Mathematical Card Magic

Fifty-Two New Effects

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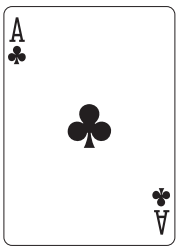
Dedicated with great fondness to the memory of departed friends
Ron Keith, Steve Sigur, Monika Decker,
Chris Orrall, Tom Rodgers, and John Beechinor,
who displayed such grace and dignity playing the cards
they were dealt over the past six or seven years,

and to
Vicki, Ann, and Molly
for being my loveliest assistants down through the decades.



Low-Down Triple Dealing

We've all met some low-down double dealing types in our lives. Have you ever considered the advantages of being a low-down *triple* dealer? You may warm to the idea once you've tried some of the items below.[1]

The recurring theme here is a reversed transfer of cards from the top to the bottom of a packet (see Figure A.1), often done under the guise of spelling out words, and generally repeated several times to interesting effect. We revisit this topic and some variations on it in Chapters 5 and 9. Throughout all dealing, we suggest that the cards be held low, so that the audience doesn't see any flashed card faces; hence, our chapter title.

Our opener is based on nontrivial mathematics (♣♣)—even though, in our experience, that's rarely suspected by onlookers. It never fails to please (♥♥♥♥), requires no setup (♠), and is easy enough to perform (♦♦) for people who can deal and count at the same time.

A♣ Three Scoop Miracle—Done Magic Before?

How it looks: *Hand about a quarter of a deck of cards to a spectator, and ask her to shuffle freely. Take those cards back, and mix them further in your hands as you ask the spectator what her favorite ice cream flavor is. Let's suppose she says, "Chocolate."*

Deal from the packet to the table, one card for each letter of "chocolate," then scoop those up with one hand, commenting that this represents one scoop of ice cream, and with the other hand, drop the remainder on top ("as a topping"). Repeat this spelling (and scooping and topping) routine twice more, for a total of three times.

[1] In Chapters 5 and 7 we'll even extol the virtues of single and double dealing.

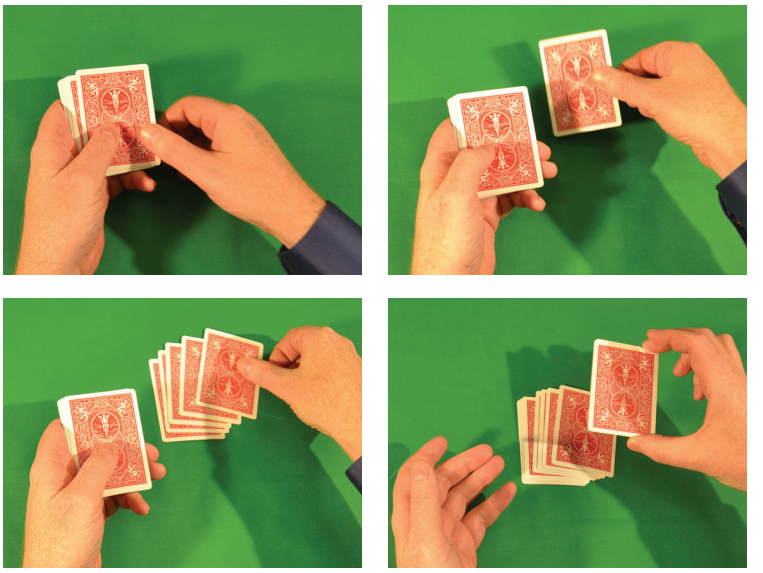


Figure A.1. Spelling "chocolate" reverse transfers nine cards.

Emphasize how random the dealing was: since the cards were shuffled repeatedly and the spectator named the ice cream flavor. Ask her if she has done magic before. Regardless of the answer, now ask her to press down hard on the top card of the packet on the table, requesting that it be miraculously turned a specific card, say, the Four of Diamonds. When the card is turned over, it is seen by all to be the desired card. Congratulate the spectator on a job well done.

How it works: There are two secrets here: (1) a key relationship between the number of letters in the word being spelled out and the size of the "quarter"-deck being used, and (2) the fact that you must know the identity of the bottom card at the start of the spelling and dropping.

There are many ways to address the second point, such as peeking at the bottom card after you get the shuffled packet back (see page 9, perhaps while tapping the cards on the table to square them up. A more sophisticated handling is suggested later.

