Revision

1. *Percentage change*

   $$\text{Percentage change} = \frac{\text{new value} - \text{original value}}{\text{original value}} \times 100\%$$

   $\text{New value} = \text{original value} \times (1 + \text{percentage change})$

2. (a) *Increase at a constant rate*

   If a value P increases at a constant rate $r\%$ in each period, then the new value A after n periods is:

   $$A = P \times (1 + r\%)^n$$

   where $(1 + r\%)$ is called the growth factor.

   (b) *Decrease at a constant rate*

   If a value P decreases at a constant rate $r\%$ in each period, then the new value A after n periods is:

   $$A = P \times (1 - r\%)^n$$

   where $(1 - r\%)$ is called the decay factor.

   (c) *Simple interest*

   Let $\$A$ be the amount, $\$P$ be the principal, $\$I$ be the simple interest, $r\%$ be the interest rate per period and t be the number of periods. We have:

   (a) $I = P \times r\% \times t$

   (b) $A = P + I$

   (c) $A = P(1 + r\% \times t)$

3. *Compound interest*

   Let $\$A$ be the amount, $\$P$ be the principal, $\$I$ be the compound interest, $r\%$ be the interest rate per period and n be the number of periods. We have:

   (a) $A = P \times (1 + r\%)^n$

   (b) $I = A - P$

Example

1. The weight of Jane was 50 kg last month. If her weight is 46 kg this month, find the percentage change in her weight.

   Solution:

   $$\text{Percentage change in her weight} = \frac{46 - 50}{50} \times 100\% = -8\%$$

2. Thomas saves $1200 this month. If he plans to save 25% more next month, how much will he save next month?

    Solution:

    Next month's saving $= \$1200 \times (1 + 25\%) = \$1200 \times 1.25 = \$1500$

3. The price of a computer is $8075 this year. As compared with last year, the price of the computer has decreased by 15%. Find the price of the computer last year.

    Solution:

    Let $\$P$ be the price of the computer last year.

    $P(1 - 15\%) = 8075$

    $0.85P = 8075$

    $P = 9500$

    Therefore, the price of the computer last year was $9500

4. Sam's salary was $9000 when he first joined the company. A year later, his salary was increased by 15%. After another year, his salary was further increased by 10%. What was his salary after the second increment?

    Solution:

    Sam's new salary $= \$9000 \times (1 + 15\%) \times (1 + 10\%) = \$9000 \times 1.15 \times 1.1 = \$11385$

5. A table is worth $697 now. Its current value is 18% lower than that of last year. Last year, its value was 15% lower than that of 2 years ago. Find the value of the table 2 years ago.

    Solution:

    Let $\$P$ be the value of the table 2 years ago.

    $P \times (1 - 15\%) \times (1 - 18\%) = 697$

    $P(0.85)(0.82) = 697$

    $P = 1000$

    Therefore, the value was $1000 2 years ago.

6. Peter rented a flat in 2007. The monthly rent was increased by 5% in 2008, and was decreased by 10% in 2009. What was the percentage change in monthly rent from 2007 to 2009?

    Solution:

    Let $\$P$ be the monthly rent of the flat in 2007

    The monthly rent in 2009 $= \$P \times (1 + 5\%) \times (1 - 10\%) = \$P(1.05)(0.9) = \$0.945P$

    The percentage change $= \dfrac{0.945P - P}{P} \times 100\% = \dfrac{-0.055P}{P} \times 100\% = -5.5\%$

7. A bottle of perfume costs $50. Of the cost, 20% is for raw materials, 30% is for wages and 50% is for advertising expenses. The costs of both raw materials and wages are now increased by 8%, but that of advertising expenses is decreased by 20%.
   (a) Find the new cost of a bottle of perfume.
   (b) Find the overall percentage change in the cost of the perfume.
   Solution:
   (a) Original cost of raw materials $= \$50 \times 20\% = \$10$
       Original cost of wages $= \$50 \times 30\% = \$15$
       Original cost of advertising expenses $= \$50 \times 50\% = \$25$
       New cost of raw materials $= \$10 \times (1+8\%) = \$10.8$
       New cost of wages $= \$15 \times (1+8\%) = \$16.2$
       New cost of advertising expenses $= \$25 \times (1-20\%) = \$20$
       The new cost of a bottle of perfume $= \$10.8 + \$16.2 + \$20 = \$47$
   (b) The overall percentage change $= \dfrac{47-50}{50} \times 100\% = -6\%$

8. The population of a town is now 20000. If the population grows at a rate of 8% per year, how many people will there be in the town after 5 years?
   (Give your answer correct to the nearest thousand.)
   Solution:
   The population after 5 years $= 20000 \times (1+8\%)^5 = 20000 \times (1.08)^5 = 29000$

9. The number of Internet users in a country increases at a rate of 20% per year. If there are 45500 Internet users in the country now, find the number of Internet users 2 years ago.
   Solution:
   Let P be the number of Internet users 2 years ago
   $P \times (1+20\%)^2 = 45500$
   $P(1.2)^2 = 45500$
   $P = 31600$
   Therefore, the number of Internet users 2 years ago was 31600

10. The value of a photocopier is $16000 now and its depreciation rate is 15% per year. Find its value 3 years ago.

    (Give your answer correct to 3 significant figures.)

