MAT 080-Algebra II
Applications of Quadratic Equations

Objectives

a *Applications involving rectangles*

b *Applications involving right triangles*

---

a *Applications involving rectangles*

One of the common applications of quadratic equations is to find the unknown length and width of a rectangle. To solve these types of problems, we need to use the formula for the area of a rectangle:

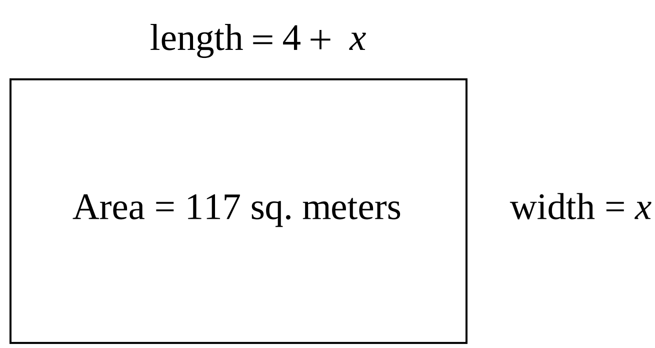
$$\boxed{\begin{array}{c}\text{length} \times \text{ width} = \text{area} \\ \text{or} \\ l \times w = A\end{array}}$$

This is a formula that is frequently used, and you will need to memorize it.

Example 1

A rectangle has a length that is 4 meters more than the width. The area of the rectangle is 117 square meters. Find the dimensions of the rectangle.

<u>*Step 1*</u> Draw a diagram of the rectangle. Label the length and the width. Since we know nothing at all about the width, we will call it *x*. In the problem we are told that the length is 4 meters more than the width, so we will let $4 + x$ represent the length.



Step 2 Write the equation using the formula for the area of a rectangle and the information from the diagram.

Formula: length $\times$ width $=$ area or $l \times w = A$

From diagram: width = x, length = $4 + x$, and area = 117 sq. meters

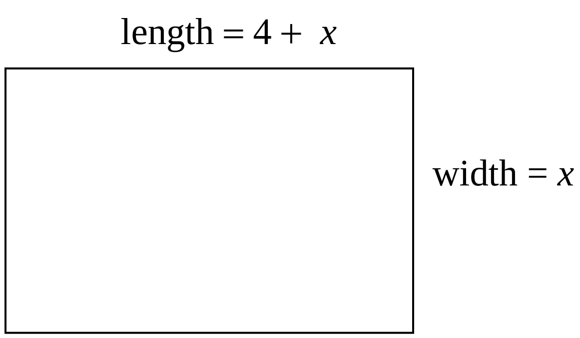
length $\times$ width $=$ area	▪ Formula
$(4+x)\times x = 117$	▪ Substitute $(4 + x)$ for length, x for width, and 117 for area.
$4x + x^2 = 117$	▪ Distribute x through $(4 + x)$.
$x^2 + 4x - 117 = 117 - 117$ $x^2 + 4x - 117 = 0$	▪ Write the equation in standard form by subtracting 117 from both sides.
$(x+13)(x-9) = 0$	▪ Factor

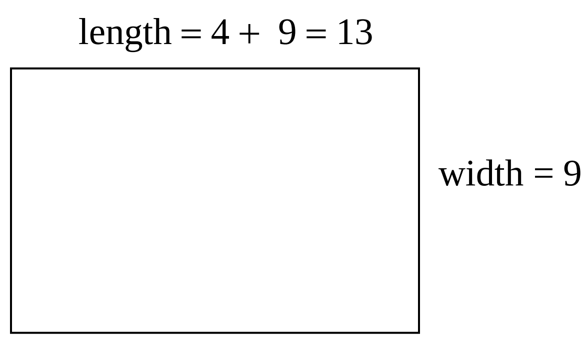
Take each factor, set it equal to 0, and solve the resulting equations:

$x + 13 = 0$	$x - 9 = 0$
$x + 13 - 13 = 0 - 13$	$x - 9 + 9 = 0 + 9$
$x = -13$	$x = 9$

Since length must be positive, the solution that fits the problem is $x = 9$. We discard the $x = -13$.

Step 3 Substitute 9 in for x in the diagram in order to determine the dimensions of the rectangle.





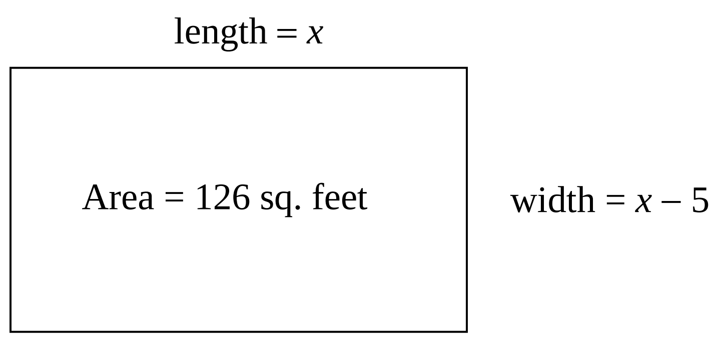
$\boxed{\text{So, the width is 9 meters, and the length is 13 meters.}}$

Practice Problem 1
A rectangle whose area is 112 square feet has a length that is 6 feet greater than the width. Find the dimensions of the rectangle.

