On the relationship between the Collatz conjecture and Mersenne prime numbers

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Abstract

The purpose of this study is to show how to get a necessary criterion for prime numbers with the help of special matrices. My special interest lies in the empirical research of these matrices and their patterns, structures and symmetries. The matrices in turn depend on an expansion of the Collatz algorithm 3n+1.

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Contents

1 Introduction

The basis of this study is an expansion of the Collatz conjecture. "The Collatz conjecture can be summarized as follows. Take any positive integer n. If n is even, divide it by 2 to get n / 2. If n is odd, multiply it by 3 and add 1 to obtain 3n + 1. Repeat the process indefinitely. The conjecture is that no matter what number you start with, you will always eventually reach 1." [**1**]

For each expansion it is possible to assign a specific structure. The structures in turn can be identified by specific matrices, and these matrices have a relation to prime numbers (Collatz matrix conjecture). Furthermore, it can be shown that there is a special relation between matrices and Mersenne prime numbers (Kaiser's conjecture).

First of all in Chapter 2, i will expand the Collatzalgorithm and define a general form of a matrix based on these Collatzalgorithms. This so-called Collatz tree matrix can be identified by other special matrices (The Collatz matrices), i will define in Chapter 3. In Chapter 4 it can be shown how the Collatz matrices deliver a necessary criterion for prime numbers (Collatz matrix conjecture), and Chapter 5 contains a conjecture about Collatz matrices and Mersenne prime numbers (Kaiser's conjecture).

The definitions of Chapter 2 (2.1, 2.2, 2.3 and 2.4) are already well known to the "Collatz community". For further Informaton see: [**2**, **3**]. All other definitions and conjectures were found by my own, if not explicitly advertised.

2 The Collatz tree matrix

2.1 Expansion of the Collatz algorithm

First i will expand the Collatzalgorithm.

$$f(n) = \begin{cases} n/2, \ if \ n \ is \ even \\ 3n+1, \ if \ n \ is \ odd \end{cases} \tag{1}$$

with respect to a

$$f_a(n) = \begin{cases} n/2, \ if \ n \ is \ even \\ an+1, \ if \ n \ is \ odd \end{cases} \tag{2}$$

$$D = \{a | a = 2n+1, \ n \in \mathbb{N}_0\}$$

2.2 The Collatz tree matrix

Definition 2.1 The Collatz tree matrix
Each Collatzalgorithm $f_a(n)$ can be shown as a Collatz tree matrix. The Collatz tree matrix consists of three different types of columns. The first column is called the knot column. In this column all results of the specific odd operation are listed. The second column is called the odd column. In this column all natural odd numbers are listed. The third and each additional column is called the even column because the numbers of these columns are results of the even operation. The Collatz tree matrices only differ from their first column.

The general form of the Collatz tree matrix of $f_a(n)$ looks like that:

Collatz tree matrix of algorithm $f_a(n)$:

$$\begin{pmatrix} b_1a+1 & b_1 & 2b_1 & 2^2b_1 & 2^3b_1 & 2^4b_1 & \dots & 2^nb_1 \\ b_2a+1 & b_2 & 2b_2 & 2^2b_2 & 2^3b_2 & 2^4b_2 & \dots & 2^nb_2 \\ b_3a+1 & b_3 & 2b_3 & 2^2b_3 & 2^3b_3 & 2^4b_3 & \dots & 2^nb_3 \\ b_4a+1 & b_4 & 2b_3 & 2^2b_4 & 2^3b_4 & 2^4b_4 & \dots & 2^nb_4 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ b_na+1 & b_n & 2b_n & 2^2b_n & 2^3b_n & 2^4b_n & \dots & 2^nb_n \end{pmatrix}$$

$$D = \{b | b = 2n+1, n \in \mathbb{N}_0\}$$

Let us have a look at the first three Collatz tree matrices:

Collatz tree matrix of $f_3(n)$:

$$\begin{pmatrix} 4 & 1 & 2 & 4 & 8 & \dots & \infty \\ 10 & 3 & 6 & 12 & 24 & \dots & \infty \\ 16 & 5 & 10 & 20 & 40 & \dots & \infty \\ 22 & 7 & 14 & 28 & 56 & \dots & \infty \\ 28 & 9 & 18 & 36 & 72 & \dots & \infty \\ 34 & 11 & 22 & 44 & 88 & \dots & \infty \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \\ \infty & \infty & \infty & \infty & \infty & & \infty \end{pmatrix}$$

Collatz tree matrix of $f_5(n)$:

$$\begin{pmatrix} 6 & 1 & 2 & 4 & 8 & \dots & \infty \\ 16 & 3 & 6 & 12 & 24 & \dots & \infty \\ 26 & 5 & 10 & 20 & 40 & \dots & \infty \\ 36 & 7 & 14 & 28 & 56 & \dots & \infty \\ 46 & 9 & 18 & 36 & 72 & \dots & \infty \\ 56 & 11 & 22 & 44 & 88 & \dots & \infty \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \\ \infty & \infty & \infty & \infty & \infty & & \infty \end{pmatrix}$$