As to the first point, the size of the "quarter"-deck used must be at least as big as the number of letters in the word being spelled, yet no larger than twice that number. For instance, using *chocolate*, you need to work with between nine and eighteen cards; we recommend a number not too close to those extremes, such as eleven to fifteen. About a quarter of a deck works well for the flavors most commonly mentioned, which seem to be *chocolate*, *vanilla*, or *strawberry*. If *rum* is desired, try to force *rum raisin*, and for selections with long names like *mint chocolate chip*, use about half of the deck.[2]

[2]That particular flavor suggests another presentation, as we'll see on page 112.

Why it works: The mathematical analysis of what is going on here is deferred to page 30.

Source: Original. The principle involved was stumbled on in the spring of 2003 while living in Las Rosas, on the northwestern outskirts of Madrid, playing around with Jim Steinmeyer's "The Nine Card Problem" [Steinmeyer 93, Steinmeyer 02] (perhaps more well known today as "Nine Card Speller"), Bob Hummer's CATO Principle [Diaconis and Graham 11], and the George Sands Prime Principle [Fulves 75] (we have more to say about these three in Chapter 9).

This was published online at MAA.org in October 2004 as the inaugural *Card Colm* "Low Down Triple Dealing" [Mulcahy 04_10], dedicated to Martin Gardner, the best friend mathematics ever had, on the occasion of his ninetieth birthday. It also appeared in print at that time [Mulcahy 04_11], and a few years later [Mulcahy 08] in slightly expanded form, with mention of a generalization that we discuss in Chapter 5.

Presentational options: You could have the spectator handle the cards throughout if you think you can get a peek at the bottom card after the first round of shuffling is done—it's surprising how careless and "revealing" some people's handling of cards can be! In such cases, the bottom quarter of the deck is used—something you must make seem natural—and the additional in-hand mixing possibility is forgone. Most importantly, you will need to direct the three rounds of spelling and dropping carefully.

One way to ensure you learn the bottom card is to first take the deck back after the initial shuffling, then talk about the coming dealing, sneaking a peak when squaring up the deck as suggested earlier, while talking about the upcoming dealing. Finally, say, "Why don't you do all the work?," handing the spectator about a quarter of the deck, taken from the bottom.

The following presentation is popular, and baffling for most audiences: ask for the ice cream flavor to be called out as you shuffle freely, peeking at the bottom card at the last minute before you set the deck on the table. Then, having explained that there are three low-calorie scoops and topping to follow, pick up the deck and set aside the top three-quarters or so. Start mixing the remaining cards in hand, being careful to keep the bottom card in place. Next, discreetly move this key card to the top, and shuffle the rest with abandon, keeping the peeked-at card on top. You can even flash many of the card faces, including the ever-changing bottom ones, saying, "Note that these cards are all different and hopelessly jumbled." Add, disingenuously, "I haven't a clue what any of them are." As you lower the cards, boldly shuffle the top card back to the bottom. You are now ready to proceed, and the audience has the impression that the cards are totally randomized, which makes the conclusion positively perplexing.

Other options are discussed in the *Card Colm* cited above [Mulcahy 04_10]. Magician and mentalist Max Maven utilized the Low-Down Triple Deal principle in the "Final Destination" routine from his "One Man Parade" in the November 2006 issue of *The Linking Ring* [Maven 06].

An extension of the principle, which was first explored by others [Sirén 08, Miller 10], is discussed in detail in Chapter 5.

♣ ♥ ♠ ♦

Our second example of mathemagic also happens in a spectator's hands, and uses the same mathematical principle as our opener. It could be performed instead of that effect, but we advise against doing both for the same audience. This one requires a little sure-footed bravado, and should not be attempted by anyone of a nervous disposition; hence, the ♦♦♦♦ rating. You need to start with a completely rigged deck, as indicated by the ♠♠♠♠ designation, so it's not repeatable. If you can pull it off, the crowd will love you (♥♥♥).

A♥ Any Card (and Any Magician)

How it looks overall: *Shuffle the deck over and over, as you ask a spectator to shout out the name of any card. Next ask for the name of any magician. You deal cards from the deck to the table, one for each letter in the magician's name, pick up those cards and do additional in-hand mixing. You then demonstrate a spelling and dropping routine twice, and hand the packet to the spectator, who now spells and drops. At the conclusion of this, the named card is miraculously found to be at the top of the packet.*

How it looks in detail: Suppose the spectator responds with "Eight of Clubs" and "Martin Gardner." Shuffle the cards one more time, and then deal cards to the table, face down, of course, one for each letter in the magician's name. Set the rest of the deck aside. Pick up the (thirteen) cards on the table and say, "Random cards, in a random order," as you mix them some more in your hands. "Let's see if Martin Gardner can help us to locate the desired card. I want to show you how to use his name to further randomize the cards."