Solution to this Practice Problem may be found on page 12.

Example 2
A rectangle has a width that is 5 feet less than the length. The area of the rectangle is 126 square feet. Find the dimensions of the rectangle.

<u>*Step 1*</u> Draw a diagram of the rectangle. Label the length and the width. Since we know nothing at all about the length, we will call it x. In the problem we are told that the width is 5 feet less than the length, so we will let $x - 5$ represent the width. (Note that $5 - x$ is *not* correct!)



<u>*Step 2*</u> Write the equation using the formula for the area of a rectangle and the information from the diagram.

Formula: length $\times$ width $=$ area or $l \times w = A$

From diagram: length = x, width = $x - 5$, and area = 126 sq. feet

length $\times$ width $=$ area	▪ Formula
$x(x-5) = 126$	▪ Substitute x for length, $x - 5$ for width, and 126 for area.

Example 2 continues on next page

Example 2-continued

$$x(x-5)=126$$

▪ Equation from last step on previous page

$$x^2-5x=126$$

▪ Distribute x through $(x - 5)$.

$$x^2-5x-126=126-126$$

$$x^2-5x-126=0$$

▪ Write the equation in standard form by subtracting 126 from both sides.

$$(x-14)(x+9)=0$$

▪ Factor

Take each factor, set it equal to 0, and solve the resulting equations:

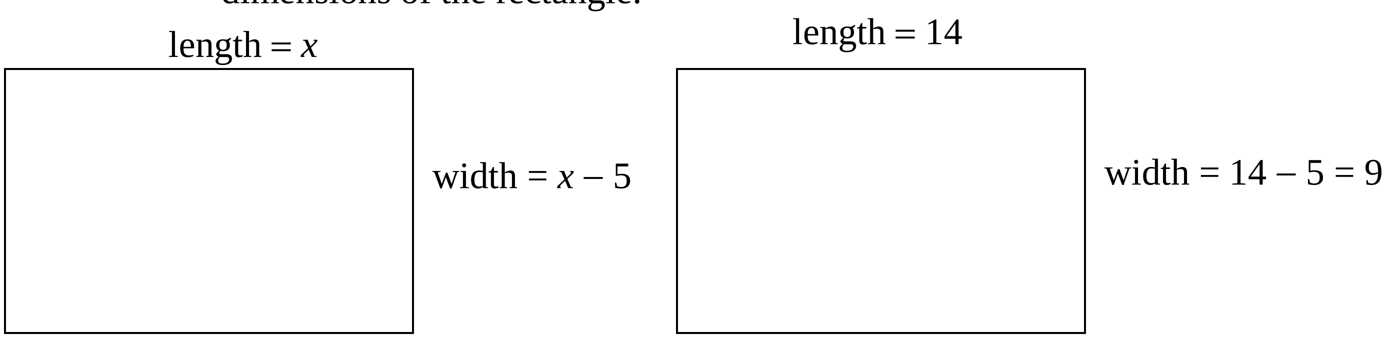
$$x-14=0 \qquad\qquad x+9=0$$

$$x-14+14=0+14 \qquad\qquad x+9-9=0-9$$

$$x=14 \qquad\qquad x=-9$$

Since length must be positive, the solution that fits the problem is $x = 14$. We discard the $x = -9$.

<u>*Step 3*</u> Substitute 14 in for x in the diagram in order to determine the dimensions of the rectangle.



$\boxed{\text{So, the length is 14 feet, and the width is 9 feet.}}$

Practice Problem 2

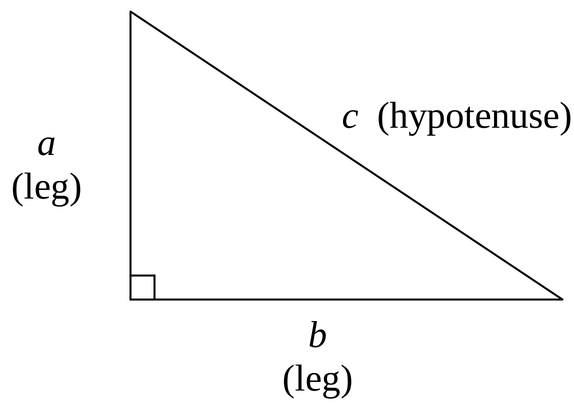
A rectangle whose area is 192 square meters has a width that is 4 meters less than the length. Find the dimensions of the rectangle.

Solution to this Practice Problem may be found on page 13.

b *Applications involving right triangles*

In your textbook in the section on Applications of Quadratic Equations you were introduced to the ***Pythagorean Theorem***: $\boxed{a^2 + b^2 = c^2}$.