Collatz tree matrix of $f_7(n)$:

$$\begin{pmatrix} 8 & 1 & 2 & 4 & 8 & \dots & \infty \\ 22 & 3 & 6 & 12 & 24 & \dots & \infty \\ 36 & 5 & 10 & 20 & 40 & \dots & \infty \\ 50 & 7 & 14 & 28 & 56 & \dots & \infty \\ 64 & 9 & 18 & 36 & 72 & \dots & \infty \\ 78 & 11 & 22 & 44 & 88 & \dots & \infty \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \\ \infty & \infty & \infty & \infty & \infty & & \infty \end{pmatrix}$$

2.3 Knot numbers

Definition 2.2 Knot numbers
Each Collatz tree matrix has knot numbers. They are even and result from the specific odd operation. Each knot number appears two times. One time they are listed in the first knot column of the Collatz tree matrix and the other time they appear right of the odd column. If a knot number appears two times in the same row, then there is a cycle. [**2**, **3**]

As can be seen each Collatz tree matrix has his own specific knot number pattern (red numbers):

Collatz tree matrix of $f_3(n)$:

$$\begin{pmatrix} \color{red}{4} & 1 & 2 & \color{red}{4} & 8 & \dots & \infty \\ \color{red}{10} & 3 & 6 & 12 & 24 & \dots & \infty \\ \color{red}{16} & 5 & \color{red}{10} & 20 & \color{red}{40} & \dots & \infty \\ \color{red}{22} & 7 & 14 & \color{red}{28} & 56 & \dots & \infty \\ \color{red}{28} & 9 & 18 & 36 & 72 & \dots & \infty \\ \color{red}{34} & 11 & \color{red}{22} & 44 & \color{red}{88} & \dots & \infty \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \\ \infty & \infty & \infty & \infty & \infty & & \infty \end{pmatrix}$$

Collatz tree matrix of $f_5(n)$:

$$\begin{pmatrix} \color{red}{6} & 1 & 2 & 4 & 8 & \color{red}{16} & \dots & \infty \\ \color{red}{16} & 3 & \color{red}{6} & 12 & 24 & 48 & \dots & \infty \\ \color{red}{26} & 5 & 10 & 20 & 40 & 80 & \dots & \infty \\ \color{red}{36} & 7 & 14 & 28 & \color{red}{56} & 112 & \dots & \infty \\ \color{red}{46} & 9 & 18 & \color{red}{36} & 72 & 144 & \dots & \infty \\ \color{red}{56} & 11 & 22 & 44 & 88 & \color{red}{176} & \dots & \infty \\ \color{red}{66} & 13 & \color{red}{26} & 52 & 104 & 208 & \dots & \infty \\ \color{red}{76} & 15 & 30 & 60 & 120 & 240 & \dots & \infty \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \\ \infty & \infty & \infty & \infty & \infty & \infty & & \infty \end{pmatrix}$$

Collatz tree matrix of $f_7(n)$:

$$\begin{pmatrix} 8 & 1 & 2 & 4 & 8 & \dots & \infty \\ 22 & 3 & 6 & 12 & 24 & \dots & \infty \\ 36 & 5 & 10 & 20 & 40 & \dots & \infty \\ 50 & 7 & 14 & 28 & 56 & \dots & \infty \\ 64 & 9 & 18 & 36 & 72 & \dots & \infty \\ 78 & 11 & 22 & 44 & 88 & \dots & \infty \\ 91 & 13 & 26 & 52 & 104 & \dots & \infty \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \\ \infty & \infty & \infty & \infty & \infty & & \infty \end{pmatrix}$$

2.4 Unbranched rows

Definition 2.3 Unbranched rows
Each Collatz tree matrix has unbranched rows. Such a row has only one knot number. [**2**, **3**]

They look like that (blue rows):

Collatz tree matrix of $f_3(n)$:

$$\begin{pmatrix} 4 & 1 & 2 & 4 & 8 & \dots & \infty \\ 10 & 3 & 6 & 12 & 24 & \dots & \infty \\ 16 & 5 & 10 & 20 & 40 & \dots & \infty \\ 22 & 7 & 14 & 28 & 56 & \dots & \infty \\ 28 & 9 & 18 & 36 & 72 & \dots & \infty \\ 34 & 11 & 22 & 44 & 88 & \dots & \infty \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \\ \infty & \infty & \infty & \infty & \infty & & \infty \end{pmatrix}$$