    Solution:

    Let $\$P$ be the value of the photocopier 3 years ago

    $P \times (1 - 15\%)^3 = 16000$

    $P(0.85)^3 = 16000$

    $P = 26100$

    Therefore, the value was $26100

11. Mr Cheung deposits $8000 in a bank at 6% p.a. Find the simple interest and the amount for each of the following deposit periods.

    (a) 2 years   (b) 18 months

    Solution:

    (a) Simple interest $= \$8000 \times 6\% \times 2 = \$960$

         Amount $= \$(8000 + 960) = \$8960$

    (b) Simple interest $= \$8000 \times 6\% \times \dfrac{18}{12} = \$720$

         Amount $= \$(8000 + 720) = \$8720$

12. Eddie invests $100000 at 8% p.a. for 3 years. Find the compound interest he will obtain if the interest is compounded

    (a) half-yearly,   (b) quarterly.

    Solution:

    (a) Interest

         $= \$100000 \times (1 + \dfrac{8\%}{2})^{3 \times 2} - \$100000 = \$100000 \times (1.04)^6 - \$100000 = \$26532$

    (b) Interest

         $= \$100000 \times (1 + \dfrac{8\%}{4})^{3 \times 4} - \$100000 = \$100000(1.02)^{12} - \$100000 = \$26824$

Exercise

1. Find the new value in each of the following questions.

   (a) 100 is increased by 8% and then increased by 5%

   (b) 80 cm is decreased by 6% and then increased by 25%

   (c) 120 g is increased by 15% and then decreased by 20%

   (d) $150 is decreased by 9% and then decreased by 30%

2. $30 000 is deposited in a bank at an interest rate of 6% p.a. compounded quarterly. Find the amount and interest after 2.5 years.

3. John deposits $200 000 in a bank and receives a compounded interest of $28 980 in 2 years. The amount received is then deposited in another bank at the same interest rate, but interest is compounded monthly. What will be the total amount received 2 more years later? (*Give the answer correct to the nearest dollar.*)

4. A certain kind of bacteria increase their number at a constant rate of 50% every 4 minutes. If there are 20 bacteria initially, how many bacteria will there be after an hour? (*Give the answer correct to the nearest integer.*)

5. In the past 3 years, the price of a flat was increased by 5% each year. If the present price of the flat is $2 083 725, what is the actual increase in the price of flat over these 3 years?

6. There were 54 students in a choral in the last school year, in which 24 were boys and 30 were girls. This school year the number of boys increases by 25% while the number of girls decreases by 10%. Find the percentage change in the number of students in the choral. (*Give the answer correct to 3 significant figures.*)

7. The value of a computer was $8250, it was then decreased to $6600 after 1 year. Find the percentage change in the value of the computer.

8. The volume of a glass of drink *A* is 300 mL. If the volume is increased by 30%, it is equal to 80% of the volume of a glass of drink *B*. Find the volume of a glass of drink *B*.

9.   A factory produced 2000 heaters in October. The number of heaters produced was decreased by 3% in November, and further decreased by 10% in December. How many heaters were produced in December?

10. If a number is increased by 25% and then further increased by 20%, the result is 30 larger than the original number. Find the original number.

11. The price of a table is $1800. If its price is increased by 10% and then decreased by 10%, find
     (a) the new price of the table,
     (b) the percentage change in the price of the table.

12. The base of a triangle is increased by 20% and its height is decreased by 20%. Find the percentage change in the area of the triangle.

13. If the lengths of all the sides of a cube are decreased by 10%, find
     (a) the percentage change in the total surface area,
     (b) the percentage change in the volume of the cube.

14. Beer A and Beer B contain 5% and 10% alcohol respectively. They are mixed to form Beer C of volume 500 mL, which contains 8% alcohol. How much Beer A and Beer B are needed to form Beer C?

15. Andy spends 20% of his salary on entertainment last month. This month, Andy's salary increases by 5% and he decides to spend 25% of his salary on entertainment. What is the percentage change in Andy's expenditure on entertainment?

16. Mr Wu has $2 300 000 worth of investments, consisting of 10% shares, 20% cash and 70% property. After 1 year, the value of his shares and the amount of cash have increased by 8% and 2% respectively, while the value of his property has fallen by 15%. Find
     (a) the new value of Mr Wu's total investments,
     (b) the percentage change in the value of Mr Wu's total investments.