This formula is used when working with the lengths of the sides of a right triangle. A right triangle is a triangle that has a right angle (90°).

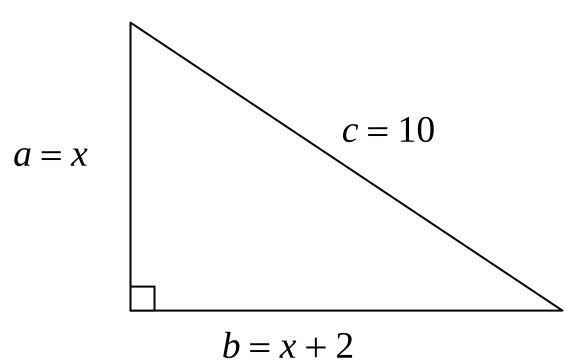


Note that the hypotenuse *must* be *c*. The hypotenuse is the longest side, and is always opposite the right (90°) angle. It makes no difference which leg is *a* and which leg is *b*.

This formula is used often, so you will need to memorize it.

Example 3
The length of one leg of a right triangle is 2 feet longer than the other leg. The length of the hypotenuse is 10 feet. Find the lengths of the two legs.

Step 1 Draw a diagram of the triangle. Label each of the sides. Since we know nothing at all about the shorter leg, we will call it x. In the problem we are told that the other leg is 2 feet longer. So, we will let $x + 2$ represent the longer leg. The hypotenuse is 10.



Step 2 Write the equation using the Pythagorean Theorem and the information from the diagram.

Formula: $a^2 + b^2 = c^2$

From diagram: side $a = x$, side $b = x + 2$, and hypotenuse = 10

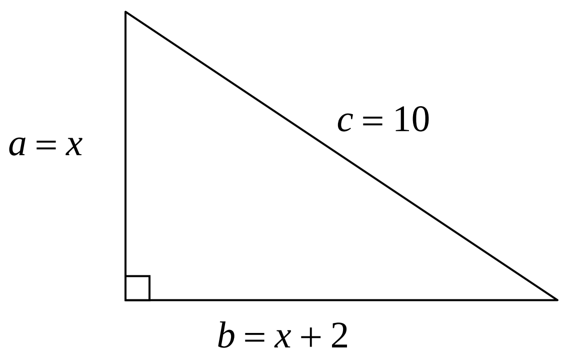
$a^2 + b^2 = c^2$	▪ Formula
$x^2 + (x+2)^2 = 10^2$	▪ Substitute x for a, $x + 2$ for b, and 10 for c.
$x^2 + (x+2)(x+2) = 100$	▪ Important: $(x+2)^2$ means $(x+2)(x+2)$, which must be multiplied using FOIL.
$x^2 + x^2 + 2x + 2x + 4 = 100$	▪ Multiply $(x+2)(x+2)$ to make $x^2 + 2x + 2x + 4$.
$2x^2 + 4x + 4 = 100$	▪ Combine like terms. $x^2 + x^2$ make $2x^2$ and $2x + 2x$ make $4x$.
$2x^2 + 4x + 4 - 100 = 100 - 100$	▪ Write the equation in standard form by subtracting 100 from both sides.
$2x^2 + 4x - 96 = 0$	▪ Equation in standard form.
$2(x^2 + 2x - 48) = 0$	▪ Factor the common factor of 2.
$\frac{\cancel{2}(x^2 + 2x - 48)}{\cancel{2}} = \frac{0}{2}$	▪ Divide both sides by 2 to cancel the common factor of 2. Note that $\frac{0}{2}$ will still equal 0.
$x^2 + 2x - 48 = 0$	
$(x+8)(x-6) = 0$	▪ Factor.

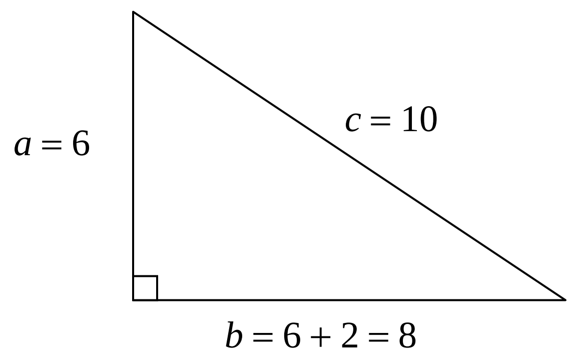
Take each factor, set it equal to 0, and solve the resulting equations:

$$\begin{aligned} x+8&=0 \\ x+8-8&=0-8 \\ x&=-8 \end{aligned} \qquad\qquad \begin{aligned} x-6&=0 \\ x-6+6&=0+6 \\ x&=6 \end{aligned}$$

Since length must be positive, the solution that fits the problem is $x = 6$. We discard the $x = -8$.

Step 3 Substitute 6 in for x in the diagram in order to determine the lengths of the legs of the right triangle.





