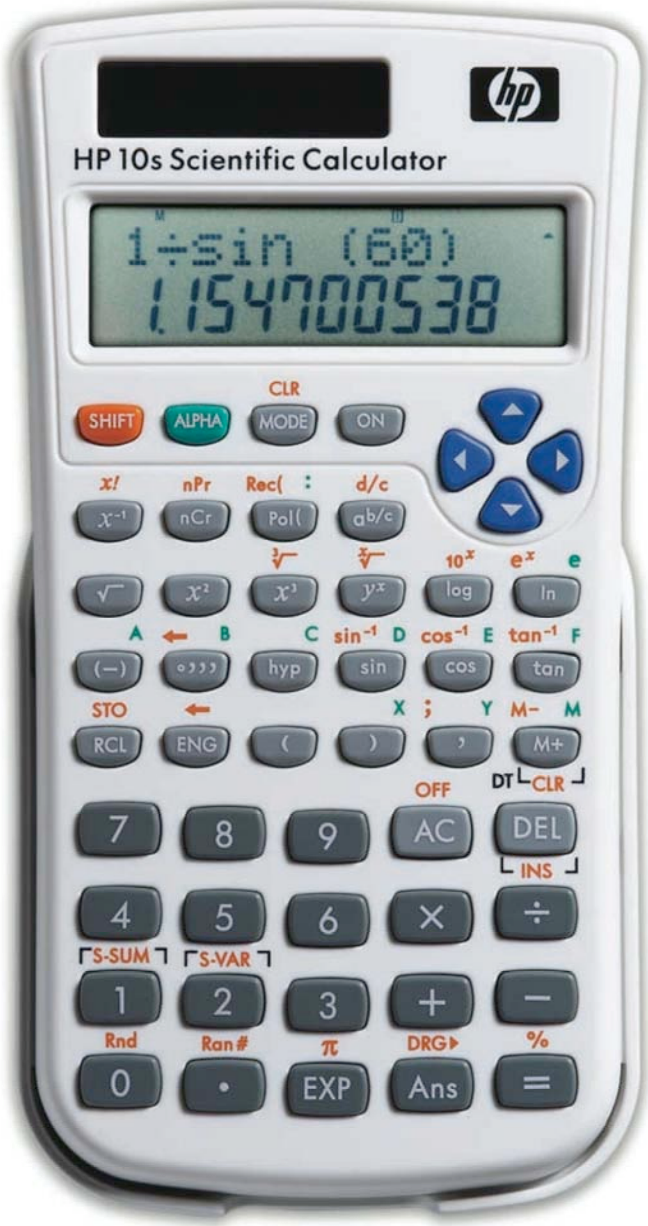
# hp calculators

**HP 10s** Logarithmic Functions

Logarithms and Antilogarithms

Practice Solving Problems Involving Logarithms



## Logarithms and antilogarithms

The logarithm of *x* to the base *a* (written as $\log_a x$) is defined as the inverse function of $x = a^y$. In other words, the logarithm of a given number is the exponent that a base number must have to equal the given number. The most usual values for *a* are 10 and e, which is the exponential constant and is defined by the infinite sum: 1+ 1/1! + 1/2! +1/3! + ...+1/n! +... Its value is approximately 2.718 and is a transcendental number, that is to say: it cannot be the solution of a polynomial equation with rational coefficients.

Logarithms to base 10 are called common logarithms and also Briggsian logarithms. They are usually symbolized as $\log_{10} x$ or simply log 10, and on the HP 10s, they correspond to the [log] key. These logarithms are used in calculations.

Logarithms to base e are called natural logarithms, Naperian logarithms and also hyperbolic logarithms. Their symbol is ln x or $\log_e x$. They are calculated with the [ln] key on the HP 10s. This kind of logarithms is most used in mathematical analysis. There is still another kind of logarithms, though somewhat unusual; they are the binary logarithms, which are logarithms with base 2 ($\log_2 x$).

The following formula is very useful to change logarithms from one base to another:

$$\log_n x = \frac{\log_m x}{\log_m n}$$

The denominator, $\log_m n$, is known as the *modulus*.

The inverse function of the logarithm is called the antilogarithm. If $y = \log_a x$, then $x = a^y$ is the antilogarithm of y. If the base is e then the inverse function is called the exponential function, $e^x$, which is also known as the compound interest function and the growth (if x > 0) or decay (if x < 0) function. Perhaps the most important property of the exponential function is that its derivative is also $e^x$, that is, it's the solution of the differential equation dy/dx = y for which y = 1 when x = 0.

On the HP 10s, the keys that carry out these calculations are [ln], [SHIFT] $e^x$, [log] and [SHIFT] $10^x$. The function $y^x$ (y [$y^x$] x [=]) can be considered the generic antilogarithm function: if $10^x$ is the inverse of $\log_{10} x$ and $e^x$ is the inverse of $\log_e x$, then $y^x$ is the inverse of $\log_y x$. Refer to the HP 10s learning module *Solving Problems Involving Powers and Roots* for more information on the $y^x$ function.

## Practice solving problems involving logarithms

Example 1: Find the common logarithm of 2

Solution: [2] [log] [=]

Answer: 0.301029995.

Example 2: What is the numerical value of the base of the natural logarithms?

Solution: Simply press:

SHIFT $e^x$ 1 =

Answer: 2.718281828. Note that the pattern 18-28-18-28 is really easy to remember!

Example 3: Calculate $\ln(8)+\ln(5)$

Solution: ln 8 + ln 5 =

Answer: 3.688879454

Example 4: Calculate $3\ln(28.34\times3.75)-\ln(6)$

Solution: The parentheses keys enable us to key in the problem as written, i.e. as it is mathematically stated from left to right:

3 × ln ( 2 8 • 3 4 × 3 • 7 5 ) – ln 6 =

Answer: 12.20633075

Example 5: Find the log to base 3 of 5.

Solution: Using the formula given above, the log to base 3 of 5 can be calculated as $\frac{\log_{10}5}{\log_{10}3}$:

log 5 ÷ log 3 =

Answer: 1.464973521

Example 6: What is the value of x in the equation $18^x$ = 324?

Solution: To solve this equation, we will use an important property of logarithms which states that the logarithm of a number raised to a power is equal to the power multiplied by the logarithm of the number. This involves taking the logarithm of both sides of the equation. The original equation would then look like this:

$$\log 18^x = \log 324 \Rightarrow x\log 18 = \log 324$$

and *x* is therefore equal to:

$$x = \frac{\log 324}{\log 18}$$

log 3 2 4 ÷ log 1 8 =

Answer: 2. Note that the same answer will be found using natural logarithms instead.