ENTHOUGHT

Introduction to Numerical Computing with Numpy

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letter

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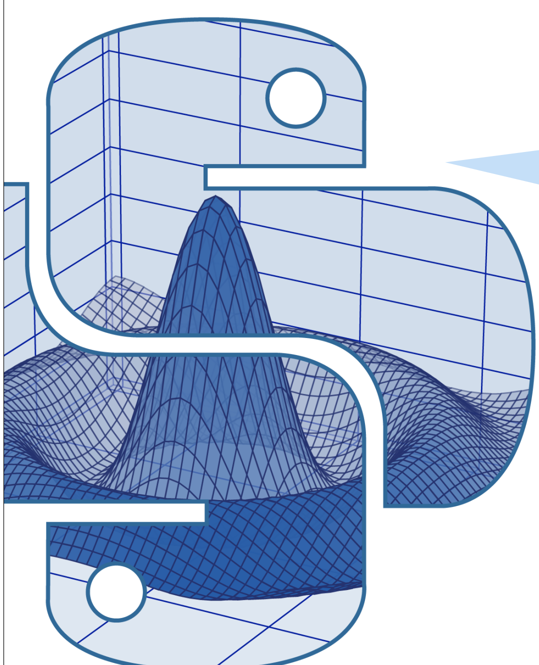


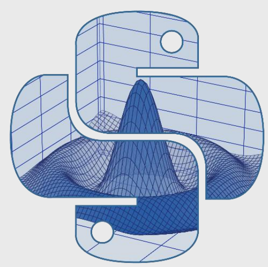
Hi there!

NumPy

The Standard Numerical Library for Python

NumPy Arrays

- Defining Arrays
- Indexing and Slicing
- Creating Arrays
- Array Calculations
- The Array Data Structure
- Advanced NumPy, Overview



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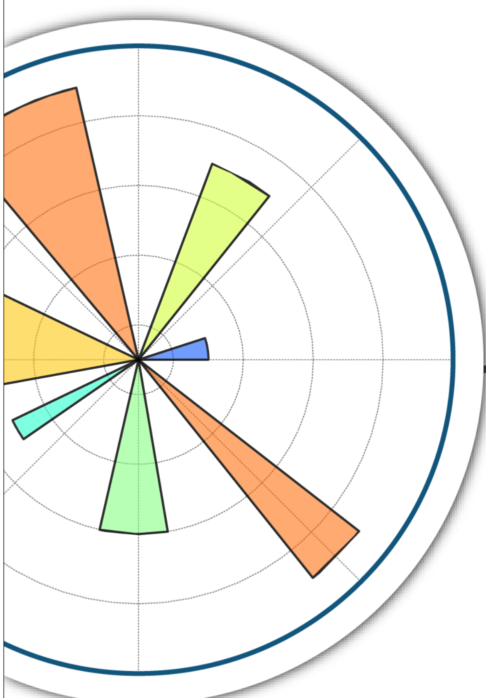
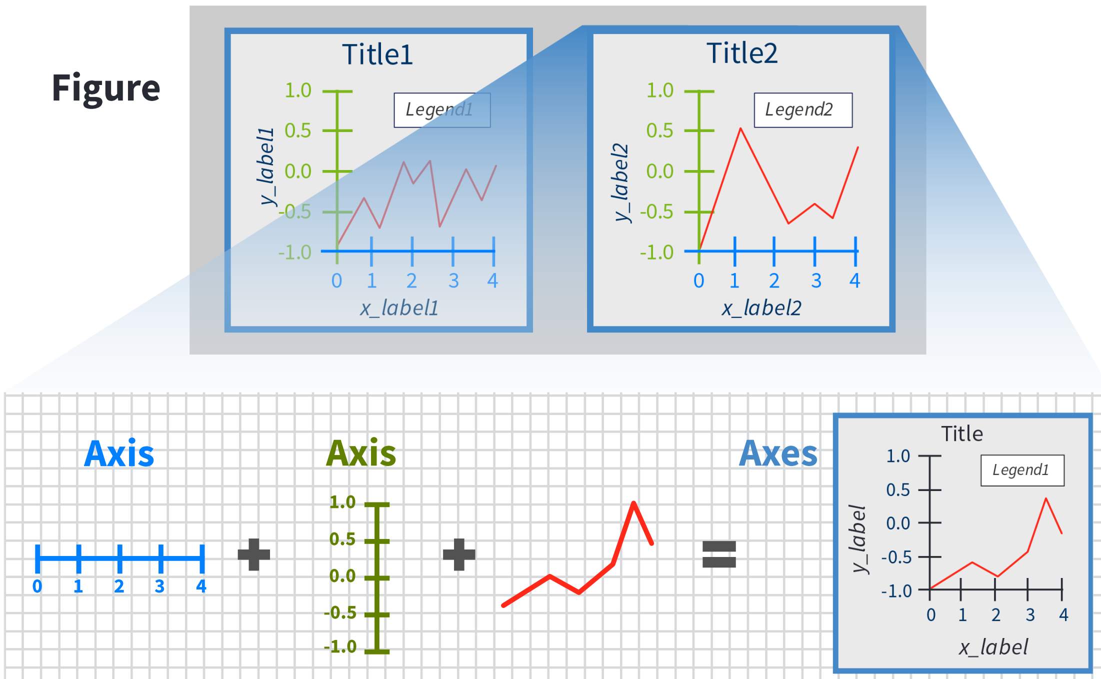


matplotlib

Matplotlib

An Interlude

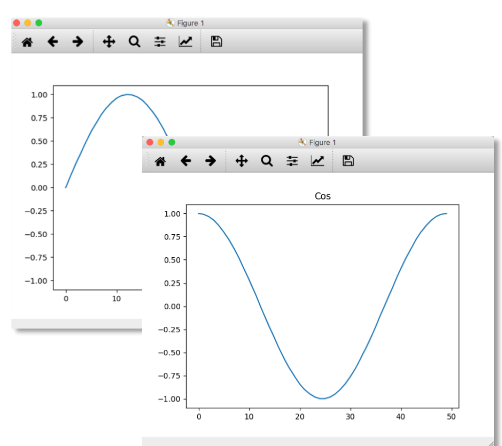
Matplotlib Object Model



Matplotlib's "State Machine"

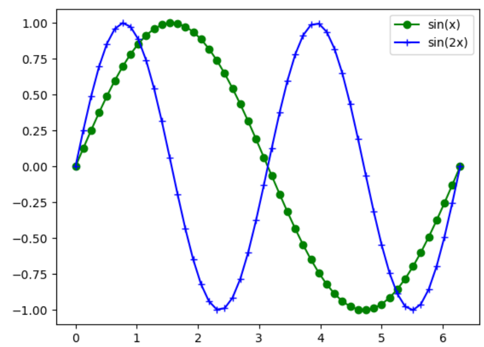
Matplotlib behaves like a state machine.
Any command is applied to the current plotting area:

```
>>> import matplotlib.pyplot as plt
>>> import numpy as np
>>> t = np.linspace(0,2*np.pi,
...                 50)
>>> x = np.sin(t)
>>> y = np.cos(t)

# Now create a figure
>>> plt.figure()
# and plot x inside it
>>> plt.plot(x)

# Now create a new figure
>>> plt.figure()
# and plot y inside it...
>>> plt.plot(y)
# ...and add a title
>>> plt.title("Cos")
```



Line Plots

```
>>> x = np.linspace(0, 2*np.pi, 50)
>>> y1 = np.sin(x)
>>> y2 = np.sin(2*x)

>>> plt.figure() # Create figure
>>> plt.plot(y1)
>>> plt.plot(x, y1)

# red dot-dash circle
>>> plt.plot(x, y1, 'r-o')

# red marker only circle
>>> plt.plot(x, y1, 'ro’)

# clear figure then plot 2 curves
>>> plt.clf()
>>> plt.plot(x, y1, 'g-o',
...             x, y2, 'b-+')
>>> plt.legend(['sin(x)',
...                   'sin(2x)'])
```



Symbol	Color
b	Blue
g	Green
r	Red
c	Cyan
m	Magenta
y	Yellow
k	Black
w	White

Symbol	Marker
.	Point
o	Circle
< > ^ v	Triangles
8	Octagon
s	Square
*	Star
+	Plus
	...and more

Scatter Plots

```
>>> N = 50 # no. of points
>>> x = np.linspace(0, 10, N)
>>> from numpy.random \
...     import rand
>>> e = rand(N)*5.0 # noise
>>> y1 = x + e

>>> areas = rand(N)*300
>>> plt.scatter(x, y1, s=areas)
>>> colors = rand(N)
>>> plt.scatter(x, y1, s=areas,
...                c=colors)
>>> plt.colorbar()
>>> plt.title("Random scatter")
```

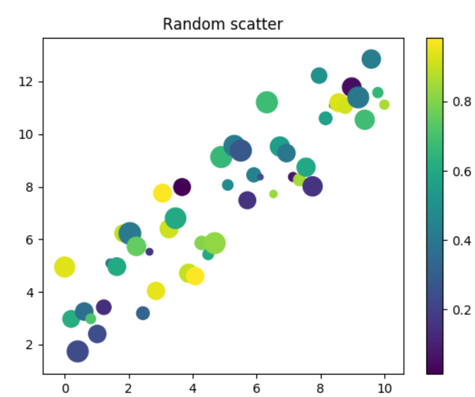
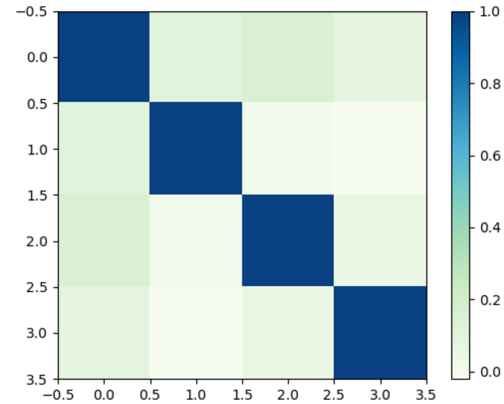


