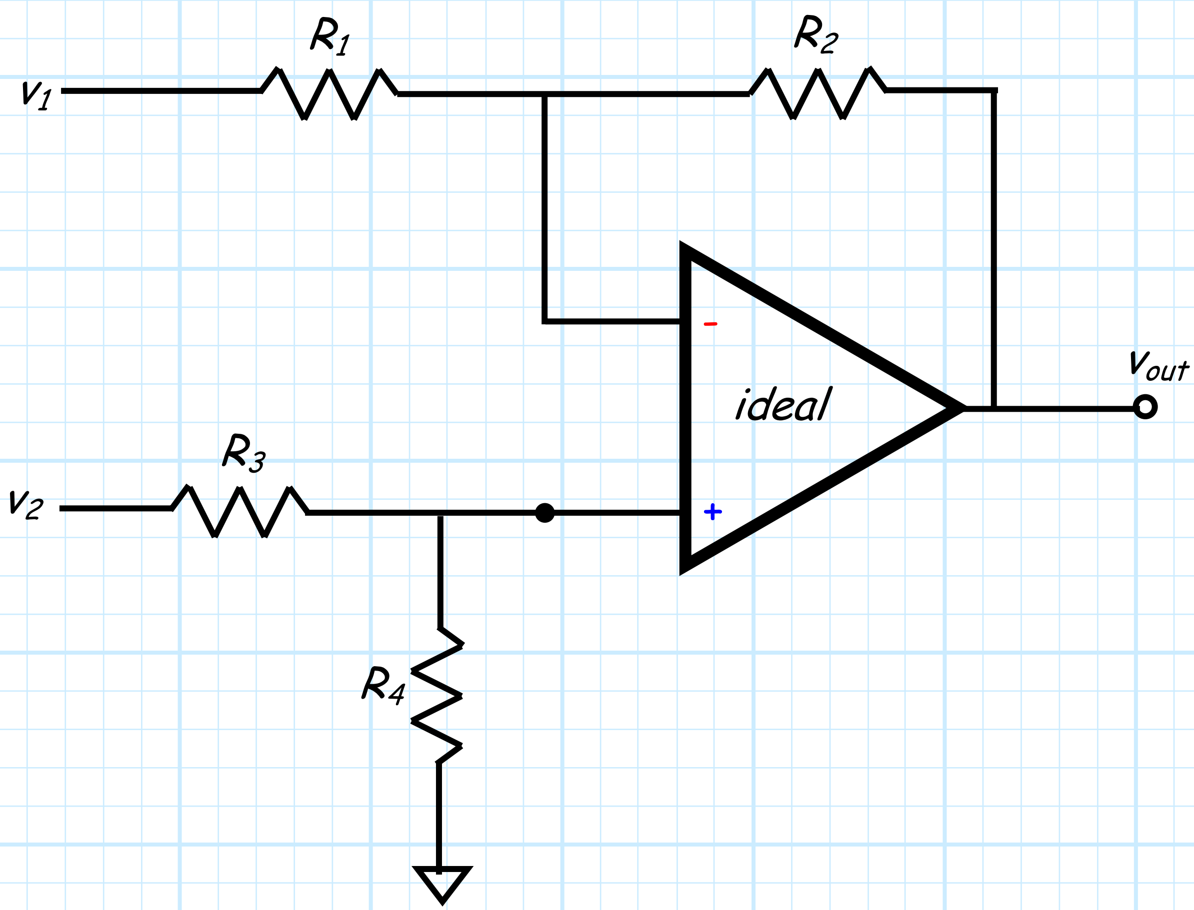
# Differential and Common-Mode Gain

Recall that in a previous handout, we analyzed **this** circuit:



# Common mode and differential mode

We found that the output is related to the inputs as:

$$v_{out} = \left(1+\frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3+R_4}\right)v_2 - \left(\frac{R_2}{R_1}\right)v_1$$

This circuit is a **weighted difference amplifier**, and typically, it is expressed in terms of its **differential** gain $A_d$ and **common-mode** gain $A_{cm}$.

To understand what these gains mean, we must first define the **difference signal** $v_d(t)$ and **common-mode signal** $v_{cm}(t)$ of two inputs $v_1(t)$ and $v_2(t)$.