Deal out cards to the table again, one for each letter in "Gardner," as you spell that word out loud. Drop the rest (there should be six) on top. Now peek at the top card and say, "No." Pick up the cards and repeat the spelling and dropping, again peeking at the top card and expressing mild disappointment at what you see. "Something tells me that it won't work until the third try, or maybe I just don't have the touch," you say,

as you give the cards to the spectator and request that the experiment be repeated one more time.

Upon completion of this last round of spelling and dropping, pause and remind the audience that you had absolutely no control over what card or magician was named. Have the top card turned over, as you tentatively say, "Third time lucky?" It is indeed the Eight of Hearts. "Congratulations! Maybe I should have let you do all the work earlier on, too!"

How it works: At the outset, the deck should be arranged in such a way that as you peek at the ever-changing bottom card while doing simple overhand shuffles, it's easy for you to get the named card to the bottom, and then pause. For instance, new deck order, in which the cards of each suit are together and in either ascending or descending order, works. The idea is that you cycle the cards around as you wait to be told the name of the desired card, and then keep "shuffling" until that card is on the bottom. Yes, it takes a little nerve to pull this off while being watched, but try it; it's a skill you can acquire.

For instance, to get the Eight of Clubs on the bottom, you could cycle the deck with overhand shuffles until some Club is on the bottom, and then either shuffle off a few more cards as necessary or move about twenty cards to the bottom twice in succession, and then perform that maneuver. This is the only tricky part, and yes, it requires practice and some nerve, but it is doable! It's also vital that nobody other than you sees this force card at the bottom.

The rest is easy enough. Shuffle the desired card to the top and do several more sloppy shuffles that keep it there. You may now safely let the audience glimpse the new bottom card, as it plays no role in what follows. Whatever magician's name is now suggested, deal out a packet as you spell out that name, one card for each letter. (The named card is now on the bottom of this packet, and care must be taken not to flash it.) Set the rest of the deck aside.

In the case of *Martin Gardner*, deal again to the table, from the thirteen-card packet, while spelling out loud the letters of Gardner. Drop the rest on top. In the case of *Ricky Jay*, deal from the resulting eight-card packet, while spelling the letters of Ricky, and then drop the rest on top. If one name is shorter than the other, it's important to use the longer of the two words when spelling and dealing before dropping.

When you peek at the new top card, as suggested, it will not be the desired one. Repeat with a second round of dealing and dropping, with similar results. Then hand the packet to the spectator, who by now should be familiar with the dealing and dropping routine. At the conclusion of the final dealing and dropping, the named card will indeed

be on top. Like the previous effect, it's a straightforward application of the Low-Down Triple Dealing principle which we are about to explain.

Source: Original. The use of the magician name spelling in conjunction with Low-Down Triple Dealing appears in the October 2004 *Card Colm* [Mulcahy 04_10], but in the context of the next highlighted effect.

A First Look at Low-Down Triple Dealing

The mechanism behind our first two effects is a *reversed transfer* of some fixed number of cards in a packet—at least half—from top to bottom, done three times in total.

The dealing out (and hence reversing) of k cards from a packet that runs $\{1, 2, \ldots, k-1, k, k+1, k+2, \ldots, n-1, n\}$ from the top down, and then dropping the rest on top as a unit, yields the rearranged packet $\{k+1, k+2, \ldots, n-1, n, k, k-1, \ldots, 2, 1\}$.

For instance, reverse transferring eight cards from the packet A♥–K♥ yields: 9♥, 10♥, J♥, Q♥, K♥, 8♥, 7♥, 6♥, 5♥, 4♥, 3♥, 2♥, A♥.

Figure A.2 shows how this would appear if the cards were face up. Of course, in actual performance, all cards would be face down.

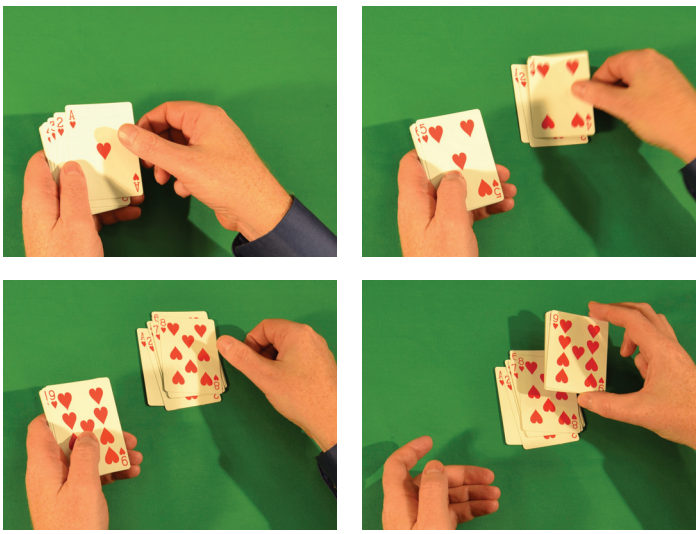


Figure A.2. Reverse transferring eight cards from A–K♥.