So, one leg is 6 feet and the other leg is 8 feet.

Practice Problem 3

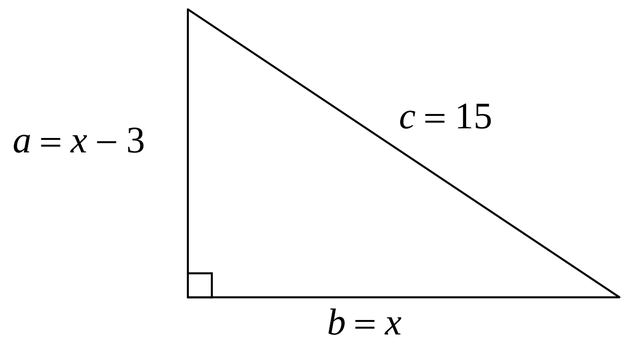
The length of one leg of a right triangle is 3 feet longer than the other leg. The length of the hypotenuse is 15 feet. Find the lengths of the two legs.

Solution to this Practice Problem may be found on page 15.

Example 4

The length of one leg of a right triangle is 3 feet shorter than the other leg. The length of the hypotenuse is 15 feet. Find the lengths of the two legs.

<u>*Step 1*</u> Draw a diagram of the triangle. Label each of the sides. Since we know nothing at all about the longer leg, we will call it x. In the problem we are told that the other leg is 3 feet shorter. So, we will let $x - 3$ represent the shorter leg (Note that $3 - x$ is *not* correct!). The hypotenuse is 15.



<u>*Step 2*</u> Write the equation using the Pythagorean Theorem and the information from the diagram.

Formula: $a^2 + b^2 = c^2$

From diagram: side $a = x - 3$, side $b = x$, and hypotenuse = 15

$a^2 + b^2 = c^2$	▪ Formula
$(x-3)^2 + x^2 = 15^2$	▪ Substitute $x - 3$ for a, x for b, and 15 for c.
$(x-3)(x-3) + x^2 = 225$	▪ Important: $(x-3)^2$ means $(x-3)(x-3)$, which must be multiplied using FOIL.
$x^2 - 3x - 3x + 9 + x^2 = 225$	▪ Multiply $(x-3)(x-3)$ to make $x^2 - 3x - 3x + 9$.
$2x^2 - 6x + 9 = 225$	▪ Combine like terms. $x^2 + x^2$ make $2x^2$ and $-3x - 3x$ make $-6x$.
$2x^2 - 6x + 9 - 225 = 225 - 225$	▪ Write the equation in standard form by subtracting 225 from both sides.

$$2x^2 - 6x - 216 = 0$$
▪ Equation in standard form.

$$2(x^2 - 3x - 108) = 0$$
▪ Factor the common factor of 2.

$$\frac{\cancel{2}(x^2 - 3x - 108)}{\cancel{2}} = \frac{0}{2}$$
▪ Divide both sides by 2 to cancel the common factor of 2. Note that $\frac{0}{2}$ will still equal 0.

$$x^2 - 3x - 108 = 0$$

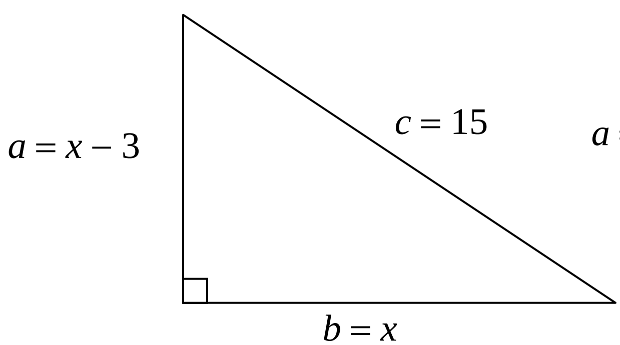
$$(x - 12)(x + 9) = 0$$
▪ Factor.

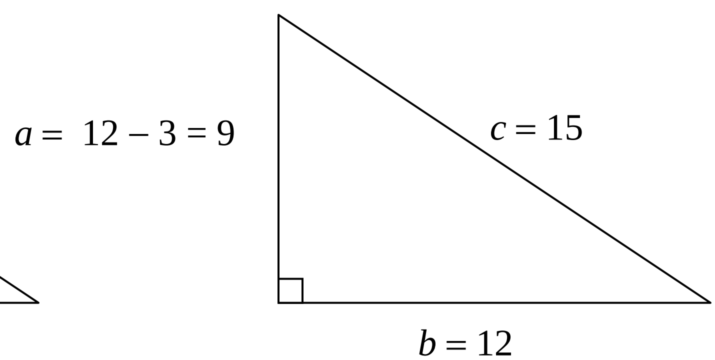
Take each factor, set it equal to 0, and solve the resulting equations:

$$\begin{aligned} x - 12 &= 0 \\ x - 12 + 12 &= 0 + 12 \\ x &= 12 \end{aligned} \qquad \begin{aligned} x + 9 &= 0 \\ x + 9 - 9 &= 0 - 9 \\ x &= -9 \end{aligned}$$

Since length must be positive, the solution that fits the problem is $x = 12$. We discard the $x = -9$.