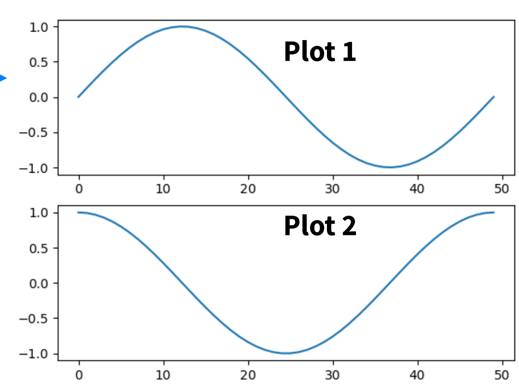
Image "Plots"

```
>>> # Create some data
>>> e1 = rand(100)
>>> e2 = rand(100)*2
>>> e3 = rand(100)*10
>>> e4 = rand(100)*100
>>> corrmatrix = \
...   np.corrcoef([e1, e2,
...                e3, e4])

>>> # Plot corr matrix as image
>>> plt.imshow(corrmatrix,
...     cmap='GnBu')
>>> plt.colorbar()
```



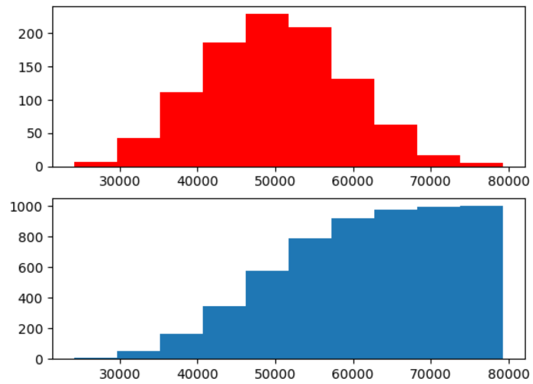
Multiple Plots Using `subplot`

```
>>> t = np.linspace(0, 2*np.pi)
>>> x = np.sin(t)
>>> y = np.cos(t)

# To divide the plotting area
                columns
>>> plt.subplot(2, 1, 1)
>>> plt.plot(x)
             rows    active plot

# Now activate a new plot
# area.
>>> plt.subplot(2, 1, 2)
>>> plt.plot(y)
```



Histogram Plots

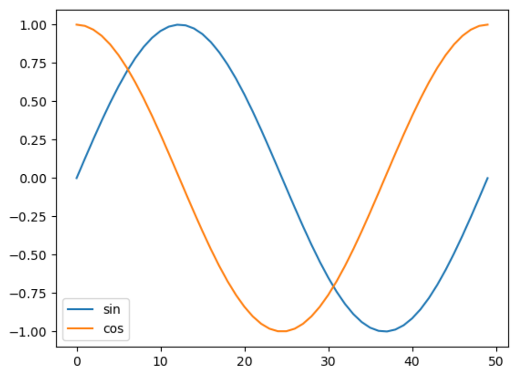
```
>>> # Create array of data
>>> from numpy.random import \
...     randint
>>> data = randint(10000,
...     size=(10,1000))

>>> # Approx norm distribution
>>> x = np.sum(data, axis=0)

>>> # Set up for stacked plots
>>> plt.subplot(2,1,1)
>>> plt.hist(x, color='r')

>>> # Plot cumulative dist
>>> plt.subplot(2,1,2)
>>> plt.hist(x, cumulative=True)

>>> # For multiple histograms:
>>> plt.hist([d1, d2, …])
```



Legends, Titles, and Axis Labels

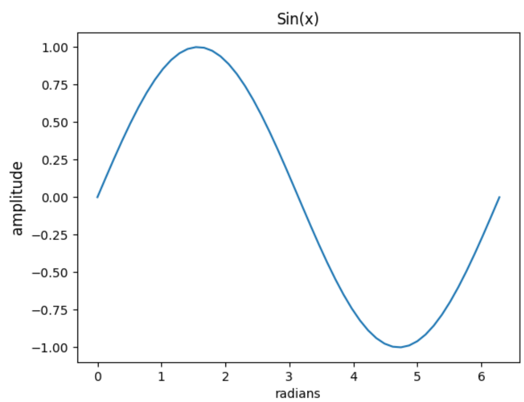
LEGEND LABELS WITH PLOT

```
# Add labels in plot command.
>>> plt.plot(np.sin(t),
...          label='sin')
>>> plt.plot(np.cos(t),
...          label='cos')
>>> plt.legend()
```



TITLES AND AXIS LABELS

```
>>> plt.plot(t, np.sin(t))
>>> plt.xlabel('radians')
# Keywords set text properties.
>>> plt.ylabel('amplitude',
...            fontsize='large')
>>> plt.title('Sin(x)')
```



Plotting from Scripts

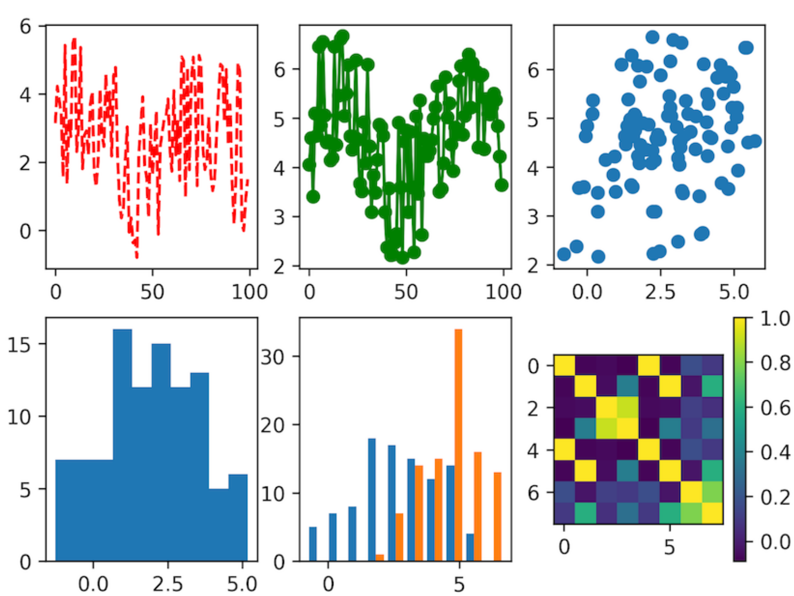
INTERACTIVE MODE

```
# In IPython, plots show up
# as soon as a plot command
# is issued:
>>> plt.figure()
>>> plt.plot(np.sin(t))
>>> plt.figure()
>>> plt.plot(np.cos(t))
```

NON-INTERACTIVE MODE

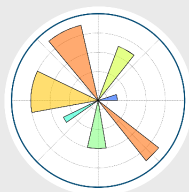
```
# In a script, you must call
# the show() command to display
# plots. Call it at the end of
# all your plot commands for
# best performance.
import matplotlib.pyplot as plt
import numpy as np
t = np.linspace(0,2*np.pi,
...                     50)
plt.figure()
plt.plot(np.sin(t))
plt.figure()
plt.plot(np.cos(t))
# Plots will not appear until
# this command is run:
plt.show()
```

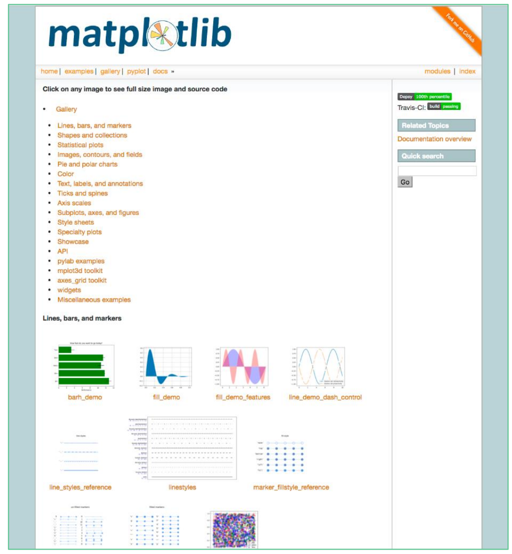

MPL Exercise: Desired Output



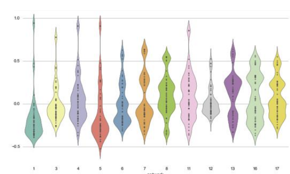
Additional Matplotlib Resources

- Simple examples with increasing difficulty
  http://matplotlib.org/examples/index.html
- Gallery (huge)
  http://matplotlib.org/gallery.html
- See appendix for reference materials
- Usage FAQ
  http://matplotlib.org/faq/usage_faq.html

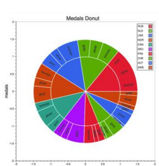




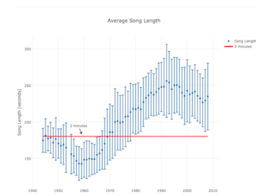
Other Visualization Libraries



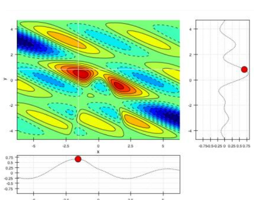
Seaborn: Better looking, high-level plots based on matplotlib



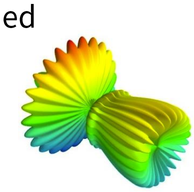
bokeh: D3-like visualization in web browsers (HTML file, or inline in IPython notebooks)