Figure A.3 shows how such a face-down packet would look before and after, if fanned face up. Note that the Ace moves from top to bottom.

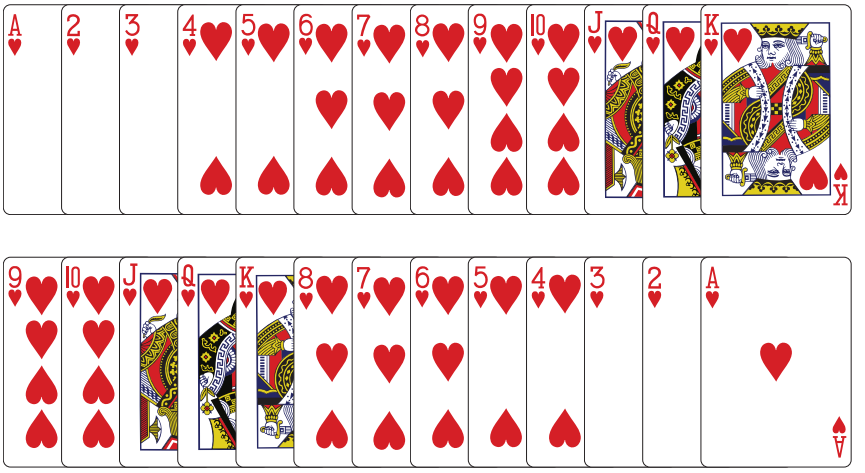


Figure A.3. Before and after reverse transferring eight cards from A–K♥.

If we reverse transfer another eight cards from this packet, face down, and then fan it face up, we obtain the first image in Figure A.4.

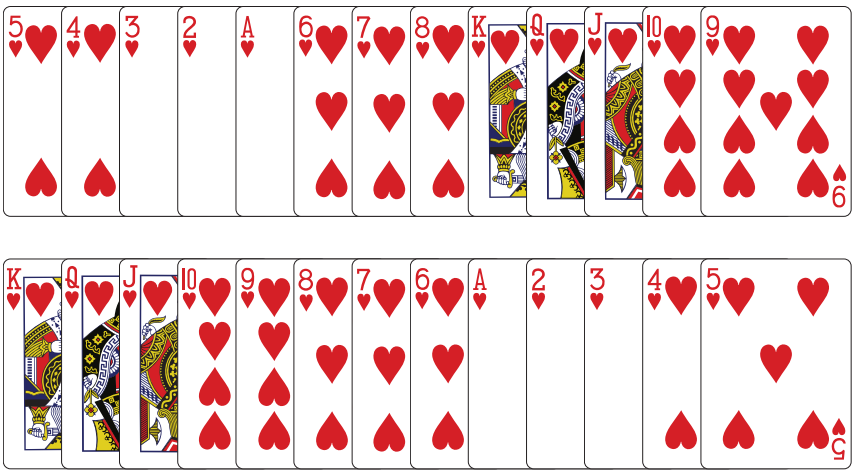


Figure A.4. After a second and third reverse transferring of eight cards.

A third such reverse transferral of eight cards yields the second image in Figure A.4.

Note that the original bottom card of our packet K♥ is on top after three such reverse transferrings of eight cards. This holds in general, and is the real secret behind our first two effects above:

Bottom to Top Principle

The original bottom card of the packet ends up on top after three such reverse transferrings of k cards from n, provided that $k \geq \frac{n}{2}$.

It is certainly easy to see how this works if $k = n$ or $k = n - 1$, since we are actually reversing all of the cards in both cases. It's almost as easy to see if $k = \frac{n}{2}$, when we're reversing exactly half of the cards.

As we later demonstrate visually, it's not so hard to *see* in all cases. Before we explain why this magic property holds, we point out something that is more obvious and has its own applications.

Low-Down Deal Deck Separation

Reverse transferring at least half of a packet preserves top and bottom halves in a certain sense. For instance, if the packet starts with five Red cards on top of five Black cards, then no matter how many ($k \geq 5$) are reverse transferred, the Red cards will end up on the bottom (reversed, but that's not our focus here), and the Black cards will end up on top of those (rearranged a little).

Specifically, reverse transferring seven cards from the packet running 2♦, 10♥, 5♥, 6♦, 9♥, A♣, K♠, 7♠, Q♣, 4♣, from the top yields: 7♠, Q♣, 4♣, K♠, A♣, 9♥, 6♦, 5♥, 10♥, 2♦. Figure A.5 shows such a face-down packet before and after, fanned face up.

