot B_m)}$$

Roots of denominator effected by values of parameters. Can be Imaginary.

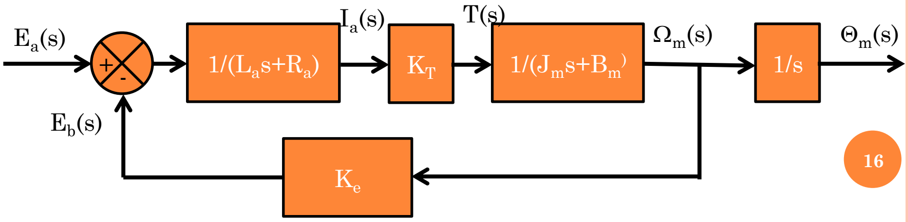
DC Motor Position Transfer Function

Motor shaft position is the integral of the motor velocity with respect to time. To find shaft position, integrate velocity

$$\frac{d\theta(t)}{dt} = \omega(t)$$

$$\int \frac{d\theta(t)}{dt}\ dt = \int \omega(t)\ dt = \theta(t)$$

To find the motor shaft position with respect to armature voltage, reduce the following block diagram



DC MOTOR POSITION TRANSFER FUNCTION

Position found by multiplying speed by 1/s (integration in time)

$$\Theta_m(s) = \left[\frac{1}{s}\right] \cdot \Omega_m(s)$$

$$\frac{\Theta_m(s)}{E_a(s)} = \left[\frac{1}{s}\right] \cdot \left[\frac{K_T}{L_m \cdot J_m \cdot s^2 + (L_a \cdot B_m + R_a \cdot J_m) \cdot s + (K_T \cdot K_E + R_a \cdot B_m)}\right]$$

$$\frac{\Theta_m(s)}{E_a(s)} = \frac{K_T}{s \cdot (L_m \cdot J_m \cdot s^2 + (L_a \cdot B_m + R_a \cdot J_m) \cdot s + (K_T \cdot K_E + R_a \cdot B_m))}$$

$$\frac{\Theta_m(s)}{E_a(s)} = \frac{K_T}{L_m \cdot J_m \cdot s^3 + (L_a \cdot B_m + R_a \cdot J_m) \cdot s^2 + (K_T \cdot K_E + R_a \cdot B_m) \cdot s}$$

T.F.

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REDUCED ORDER MODEL

Define motor time constants

$$\frac{J_m}{B_m} = \tau_m \quad \text{and} \quad \frac{L_a}{R_a} = \tau_e$$

Where: τ_m = mechanical time constant
τ_e = electrical time constant

Electrical time constant is much smaller than mechanical time constant. Usually neglected. Reduced transfer function becomes…

$$\frac{\Omega_m(s)}{E_a(s)} = \frac{K_s}{1 + \tau_s \cdot s}$$

Where $K_s = \frac{K_T}{K_T \cdot K_E + R_a \cdot B_m}$ and $\tau_s = \frac{R_a \cdot J_m}{K_T \cdot K_E + R_a \cdot B_m}$

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Motor with Load

Consider a motor with load connected through a speed reducer.
Load inertia $= J_L$
Load viscous friction $= B_L$

Motor coupled to speed reducer, motor shaft coupled to smaller gear with N_1 teeth. Load connected to larger gear with N_2 teeth.

$$\omega_L = \left[\frac{N_1}{N_2}\right] \cdot \omega_m \ \text{rad/sec} \quad N_1 < N_2$$

$$T_L = \left[\frac{N_2}{N_1}\right] \cdot T_m \ \text{N-m} \quad N_1 < N_2$$

Gear reduction decreases speed but increases torque
P_{mech}=constant. Similar to transformer action

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Motor with Load

Speed changer affects on load friction and rotational inertia

Without speed changer (direct coupling)

$$B_T = B_m + B_L \quad \text{N-m-s/rad}$$
$$J_T = J_m + J_L \quad \text{N-m-s}^2\text{/rad}$$

With speed changer

$$B_T = B_m + \left[\frac{N_1}{N_2}\right]^2 \cdot B_L \quad \text{N-m-s/rad}$$
$$J_T = J_m + \left[\frac{N_1}{N_2}\right]^2 \cdot J_L \quad \text{N-m-s}^2\text{/rad}$$

Where: B_T = total viscous friction
J_T = total rotational inertia
B_L = load viscous friction
B_m = motor viscous friction
J_m = motor rotational inertia
J^L = load rotational inertia

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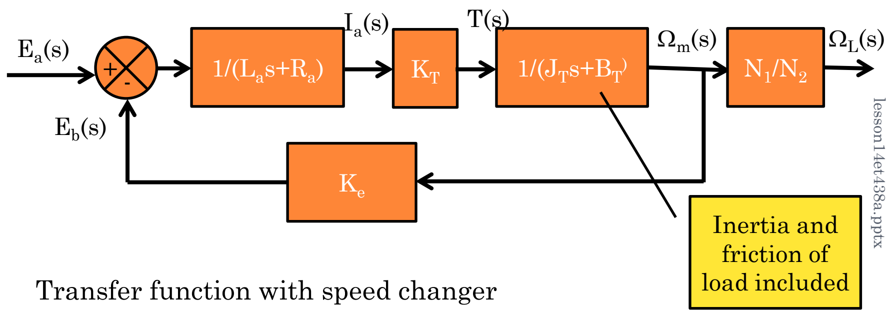
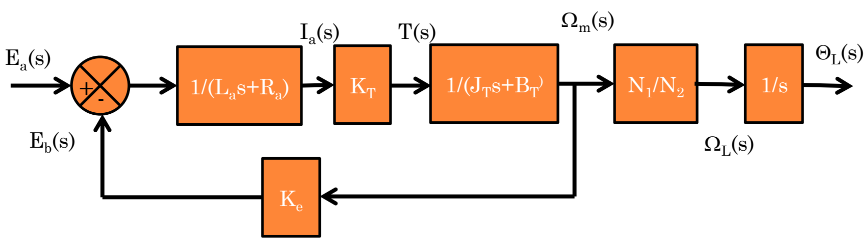
Motor with Load Block Diagram