# Definitions

The **difference**, as we might expect, is defined as:

$$v_d(t) \doteq v_2(t) - v_1(t)$$

whereas the **common-mode** signal is simply the **average** of the two inputs:

$$v_{cm}(t) \doteq \frac{v_2(t) + v_1(t)}{2}$$

Using these definitions, we can express the two input signals as:

$$v_2(t) = v_{cm}(t) + \frac{v_d(t)}{2}$$

$$v_1(t) = v_{cm}(t) - \frac{v_d(t)}{2}$$

# A new way to express the output

Thus, the differential signal $v_d(t)$ and the common-mode signal $v_{cm}(t)$ provide another way to **completely** specify input signals $v_1(t)$ and $v_2(t)$—if you know $v_d(t)$ and $v_{cm}(t)$, you know $v_1(t)$ and $v_2(t)$.

Moreover, we can express the behavior of our **differential amplifier** in terms of $v_d(t)$ and $v_{cm}(t)$. Inserting these functions into the expression of the amplifier output $v_o(t)$, we find:

$$
\begin{aligned}
v_{out} &= \left(1+\frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3+R_4}\right)v_2 - \left(\frac{R_2}{R_1}\right)v_1 \\
&= \left(1+\frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3+R_4}\right)\left(v_{cm}(t)+\frac{v_d(t)}{2}\right) - \left(\frac{R_2}{R_1}\right)\left(v_{cm}(t)-\frac{v_d(t)}{2}\right) \\
&= \frac{1}{2}\left[\left(1+\frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3+R_4}\right)+\left(\frac{R_2}{R_1}\right)\right]v_d(t) \\
&\quad + \left[\left(1+\frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3+R_4}\right)-\left(\frac{R_2}{R_1}\right)\right]v_{cm}(t) \\
&= \frac{R_1R_4+R_2R_3+2R_2R_4}{2R_1\left(R_3+R_4\right)}v_d(t) + \frac{R_1R_4-R_2R_3}{R_1\left(R_3+R_4\right)}v_{cm}(t)
\end{aligned}
$$

# A more "common" form

Thus, we now have an expression for the **open-circuit** output in the form:

$$v_{out}(t) = A_d\, v_d(t) + A_{cm}\, v_{cm}(t)$$

where:

$$A_d \doteq \text{differential gain}$$

$$A_{cm} \doteq \text{common-mode gain}$$

# Difference amplifiers should have no common-mode gain

Note that each of these gains are **open-circuit voltage** gains.

* An **ideal** differential amplifier has **zero** common-mode gain (i.e., $A_{cm}=0$)!

* In other words, the output of an **ideal** differential amplifier is **independent** of the **common-mode** (i.e., average) of the two input signals.

* We refer to this characteristic as **common-mode suppression**.

Typically, **real** differential amplifiers exhibit **small**, but non-zero common mode gain.

# Common-Mode Rejection ratio

The **Common-Mode Rejection Ratio** (CMRR) is therefore used to indicate the **quality** of a differential amplifier:

$$\text{CMRR} = 10 \log_{10} \frac{|A_d|^2}{|A_{cm}|^2} \qquad (dB)$$

Note the CMRR of a **good** differential amplifier is very **large** (e.g., > 40 dB).

For our **example** circuit, we find that the differential and common-mode gain are:

$$A_d = \frac{R_1 R_4 + R_2 R_3 + 2 R_2 R_4}{2 R_1 \left(R_3 + R_4\right)}$$

$$A_{cm} = \frac{R_1 R_4 - R_2 R_3}{R_1 \left(R_3 + R_4\right)}$$

# Common-mode gain depends on design

The **ratio** of these two gains is thus:

$$\frac{A_d}{A_{cm}} = \left(\frac{R_1R_4 + R_2R_3 + 2R_2R_4}{2R_1\left(R_3 + R_4\right)}\right)\left(\frac{R_1R_4 - R_2R_3}{R_1\left(R_3 + R_4\right)}\right)^{-1}$$

$$= \frac{1}{2}\frac{R_1R_4 + R_2R_3 + 2R_2R_4}{R_1R_4 - R_2R_3}$$

and therefore CMRR is:

$$CMRR = 10\log_{10}\left|\frac{1}{2}\frac{R_1R_4 + R_2R_3 + 2R_2R_4}{R_1R_4 - R_2R_3}\right|^2 \qquad (dB)$$

It is evident that for **this** example, the common-mode gain $A_{cm}$ is **minimized**, and thus the CMRR is **maximized**, when:

$$R_1R_4 = R_2R_3$$

so that $R_1R_4 - R_2R_3 = 0$.