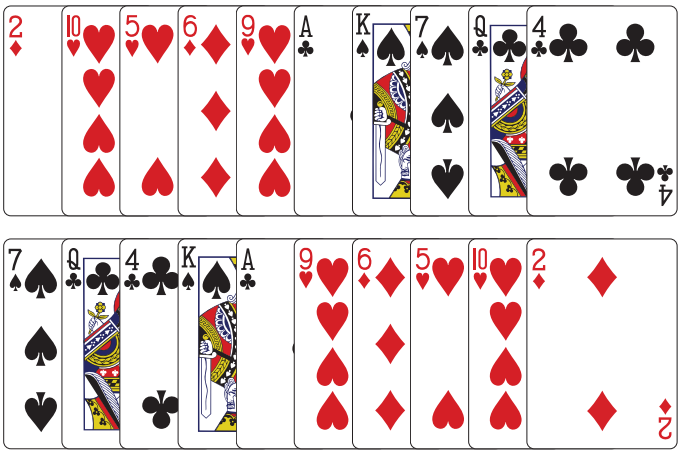


Figure A.5. Color separation if reverse transferring seven from ten.

This generalizes as follows.

> **Low-Down Deal Packet Separation Principle**
> *If k cards are reverse transferred from the top to the bottom of a packet of size n, and $k \geq \frac{n}{2}$, then the top and bottom halves switch places, subject to some internal reordering.*

If the packet size is an odd number, then the middle card is effectively fixed, and the sets of cards initially above and below this fulcrum are the ones that exchange places, getting rearranged in the process.

Odd Location Method

If you're feeling cheated because our first two effects are so similar, here's an extra offering that is easy to master. It's based on the fact that the middle card in an odd-sized packet is doomed to return to that location over and over when at least half of the cards are reverse transferred.

A spectator picks a card at random from a shuffled deck, noting and remembering its face. The selected card is sandwiched between two groups of cards that you offer, following which the spectator mixes the cards repeatedly using words chosen at random from a list you have left in full view. These are words that might describe you, the mathemagician, such as "affable," "engaging," "persistent," "bewildering," and so on.

Remind the audience that you have never touched the selected card. Announce that the spectator can now find it herself, without seeing any card face, "Using an odd method of location I have devised."

Have the cards dealt into two new piles, left to right. The one with an odd number of cards in it is picked up, the other being set aside. Repeat. In short order, the spectator's packet of cards will reduce to just one. Ask what her selection was, and have the surviving card turned over to confirm that she found it successfully despite all her mixing.

The only thing required for this to work is that the two groups of cards you offer be of the same size. We suggest six of one and half a dozen of the other, resulting in the selected card being in the middle of a thirteen card packet. You may offer to have the selected card sandwiched between groups of four and eight cards if you don't mind handling them yourself, later casually shuffling two cards from front to back while distracting the audience with the word list.

An arbitrary number of reverse transfers of at least half of the cards can now be done without altering the fact that the selected card is in position 7, provided that each word spelled out has between seven and thirteen letters. We suggest providing humorously immodest words of no more than eleven letters.

The elimination deal is well known: starting with thirteen cards we then get piles of size seven, then three, then a lonesome card, and that final card is the one that started out as the middle card of the thirteen.

It can be modified to work for other-sized packets. You can hide the selected card between two groups of size seven if the words on the list are adjusted appropriately (e.g., omit "affable" and throw in "infuriating"). This could be the basis of a repeat performance.

Two months after this was conceived, a more romantic take on the idea appeared as the *Huffington Post* blog "In My Heart of Hearts: Valentine's Day Special" [Mulcahy 13_02b].

♣ ♥ ♠ ♦

The Top, Middle, Bottom Decomposition

Let's do some bookkeeping. It turns out that when reverse transferring the same number of cards over and over, you have to keep track of three portions of the packet. These three portions move around intact, subject at most to some internal reversals.

Note that since $k \geq \frac{n}{2}$, we have $k - (n-k) = 2k - n \geq 0$.

Gallia est omnis divisa in partes tres[3]

Writing $n = k + (n-k) = [(n-k) + (k-(n-k))] + (n-k) = (n-k) + (2k-n) + (n-k)$, *we see that a packet of* n *cards naturally breaks symmetrically into three pieces* T, M, B, *(top, middle, and bottom) of sizes* $n-k$, $2k-n$, $n-k$, *respectively.*

Starting with the packet $\{1, 2, \ldots, n\}$, *we thus get*

$$\begin{aligned} T &= \{1, 2, \ldots, n-k\}, \\ M &= \{n-k+1, n-k+2, \ldots, k\}, \\ B &= \{k+1, k+2, \ldots, n\}. \end{aligned}$$

Needless to say, if $n = 2k$, i.e., exactly half of the packet is dealt each time, then M is nonexistent (that is not a problem).