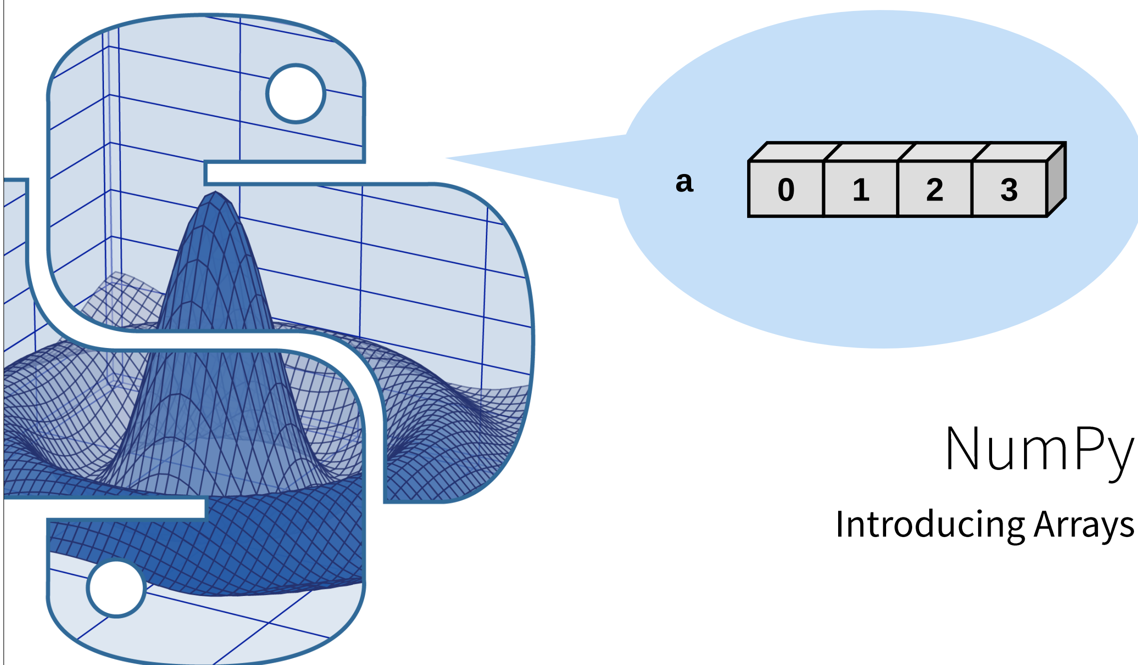
plot.ly: D3-based visualization with support for multiple languages (R, Python, Matlab, Julia and more.)



chaco: For interactive, custom plots embedded in user applications



mayavi: 3D visualization, based on VTK



NumPy

Introducing Arrays

Introducing NumPy Arrays

SIMPLE ARRAY CREATION

```
>>> a = np.array([0, 1, 2, 3])
>>> a
array([0, 1, 2, 3])
```

CHECKING THE TYPE

```
>>> type(a)
numpy.ndarray
```

NUMERIC "TYPE" OF ELEMENTS

```
>>> a.dtype
dtype('int32')
```

NUMBER OF DIMENSIONS

```
>>> a.ndim
1
```

ARRAY SHAPE

```
# Shape returns a tuple
# listing the length of the
# array along each dimension.
>>> a.shape
(4,)
```

BYTES PER ELEMENT

```
>>> a.itemsize
4
```

BYTES OF MEMORY USED

```
# Return the number of bytes
# used by the data portion of
# the array.
>>> a.nbytes
16
```

Array Operations

SIMPLE ARRAY MATH

```
>>> a = np.array([1, 2, 3, 4])
>>> b = np.array([2, 3, 4, 5])
>>> a + b
array([3, 5, 7, 9])

>>> a * b
array([ 2, 6, 12, 20])

>>> a ** b
array([  1,   8,   81,   1024])
```

NumPy defines these constants:
pi = 3.14159265359
e = 2.71828182846

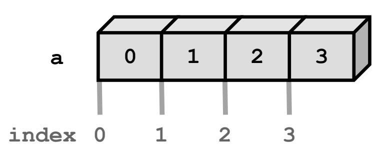
MATH FUNCTIONS

```
# create array from 0. to 10.
>>> x = np.arange(11.)

# multiply entire array by
# scalar value
>>> c = (2 * np.pi) / 10.
>>> c
0.62831853071795862
>>> c * x
array([ 0.,0.628,…,6.283])

# in-place operations
>>> x *= c
>>> x
array([ 0.,0.628,…,6.283])

# apply functions to array
>>> y = np.sin(x)
```

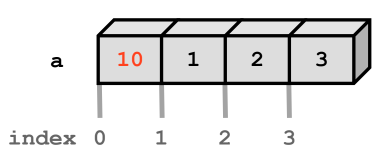

Setting Array Elements

ARRAY INDEXING

```
>>> a[0]
0
```




```
>>> a[0] = 10
>>> a
array([10, 1, 2, 3])
```



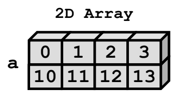
BEWARE OF TYPE COERCION

```
>>> a.dtype
dtype('int32')

# assigning a float into
# an int32 array truncates
# the decimal part
>>> a[0] = 10.6
>>> a
array([10, 1, 2, 3])

# fill has the same behavior
>>> a.fill(-4.8)
>>> a
array([-4, -4, -4, -4])
```

Multi-Dimensional Arrays

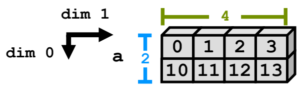
MULTI-DIMENSIONAL ARRAYS

```
>>> a = np.array([[ 0, 1, 2, 3],
...               [10,11,12,13]])
>>> a
array([[ 0,  1,  2,  3],
       [10,11,12,13]])
```



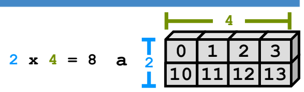
SHAPE = (ROWS, COLUMNS)

```
>>> a.shape
(2,  4)
```



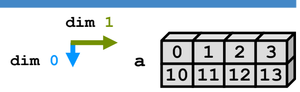
ELEMENT COUNT

```
>>> a.size
8
```



NUMBER OF DIMENSIONS

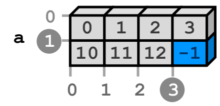
```
>>> a.ndim
2
```



GET / SET ELEMENTS

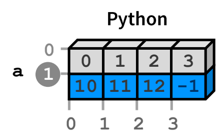
```
>>> a[1, 3]
13
```

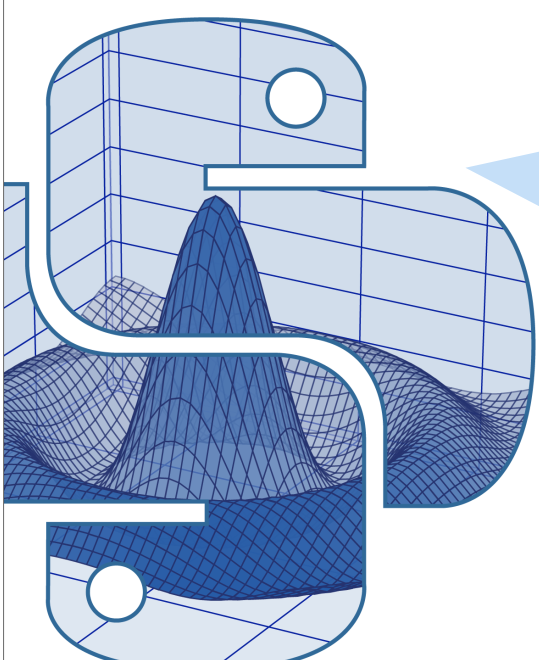

```
>>> a[1, 3] = -1
>>> a
array([[ 0,  1,  2,  3],
       [10,11,12,-1]])
```

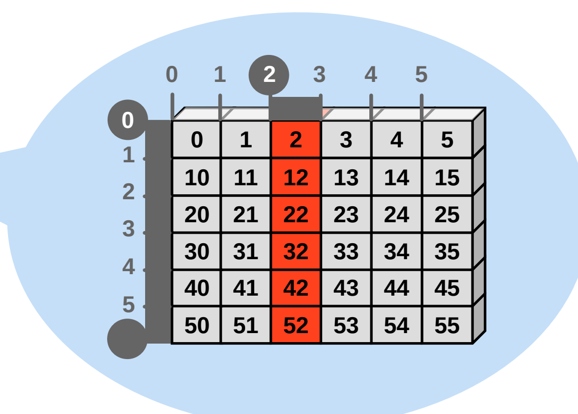


ADDRESS SECOND (ONETH) ROW USING SINGLE INDEX

```
>>> a[1]
array([10,  11,  12,  -1])
```







NumPy

Indexing and Slicing

Slicing

`var[lower:upper:step]`

Extracts a portion of a sequence by specifying a lower and upper bound.
The lower-bound element is included, but the upper-bound element is **not** included.
Mathematically: [lower, upper). The step value specifies the stride between elements.