17. If 625 cm is decreased by y% and then further decreased by y%, the result is 400 cm. Find y.

18. The population of a city grows at a constant rate per year. If the population of the city in 2000 and 2002 were 165 000 and 206 976 respectively, find
     (a) the growth rate per year of the population,
     (b) the population of the city in 2005.

19. The values of a washing machine were $4000 and $3610 in 2002 and 2004 respectively. It is known that the value of the washing machine depreciates at a constant rate per year.
    (a) Find the depreciation rate per year of the washing machine.
    (b) Find the value of the washing machine in 2008.
    (Give your answer correct to the nearest $1.)

20. A ball falls vertically at a height of 300 cm from the ground. Each time the ball hits the ground, it will rebound to a height of 40% lower than the previous time. Find the height that the ball rebounds after hitting the ground for the fourth time.

21. The value of a plasma TV is $n now. It is known that the plasma TV depreciates at a rate of 10% per year, and the depreciation of the TV is $7046 after 3 years.
    (a) Find the value of n.
    (b) Find the value of the plasma TV 5 years later.
    (Give your answer correct to the nearest $10.)

22. The depreciation rate of a mobile phone is 30% per year. If the price of the mobile phone was $2800 in 2010,
    (a) find the price of the mobile phone in 2013,
    (b) find the depreciation of the mobile phone from 2010 to 2013.

23. The volume of water in a lake is 6000 m^3 in a certain year. In the coming 3 consecutive years, the volume of water in the lake decreases at a rate of 5% per year. In the 4th year, the volume of water in the lake increases by 8%. Find the overall percentage change in the volume of water in the lake after 4 years.
    (Give your answer correct to the nearest 0.1%.)

24. There are 30 100 bacteria in a test tube at 1 pm. The number of bacteria in the test tube increases at a rate of 40% per hour. Find the number of bacteria in the test tube
    (a) 3 hours later,   (b) at 8 am on the same day.
    (Give your answers correct to the nearest hundred.)

25. The value of a factory's equipment reduced from $750 000 in Dec 2010 to $541 875 in Dec 2011 at a constant depreciation rate per 6 months.
    (a) Find the depreciation rate per 6 months of the equipment.
    (b) If the depreciation rate per 6 months was constant from Jun 2010 to Dec 2011, find the value of the equipment in Jun 2010. (Give your answer correct to the nearest dollar.)

26. Mary deposits $1500 in bank A at a simple interest rate of 4% p.a., and $2500 in bank B at a simple interest rate of 6% p.a. Find the total amount she will receive 2 years later.

27. Mr Wong deposits a sum of $200 000 in banks *A* and *B*. For bank *A*, the simple interest rate is $3\frac{1}{2}\%$; for bank *B*, the simple interest rate is $4\frac{1}{2}\%$. After 2 years, the simple interests received from these 2 banks are the same, how much does he deposit in bank *A* and bank *B* respectively?

28. Simon deposits a sum of money in a bank at a simple interest rate for 5 years. If the annual interest rate is increased by 3%, he will receive $1650 more interest. Find the principal that he deposits.

29. Mr Lee deposited a sum of money in a bank at a simple interest rate of 5% p.a. for 10 years. If he received an amount of $240 000 after 10 years, find the principal that Mr Lee deposited.

30. Andrew deposits $8000 in a bank at 4% p.a. Find the interest he will obtain 3 years later if
    (a) the interest is compounded yearly,
    (b) the interest is compounded half-yearly.
    (Give your answers correct to the nearest $1.)

31. $50 000 is invested for 3 years at 8% p.a. Find the difference between the interests calculated at a simple interest rate and at a compound interest rate, compounded quarterly. (Give your answer correct to the nearest $1.)

32. Sally wants to deposit $9000 in a bank for 3 years. Bank *A* offers an interest rate of 5% p.a. compounded half-yearly while bank *B* offers an interest rate of 4.5% p.a. compounded monthly. Which bank should Sally deposit the money in order to get larger interest?

33. Miss Chow deposits $30 000 in a bank and the interest is compounded yearly. After 3 years, she receives an amount of $37 791.36. Find the annual interest rate.

34. On 1 January 2011, Jack deposits $25 000 in a bank at 3% p.a. compounded monthly. The bank charges a monthly administration fee of $50 at the end of each month.
    (a) Find the balance in Jack's account on 31 March 2011. (Give your answer correct to the nearest dollar.)
    (b) If Jack draws $15 000 from his account on 1 April 2011, should Jack still keep the balance in this bank? Explain your answer.

35. Mr Johnson invests a total of $50 000 in two banks. Bank *A* pays simple interest at 10.5% p.a. Bank *B* pays 10% p.a. interest compounded yearly. After 2 years, the interest obtained from bank *A* is 50% more than that obtained from bank *B*. How much does Mr Johnson invest in bank *A* and bank *B* respectively?