Transfer function with speed changer

$$\frac{\Omega_m(s)}{E_a(s)} = \frac{K_T \cdot \left[\frac{N_1}{N_2}\right]}{L_a \cdot J_m \cdot s^2 + (R_a \cdot J_m + B_m \cdot L_a) \cdot s + (K_T \cdot K_E + R_a \cdot B_m)}$$

Motor Position with Load Block Diagram



Motor position transfer function with speed changer. Note: multiplication by s

$$\frac{\Theta_L(s)}{E_a(s)} = \frac{K_T \cdot \left[\frac{N_1}{N_2}\right]}{L_a \cdot J_m \cdot s^3 + (R_a \cdot J_m + B_m \cdot L_a) \cdot s^2 + (K_T \cdot K_E + R_a \cdot B_m) \cdot s}$$

DC MOTOR TRANSFER FUNCTION EXAMPLE

Example 14-2: A permanent magnet dc motor has the following specifications.

Maximum speed = 500 rad/sec
Maximum armature current = 2.0 A
Voltage constant (K_e) = 0.06 V-s/rad
Torque constant (K_T) = 0.06 N-m/A
Friction torque = 0.012 N-m
Armature resistance = 1.2 ohms
Armature inductance = 0.020 H
Armature inertia = 6.2×10^{-4} N-m-s^2/rad
Armature viscous friction = 1×10^{-4} N-m-s/rad

a) Determine the voltage/velocity and voltage/position transfer functions for this motor
b) Determine the voltage/velocity and voltage/position transfer functions for the motor neglecting the electrical time constant.

EXAMPLE 14-2 SOLUTION (1)

Define all motor parameters

$\omega_m = 500$ rad/s
$i_{a\,max} = 2.0$ A
$T_f = 0.012$ N-m
$J_m = 6.2 \times 10^{-4}$ N-m-s^2/rad
$B_m = 1 \times 10^{-4}$ N-m-s/rad

$K_e = 0.06$ V-s/rad
$K_T = 0.06$ N-m/A
$R_a = 1.2\,\Omega$
$L_a = 0.02$ H

a) Full transfer function model

$$\frac{\Omega_m(s)}{E_a(s)} = \frac{K_T}{(R_a B_m + K_e K_T) + (R_a J_m + B_m L_a)s + L_a J_m s^2}$$

Example 14-2 Solution (2)

Compute denominator coefficients from parameter values

$$R_a B_m + K_e K_T = (1.2)(1\times10^{-4}) + (0.06)(0.06) = 0.00372$$
$$R_a J_m + B_m L_a = (1.2)(6.2\times10^{-4}) + (1\times10^{-4})(0.02) = 7.46\times10^{-4}$$
$$L J_m = 0.02(6.2\times10^{-4}) = 1.24\times10^{-5}$$

$$\frac{\Omega_m(s)}{E_a(s)} = \frac{0.06}{0.00372 + 7.46\times10^{-4}s + 1.24\times10^{-5}s^2}$$

Can normalize constant by dividing numerator and denominator by 0.00372

$$\boxed{\frac{\Omega_m(s)}{E_a(s)} = \frac{16.13}{1 + 0.201s + 0.00333s^2}}$$

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Example 14-2 Solution (3)

To covert this to a position transfer function, multiple it by 1/s

$$\theta_m(s) = \frac{1}{s}\Omega_m(s)$$

$$\frac{\theta_m(s)}{E_a(s)} = \frac{1}{s}\left[\frac{16.13}{1+0.201s+0.00333s^2}\right] = \boxed{\frac{16.13}{s+0.201s^2+0.00333s^3}}$$

b) Compute the transfer functions ignoring the electrical time constant

$$\frac{\Omega_m(s)}{E_a(s)} = \frac{K_s}{1+\tau_s s}$$

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Example 14-2 Solution (4)

Compute parameter values

$$K_s = \frac{K_T}{R_a B_m + K_e K_T} = \frac{0.06}{1.2(1\times10^{-4}) + 0.06(0.06)}$$

$$K_s = 16.13$$

$$\tau_s = \frac{R_a J_m}{R_a B_m + K_e K_T} = \frac{(1.2)(6.2\times10^{-4})}{1.2(1\times10^{-4}) + 0.06(0.06)} \qquad \tau_s = 0.2$$

$$\frac{\Omega_m(s)}{E_a(s)} = \boxed{\frac{16.13}{1+0.2s}} \qquad \Theta_m(s) = \frac{1}{s}\Omega_m(s) \qquad \boxed{\frac{\Theta_m(s)}{E_a(s)} = \frac{16.13}{s+0.2s^2}}$$

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