SLICING ARRAYS

```
#                  -5 -4 -3 -2 -1
# indices:          0  1  2  3  4
>>> a = np.array([10,11,12,13,14])

# [10, 11, 12, 13, 14]
>>> a[1:3]
array([11, 12])

# negative indices work also
>>> a[1:-2]
array([11, 12])
>>> a[-4:3]
array([11, 12])
```

OMITTING INDICES

```
# omitted boundaries are
# assumed to be the beginning
# (or end) of the list

# grab first three elements
>>> a[:3]
array([10, 11, 12])

# grab last two elements
>>> a[-2:]
array([13, 14])

# every other element
>>> a[::2]
array([10, 12, 14])
```

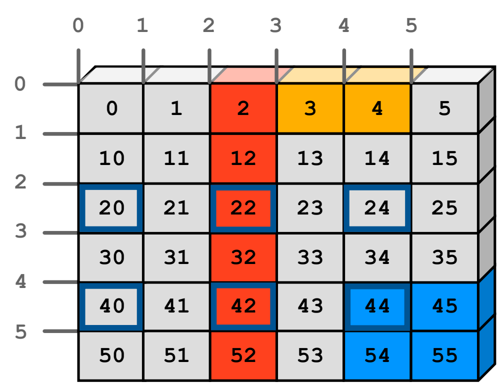

Array Slicing

SLICING WORKS MUCH LIKE STANDARD PYTHON SLICING

```
>>> a[0, 3:5]
array([3, 4])

>>> a[4:, 4:]
array([[44, 45],
       [54, 55]])

>>> a[:, 2]
array([2, 12, 22, 32, 42, 52])
```

STRIDED ARE ALSO POSSIBLE

```
>>> a[2::2, ::2]
array([[20, 22, 24],
       [40, 42, 44]])
```



Slices are References

Slices are references to locations in memory.
These memory locations can be used in assignment operations.

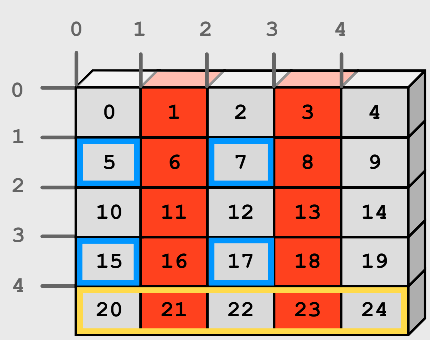
```
>>> a = np.array([0, 1, 2, 3, 4])
# slicing the last two elements returns the data there
>>> a[-2:]
array([3, 4])
# we can insert an iterable of length two
>>> a[-2:] = [-1, -2]
>>> a
array([ 0,  1,  2, -1, -2])
# or a scalar value
>>> a[-2:] = 99
>>> a
array([ 0,  1,  2, 99, 99])
```


Give it a try!

Create the array below with the command

`a = np.arange(25).reshape(5, 5)`

and extract the slices as indicated.



Sliced Arrays Share Data

Arrays created by slicing share data with the originating array.
Changing values in a slice also changes the original array.

```
>>> a = np.array([0, 1, 2, 3, 4])

# create a slice containing two elements of a
>>> b = a[2:4]
>>> b
array([2, 3])
>>> b[0] = 10

# changing b changed a!
>>> a
array([ 0,  1, 10, 3, 4])
```

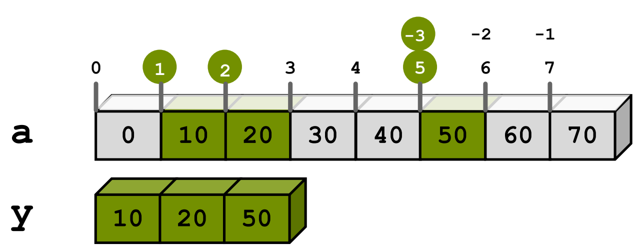

Fancy Indexing

INDEXING BY POSITION

```
>>> a = np.arange(0, 80, 10)

# fancy indexing
>>> indices = [1, 2, -3]
>>> y = a[indices]
>>> print(y)
[10 20 50]

# this also works with setting
>>> a[indices] = 99
>>> a
array([ 0, 99, 99, 30, 40, 99,
60, 70])
```

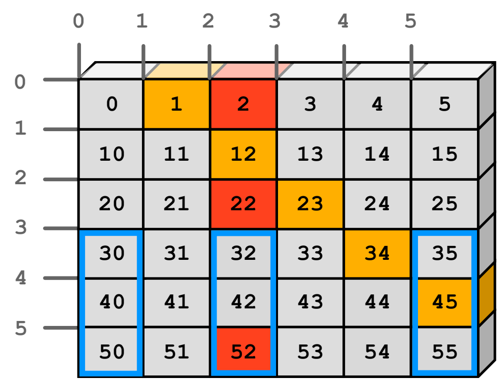
INDEXING WITH BOOLEANS

```
# manual creation of masks
>>> mask = np.array(
...       [0, 1, 1, 0, 0, 1, 0, 0],
...       dtype=bool)

# fancy indexing
>>> y = a[mask]
>>> print(y)
array([99, 99, 99])
```



Fancy Indexing in 2-D

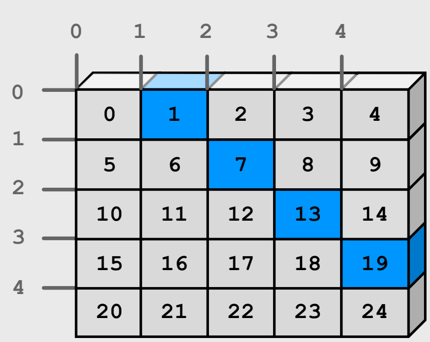
```
>>> a[[0, 1, 2, 3, 4],
...   [1, 2, 3, 4, 5]]
array([ 1, 12, 23, 34, 45])

>>> a[3:, [0, 2, 5]]
array([[30, 32, 35],
       [40, 42, 45],
       [50, 52, 55]])

>>> mask = np.array(
...       [1, 0, 1, 0, 0, 1],
...       dtype=bool)
>>> a[mask, 2]
array([2, 22, 52])
```



Unlike slicing, fancy indexing creates copies instead of a view into original array.

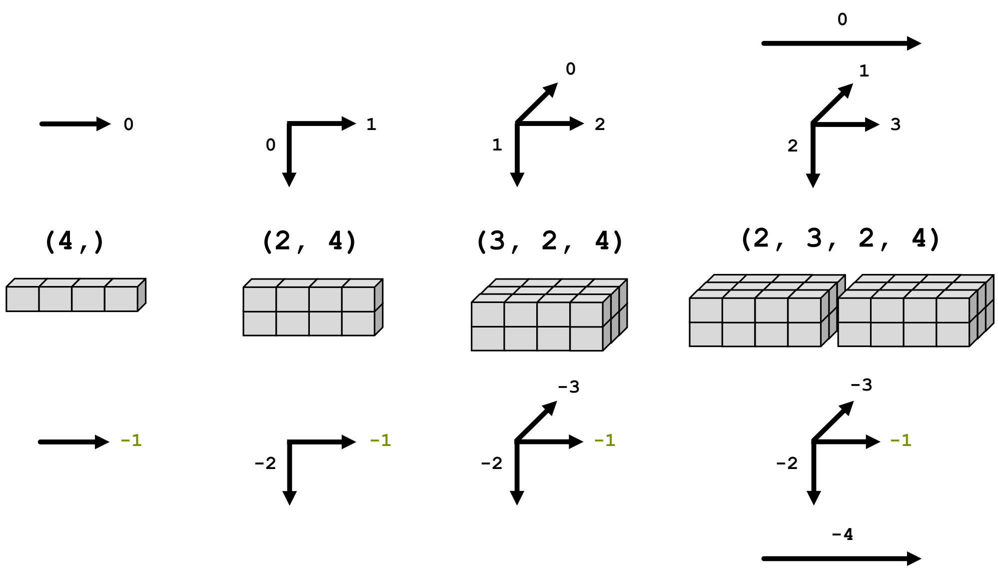
Give it a try!

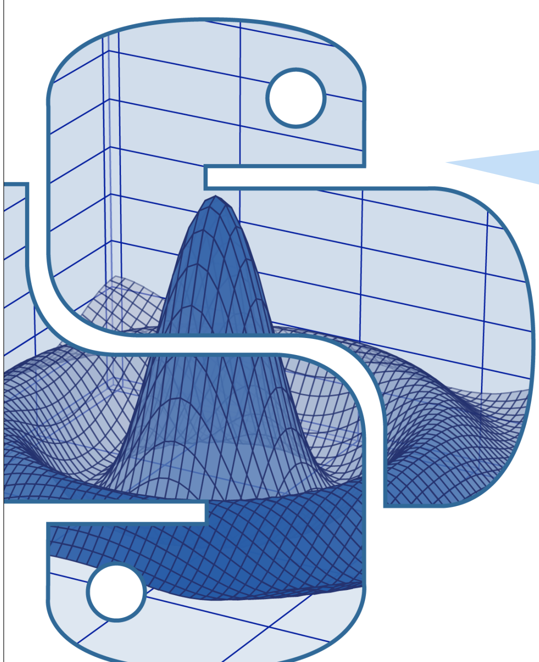
1. Create the array below with
   `a = np.arange(25).reshape(5, 5)`
   and extract the elements indicated in blue.
2. Extract all the numbers divisible by 3 using a boolean mask.



Multi-Dimensional Arrays

VISUALIZING MULTI-DIMENSIONAL ARRAYS





arange()
linspace()
array()
zeros()
ones()

NumPy

Creating Arrays

Array Constructor Examples

FLOATING POINT ARRAYS

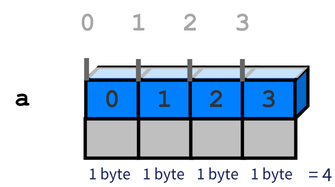
```
# Default to double precision
>>> a = np.array([0,1.0,2,3])
>>> a.dtype
dtype('float64')
>>> a.nbytes
32
```

REDUCING PRECISION

```
>>> a = np.array([0,1.,2,3],
...               dtype='float32')
>>> a.dtype
dtype('float32')
>>> a.nbytes
16
```

UNSIGNED INTEGER BYTE

```
>>> a = np.array([0,1,2,3],
...               dtype='uint8')
>>> a.dtype
dtype('uint8')
>>> a.nbytes
4
```




```
Base 2                  Base 10

00000000    ->  0 = 0*2**0 + 0*2**1 + ... + 0*2**7
00000001    ->  1 = 1*2**0 + 0*2**1 + ... + 0*2**7
00000010    ->  2 = 0*2**0 + 1*2**1 + ... + 0*2**7
...
11111111    ->  255 = 1*2**0 + 1*2**1 + ... + 1*2**7
```


Array Creation Functions

ARANGE

```
arange([start,] stop[, step],
                   dtype=None)
```

Nearly identical to Python's **range()**. Creates an array of values in the range [start,stop) with the specified step value. Allows non-integer values for start, stop, and step. Default **dtype** is derived from the start, stop, and step values.