Collatz tree matrix of $f_5(n)$:

$$\begin{pmatrix} 6 & 1 & 2 & 4 & 8 & 16 & \dots & \infty \\ 16 & 3 & 6 & 12 & 24 & 48 & \dots & \infty \\ 26 & 5 & 10 & 20 & 40 & 80 & \dots & \infty \\ 36 & 7 & 14 & 28 & 56 & 112 & \dots & \infty \\ 46 & 9 & 18 & 36 & 72 & 144 & \dots & \infty \\ 56 & 11 & 22 & 44 & 88 & 176 & \dots & \infty \\ 66 & 13 & 26 & 52 & 104 & 208 & \dots & \infty \\ 76 & 15 & 30 & 60 & 120 & 240 & \dots & \infty \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \\ \infty & \infty & \infty & \infty & \infty & \infty & & \infty \end{pmatrix}$$

Collatz tree matrix of $f_7(n)$:

$$\begin{pmatrix} 8 & 1 & 2 & 4 & 8 & \dots & \infty \\ 22 & 3 & 6 & 12 & 24 & \dots & \infty \\ 36 & 5 & 10 & 20 & 40 & \dots & \infty \\ 50 & 7 & 14 & 28 & 56 & \dots & \infty \\ 64 & 9 & 18 & 36 & 72 & \dots & \infty \\ 78 & 11 & 22 & 44 & 88 & \dots & \infty \\ 91 & 13 & 26 & 52 & 104 & \dots & \infty \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \\ \infty & \infty & \infty & \infty & \infty & & \infty \end{pmatrix}$$

2.5 Perfect knot numbers

Definition 2.4 Perfect knot numbers
Perfect knot numbers are rooted in unbranched rows. [**2**, **3**]

Let us have a look at the perfect knot numbers (frameboxed numbers) of the Collatz tree matrix of $f_3(n)$:

Collatz tree matrix of $f_3(n)$

$$
\begin{pmatrix}
4 & 1 & 2 & 4 & 8 & 16 & 32 & \boxed{64} & \dots & \infty \\
\boxed{10} & 3 & 6 & 12 & 24 & 48 & 96 & 192 & \dots & \infty \\
16 & 5 & \boxed{10} & 20 & 40 & 80 & 160 & 320 & \dots & \infty \\
22 & 7 & 14 & \boxed{28} & 56 & 112 & 224 & 448 & \dots & \infty \\
\boxed{28} & 9 & 18 & 36 & 72 & 144 & 288 & 576 & \dots & \infty \\
34 & 11 & 22 & 44 & 88 & 176 & \boxed{352} & 704 & \dots & \infty \\
40 & 13 & 26 & 52 & 104 & \boxed{208} & 416 & 832 & \dots & \infty \\
\boxed{46} & 15 & 30 & 60 & 120 & 240 & 480 & 960 & \dots & \infty \\
52 & 17 & 34 & 68 & \boxed{136} & 272 & 544 & 1088 & \dots & \infty \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & & \infty
\end{pmatrix}
$$

3 The Collatz matrix

3.1 The three Collatz matrices

With these knot numbers, unbranched rows and perfect knot numbers each Collatz tree matrix can be identified by three Collatz matrices i will define in the next sections. They are called *Standard Collatz matrix*, *Little Collatz matrix* and *Big Collatz matrix*.

3.2 The standard Collatz matrix

Definition 3.1 The standard Collatz matrix
Each Collatz tree matrix can be identified by a specific standard Collatzmatrix. The standard Collatz matrix is the area (of the Collatz tree matrix), whose structure (pattern of knot numbers and unbranched rows) repeats. a_{11} of the standard Collatz matrix is always $2b_1$ of the general Collatz tree matrix. The number of rows of a standard Collatz matrix (of the Collatz tree matrix) of algorithm $f_a(n)$ is always a. The number of columns depends on the first knot number in the first row. In other words: a_{1n} of the standard Collatz matrix is always the first knot number of that row.

The number of rows m_C is in the following relationship to the number of columns n_C:

$$(2^{n_C} - 1) \; mod \; m_C = 0$$

$$m_C > n_C$$

The first three standard Collatz matrices are:

Standard Collatz matrix 3 x 2:

$$\begin{pmatrix} 2 & {\color{red}4} \\ {\color{blue}6} & {\color{blue}12} \\ {\color{red}10} & 20 \end{pmatrix}$$

Standard Collatz matrix 5 x 4:

$$\begin{pmatrix} 2 & 4 & 8 & {\color{red}16} \\ {\color{red}6} & 12 & 24 & 48 \\ {\color{blue}10} & {\color{blue}20} & {\color{blue}40} & {\color{blue}80} \\ 14 & 28 & {\color{red}56} & 112 \\ 18 & {\color{red}36} & 72 & 144 \end{pmatrix}$$

Standard Collatz matrix 7 x 3:

$$\begin{pmatrix} 2 & 4 & {\color{red}8} \\ {\color{blue}6} & {\color{blue}12} & {\color{blue}24} \\ {\color{blue}10} & {\color{blue}20} & {\color{blue}40} \\ {\color{blue}14} & {\color{blue}28} & {\color{blue}56} \\ 18 & {\color{red}36} & 72 \\ {\color{red}22} & 44 & 88 \\ {\color{blue}26} & {\color{blue}52} & {\color{blue}104} \end{pmatrix}$$

