Decibels

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Introduction

The Bel is a logarithmic measure of a ratio. To make it easier for us to understand, it's a base–10 logarithm. Typically we use this unit for power ratios. For example the ratio of P_2 to P_1 in Bels is

$$\log_{10} \frac{P_2}{P_1}.$$

For example, if $P_2 = 10P_1$ the ratio is 1 Bel. This is a rather large unit, so we use decibels instead. One decibel is one tenth of a Bel, so now the ratio becomes

$$A_{dB} = 10 \log_{10} \frac{P_2}{P_1},$$

and the example above the answer is 10 decibels. We use the abbreviation, lower–case "d" for "deci–" and Capital "B" for "Bels;" 10 dB.

Converting Voltage Ratios TO dB

Often we work with voltages rather than currents. The power in a load resistor is proportional to the square of voltage, so

$$A_{dB} = 10 \log_{10} \frac{|V_2|^2}{|V_1|^2}$$

$$A_{dB} = 20 \log_{10} \frac{|V_2|}{|V_1|}.$$

Positive dB values indicate "gain" while negative ones indicate "loss."

Examples

For example, if the gain of an amplifier is $A_v = -25$, then

$$A_{dB} = 20 \log_{10} 25 = 28 \,\text{dB}.$$

We drop the minus sign on the gain because the logarithmic gain doesn't keep the phase information.

If I measure 100 mV at the input of a long cable and measure 43 mV at the output then I say the loss along the cable is

$$A_{dB} = 20 \log_{10} \frac{|43 \,\text{mV}|}{|100 \,\text{mV}|} = -7.3 \,\text{dB}.$$

Converting FROM dB

To invert, simply remember that the inverse of the logarithm is the exponential;

$$|A_v| = 10^{A_{db}/20}.$$

Remember that this doesn't give you the phase of A_v. We have to keep track of that separately.

Chains of Components

Because cascaded systems multiply the gain or loss, decibels add. For example, if I have a 28 dB amplifier with the output connected to the cable with -7.3 dB loss, which is in turn connected to the input of another amplifier with a gain of 10 dB, then the gain of the whole system is

$$28 - 7.3 + 10 = 30.7\,\text{dB}.$$

Frequently Used Values

Here are some frequently used values. You can do a lot in your head by combining these. For example, a gain of $A_v = 20$ is 2×10 or a power gain of $20^2 = 400 = 4 \times 100$ or $6 + 20 = 26$ dB.

Voltage Ratio	Power Ratio	dB
10^{-2}	10^{-4}	-40
10^{-1}	10^{-2}	-20
$\sqrt{10^{-1}} = 0.3162$	10^{-1}	-10
$1/2$	$1/4$	-6
$\sqrt{1/2} = 0.707$	$1/2$	-3
1	1	0
$\sqrt{2} = 1.414$	2	3
2	$1/4$	6
$\sqrt{10} = 3.162$	10^1	10
10	100	20
100	10^4	40