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Learners' Difficulties and Types of Errors in Topic of Limit

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***Abstract*—The problem of learning difficulties in calculus courses has become a hot topic in universities. Calculus is a compulsory course, which is not only studied by learners majoring in mathematics but also physics, chemistry, biology, engineering, etc. However, learner's achievement in this course are still very concerning. Many learners fail in this course. Considering this problem, this study attempts to investigate difficulties and errors made by learners in understanding and solving calculus problem, with a particular focus on research topics associated to limit. To this aim, observation and documentation were conducted to reveal the data. The subjects of this study were learners of biology and science study program. The subjects were selected through certain consideration, purposive sampling technique. The research results about learner's difficulties and errors made by learners are classified based on the steps of problem solving process, namely understanding the problem, planning a strategy, select a strategy, performing the strategy and verifying.**

Keywords—learners'difficulties; calculus; limit

I. INTRODUCTION

The problem of learning difficulties in calculus courses has become a hot topic in universities. Evidences as in Tall shows that this focus of study are considered as very important since years ago [1]. Study in Tarmizi, Muzangwa, Carison, and Bressoud, show how many researchers are still enthusiastic to study the problem nowadays [2-5]. They study learners difficulties in calculus course.

According to the experience of researchers in teaching calculus course and the results of informal interviews with several calculus lecturers, the final grades obtained by learners are still very concerning, especially learners of biology and sciences major. Many students fail in this course. This failure was caused by, among others, the characteristics of students who are different from students majoring in mathematics, including the input of early mathematics abilities, beliefs in learning mathematics and also anxiety in mathematics. If learners have positive beliefs about mathematics than it will come to better achievement, this was proved by Matic whose research shown that of non-mathematics students who believe that mathematics is important in technical and natural sciences, their procedural knowledge get better than conceptual knowledge [6]. Another study conducted by Adamu revealed that learners'anxiety in learning mathematics is negatively correlated with their performance in calculus [7]. Later, Hart in his research also provided literature sources which proved that math anxiety correlate negatively with mathematics achievement for several reasons such as low mathematical ability and a fear to take mathematics examination [8].

One of the materials taught in calculus course is the limit. In this material there are still many students who solve the limit problem incorrectly. Cornu as in Bressoud states that the concept of limit is a basis of theory of other mathematical concepts, such as analysis, continuity, derivative and integral [5]. He further states that learners experience difficulties to learn limit particulary in the topic of intuitive understanding of limit. Limit lies in cognitive aspects that cannot be derived only from definitions and theorems.

The concept of limits has long been considered as a basis for comprehending other topics such as Real Analysis and Geometry. In other disciplines, limits also play an important role in solving many practical problem, such as determine the limit for a car to speed about an hour, velocity, to approximate an area of wiggles curve, to predict the genotypes inheritance, to estimate the growth of plants to be harvested, or to calculate growth of bacteria. Eventhough, there are many evidences show how important limit is, but recent studies have confirmed that learners' complete understanding of the concept of limits is relatively lacking, misconceptions of limit concept, and learners make many mistakes in solving problems related to limit [3,5,9]. Since, those several previous researches have limited discussion in presenting detailed difficulties related to problem solving process experienced by learners, this present research is an attempt to fill the gap in this issue. Therefore, this study aimed to investigate the types of difficulties and error made by learners in understanding and solving limit problem related to the steps of problem solving process.

In line with the purposes of the study, this study attempted to answer research question, "what types of difficulties and error made by learners in understanding and solving limit problem?".

The results of this study attempt to give some advantages. This study is expected to enrich literature on mathematics

learning in college specifically related to the learners difficulties. It is also expected to give a portrayal of how lecturer can anticipate learners' problems and overcome them and later will enhance the teaching practices in Calculus class.

II. METHODS

This section explains a detailed outlining of the methods of the study.

A. Research Design

A case study was conducted to reveal the data qualitatively. Since case study focuses on particular group or an individual [10]. This study focuses on learners of study programs other than mathematics. To ensure the validity of this research, triangulation was used in order to obtain a more complete picture of what is being studied and to cross-check information [11]. This study used triangulation of the data collection.

B. Site and Participants

Learners of study programs other than mathematics involved as 'key respondent' in this study. The learners were selected by purposive sampling considering that they have low interest in math, but have to study it in order to solve practical problems of their interest related to math.

C. Data Collection

The data were collected through observation and documentation. First, observation was employed to observe the leaners during learning process, they were in biology and science study programs. An observation sheet was used to guide the observation. Documentation of learners answer sheets of cognitive test was used to analyze learners difficulties.