To get a better view on the standard Collatz matrix let us have a look in context of the Collatz tree matrix (Figure 1):

Standard Collatz matrix 3 x 2 (green box) as part of the

Collatz tree matrix of $f_3(n)$

$$
\begin{pmatrix}
4 & 1 & 2 & 4 & 8 & 16 & 32 & 64 & . & \infty \\
10 & 3 & 6 & 12 & 24 & 48 & 96 & 192 & . & \infty \\
16 & 5 & 10 & 20 & 40 & 80 & 160 & 320 & . & \infty \\
22 & 7 & 14 & 28 & 56 & 112 & 224 & 448 & . & \infty \\
28 & 9 & 18 & 36 & 72 & 144 & 288 & 576 & . & \infty \\
34 & 11 & 22 & 44 & 88 & 176 & 352 & 704 & . & \infty \\
40 & 13 & 26 & 52 & 104 & 208 & 416 & 832 & . & \infty \\
46 & 15 & 30 & 60 & 120 & 240 & 480 & 960 & . & \infty \\
52 & 17 & 34 & 68 & 136 & 272 & 544 & 1088 & . & \infty \\
. & . & . & . & . & . & . & . & . & . \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & . & .
\end{pmatrix}
$$

Figure 1: Collatz tree matrix of $f_3(n)$

As can be seen the structure (position of knot numbers and unbranched rows) of the standard Collatz matrix (green box) repeats downwards and to the right.

Important Notice: When i write of the Collatz matrix in the next sections i always mean the standard Collatz matrix.

3.3 Little Collatz matrix

Definition 3.2 The little Collatz matrix
The little Collatz matrix is part of the Collatz matrix. It has the same value for a_{11} like the Collatz matrix but it ends with the first (lowest) knot number.

For example the Collatz tree matrix of $f_3(n)$ has the little Collatz matrix:

little collatz matrix 1 x 2:

$$\begin{pmatrix} 2 & 4 \end{pmatrix}$$

The Collatz tree matrix of $f_5(n)$ has the little Collatz matrix:

Little collatz matrix 2 x 1:

$$\begin{pmatrix} 2 \\ 6 \end{pmatrix}$$

The Collatz tree matrix of $f_{23}(n)$ has the little Collatz matrix:

Little Collatz matrix 2 x 3:

$$\begin{pmatrix} 2 & 4 & 8 \\ 6 & 12 & 24 \end{pmatrix}$$

The little Collatz matrix is related to Collatz matrix like this :

$$\frac{1 + m_C + 2^{n_L}}{2^{n_L+1}} = m_L$$

$m_C = m$-value of the standard Collatz matrix

$m_L = m$-value of the little Collatz matrix

$n_L = n$-value of the little Collat zmatrix

3.4 Big Collatz matrix

Definition 3.3 The big Collatz matrix
Each Collatz tree matrix can be identified by a specific big Collatz matrix. The big Collatz matrix is the area (of the Collatz tree matrix), whose structure (pattern of perfect knot numbers) repeats. a_{11} of the big Collatz matrix is always $2b_1$ of the general Collatz tree matrix. The number of rows of a big Collatz matrix (of the Collatz tree matrix) of algorithm $f_a(n)$ is always a^2.

The big Collatz matrix of $f_3(n)$ with the perfect knot numbers (frameboxed numbers):

Big Collatz matrix 9 x 6

$$\begin{pmatrix} 2 & 4 & 8 & 16 & 32 & \boxed{64} \\ 6 & 12 & 24 & 48 & 96 & 192 \\ \boxed{10} & 20 & 40 & 80 & 160 & 320 \\ 14 & \boxed{28} & 56 & 112 & 224 & 448 \\ 18 & 36 & 72 & 144 & 288 & 576 \\ 22 & 44 & 88 & 176 & \boxed{352} & 704 \\ 26 & 52 & 104 & \boxed{208} & 416 & 832 \\ 30 & 60 & 120 & 240 & 480 & 960 \\ 34 & 68 & \boxed{136} & 272 & 544 & 1088 \end{pmatrix}$$

The m- and n-values of the big Collatz matrix are given by that:

$$m_B \ x \ n_B = (m_C * m_C)x(m_C * n_C)$$

$m_B = m$-value of the big Collatz matrix

$n_B = n$-value of the big Collatz matrix

$m_C = m$-value of the Standard Collatz matrix

$n_C = n$-value of the Standard Collatz matrix

3.5 The three Collatz matrices as part of the Collatz tree matrix

Here again pictures of the Collatz tree matrices of $f_3(n)$ (Figure 2),$f_5(n)$ (Figure 3) and $f_7(n)$ (Figure 4) with their three matrices (blue: Little Collatz matrix, green: Standard Collatz matrix and red: Big Collatz matrix):

4	1	2	4	8	16	32	64	.	∞
10	3	6	12	24	48	96	192	.	∞
16	5	10	20	40	80	160	320	.	∞
22	7	14	28	56	112	224	448	.	∞
28	9	18	36	72	144	288	576	.	∞
34	11	22	44	88	176	352	704	.	∞
40	13	26	52	104	208	416	832	.	∞
46	15	30	60	120	240	480	960	.	∞
52	17	34	68	136	272	544	1088	.	∞
.	.	.	.	.	.	.	.	.	.
∞	∞	∞	∞	∞	∞	∞	∞	.	.