<u>*Step 3*</u> Substitute 12 in for x in the diagram in order to determine the lengths of the legs of the right triangle.





$\boxed{\text{So, one leg is 12 feet and the other leg is 9 feet.}}$

Practice Problem 4

The length of one leg of a right triangle is 4 feet shorter than the other leg. The length of the hypotenuse is 20 feet. Find the lengths of the two legs.

Solution to this Practice Problem may be found on page 17.

Homework Problems

Answers to Homework Problems are on page 19

a Applications involving rectangles

1. A rectangle whose area is 180 square feet has a width that is 3 feet less than the length. Find the dimensions of the rectangle.

2. A rectangle has a length that is 2 meters more than the width. The area of the rectangle is 288 square meters. Find the dimensions of the rectangle.

3. A rectangle has a width that is 6 meters less than the length. The area of the rectangle is 280 square meters. Find the dimensions of the rectangle.

4. A rectangle whose area is 336 square feet has a length that is 4 meters less than the twice the width. Find the dimensions of the rectangle.

5. A rectangle has a length that is 3 meters more than twice the width. The area of the rectangle is 44 square meters. Find the dimensions of the rectangle.

b Applications involving right triangles

6. The hypotenuse of a right triangle is 25 meters in length. One leg is 17 meters longer than the other. Find the length of each leg.

7. The hypotenuse of a right triangle is 15 yards in length. One leg is 3 yards shorter than the other. Find the length of each leg.

8. The hypotenuse of a right triangle is 40 yards in length. One leg is 8 yards longer than the other. Find the length of each leg.

9. The sporting goods store at the mall is a perfect rectangle. Its diagonal is 13 meters and the width of the rectangle is 7 meters shorter than its length. Find the length and the width of the store.

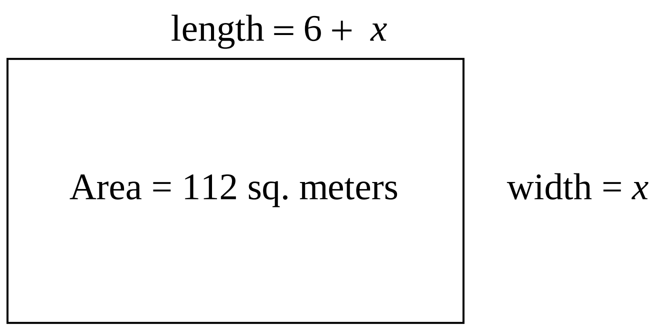
10. Manual placed a ladder 20 feet long against the side of his house. The distance from the top of the ladder to the bottom of the house is 4 feet greater than the distance from the bottom of the house to the foot of the ladder. How far is the foot of the ladder from the house? How far up on the house does the top of the ladder touch the building?

Solutions to Practice Problems

Practice Problem 1

A rectangle whose area is 112 square feet has a length that is 6 feet greater than the width. Find the dimensions of the rectangle.

<u>*Step 1*</u> Draw a diagram of the rectangle. Label the length and the width. Since we know nothing at all about the width, we will call it *x*. In the problem we are told that the length is 6 meters more than the width, so we will let $6+x$ represent the length.



<u>*Step 2*</u> Write the equation using the formula for the area of a rectangle and the information from the diagram.

Formula: length $\times$ width $=$ area or $l \times w = A$

From diagram: width = *x*, length = 6 + *x*, and area = 112 sq. meters

$$\text{length} \times \text{width} = \text{area}$$

▪ Formula

$$(6+x)\times x = 112$$

▪ Substitute (6 + *x*) for length, *x* for width, and 112 for area.

$$6x + x^2 = 112$$

▪ Distribute *x* through (6 + *x*).

$$x^2 + 6x - 112 = 112 - 112$$

$$x^2 + 6x - 112 = 0$$

▪ Write the equation in standard form by subtracting 112 from both sides.

$$(x+14)(x-8) = 0$$

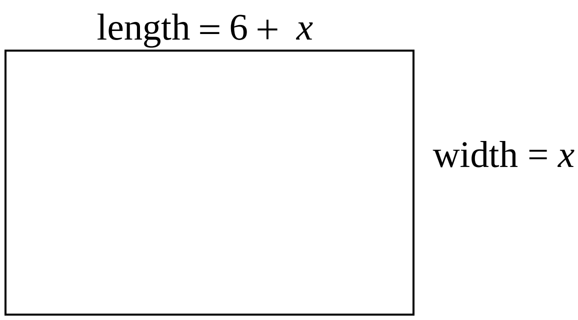
▪ Factor

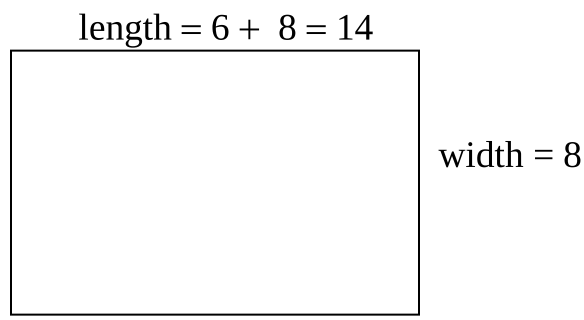
Take each factor, set it equal to 0, and solve the resulting equations:

$$x+14=0 \qquad\qquad x-8=0$$
$$x+14-14=0-14 \qquad\qquad x-8+8=0+8$$
$$x=-14 \qquad\qquad x=8$$

Since length must be positive, the solution that fits the problem is $x = 8$. We discard the $x = -14$.

<u>*Step 3*</u> Substitute 8 in for x in the diagram in order to determine the dimensions of the rectangle.



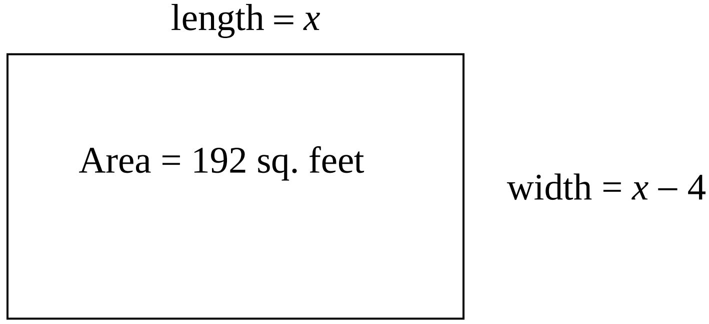


$\boxed{\text{So, the width is 8 meters, and the length is 14 meters.}}$

Practice Problem 2

A rectangle whose area is 192 square feet has a width that is 4 feet less than the length. Find the dimensions of the rectangle.

<u>*Step 1*</u> Draw a diagram of the rectangle. Label the length and the width. Since we know nothing at all about the length, we will call it x. In the problem we are told that the width is 4 feet less than the length, so we will let $x - 4$ represent the width. (Note that $4 - x$ is *not* correct!)



Step 2 Write the equation using the formula for the area of a rectangle and the information from the diagram.

Formula: length $\times$ width $=$ area or $l \times w = A$

From diagram: length $= x$, width $= x - 4$, and area $= 192$ sq. feet

length $\times$ width $=$ area	▪ Formula
$x(x-4) = 192$	▪ Substitute x for length, $x - 4$ for width, and 192 for area.
$x^2 - 4x = 192$	▪ Distribute x through $(x - 4)$.
$x^2 - 4x - 192 = 192 - 192$	▪ Write the equation in standard form by subtracting 192 from both sides.
$x^2 - 4x - 192 = 0$	
$(x-16)(x+12) = 0$	▪ Factor

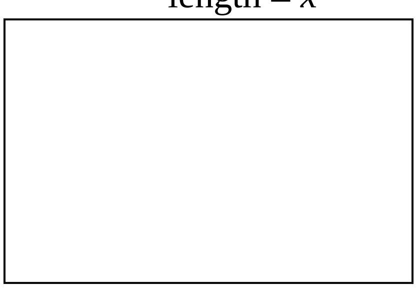
Take each factor, set it equal to 0, and solve the resulting equations:

$x - 16 = 0$	$x + 12 = 0$
$x - 16 + 16 = 0 + 16$	$x + 12 - 12 = 0 - 12$
$x = 16$	$x = -12$

Since length must be positive, the solution that fits the problem is $x = 16$. We discard the $x = -12$.

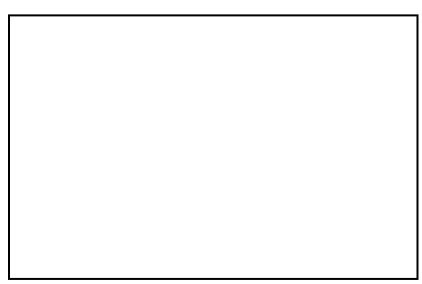
Step 3 Substitute 16 in for x in the diagram in order to determine the dimensions of the rectangle.

length $= x$



width $= x - 4$

length $= 16$

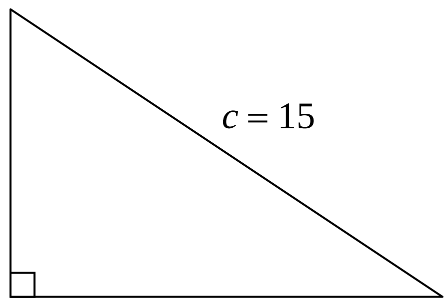


width $= 16 - 4 = 12$

$\boxed{\text{So, the length is 16 feet, and the width is 12 feet.}}$

Practice Problem 3
The length of one leg of a right triangle is 3 feet longer than the other leg. The length of the hypotenuse is 15 feet. Find the lengths of the two legs.