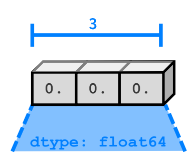
```
>>> np.arange(4)
array([0, 1, 2, 3])
>>> np.arange(0, 2*pi, pi/4)
array([ 0.000, 0.785, 1.571,
2.356, 3.142, 3.927, 4.712,
5.497])

# Be careful…
>>> np.arange(1.5, 2.1, 0.3)
array([ 1.5, 1.8, 2.1])
```

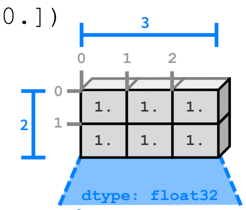
ONES, ZEROS

```
ones(shape, dtype='float64')
zeros(shape, dtype='float64')
```

shape is a number or sequence specifying the dimensions of the array. If **dtype** is not specified, it defaults to **float64.**

```
>>> np.ones((2, 3),
...         dtype='float32')
array([[ 1.,  1.,  1.],
       [ 1.,  1.,  1.]],
      dtype=float32)
>>> np.zeros(3)
array([ 0., 0., 0.])
```





zeros(3) is equivalent to **zeros((3,))**

Array Creation Functions (cont'd)

IDENTITY

```
# Generate an n by n identity
# array. The default dtype is
# float64.
>>> a = np.identity(4)
>>> a
array([[ 1.,  0.,  0.,  0.],
       [ 0.,  1.,  0.,  0.],
       [ 0.,  0.,  1.,  0.],
       [ 0.,  0.,  0.,  1.]])
>>> a.dtype
dtype('float64')
>>> np.identity(4, dtype=int)
array([[ 1, 0, 0, 0],
       [ 0, 1, 0, 0],
       [ 0, 0, 1, 0],
       [ 0, 0, 0, 1]])
```

EMPTY AND FILL

```
# empty(shape, dtype=float64,
#       order='C')
>>> np.empty(2)
array([1.78021120e-306,
 6.95357225e-308])

# array filled with 5.0
>>> a = np.full(2, 5.0)
array([5., 5.])

# alternative approaches
# (slower)
>>> a = np.empty(2)
>>> a.fill(4.0)
>>> a
array([4., 4.])
>>> a[:] = 3.0
>>> a
array([3., 3.])
```


Array Creation Functions (cont'd)

LINSPACE

```
# Generate N evenly spaced
# elements between (and including)
# start and stop values.
>>> np.linspace(0, 1, 5)
array([0., 0.25, 0.5, 0.75, 1.0])
```

LOGSPACE

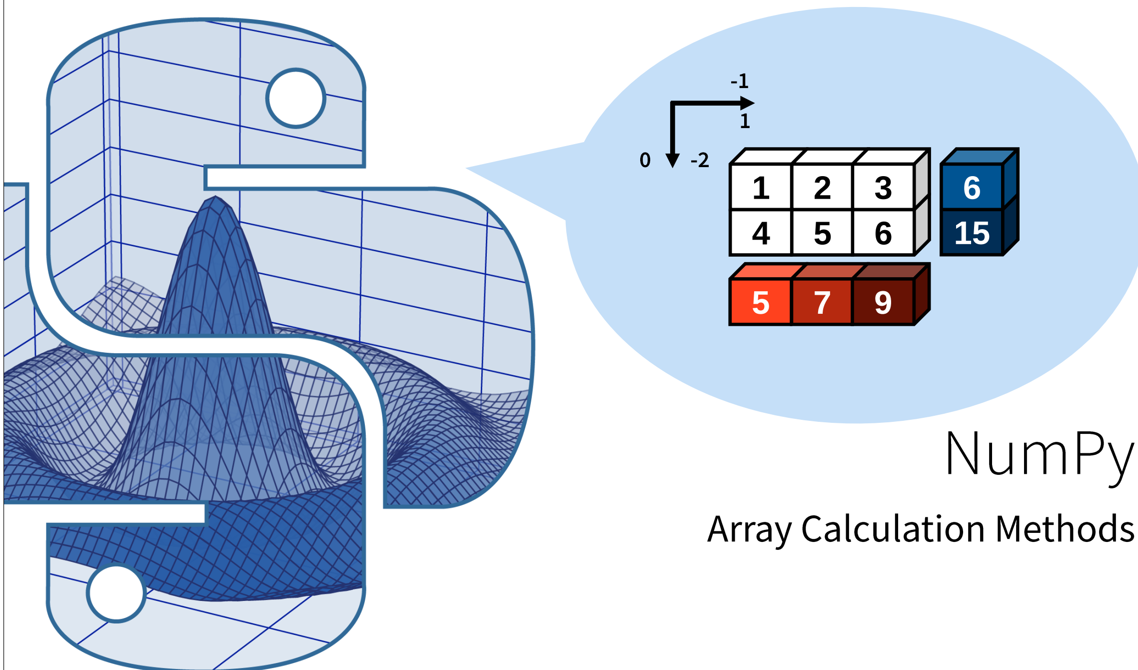
```
# Generate N evenly spaced
# elements on a log scale
# between base**start and
# base**stop (default base=10)
>>> np.logspace(0, 1, 5)
array([1., 1.78, 3.16, 5.62, 10.])
```

ARRAYS FROM/TO TXT FILES

```
BEGINNING OF THE FILE
% Day, Month, Year, Skip, Avg Power
01, 01, 2000, x876, 13 % crazy day!
% we don't have Jan 03rd
04, 01, 2000, xfed, 55
```

Data.txt

```
# loadtxt() automatically
# generates an array from the
# txt file
arr = np.loadtxt('Data.txt',
...       skiprows=1,
...       dtype=int, delimiter=",",
...       usecols = (0,1,2,4),
...       comments = "%")

# Save an array into a txt file
np.savetxt('filename', arr)
```



NumPy

Array Calculation Methods

Computations with Arrays

Rule 1: Operations between multiple array objects are first checked for proper shape match.

Rule 2: Mathematical operators (+ - * / exp, log, ...) apply element by element, on the values.

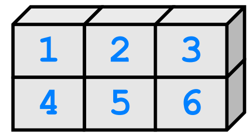
Rule 3: Reduction operations (mean, std, skew, kurt, sum, prod, ...) apply to the whole array, unless an axis is specified.

Rule 4: Missing values propagate unless explicitly ignored (nanmean, nansum, ...).

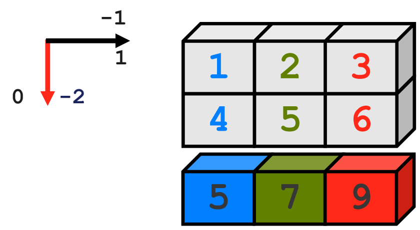
Array Calculation Methods

SUM METHOD

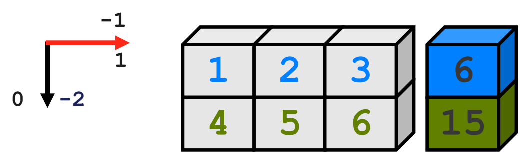
```
# Methods act on data stored
# in the array
>>> a = np.array([[1,2,3],
                  [4,5,6]])

# .sum() defaults to adding up
all the values in an array.
>>> a.sum()
21

# supply the keyword axis to
# sum along the 0th axis
>>> a.sum(axis=0)
array([5, 7, 9])

# supply the keyword axis to
# sum along the last axis
>>> a.sum(axis=-1)
array([ 6, 15])
```



sum(a)



sum(a, axis=0)



sum(a, axis=-1)

Other Operations on Arrays

SUM FUNCTION

```
# Functions work on data
# passed to it
>>> a = np.array([[1,2,3],
                  [4,5,6]])

# sum() defaults to adding
# up all values in an array.
>>> np.sum(a)
21

# supply an axis argument to
# sum along a specific axis
>>> np.sum(a, axis=0)
array([5, 7, 9])
```

OTHER METHODS AND FUNCTIONS

Mathematical functions

- `sum, prod`
- `min, max, argmin, argmax`
- `ptp (max – min)`

Statistics

- `mean, std, var`

Truth value testing

- `any, all`

See the Numpy appendix for more.