D. Data Analysis

The collected data were then analyzed in a qualitative method that involves analyzing, synthesizing, and reducing the information that are not needed [11].

III. RESULTS AND DISCUSSION

This section will present the results of the research from three data collection techniques elaborately. The observation revealed learners' activities in Calculus class. Documentation of answer sheets was performed to check the result of cognitive test in limit topic and analyzed the errors made.

A. Learners' activities in Calculus class

There were two observers that observed learners activities during learning process. This was in order to reveal how learners give attention, feedback, and also questions related to limit topic. Observations were done three times. First, observation in the first meeting, topics were about understanding limit concept, right-hand and left-hand limit. Second, observation in the topic of limit theorems. Last, observation in the topic of continuity. In each meeting, observers used an observation sheet that consist of rubric and space to take note. Results of the observation from those two observers were then combined and analyzed. The result shows that there are 7 findings during learning process. Those are presented below :

1. A few learners give a meaningful attention to the material given, but some other did not.
2. At the part of introduction to limit, learners need longer time to understand but show enthusiasm while lecturer give an example of limit in contextual problem.
3. They who are in a category of having high cognitive ability actively ask many question during the learning process if they experienced difficulties. While the lowers are passive to ask.
4. Question are about how to draw a graph from given value of a limit, how to determine the limit value from a graph, how to predict in what point that a given function is continue or discontinue, and how to do algebraic manipulation for calculating limit value.
5. Learners do exercises asked by lecturer but had not finished by the time allocated, they need much more time to complete the problem. Learners make errors in choosing appropriate method for solving certain problem, especially caused by confusion of different method for different problem.
6. Most leaners do not checking back their answer of the exercises, tend to wait for lecturer'explanation.
7. Learners are more active when doing a discussion session in a small group. The lower is eager to ask a question about the material to his friends.

B. Result of cognitive test and errors made by learners

After the lecture was completed, learners were given a written test in the topic of limit. The result of the test was documented and then analyzed to describe errors in solving problems. The errors were discussed by relate it to the steps of problem solving process namely: understanding the problem, planning a strategy, select a strategy, performing the strategy and verifying. Researcher took this method because it can give detailed information about errors in the step by step action taken by learners. The result is explained in the following paragraphs.

At the first step, understanding the problem, some learners just re-write the problem given without any further attempt to solve the problem. Fig.1 is an example of their answer.

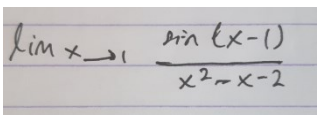

Fig. 1. Example of understanding the problem.

It is assumed that learners understand the problem but do not have any idea about appropriate strategy. According to observation result, they are confused by different sub-topics in limit that need different strategy so that they just stuck after re-write the problem.

Second, in selecting a strategy learners frequently chosen inappropriate strategy. Topic of continuity and limit involving trigonometric function are the topic that most learners made an error. Fig.2 and Fig.3 are the example of their answer.

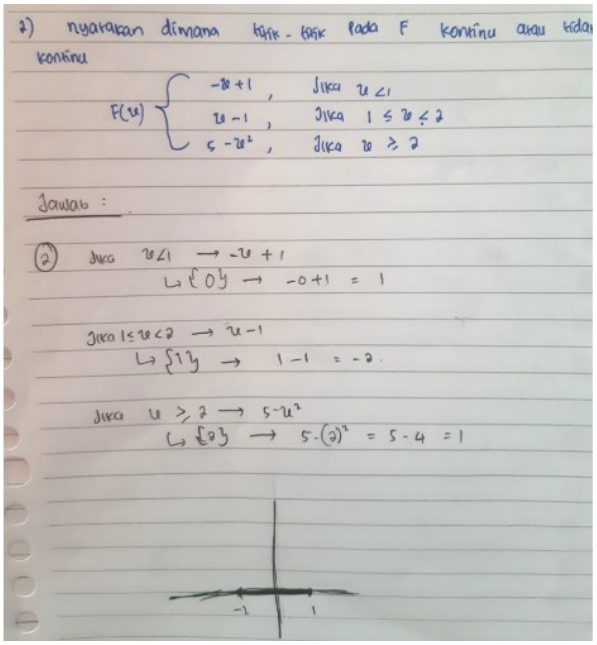

Fig. 2. Example of selecting a strategy.