For instance, if $n = 13$, and $k = 8$, and we start with $\{1, 2, \ldots, 12, 13\}$, we have $T = \{1, 2, 3, 4, 5\}$, $M = \{6, 7, 8\}$, and $B = \{9, 10, 11, 12, 13\}$, as seen in Figure A.6. Counting out eight cards (i.e., T and M together) and dropping the rest on top yields $\{9, 10, 11, 12, 13, 8, 7, 6, 5, 4, 3, 2, 1\}$, that is B followed by M reversed followed by T reversed. Note that the middle card, in position 7 in this case, remains fixed throughout.

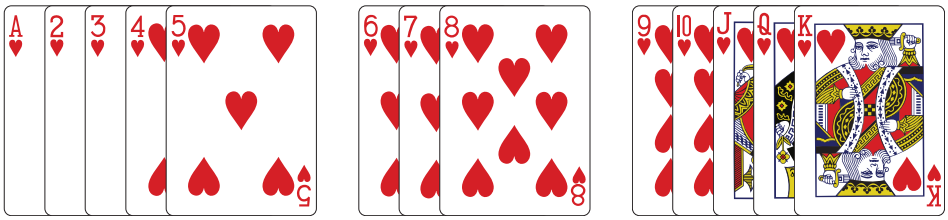


Figure A.6. *T, M, B* when reverse transferring eight cards from thirteen.

In general, counting out (hence reversing) the first k cards (i.e., T and M together) and dropping the rest on top leads to B followed by M reversed followed by T reversed. We denote this basic count out and transfer operation by using the notation $X, Y, Z \to Z, \overline{Y}, \overline{X}$, where the bar indicates a complete subpacket reversal.

[3] Classical Roman Empire references are divided into three parts.

So under the first count out and drop, we find that $T, M, B \to B, \overline{M}, \overline{T}$. Note that the middle card in the packet is fixed if n is odd, and the middle two cards trade places if n is even. In fact, cards equidistant from the center trade places, as long as they are not too close to either end.

The second round of counting and dropping yields $B, \overline{M}, \overline{T} \to \overline{T}, M, \overline{B}$, since two reversals of a subpacket restore it to its initial order. The third round results in $\overline{T}, M, \overline{B} \to \overline{B}, \overline{M}, T$, and hence the original bottom card is now on top, just as we wished to show.

We can actually say a lot more, and we will shortly.

A COAT by Any Other Name

Before returning to the analysis of Low-Down Triple Dealing, let's give the move involved a new name. Note that reverse transferring is different from simply *cutting* cards from top to bottom, without altering their order. (Cutting k cards—either individually or as a group—merely cycles everything around, changing $\{1, 2, \ldots, k-1, k, k+1, k+2, \ldots, n-1, n\}$ into $\{k+1, k+2, \ldots, n-1, n, 1, 2, \ldots, k-1, k\}$.)

The key order reversal in a reverse transfer amounts to **C**ounting **O**ut **A**nd **T**ransferring cards from top to bottom, which suggests an easy-to-remember acronym, COAT. It is vital to note that the first two letters stand for Count Out, not Cut Off! (In Chapter 9, we'll examine other situations in which we *do* cut off and transfer.)

> **COAT (Count Out And Transfer)**
> *Given a packet of n cards, COATing k cards refers to counting out that many from the top into a pile, thus reversing their order, and transferring those as a unit to the bottom.*

Up until now, we have implied that the counting out is done to a table, with the rest of the packet then being dropped on top. An alternative handling is to **C**ount **O**ut (or push off with a thumb) cards from one hand to a second, one by one, to form an ever-growing new pile, **A**nd then **T**uck that behind the remainder of the initial packet, as shown in Figure A.7. Any way you COAT it, the result is the same.

Since the focus in this chapter is on the case where $k \geq \frac{n}{2}$, we often refer to that as *overCOATing*. If $k = n - 1$, we end up reversing the whole packet, just as when $k = n$, and doing this twice puts everything back where it started. Also, if $k = 1$, we are just cutting a single card to the bottom. These borderline cases are mostly ignored in what follows, as they are a bit confusing when handling cards in the reverse transfer context.