Figure 2: Collatz tree matrix of $f_3(n)$

blue: Little Collatz matrix 1 x 2
green: Standard Collatz matrix 3 x 2
red: Big Collatz matrix 9 x 6

$$\begin{pmatrix}
6 & 1 & 2 & 4 & 8 & 16 & 32 & 64 & . & 1048576 & . & \infty \\
16 & 3 & 6 & 12 & 24 & 48 & 96 & 192 & . & 3145728 & . & \infty \\
26 & 5 & 10 & 20 & 40 & 80 & 160 & 320 & . & 5242880 & . & \infty \\
36 & 7 & 14 & 28 & 56 & 112 & 224 & 448 & . & 7340032 & . & \infty \\
46 & 9 & 18 & 36 & 72 & 144 & 288 & 576 & . & 9437184 & . & \infty \\
56 & 11 & 22 & 44 & 88 & 176 & 352 & 704 & . & 11534336 & . & \infty \\
66 & 13 & 26 & 52 & 104 & 208 & 416 & 832 & . & 13631488 & . & \infty \\
76 & 15 & 30 & 60 & 120 & 240 & 480 & 960 & . & 15728640 & . & \infty \\
. & . & . & . & . & . & . & . & . & . & . & . \\
246 & 49 & 98 & 196 & 392 & 784 & 1568 & 3136 & . & 51380224 & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & . & \infty & . & .
\end{pmatrix}$$

Figure 3: Collatz tree matrix of $f_5(n)$

blue: Little Collatz matrix 2 x 1
green: Standard Collatz matrix 5 x 4
red: Big Collatz matrix 25 x 20

8	1	2	4	8	16	32	64	.	2097152	.	∞
22	3	6	12	24	48	96	192	.	6291456	.	∞
36	5	10	20	40	80	160	320	.	10485760	.	∞
50	7	14	28	56	112	224	448	.	14680064	.	∞
64	9	18	36	72	144	288	576	.	18874368	.	∞
78	11	22	44	88	176	352	704	.	23068672	.	∞
92	13	26	52	104	208	416	832	.	27262976	.	∞
106	15	30	60	120	240	480	960	.	31457280	.	∞
.	.	.	.	.	.	.	.	.	.	.	.
680	97	194	388	776	1552	3104	6208	.	203423744	.	.
.	.	.	.	.	.	.	.	.	.	.	.
∞	∞	∞	∞	∞	∞	∞	∞	.	∞	.	.

Figure 4: Collatz tree matrix of $f_7(n)$

blue: Little Collatz matrix 1 x 3
green: Standard Collatz matrix 7 x 3
red: Big Collatz matrix 49 x 21

3.6 Symmetries and structures

When comparing the standard Collatz matrices i have found the following properties:

Each column of a standard Collatz matrix has exactly one knot number.

The first row of each standard Collatz matrix is never unbranched.

The Value n is always smaller than m. There are no m x m matrices.

Each standard Collatz matrix has a kind of mirror axis (the middle row of the matrix which is always unbranched) to which I find the following 5 main symmetries:

SM: Singlematrix
UM: Uppermatrix
MM: Mirrormatrix
IMM: Inverted mirrormatrix
USM: Unsymmetrical matrix

The standard Collatzmatrix 11 x 10 as example for a singlematrix. Singlematrix means that there is only one unbranched row in the middle:

$$\begin{pmatrix} 2 & 4 & 8 & 16 & 32 & 64 & 128 & 256 & 512 & 1024 \\ 6 & 12 & 24 & 48 & 96 & 192 & 384 & 768 & 1536 & 3072 \\ 10 & 20 & 40 & 80 & 160 & 320 & 640 & 1280 & 2560 & 5120 \\ 14 & 28 & 56 & 112 & 224 & 448 & 896 & 1792 & 3584 & 7168 \\ 18 & 36 & 72 & 144 & 288 & 576 & 1152 & 2304 & 4608 & 9216 \\ 22 & 44 & 88 & 176 & 352 & 704 & 1408 & 2816 & 5632 & 11264 \\ 26 & 52 & 104 & 208 & 416 & 832 & 1664 & 3328 & 6656 & 13312 \\ 30 & 60 & 120 & 240 & 480 & 960 & 1920 & 3840 & 7680 & 15360 \\ 34 & 68 & 136 & 272 & 544 & 1088 & 2176 & 4352 & 8704 & 17408 \\ 38 & 76 & 152 & 304 & 608 & 1216 & 2432 & 4864 & 9728 & 19456 \\ 42 & 84 & 168 & 336 & 672 & 1344 & 2688 & 5376 & 10752 & 21504 \end{pmatrix}$$