<u>*Step 1*</u> Draw a diagram of the triangle. Label each of the sides. Since we know nothing at all about the shorter leg, we will call it x. In the problem we are told that the other leg is 3 feet longer than the shorter leg. So, we will let $x + 3$ represent the longer leg. The hypotenuse is 15.



<u>*Step 2*</u> Write the equation using the Pythagorean theorem and the information from the diagram.

Formula: $a^2 + b^2 = c^2$

From diagram: side $a = x$, side $b = x + 3$, and hypotenuse = 15

$a^2 + b^2 = c^2$	▪ Formula
$x^2 + (x+3)^2 = 15^2$	▪ Substitute x for a, $x + 3$ for b, and 15 for c.
$x^2 + (x+3)(x+3) = 225$	▪ Important: $(x+3)^2$ means $(x+3)(x+3)$, which must be multiplied using FOIL.
$x^2 + x^2 + 3x + 3x + 9 = 225$	▪ Multiply $(x+3)(x+3)$ to make $x^2 + 3x + 3x + 9$.
$2x^2 + 6x + 9 = 225$	▪ Combine like terms. $x^2 + x^2$ make $2x^2$ and $3x + 3x$ make $6x$.
$2x^2 + 6x + 9 - 225 = 225 - 225$	▪ Write equation in standard form by subtracting 225 from both sides.

$$2x^2 + 6x - 216 = 0$$

- Equation in standard form.

$$2(x^2 + 3x - 108) = 0$$

- Factor the common factor of 2.

$$\frac{\cancel{2}(x^2 + 3x - 108)}{\cancel{2}} = \frac{0}{2}$$

- Divide both sides by 2 to cancel the common factor of 2. Note that $\frac{0}{2}$ will still equal 0.

$$x^2 + 3x - 108 = 0$$

$$(x + 12)(x - 9) = 0$$

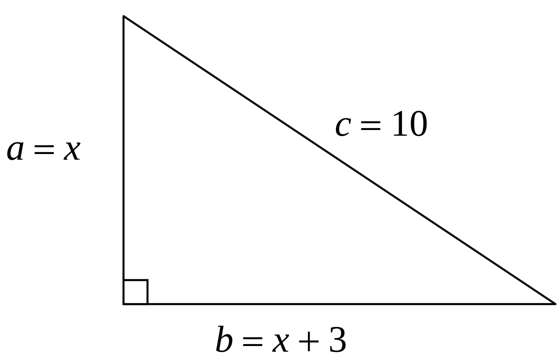
- Factor.

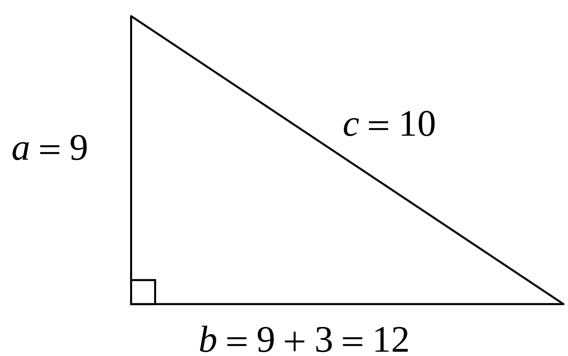
Take each factor, set it equal to 0, and solve the resulting equations:

$$\begin{aligned} x + 12 &= 0 \\ x + 12 - 12 &= 0 - 12 \\ x &= -12 \end{aligned} \qquad\qquad \begin{aligned} x - 9 &= 0 \\ x - 9 + 9 &= 0 + 9 \\ x &= 9 \end{aligned}$$

Since length must be positive, the solution that fits the problem is $x = 9$. We discard the $x = -12$.

<u>*Step 3*</u> Substitute 9 in for x in the diagram in order to determine the lengths of the legs of the right triangle.



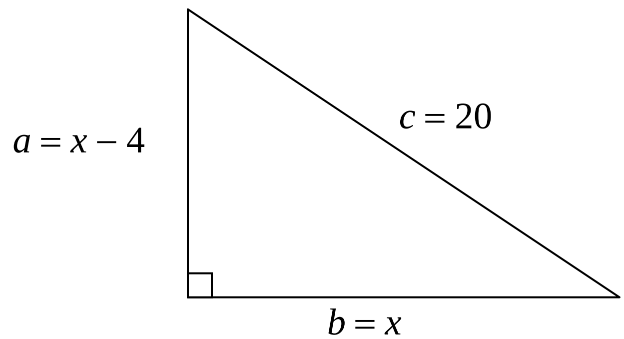


$\boxed{\text{So, one leg is 9 feet and the other leg is 12 feet.}}$

Practice Problem 4

The length of one leg of a right triangle is 4 feet shorter than the other leg. The length of the hypotenuse is 20 feet. Find the lengths of the two legs.