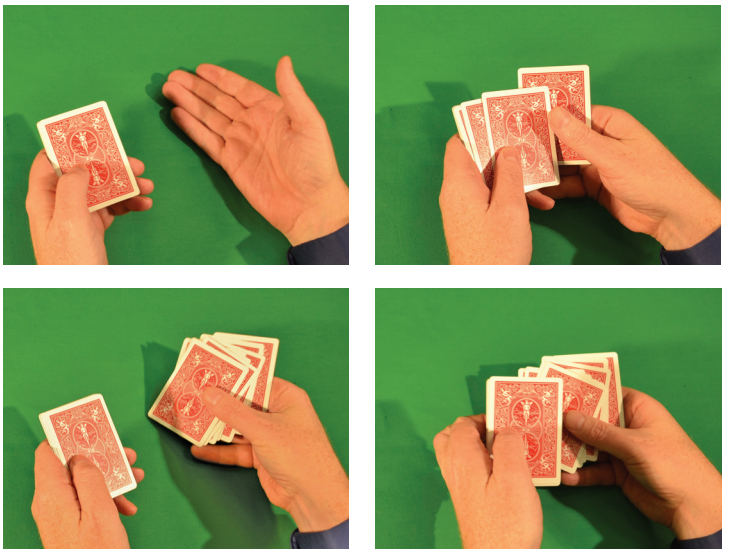


Figure A.7. In-hand COATing.

A Second Look at Low-Down Triple Dealing

As hinted above, overCOATing the same number of cards three times yields more than we have highlighted so far. Not only is the bottom card moved to the top, the entire bottom half of the packet (and then some) is moved intact to the top, in reverse order.

> **Save at Least 50% Principle**
> *If k cards from n cards are dealt out into a pile, reversing their order, and the remaining $n-k$ are dropped on top, and this process is repeated twice more, then provided that $k \geq \frac{n}{2}$, the original k bottom cards become the top k cards, in exact reverse order. That is to say, three overCOATs preserve at least half the packet—the bottom half—only in reversed order, at the top.*

A close examination of our earlier analysis reveals why this holds. We know that three rounds of overCOATing transforms T, M, B to $\overline{B}, \overline{M}, T$, and since the subpacket M, B is the original k bottom cards, they end up on top here, reversed, as the subpacket $\overline{B}, \overline{M}$.

For example, overCOATing $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ three times, seven cards at a time, transforms it into $\{10, 9, 8, 7, 6, 5, 4, 1, 2, 3\}$, yielding what one might call a (reversed) saving of 70%, on top of everything else.

To put it another way, if you want to get the bottom five cards of a thirteen-card packet to the top, in reverse order, there is an alternative to the obvious method of counting out all of the cards into a pile to reverse

their order. Simply select a number between seven and eleven, and COAT that many cards, three times over. This can be done casually in-hand, to give the illusion of mixing the cards (see Figure A.7).

Shortly, we present a visual proof of all of the above. It is instructive to experiment with a face-up packet of about a dozen cards in a known order (e.g., numerical), and COAT them, say, eight at a time.

The next effect requires no setup, and is particularly impressive if you know some sleight-of-hand card control. If that sounds daunting, don't worry, we also suggest a more open handling that is perfectly acceptable.

A♠ Triple Revelation

How it looks: *Have three spectators each pick one card at random, look at it, and memorize it. Return those three cards to the deck, and shuffle.*

Ask a fourth person to name their favorite magician; let's suppose Bill Simon is selected. Hold the deck in one hand and peel cards off the bottom into the other hand without altering their order, one for each letter, as you spell out the whole name.

Hand the stack of cards (here, nine) to the first spectator and ask that "Simon" be spelled out while dealing five cards to the table, then dropping the other four on top. Give the cards to the second spectator, with the same directions, and finally to the third spectator for one last deal and drop. Take the cards behind your back and immediately produce three cards, handing one to each spectator, face down.

Have the chosen cards named, as they are turned over, to reveal that you have correctly located each one.

How it works: The selected cards must first be returned to the bottom of the deck, in a known order. You can either do this openly, or, if you have some magic chops, "control cards to the bottom"; this means that you seemingly return each card to random parts of the deck but secretly get them all to the bottom in due course. The pinky break (see page 11) may help. However you pull it off, these three key cards will then remain in position throughout some riffle shuffling, if you are careful first to drop at least three cards from the bottom of the deck each time.

Let's assume that, as a result, the third spectator's card is at the bottom of the deck, the second spectator's card is one up from the bottom, and the first spectator's card is two up from the bottom. Peel cards off the bottom of the deck without altering their order, one for each letter of the name of the magician called out, as you spell out both words in full. Hand the resulting packet of cards to the first spectator and ask that the longer of the two names (Simon in our example) be spelled out, as cards

are dealt into a pile, before dropping the remainder on top. Now give the cards to the second spectator and finally to the third spectator for two more deals. The three chosen cards are now on the top of the packet of cards, with the order reversed, and you are all set to conclude in triumph.