The standard Collatzmatrix 31 x 5 as Example for an uppermatrix. Uppermatrix means that every row above the middle axis is unbranched with the exception of the first row:

2	4	8	16	32
6	12	24	48	96
10	20	40	80	160
14	28	56	112	224
18	36	72	144	288
22	44	88	176	352
26	52	104	208	416
30	60	120	240	480
34	68	136	272	544
38	76	152	304	608
42	84	168	336	672
46	92	184	368	736
50	100	200	400	800
54	108	216	432	864
58	116	232	464	928
62	124	248	496	992
66	132	264	528	1056
70	140	280	560	1120
74	145	296	592	1184
78	156	312	624	1248
82	164	328	656	1312
86	172	344	688	1376
90	180	360	720	1440
94	188	376	752	1504
98	196	392	784	1568
102	204	408	816	1632
106	212	424	848	1696
110	220	440	880	1760
114	228	456	912	1824
118	236	472	944	1888
122	244	488	976	1952

The standard Collatzmatrix 17 x 8 as a mirrormatrix. It means that the property (branched or unbranched) of each row is mirrored to the other side of the axis. For example: If the second row below the axis is unbranched then the second row above the axis is also unbranched:

2	4	8	16	32	64	128	256
6	12	24	48	96	192	384	768
10	20	40	80	160	320	640	1280
14	28	56	112	224	448	896	1792
18	36	72	144	288	576	1152	2304
22	44	88	176	352	704	1408	2816
26	52	104	208	416	832	1664	3328
30	60	120	240	480	960	1920	3840
34	68	136	272	544	1088	2176	4352
38	76	152	304	608	1216	2432	4864
42	84	168	336	672	1344	2688	5376
46	92	184	368	736	1472	2944	5888
50	100	200	400	800	1600	3200	6400
54	108	216	432	864	1728	3456	6912
58	116	232	464	928	1856	3712	7424
62	124	248	496	992	1984	3968	7936
66	132	264	528	1056	2112	4224	8448

The standard Collatzmatrix 23 x 11 as an inverted mirrormatrix. The contrary of a mirrormatrix is the inverted mirrormatrix. If the second row below the axis is unbranched then the second row above the matrix is not unbranched:

2	4	8	16	32	64	128	256	512	1024	2048
6	12	24	48	96	192	384	768	1536	3072	6144
10	20	40	80	160	320	640	1280	2560	5120	10240
14	28	56	112	224	448	896	1792	3584	7168	14336
18	36	72	144	288	576	1152	2304	4608	9216	18432
22	44	88	176	352	704	1408	2816	5632	11264	22528
26	52	104	208	416	832	1664	3328	6656	13312	26624
30	60	120	240	480	960	1920	3840	7680	15360	30720
34	68	136	272	544	1088	2176	4352	8704	17408	34816
38	76	152	304	608	1216	2432	4864	9728	19456	38912
42	84	168	336	672	1344	2688	5376	10752	21504	43008
46	92	184	368	736	1472	2944	5888	11776	23552	47104
50	100	200	400	800	1600	3200	6400	12800	25600	51200
54	108	216	432	864	1728	3456	6912	13824	27648	55296
58	116	232	464	928	1856	3712	7424	14848	29696	59392
62	124	248	496	992	1984	3968	7936	15872	31744	63488
66	132	264	528	1056	2112	4224	8448	16896	33792	76584
70	140	280	560	1120	2240	4480	8960	17920	35840	71680
74	145	296	592	1184	2368	4736	9472	18944	37888	75776
78	156	312	624	1248	2496	4992	9984	19968	39936	79872
82	164	328	656	1312	2624	5248	10496	20992	41984	83968
86	172	344	688	1376	2752	5504	11008	22016	44032	88064
90	180	360	720	1440	2880	5760	11520	23040	46080	92160