Min/Max

MIN

```
>>> a = np.array([[2, 3], [0, 1]])
# Prefer NumPy functions to
# builtins when working with
# arrays
>>> np.min(a)
0
# Most NumPy reducers can be used
# as methods as well as functions
>>> a.min()
0
```

MAX

```
# Use the axis keyword to find
# max values for one dimension
>>> a.max(axis=0)
array([2, 3])
# as a function
>>> np.max(a, axis=1)
array([3, 1])
```

ARGMIN/MAX

```
# Many tasks (like optimization)
# are interested in the location
# of a min/max, not the value
>>> a.argmax()
1

# arg methods return the
# location in 1D, on a raveled
# index of the original array
>>> np.argmin(a)
2
```

UNRAVELING

```
# NumPy includes a function
# to un-flatten 1D locations
>>> np.unravel_index(
...     a.argmax(), a.shape)
(0, 1)
```


Where

COORDINATE LOCATIONS

```
# NumPy's where function has two
# distinct uses. One is to
# provide coordinates from masks
>>> a = np.arange(-2, 2) ** 2
>>> a
array([4, 1, 0, 1])
>>> mask = a % 2 == 0
array([ True, False,  True,
False], dtype=bool)
# Coordinates are returned as
# a tuple of arrays, one for
# each axis
>>> np.where(mask)
(array([0, 2]),)
```

CONDITIONAL ARRAY CREATION

```
# Where can also be used to
# construct a new array by
# choosing values from other
# arrays of the same shape
>>> positives = np.arange(4)
>>> negatives = -positives
>>> np.where(mask, positives,
...      negatives)
array([ 0, -1,  2, -3])
# Or from scalar values.
# This can be useful for
# recoding arrays
>>> np.where(mask, 1, 0)
array([1, 0, 1, 0])
# Or from both
>>> np.where(mask, positives, 0)
array([0, 0, 2, 0])
```

Statistics Array Methods

MEAN

```
>>> a = np.array([[1,2,3],
...               [4,5,6]])

# mean value of each column
>>> a.mean(axis=0)
array([ 2.5,  3.5,  4.5])
>>> np.mean(a, axis=0)
array([ 2.5,  3.5,  4.5])
```

STANDARD DEV./VARIANCE

```
# Standard Deviation
>>> a.std(axis=0)
array([ 1.5,  1.5,  1.5])
# For sample, set ddof=1
>>> a.std(axis=0, ddof=1)
array([ 2.12,  2.12,  2.12])

# variance
>>> a.var(axis=0)
array([2.25, 2.25, 2.25])
>>> np.var(a, axis=0)
array([2.25, 2.25, 2.25])
```


Give it a try!

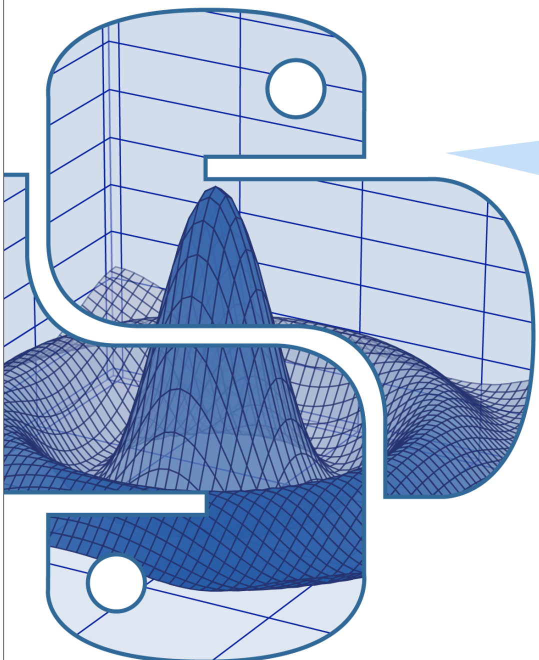
Create the array below with

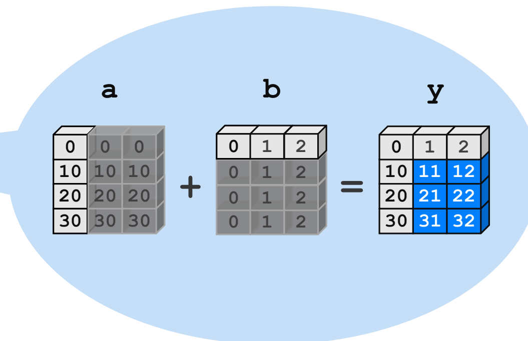
```
a = np.arange(-15, 15).reshape(5, 6) ** 2
```

and compute:

1. The maximum of each row (one max per row)
2. The mean of each column (one mean per column)
3. The position of the overall minimum (requires 2-3 steps)

	0	1	2	3	4	5
0	225	196	169	144	121	100
1	81	64	49	36	25	16
2	9	4	1	0	1	4
3	9	16	25	36	49	64
4	81	100	121	144	169	196





NumPy

Array Broadcasting

Array Broadcasting

NumPy arrays of different dimensionality can be combined in the same expression. Arrays with smaller dimension are **broadcasted** to match the larger arrays, *without copying data*. Broadcasting has **two rules**.

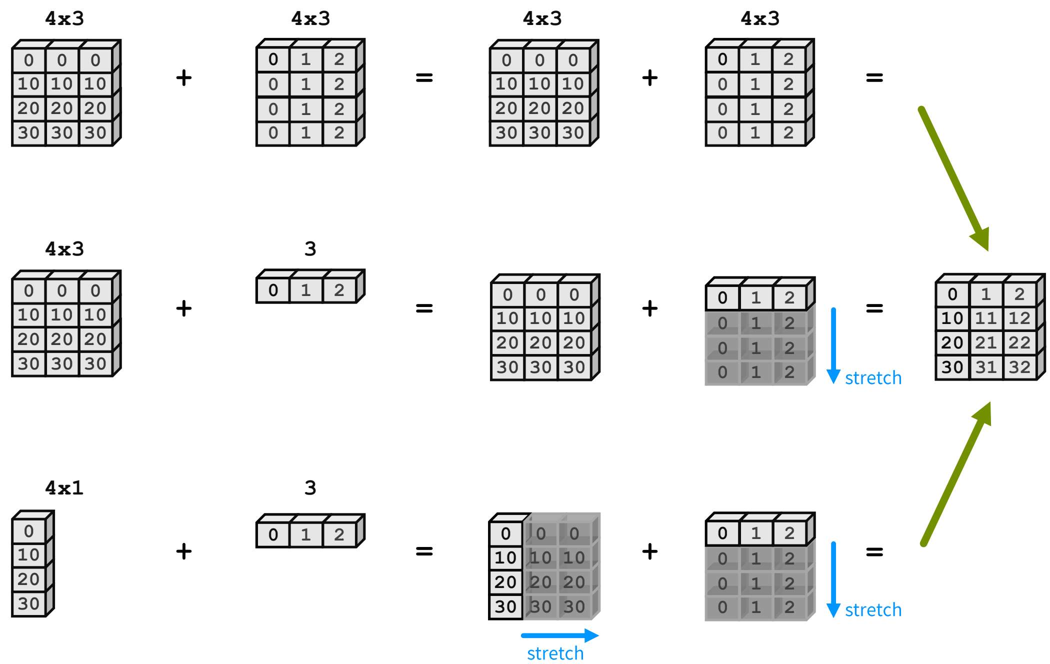
RULE 1: PREPEND ONES TO SMALLER ARRAY'S SHAPE

```
>>> import numpy as np
>>> a = np.ones((3, 5)) # a.shape == (3, 5)
>>> b = np.ones((5,)) # b.shape == (5,)
>>> b.reshape(1, 5) # result is a (1,5)-shaped array.
>>> b[np.newaxis, :] # equivalent, more concise.
```

RULE 2: DIMENSIONS OF SIZE 1 ARE REPEATED WITHOUT COPYING

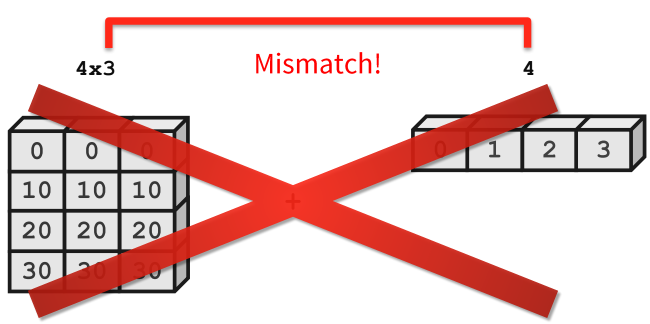
```
>>> c = a + b # c.shape == (3, 5)
# is logically equivalent to...
>>> tmp_b = b.reshape(1, 5)
>>> tmp_b_repeat = tmp_b.repeat(3, axis=0)
>>> c = a + tmp_b_repeat
# But broadcasting makes no copies of "b"s data!
```

Array Broadcasting



Broadcasting Rules

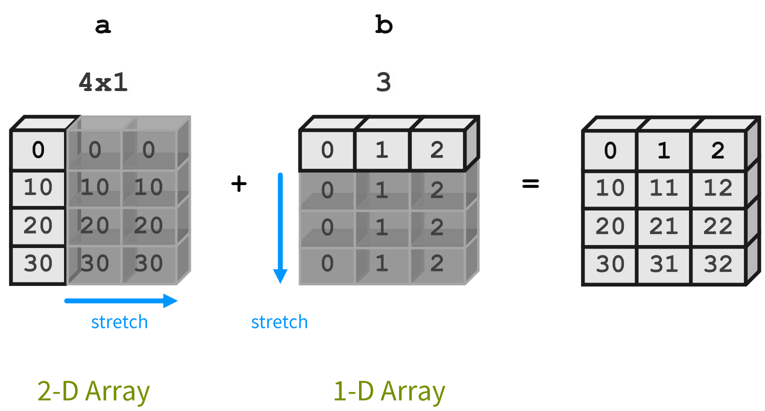
The trailing axes of either arrays must be 1 or both must have the same size for broadcasting to occur. Otherwise, a `"ValueError: shape mismatch: objects cannot be broadcast to a single shape"` exception is thrown.



