The Speed of Sound
Notes to Supplement Lab #1

Sound is a longitudinal mechanical/pressure wave that propagates in a material medium. When part of the medium, which could be a gas like air, or a liquid like water, or a solid such as wood or steel, is displaced or moved from its resting position, a mechanical disturbance (and a pressure wave) moves through the medium from that spot producing a backward and forward motion. Two properties of the medium determine how fast the sound wave moves: the density (the mass per volume, represented by the symbol ?) and the elasticity (the change in the pressure for a given change in volume), which is measured by the "modulus" of elasticity represented by the symbols E (for solids) or B (for gases or liquids). The elasticity is a way of quantifying the stiffness of a medium: higher E (or B) means a stiffer medium. If we consider a simplified model of the medium, these properties and their significance will become clearer.

The Model

Let us think about a medium as made up of small blocks of material, each with a mass of m, that are connected to each other by spring-like forces, as in the figure. When a mass is displaced from its resting position, it pulls and pushes on its neighbors, alternately crowding one way then the other and increasing or decreasing the pressure from the equilibrium value. The crowding moves to the next pair of blocks, that then moves to the next nearest pair, and so on as the wave moves away. If the blocks as very heavy, then they have significant inertia and will respond only sluggishly to any push or pull. If the blocks are lighter than in the first case, then the response is quicker than before and the wave will travel relatively more quickly than in a denser medium. Thus, the speed of sound in air will be slower than it is in helium because air is significantly denser than is helium. This fact accounts for the quacking sound of divers' voices when they breathe a helium and oxygen mixture instead of air, which is mostly nitrogen and oxygen.

If the density of the medium were the only important factor in determining the speed of sound, then one could reasonably expect the speed of sound in air

to be greater than the speed of sound in water, since water is about eight hundred times denser than air. But surprisingly, this is not the case; the speed of sound in water is greater by about three times that it is in air. In this case, the elasticity of the medium is more important than the density. In our model, if the springs are stiff then a small displacement produces a large force that more effectively overcomes the inertia of its neighbors than does a weaker spring. While air is very compressible, that is, only a small increase in pressure is produced by a change in its volume, water is very stiff. The pressure rise when water is compressed is huge (about 7,000 times that of air). The same effect holds for solids: wood is slightly less dense than water and about as stiff, so the speed of sound in wood is larger than it is in water. Steel, however, is denser than wood but is stiffer still. Thus, the speed of sound in steel is greater than it is in wood.

The mathematical relationship connecting the speed of sound to the density (?, "rho"), the "Modulus" B (for gases and liquids) or E (for solids) is

$$\text{Velocity of Sound} = \mathbf{V = v\ (B/\ ?)}\text{ or } = v\ (E/\ ?).$$

Therefore, it is the competition between the stiffness, measured by B (or E), and the inertia, measured by the density ?, that determines the speed of sound in a medium.

Variation with Temperature

One may wonder what happens as one changes the density of a gas by changing the temperature or the pressure. Reality has a surprise: changing the temperature changes the density of air more than it changes the elasticity, so that higher temperatures produce lower densities—hot air is lighter than cold air—and, thus, the speed of sound increases with higher temperature. On the other hand, when one changes only the pressure or relative humidity, the bulk modulus of the gas and its density change in compensating ways: there is no net change in the speed of sound with pressure or humidity if the temperature is

constant, until there is practically nothing left to carry the disturbance. An approximate expression for the speed of sound in air versus temperature is

V = 343 m/s + 0.6 (T –20 C).

The speed of sound in air at 20 C is very nearly 343 m/sec and increases (decreases) by 0.6 m/sec for every degree above (below) 20 C.

One can understand the dependence of the velocity of sound also from a microscopic point of view. When the air is heated, the molecules move faster. That means that the disturbance can more quickly move from one region to another. Indeed, the speed of sound is only about 1/3 the average speed of the molecules. If the pressure is reduced, the number of molecules per unit volume decreases but they cover the same distance in the same time if the temperature is the same. Thus, the speed of sound is not affected by a change in pressure. That is until the gas becomes so tenuous that the molecules fail to interact with each other; then the disturbance cannot propagate, and there is no sound at all.

Non-dispersive Property

One of the happy facts about the speed of sound is that it does not depend on the frequency in most media. A small volume of air shaken back and forth by a loud speaker transmits its motion and pressure to the neighboring volume of air just as fast if it is oscillating at 20 times per second (Hertz, abbreviated Hz) or 20,000 times per second (20 kilohertz, abbreviated 20 kHz). Otherwise, the sound of a band would change very dramatically as you got farther away. The timbre or tone of a voice would be different at one meter and ten meters. This property of sound is called the **non-dispersive property of sound**, all frequencies have the same speed.

Applications

The speed of sound is useful to us. While not as direct or as keen an effect as is the case with bats, we get clues to the distance to objects from echoes or reflected sound. As we approach an object, the quality of the sound that our steps make on the pavement changes because of the time between the

sound from the ground and the sound from the reflection. In Sonar, which stands for **SO**und **N**avigation **A**nd **R**anging, the distance to objects can be computed by measuring the time for the round trip of a sound wave. This technique finds application also in a sonic ruler (available at hardware stores) that uses high frequency sound (ultrasound) to measure the distance across a room. Of course, a small correction should be made for the temperature of the air in the room (about ½ % per degree). We realize the fast but nevertheless finite speed of sound when we see lightning but hear the thunder produced by the rapid heating of the air in the lighting bolt some seconds later. Counting off seconds between the flash and the rumble tells us the distance to the lightning: 3 seconds is about a kilometer, five seconds about a mile. The finite speed of sound becomes noticeable when the distance is greater than about 20 to 30 meters. An organist who sits before a console that is some distance from the pipes must learn to ignore the delay if he is to be successful. A significant part of the delay in the time between when he presses the key and the sound is heard is simply the time required for the sound to get to the player from the pipe that may be buried far back in the organ chamber. Moreover, the speed of sound is one parameter that determines the pitch of notes played in wind instruments. Thus, a flute player will "warm up" by blowing air through her instrument. The frequency will otherwise rise as the air in the pipe warms up to body temperature.

The speed of sound is an important and understandable characteristic of acoustic experience. I hope that you find this note helpful.

Happy listening,

Dr. M.