Fig.2 illustrates that learner already know how to interpret piecewise function, how to use the definition given, but he can not select an appropriate strategy. Thus, he can not choose the predicted points that lead to continuity or discontinuity. He just straightly try to draw the graph and then get stuck.

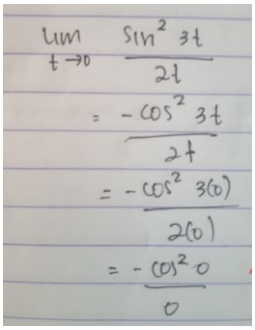

Fig. 3. Example of selecting a strategy.

Fig.3 shows that learner is likely try to use L'Hopital method for solving the problem, but he make error. The derivative of sinus is wrong. He should use another method, that is manipulating the number and the form.

Third, the results of cognitive test also reveal that learners has planned and selected a strategy, nearly true, but not be able to perform their plan, as presented in the Fig.4,

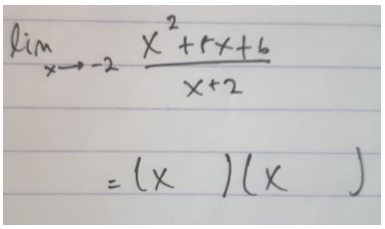

Fig. 4. Example of performing the strategy.

Error in performing algebraic strategy is the most frequently observed. Learners are unable to manipulate the algebraic form and they combine the operation in trigonometric form with algebraic form without realizing that those form cannot be combined directly. Fig.5 gives a portrait of the error.

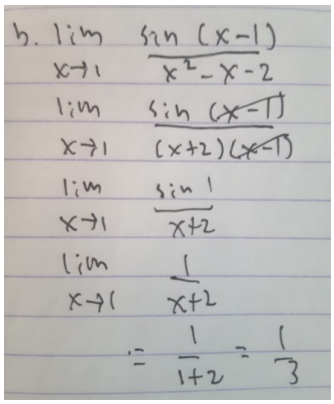

Fig. 5. Example of performing the strategy.

And other observed error concerns in interpreting a piecewise function and decide whether it is continue or discontinue in a point as presented in Fig.6.

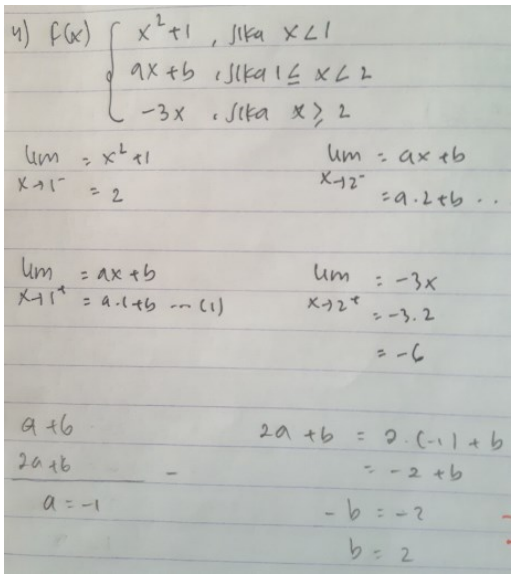

Fig. 6. Example of performing the strategy.

Fig.6 give an image that learner has known how to interpret the piecewise function, choose the correct predicted

points to reveal whether it is continu or discontinu but unfortunately false in finding the value of *a* and *b*.

Lastly, according to the observation in class, when doing an exercise, learners tend to move forward to the next problem given rather than checking back the step they had performed. They think that it will take much more time to do that and they don't want to trouble himself.

IV. CONCLUSION

According to the purpose of this study and also result and discussion, learners difficulties and types of errors made by them can be categorized as follow:

- Errors in understanding the problem: difficult to find idea that is related to the problem given
- Errors in planning a strategy: difficult to differentiate strategy that appropriated to certain sub-topic
- Errors in selecting a strategy: choose inappropriate strategy because they are confused with sub-topics of limit. Continuity and limit involving trigonometric function are the sub-topic that most learners made an error, Hard to understand the material given, in the sub-topics of intuitive understanding of limit, limit theorems and continuity
- Errors in performing the strategy: false to perform changing form of trigonometric function. They had selected a strategy but made error in performing. Eventhough they succeed in factorizing quadratic function, they did not understand how to combine strategy for limit involving trigonometric function and algebraic problem. Observed errors concerning interpreting a piecewise function whether it is continue or discontinue in a point is learners perform false step in eliminating the unknown variable.
- Learners do not verify the process of solving a problem. Time consuming is the reason.

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