Broadcasting in Action

ENTHOUGHT

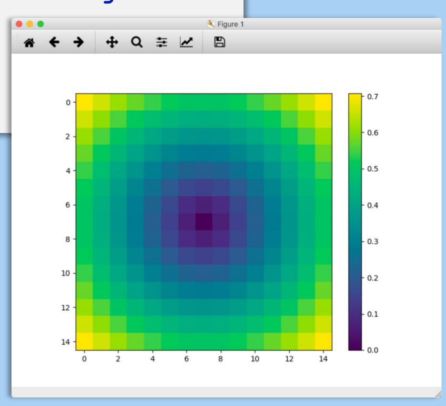
```
>>> a = array([0, 10, 20, 30])
>>> b = array([0, 1, 2])
>>> y = a[:, newaxis] + b
```



Application: Distance from Center

ENTHOUGHT

```
>>> import matplotlib.pyplot as plt
>>> a = np.linspace(0, 1, 15) - 0.5
>>> b = a[:, np.newaxis] # b.shape == (15, 1)
>>> dist2 = a**2 + b**2 # broadcasting sum.
>>> dist = np.sqrt(dist2)
>>> plt.imshow(dist)
>>> plt.colorbar()
```



Broadcasting's Usefulness

ENTHOUGHT

Broadcasting can often be used to replace needless data replication inside a NumPy array expression.

`np.meshgrid()` – use **`newaxis`** appropriately in broadcasting expressions.

`np.repeat()` – broadcasting makes repeating an array along a dimension of size 1 unnecessary.

MESHGRID: COPIES DATA

```
>>> x, y = \
...     np.meshgrid([1,2],
...                 [3,4,5])
>>> z = x + y
```

BROADCASTING: NO COPIES

```
>>> x = np.array([1, 2])
>>> y = np.array([3, 4, 5])
>>> z = x[np.newaxis, :] \
...     + y[:, np.newaxis]
```


Broadcasting Indices

ENTHOUGHT

Broadcasting can also be used to slice elements from different "depths" in a 3-D (or any other shape) array. This is a very powerful feature of indexing.

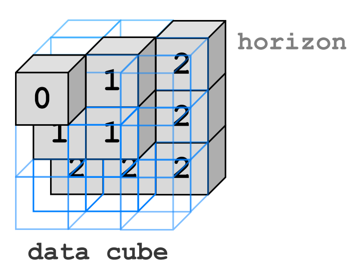
```
>>> data_cube = np.arange(27).reshape(3, 3, 3)
>>> yi, xi = np.meshgrid(np.arange(3), np.arange(3),
...                     sparse=True)
>>> zi = np.array([[0, 1, 2],
...                [1, 1, 2],
...                [2, 2, 2]])
>>> horizon = data_cube[xi, yi, zi]
```

Indices

	yi	0	1	2
xi	0	0	1	2
	1	0	1	2
	2	0	1	2

zi

Selected Data



Universal Function Methods

The mathematical, comparative, logical, and bitwise operators *op* that take two arguments (binary operators) have special methods that operate on arrays:

```
op.reduce(a,axis=0)
op.accumulate(a,axis=0)
op.outer(a,b)
op.reduceat(a,indices)
```


op.reduce()

`op.reduce(a)` applies **op** to all the elements in a 1-D array **a** reducing it to a single value.

For example:

$$y = \mathrm{add.reduce}(a) = \sum_{n=0}^{N-1} a[n] = a[0]+a[1]+\ldots+a[N-1]$$

ADD EXAMPLE

```
>>> a = np.array([1,2,3,4])
>>> np.add.reduce(a)
10
```

STRING LIST EXAMPLE

```
>>> a = np.array(
     ['ab','cd','ef'],
     dtype='object')
>>> np.add.reduce(a)
'abcdef'
```

LOGICAL OP EXAMPLES

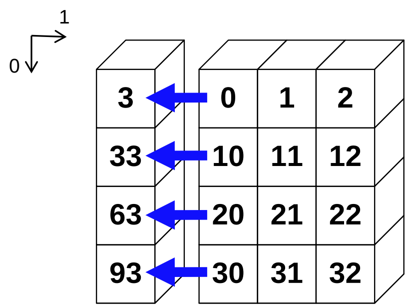
```
>>> a = np.array([1,1,0,1])
>>> np.logical_and.reduce(a)
False
>>> np.logical_or.reduce(a)
True
```

`op.reduce()`

For multidimensional arrays, **`op.reduce(a,axis)`** applies **`op`** to the elements of **`a`** along the specified **`axis`**. The resulting array has dimensionality one less than `a`. The default value for **`axis`** is 0.

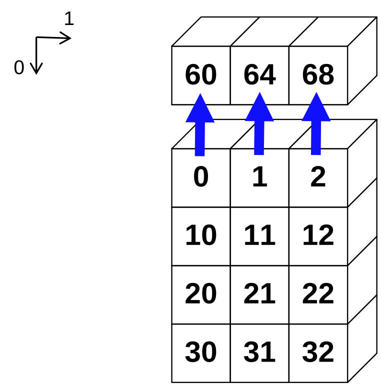
SUMMING UP EACH ROW

```
>>> a = np.arange(3) + np.arange(0, 40,
...                  10).reshape(-1, 1)
>>> np.add.reduce(a,1)
array([ 3, 33, 63, 93])
```



SUM COLUMNS BY DEFAULT

```
>>> np.add.reduce(a)
array([60, 64, 68])
```



`op.accumulate()`

`op.accumulate(a)` creates a new array containing the intermediate results of the **`reduce`** operation at each element in **`a`**.

For example:

`y=add.accumulate(a)`

$$= \left[\sum_{n=0}^{0} a[n], \sum_{n=0}^{1} a[n], \ldots, \sum_{n=0}^{N-1} a[n] \right]$$

ADD EXAMPLE

```
>>> a = np.array([1,2,3,4])
>>> np.add.accumulate(a)
array([ 1,  3,  6, 10])
```

STRING LIST EXAMPLE

```
>>> a = np.array(
     ['ab','cd','ef'],
     dtype='object')
>>> np.add.accumulate(a)
array([ab,abcd,abcdef],
       dtype=object)
```

LOGICAL OP EXAMPLES

```
>>> a = np.array([1,1,0])
>>> np.logical_and.accumulate(a)
array([True, True, False])
>>> np.logical_or.accumulate(a)
array([True, True, True])
```

op.reduceat()

op.reduceat(a,indices) applies `op` to ranges in the 1-D array `a` defined by the values in `indices`. The resulting array has the same length as `indices`.

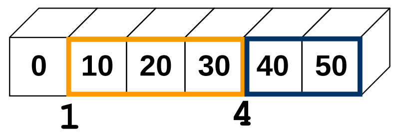
For example:

`y = add.reduceat(a, indices)`

$$\mathtt{y[i]} = \sum_{n=indices[i]}^{indices[i+1]} a[n]$$

EXAMPLE

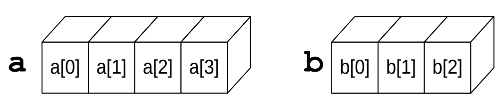
```
>>> a = np.array(
     [0,10,20,30,40,50])
>>> indices = np.array([1,4])
>>> np.add.reduceat(a,indices)
array([60, 90])
```



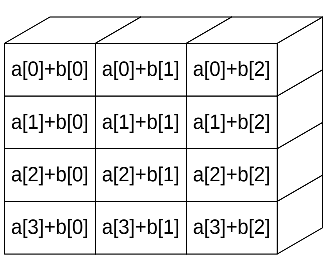
For multidimensional arrays, `reduceat()` is always applied along the *last* axis (sum of rows for 2-D arrays). This is different from the default for `reduce()` and `accumulate()`.

op.outer()

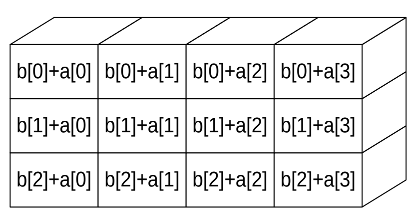
op.outer(a,b) forms all possible combinations of elements between `a` and `b` using `op`. The shape of the resulting array results from concatenating the shapes of `a` and `b`. (Order matters.)

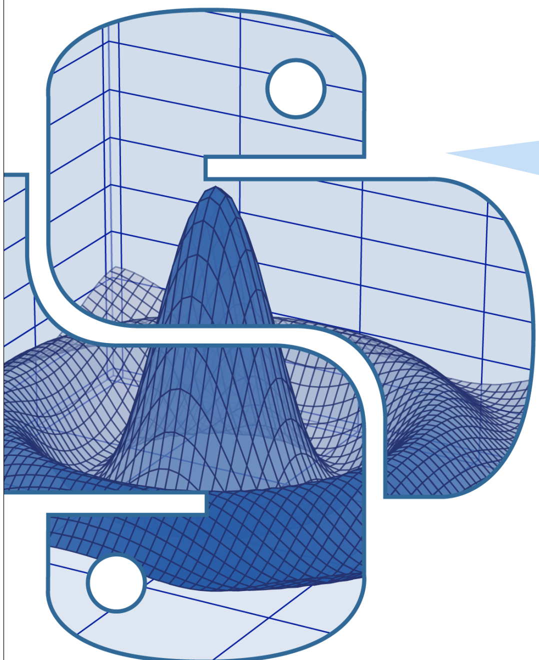


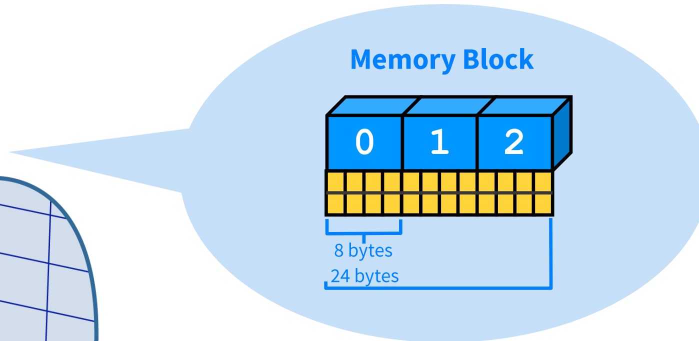

```
>>> np.add.outer(a,b)
```




```
>>> np.add.outer(b,a)
```