Why it works: This works because of the Save at Least 50% Principle.

Source: Original. It appeared online as the October 2004 *Card Colm* [Mulcahy 04_10].

Low-Down Quadruple Dealing

We have one further confession to make. The Low-Down Triple Deal explored above is actually 75% of a special quadruple deal. Note that since three overCOATs takes the bottom part of a packet to the top, reversed, one more overCOAT will restore those cards to their original positions at the bottom. Hence at least half of the packet is restored to its original order. Here's the full scoop (or real low down):

> Four OverCOATs Principle
> *If four reversed transfers (overCOATs) of k cards are done to a packet of size n, where $k \geq \frac{n}{2}$, then every card in the packet is returned to its original position.*

In other words, you'll never notice the effect of four overCOATs. In a sense, they cancel each other out. Thus, we have a false shuffle worth adding to one's portfolio.

Casually running off a number of cards gives the appearance of shuffling (see page 10). If done with purpose, it does the opposite.

> Quad False Shuffle Principle
> *Four applications of the following move restores a packet of size n to its original order, assuming $k \geq \frac{n}{2}$. Run off k cards from the top of the packet into a waiting hand, thus reversing their order, and then tuck them as a unit behind the remainder (or just drop the rest on top).*

Done casually four times in-hand (as shown on page 10) to a small packet (e.g., COATing six cards out of ten each time) gives the illusion of mixing the cards. As noted earlier, three such COATs moves the original bottom stock to the top, in reversed order. *Top* or *bottom stock* simply refers to a clump of adjacent cards of interest at one end of the deck (see page 10).

One way to see why this quad property holds is to go back to our earlier analysis: we already saw that the symmetric breakdown of the packet into subpackets T, M, B (top, middle, bottom) of respective sizes $n-k$, $2k-n$, $n-k$, results in $\overline{B}, \overline{M}, T$ after three overCOATs. After a fourth overCOAT, $\overline{B}, \overline{M}, T \to T, M, B$, which is indeed back to where we started, as desired.

A Visual Approach to Low-Down Dealing

Perhaps the best way to *see* what really is going on here is with pictures. Suppose, for the sake of concreteness, that $n = 13$ and $k = 8$. Let's represent a packet of thirteen cards by a vertical strip of gray-scale panels in decreasing order of brightness, from white for the top card to black for the bottom card, as depicted in the leftmost strip of Figure A.8.

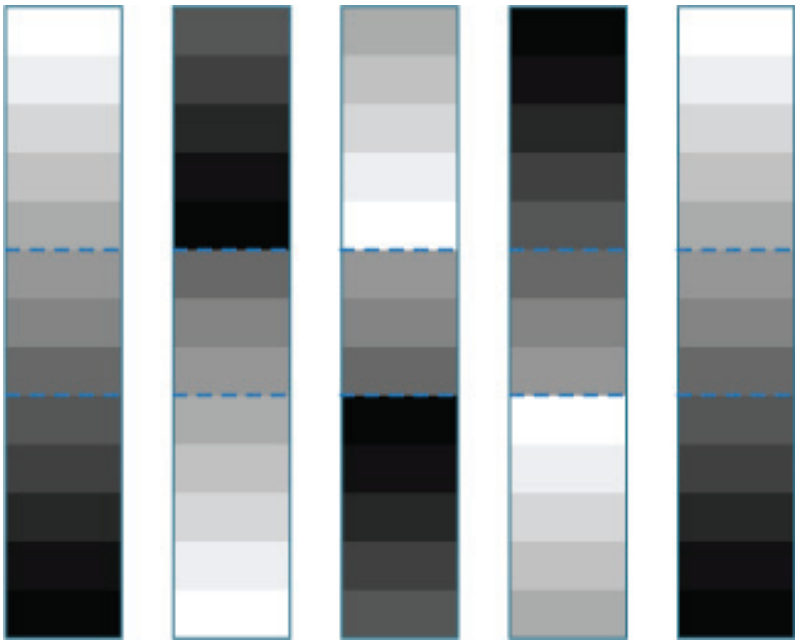


Figure A.8. Proof without words.

The results of the four overCOATs of eight cards are given by the successive vertical strips. The last strip shows a fully restored packet, so this overCOAT sequence has a period of four: after four reverse transfers, we are back to where we started.

It is also clear from these images why the original bottom card (represented as a black panel here) has risen to the top after three overCOATs, in preparation for its final journey back to the bottom under one more overCOAT.