Compound Interest

$$A = P(1 + \frac{r}{n})^{nt}$$

$A =$ Amount you have at a certain time
$P =$ Principal (the initial amount, or how much you had at the start)
$r =$ Annual interest rate (remember to turn percents into decimals, so 5% = .05)
$t =$ Years (it's still time, but it must be years)
$n =$ Number of times the interest is compounded per year

If the interest is compounded annually, $n = 1$
If the interest is compounded quarterly, $n = 4$
If the interest is compounded monthly, $n = 12$

Example 1- Solving for *A* (the amount you want to have at a certain time)

You invest $4500 in a savings account that pays 3% annual interest compounded monthly. How much money will be in the account after 5 years?

Step 1- Let's see what we have.

$A =$ That's what we want to know!
$P = 4500$
$r = .03$
$t = 5$
$n = 12$

Step 2- Substitute this stuff into the compound interest formula.

$$A = P(1 + \frac{r}{n})^{nt}$$

$$A = 4500(1 + \frac{.03}{12})^{(12)(5)}$$

Step 3- Solve!

$$A = 4500(1 + 0.0025)^{60}$$
$$A = 4500(1.0025)^{60}$$
$$A = 5227.28$$

$$\$\mathbf{5227.28}$$

<u>Example 2- Solving for *P* (the amount you had at the start / your initial value)</u>

How much money must be invested in a savings account that pays 3% annual interest compounded quarterly if you want to have $7000 after 8 years?

Step 1- Let's see what we have.

$$A = 7000$$
$$P = \text{That's what we want to know!}$$
$$r = .03$$
$$t = 8$$
$$n = 4$$

Step 2- Substitute this stuff into the compound interest formula.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$
$$7000 = P\left(1 + \frac{.03}{4}\right)^{(4)(8)}$$

Step 3- Solve!

$$7000 = P(1 + 0.0075)^{32}$$
$$7000 = P(1.0075)^{32}$$
$$7000 = P(1.2701112243174021254623736588438)$$
$$5511.3283513906595216007357382196 = P$$

$$\mathbf{\$5511.33}$$

Example 3- Solving for *r* (the interest rate you need)

You want to have $20,000 ten years from now. You currently have $16,000. What annual interest rate (compounded monthly) is necessary for you to reach your goal?

Step 1- Let's see what we have.

$$A = 20{,}000$$
$$P = 16{,}000$$
$$r = \text{That's what we want to know}$$
$$t = 10$$
$$n = 12$$

Step 2- Substitute this stuff into the compound interest formula.

$$A = P(1 + \frac{r}{n})^{nt}$$
$$20{,}000 = 16{,}000(1 + \frac{r}{12})^{(12)(10)}$$

Step 3- Solve!

$$20{,}000 = 16{,}000(1 + \frac{r}{12})^{120}$$

Divide both sides by 16,000

$$1.25 = (1 + \frac{r}{12})^{120}$$

Take the 120th root of both sides (that means finding $\sqrt[120]{1.25}$)

$$1.0018612595916019726587729883803 = 1 + \frac{r}{12}$$

Subtract 1 from both sides

$$0.0018612595916019726587729883803 = \frac{r}{12}$$

Multiply both sides by 12

$$0.022335115099223671905275860563449 = r$$

The annual interest rate you need is **2.23%**

<u>Example 4- Solving for *t* (how many years will it take)</u>
You want have $5,000 that you would like to see grow to $8000 in an account with an annual interest rate of 5% (compounded quarterly). How long will it take for your money to reach $8000?

Step 1- Let's see what we have.

$$A = 8000$$
$$P = 5000$$
$$r = .05$$
$$t = \text{That's what we want to know!}$$
$$n = 4$$

Step 2- Substitute this stuff into the compound interest formula.

$$A = P(1 + \frac{r}{n})^{nt}$$
$$8000 = 5000(1 + \frac{.05}{4})^{4t}$$

Step 3- Solve! Since the variable *t* is in an exponent position, we are probably going to have to use logarithms (so we can then put it in front)

$$8000 = 5000(1 + \frac{.05}{4})^{4t}$$
$$8000 = 5000(1 + 0.0125)^{4t}$$
$$8000 = 5000(1.0125)^{4t}$$

Divide both sides by 5000

$$1.6 = (1.0125)^{4t}$$

Use log on both sides

$$\log\ 1.6 = \log(1.0125)^{4t}$$

Now we can move the *4t*

$$\log\ 1.6 = 4t \log\ 1.0125$$

Divide both sides by log 1.0125 and 4

$$\frac{\log 1.6}{4(\log\ 1.0125)} = t$$

$$\frac{0.204119982655924780854955578897}{4(0.00539503188670616353889492884698 22)} = t$$

$9.45870140076903433508667362472 12 = t$ **It will take 9.46 years**