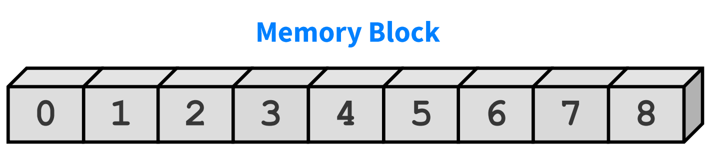




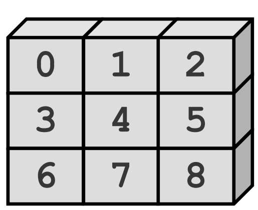
NumPy

The Array Data Structure

Array Data Structure

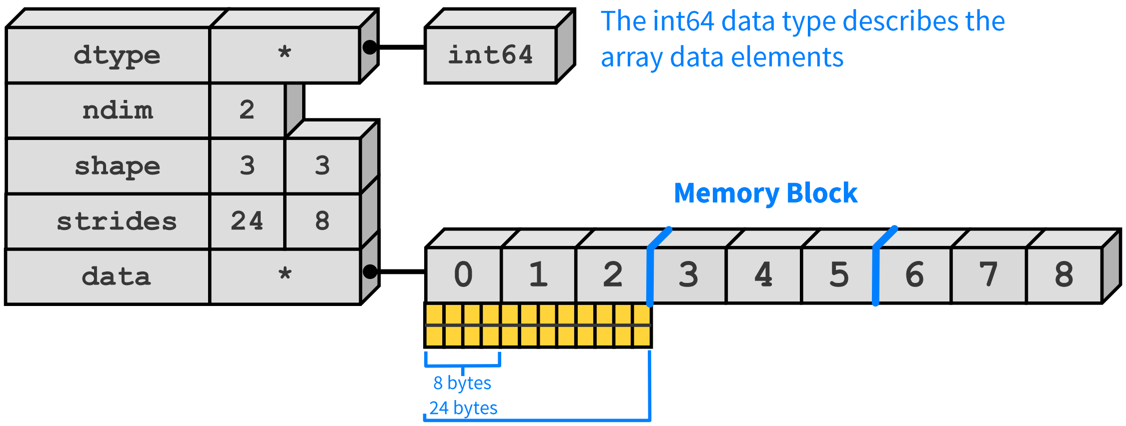


Python View:



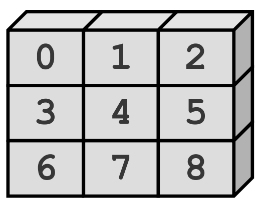
Array Data Structure

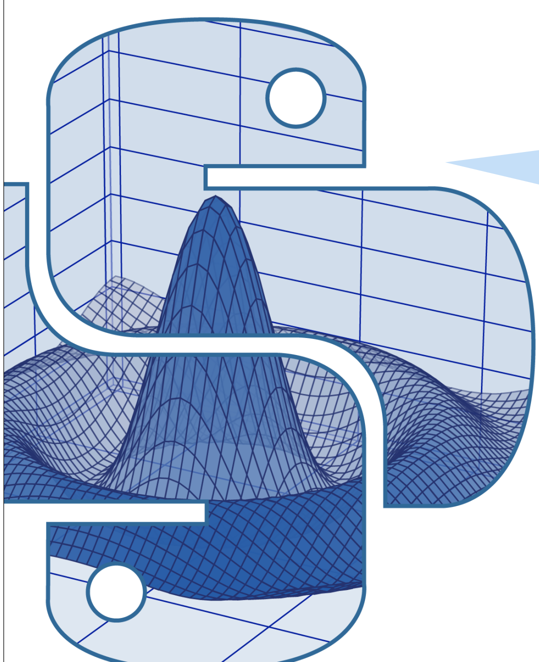
NDArray Data Structure

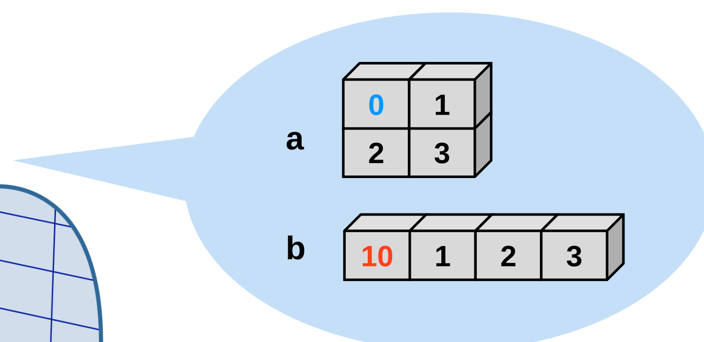


The int64 data type describes the array data elements

Python View:







NumPy

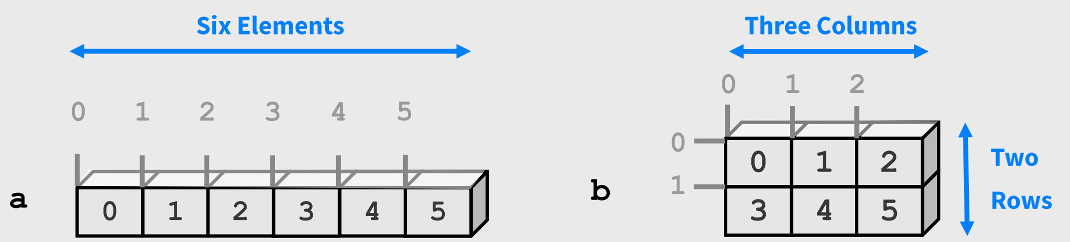
Structure Operations

Operations on the Array Structure

Operations that only affect the array structure, not the data, can usually be executed without copying memory.

```
>>> a = np.arange(6)
>>> a
```

```
>>> b = a.reshape(2, 3)
>>> b
```



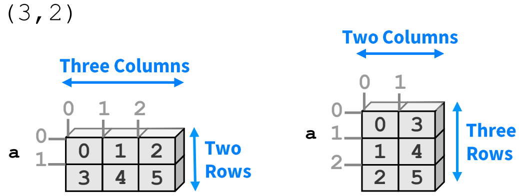
This is not a new copy of the data.

The original data does not get reordered.

Transpose

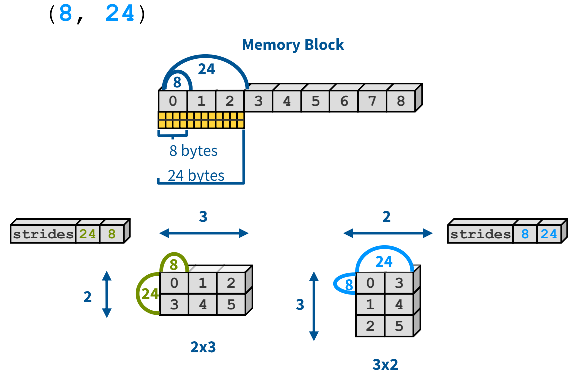
TRANSPOSE

```
>>> a = np.array([[0,1,2],
...               [3,4,5]])
>>> a.shape
(2,3)

# Transpose swaps the order
# of axes.

>>> a.T
array([[0, 3],
       [1, 4],
       [2, 5]])
>>> a.T.shape
(3,2)
```



TRANSPOSE RETURNS VIEWS

```
# Transpose does not move
# values around in memory.
# It only changes the order
# of "strides" in the array
>>> a.strides
(24, 8)
>>> a.T.strides
(8, 24)
```



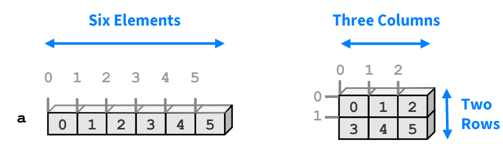
Reshaping Arrays

RESHAPE

```
>>> a = np.array([[0,1,2],
...               [3,4,5]])

# Return a new array with a
# different shape (a view
# where possible)
>>> a.reshape(3,2)
array([[0, 1],
       [2, 3],
       [4, 5]])

# Reshape cannot change the
# number of elements in an
# array
>>> a.reshape(4,2)
ValueError: total size of new
array must be unchanged
```

SHAPE

```
>>> a = np.arange(6)
>>> a
array([0, 1, 2, 3, 4, 5])
>>> a.shape
(6,)

# Reshape array in-place to
# 2x3

>>> a.shape = (2,3)
>>> a
array([[0, 1, 2],
       [3, 4, 5]])
```

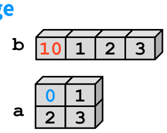


Flattening Arrays

FLATTEN (SAFE)

`a.flatten()` converts a multi-dimensional array into a 1-D array. The new array is a *copy* of the original data.

```
# Create a 2D array
>>> a = np.array([[0,1],
...               [2,3]])

# Flatten out elements to 1D
>>> b = a.flatten()
>>> b
array([0,1,2,3])

# Changing b does not change a
>>> b[0] = 10
>>> b
array([10,1,2,3])
>>> a                    no change
array([[0, 1],
       [2, 3]])
```



RAVEL (EFFICIENT)

`a.ravel()` is the same as `a.flatten()`, but returns a *reference (or view)* of the array if possible (i.e., the memory is contiguous). Otherwise the new array copies the data.

```
# Flatten out elements to 1-D
>>> b = a.ravel()
>>> b
array([0,1,2,3])

# Changing b does change a
>>> b[0] = 10
>>> b
array([10,1,2,3])
>>> a                    changed!
array([[10, 1],
       [ 2, 3]])
```