<u>*Step 1*</u> Draw a diagram of the triangle. Label each of the sides. Since we know nothing at all about the longer leg, we will call it x. In the problem we are told that the other leg is 4 feet shorter. So, we will let $x - 4$ represent the shorter leg (Note that $4 - x$ is *not* correct!). The hypotenuse is 20.



<u>*Step 2*</u> Write the equation using the Pythagorean theorem and the information from the diagram.

Formula: $a^2 + b^2 = c^2$

From diagram: side $a = x - 4$, side $b = x$, and hypotenuse = 20

$$a^2 + b^2 = c^2$$
▪ Formula

$$(x-4)^2 + x^2 = 20^2$$
▪ Substitute $x - 4$ for a, x for b, and 20 for c.

$$(x-4)(x-4) + x^2 = 400$$
▪ Important: $(x-4)^2$ means $(x-4)(x-4)$, which must be multiplied using FOIL.

$$x^2 - 4x - 4x + 16 + x^2 = 400$$
▪ Multiply $(x-4)(x-4)$ to make $x^2 - 4x - 4x + 16$.

$$2x^2 - 8x + 16 = 400$$
▪ Combine like terms. $x^2 + x^2$ make $2x^2$ and $-4x - 4x$ make $-8x$.

$$2x^2 - 8x + 16 - 400 = 400 - 400$$
▪ Write the equation in standard form by subtracting 400 from both sides.

$$2x^2 - 8x - 384 = 0$$

▪ Equation in standard form.

$$2(x^2 - 4x - 192) = 0$$

▪ Factor the common factor of 2.

$$\frac{\cancel{2}(x^2 - 4x - 192)}{\cancel{2}} = \frac{0}{2}$$

▪ Divide both sides by 2 to cancel the common factor of 2. Note that $\frac{0}{2}$ will still equal 0.

$$x^2 - 4x - 192 = 0$$

$$(x - 16)(x + 12) = 0$$

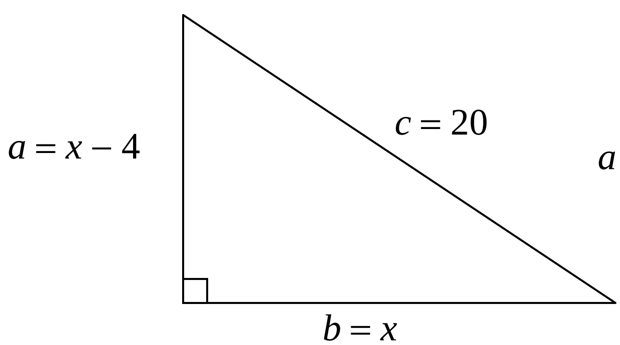
▪ Factor.

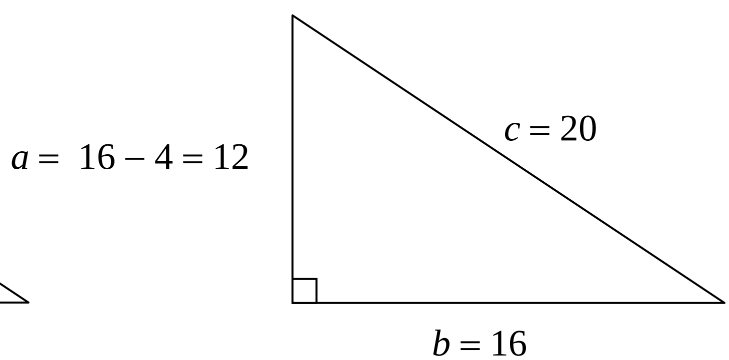
Take each factor, set it equal to 0, and solve the resulting equations:

$$\begin{aligned} x - 16 &= 0 \\ x - 16 + 16 &= 0 + 16 \\ x &= 16 \end{aligned} \qquad\qquad \begin{aligned} x + 12 &= 0 \\ x + 12 - 12 &= 0 - 12 \\ x &= -12 \end{aligned}$$

Since length must be positive, the solution that fits the problem is $x = 16$. We discard the $x = -12$.

Step 3 Substitute 16 in for x in the diagram in order to determine the lengths of the legs of the right triangle.





$\boxed{\text{So, one leg is 16 feet and the other leg is 12 feet.}}$

Answers to Homework Problems

1. length = 15 feet and width = 12 feet

2. length = 18 meters and width = 16 meters

3. length = 20 meters and width = 14 meters

4. length = 24 feet and width = 14 feet

5. length = 11 meters and width = 4 meters

6. one leg is 7 meters; the other leg is 24 meters

7. one leg is 12 yards; the other leg is 9 yards

8. one leg is 32 yards; the other leg is 24 yards

9. length is 12 meters; width is 5 meters

10. the foot of the ladder is 12 feet from the bottom of the house;

    the top of the ladder touches the building at a point 16 feet above the ground