And finally the standard Collatzmatrix 21 x 6 as an seemingly unsymmetrical matrix:

$$\begin{pmatrix}
2 & 4 & 8 & 16 & 32 & 64 \\
6 & 12 & 24 & 48 & 96 & 192 \\
10 & 20 & 40 & 80 & 160 & 320 \\
14 & 28 & 56 & 112 & 224 & 448 \\
18 & 36 & 72 & 144 & 288 & 576 \\
22 & 44 & 88 & 176 & 352 & 704 \\
26 & 52 & 104 & 208 & 416 & 832 \\
30 & 60 & 120 & 240 & 480 & 960 \\
34 & 68 & 136 & 272 & 544 & 1088 \\
38 & 76 & 152 & 304 & 608 & 1216 \\
42 & 84 & 168 & 336 & 672 & 1344 \\
46 & 92 & 184 & 368 & 736 & 1472 \\
50 & 100 & 200 & 400 & 800 & 1600 \\
54 & 108 & 216 & 432 & 864 & 1728 \\
58 & 116 & 232 & 464 & 928 & 1856 \\
62 & 124 & 248 & 496 & 992 & 1984 \\
66 & 132 & 264 & 528 & 1056 & 2112 \\
70 & 140 & 280 & 560 & 1120 & 2240 \\
74 & 148 & 296 & 592 & 1184 & 2386 \\
78 & 156 & 312 & 624 & 1248 & 2496 \\
82 & 164 & 328 & 656 & 1312 & 2624
\end{pmatrix}$$

Table 1 is a summary of the first symmetries and matrices. It can be seen that there are integer sequences (A003602, A001511, A005408, A002326). [**4**]

Table 1: Summary Symmetries

Algo.	**Standard Collatz ma.**				**Little Collatz ma.**				**Big Collatz ma.**			**Sym.**
	A5408		A2326		A3602		A1511		A16754			
$f_a(n)$	m_C	x	n_C		m_L	x	n_L		m_B	x	n_B	
$f_1(n)$	1	x	1		1	x	1		1	x	1	UM
$f_3(n)$	3	x	2		1	x	2		9	x	6	UM
$f_5(n)$	5	x	4		2	x	1		25	x	20	SM
$f_7(n)$	7	x	3		1	x	3		49	x	21	UM
$f_9(n)$	9	x	6		3	x	1		81	x	54	MM
$f_{11}(n)$	11	x	10		2	x	2		121	x	110	SM
$f_{13}(n)$	13	x	12		4	x	1		169	x	156	SM
$f_{15}(n)$	15	x	4		1	x	4		225	x	60	UM
$f_{17}(n)$	17	x	8		5	x	1		289	x	136	MM
$f_{19}(n)$	19	x	18		3	x	2		361	x	342	SM
$f_{21}(n)$	21	x	6		6	x	1		441	x	126	USM
$f_{23}(n)$	23	x	11		2	x	3		529	x	253	IMM
$f_{25}(n)$	25	x	20		7	x	1		625	x	500	MM
$f_{27}(n)$	27	x	18		4	x	2		729	x	486	MM
$f_{29}(n)$	29	x	28		8	x	1		841	x	812	SM
$f_{31}(n)$	31	x	5		1	x	5		961	x	155	UM
$f_{33}(n)$	33	x	10		9	x	1		1089	x	330	MM
$f_{35}(n)$	35	x	12		5	x	2		1225	x	420	USM
$f_{37}(n)$	37	x	36		10	x	1		1369	x	1332	SM
$f_{39}(n)$	39	x	12		3	x	3		1521	x	468	USM
$f_{41}(n)$	41	x	20		11	x	1		1681	x	820	MM
$f_{43}(n)$	43	x	14		6	x	2		1849	x	602	MM
$f_{45}(n)$	45	x	12		12	x	1		2025	x	540	USM
$f_{47}(n)$	47	x	23		2	x	4		2209	x	1081	IMM
$f_{49}(n)$	49	x	21		13	x	1		2401	x	1029	USM
$f_{51}(n)$	51	x	8		7	x	2		2601	x	408	USM
$f_{53}(n)$	53	x	52		14	x	1		2809	x	2756	SM
$f_{55}(n)$	55	x	20		4	x	3		3025	x	1100	USM
$f_{57}(n)$	57	x	18		15	x	1		3249	x	1026	MM
$f_{59}(n)$	59	x	58		8	x	2		3481	x	3422	SM
$f_{61}(n)$	61	x	60		16	x	1		3721	x	3660	SM

3.7 Symmetry and prime numbers

There are some interesting correlations between symmetry and prime numbers: [**4**]

Each singlematrix is seemingly a prime. (A001122)

Each inverted mirrormatrix is seemingly a prime. (A139035)

Each uppermatrix is seemingly a Mersenne number. (A000225).

4 The Collatz matrix conjecture

4.1 The Collatz matrix conjecture

The Collatz matrix conjecture establishes a connection between the standard Collatz matrix and primes like this:

Conjecture 1. The Collatz matrix conjecture

$$(m_C - 1)\ mod\ n_C = 0,\ m_C\ is\ a\ prime\ or\ a\ fermat\ pseudoprime\ base\ 2$$

$$(m_C - 1)\ mod\ n_C \neq 0,\ m_C\ is\ not\ a\ prime$$

m_C = m-value of the standard Collatz matrix

n_C = n-value of the standard Collatz matrix

4.2 The rank of primes

Definition 4.1 The rank of primes
With the Collatz matrix conjecture it is possible to classify primes by a rank:

$$p_r = \frac{m_C - 1}{n_C}$$

p_r = rank of a prime

m_C = m-value of the standard Collatz matrix

n_C = n-value of the standard Collatz matrix

As can be seen in Table 2 all primes are classified by rank in the right column. In Table 3 the frequency of this rank 1-18 is listed.

Table 2: Rank of a prime

	Collatz matrix			Rank
	A5408		A002326	
	m_C	x	n_C	p_r
	1	x	1	0
Prime	3	x	2	1
Prime	5	x	4	1
Prime	7	x	3	2
	9	x	6	1,333
Prime	11	x	10	1
Prime	13	x	12	1
	15	x	4	3,5
Prime	17	x	8	2
Prime	19	x	18	1
	21	x	6	3,33
Prime	23	x	11	2
	25	x	20	1,2
	27	x	18	1,444
Prime	29	x	28	1
Prime	31	x	5	6
	33	x	10	3,2
	35	x	12	2,83
Prime	37	x	36	1
	39	x	12	3,16
Prime	41	x	20	2
Prime	43	x	14	3
	45	x	12	3,66
Prime	47	x	23	2
	49	x	21	2,285
	51	x	8	6,25

Table 3: Primefrequency by rank (1-18) up to number 1,000,000

Rank	Frequency		Rank	Frequency
1	29341		10	1089
2	22092		11	278
3	5233		12	628
4	3655		13	195
5	1477		14	547
6	3931		15	248
7	694		16	686
8	2781		17	115
9	579		18	432

4.3 Rank and symmetry

With the rank of a prime it is possible to make propositions about the symmetry of the Collatz matrices:

Singlematrix:

Each Collatz matrix with the rank 1 are singlematrices.

Inverted mirrormatrix:

Each Collatz matrix with the rank 2 and an odd n-value of the Collatz-matrix are inverted mirrormatrices.

Uppermatrices:

Each Collatz matrix with 1 as an m-value of the little Collatzmatrix are uppermatrices.

Table 4: Classified Pseudoprimes

Collatz matrix				**Little Collatz matrix**			**Rank**
341	x	10		86	x	1	34
561	x	40		141	x	1	14
645	x	28		162	x	1	23
1105	x	24		277	x	1	46
1387	x	18		174	x	2	77
1729	x	36		433	x	1	48
1905	x	28		477	x	1	68
2047	x	11		1	x	11	186
2465	x	56		617	x	1	44
2701	x	36		676	x	1	75
2821	x	60		706	x	1	47
3277	x	28		820	x	1	117
4033	x	36		1009	x	1	112
4369	x	16		1093	x	1	273
4371	x	230		547	x	2	19
4681	x	15		1171	x	1	312
5461	x	14		1366	x	1	390

4.4 Fermat pseudoprimes (base 2) classified by the little Collatz matrix

As mentioned in 4.1, the Collatz matrix conjecture shows a necessary criterion for a prime. The numbers which have this criterion but are no primes are Fermat pseudoprimes to base 2. With the little Collatz matrix it is now possible to get a new classification for this pseudoprimes (Table 4). As can be seen there are many pseudos with 1, a few with 2 and so on...

5 Kaiser's conjecture

5.1 Mersenne primes

A Mersenne number is a number of the form $2^n - 1$. Mersenne primes are Mersenne numbers which are also primes. The exponents n of such a Mersenne prime are 2,3,5,7,13,17,19,31,61... (A000043) [**4**, **5**]

5.2 Kaisers conjecture

When comparing Collatz matrices with Mersenne prime exponents the following context can be seen:

Conjecture 2. Kaiser's conjecture
The exponent n of a Mersenne prime of the form $2^n - 1$ has to be singular. Singular means that it exists only one Collatz matrix with this exponent n as n-value n_C.

For example:

There is one Collatz matrix with the value $n_C = 3$. It is 7x3...

There is one Collatz matrix with the value $n_C = 5$. It is 31x5...

There is one Collatz matrix with the value $n_C = 7$. It is 127x7...

But there are 3 Collatzmatrices with the value $n_C = 11$! These Collatz matrices are 23x11,89x11 and 2047x11. So 11 is not a Mersenne prime exponent.

Table 5: Frequency n-value

n_C	Frequency
1	1
2	1
3	1
4	2
5	1
6	3
7	1
8	4
9	2
10	5
11	3
12	16
13	1
14	5
15	5
16	8
17	1
18	24
19	1

In Table 5 you can see the frequency of the n-values n_C 1-19 for Collatz matrices up to the Collatz matrix 1.999.999 x 6440.

6 Summary

I have come to the conclusion that there is a deep relationship between the Collatz matrices and primes . As can be seen there is a way to test Mersenne primes theoreticaly. Furthermore, there are many structures and symmetries to find in these Collatz matrices.

7 Acknowledgement

Thanks to the internet and to the books of Lothar Papula.

References

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