Rational Numbers

next generation textbooks

To access a customizable version of this book, as well as other interactive content, visit www.ck12.org

CK-12 Foundation is a non-profit organization with a mission to reduce the cost of textbook materials for the K-12 market both in the U.S. and worldwide. Using an open-content, web-based collaborative model termed the **FlexBook**®, CK-12 intends to pioneer the generation and distribution of high-quality educational content that will serve both as core text as well as provide an adaptive environment for learning, powered through the **FlexBook Platform**®.

Printed: July 9, 2013

flexbook
next generation textbooks

CONCEPT 1 Rational Numbers

Introduction

Comparing Distances

On the dive boat one morning, Cameron began talking with another boy named Chet. Chet and his family were from Colorado and Chet was just two years older than Cameron. The boys struck up a great conversation about diving and fish and the things that they had seen on their dives.

After a little while, they spotted some dolphins swimming with the boat. This is something that often happens as dolphins love the rushing water generated by the motor on the back of the boat.

"Did you know that they can swim .83 miles in one minute?" Chet asked Cameron.

"Really, no I didn't know that. I do know that a swordfish can swim almost one-half mile in a minute. I think the exact number is $\frac{9}{20}$ of a mile."

"Wow, which one can swim the farthest in one minute?" Chet asked thinking carefully through the math.

By the time they reached the dive site, Cameron had figured out which one can swim the farthest in one minute.

Have you? The numbers that the boys are using are called rational numbers. When you understand rational numbers, you will also understand how to figure out which one can swim the farthest in one minute. Pay attention, and this lesson on rational numbers will teach you all that you need to know.

What You Will Learn

In this lesson, you will learn how to do the following:

- Identify a rational number as the ratio of two integers.
- Compare and order rational numbers on a number line.
- Identify commutative, associative, inverse and identity properties of addition and multiplication for rational numbers.
- Apply properties and use order of operations to evaluate numerical and variable expressions.

Teaching Time

I. **Identify a Rational Number as the Ratio of Two Integers**

Some numbers are considered ***rational numbers***. **A rational number is a number that can be written as a ratio.**

What is a ratio?

A *ratio* is a comparison of two numbers. For example, you might discover that the ratio of boys to girls in your class is 12 to 13. That same ratio could be also be expressed using a colon, 12 : 13, or as a fraction, $\frac{12}{13}$.

In fact, any number that can be written as a ratio of two integers is classified as a rational number. Let's take a closer look at how to identify rational numbers.

How can we determine if an integer is a rational number?

That is a good question. Let's look at an example and see if we can write it as a ratio.

Example

10

This number can be written as a ratio. Every whole number can be written over 1. That means that it can be written in the form of a ratio. Notice that the fraction bar is a way to tell if the integer can be written as a ratio. In other words, if it can be written as a fraction, it is a rational number.

10 is a rational number.

Example

$-\frac{2}{3}$

This fraction is a rational number. Notice that it is written as a ratio already. We are comparing the numerator and the denominator. Yes, it is negative. That is okay, because we can have negative fractions and they are still considered rational numbers.

$-\frac{2}{3}$ **is a rational number.**

Example

.687

This decimal can be written as a rational number over 1000. This is a rational number too.

.687 is a rational number.

Are there any others?

Yes. ***Terminating decimals*** **and** ***repeating decimals*** **are also rational numbers.**

- **Terminating decimals**, which are decimals with a set number of digits, are always rational. For example, 0.007 is a terminating decimal, so it is rational.
- **Repeating decimals**, which are decimals in which one or more digits repeat, are always rational. For example, $0.\bar{3}$ is a repeating decimal in which the digit 3 repeats forever, so it is rational.

Are there any numbers that are not rational?

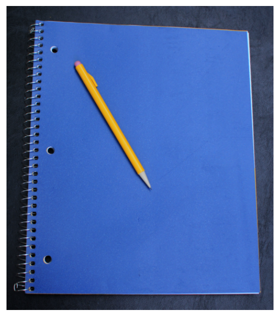
Yes. Some decimals don't terminate and they don't repeat. They just go on and on and on forever. These are a special group of numbers called *irrational numbers*. They are not rational numbers. You will learn more about them in a later lesson.

4N. Lesson Exercises

Determine whether each is a rational number.

1. **-4**
2. $\frac{1}{3}$
3. **.89765....**

Take a few minutes to check your work with a partner.



Write down the definition of a rational number and how you can tell if a number is rational or not. Be sure to include this information in your notebook and then continue.

II. **Compare and Order Rational Numbers on a Number Line**

Now that you know how to identify a rational number, you may need to compare or order them from time to time. For example, what if you have a loss of $\frac{1}{2}$ compared to a loss of .34. You would need to determine which loss is greater.

Placing the numbers on a number line can help you do this.

Let's review the inequality symbols which can help us compare and order rational numbers:

>means *is greater than*.

<means *is less than*.

= means *is equal to*.

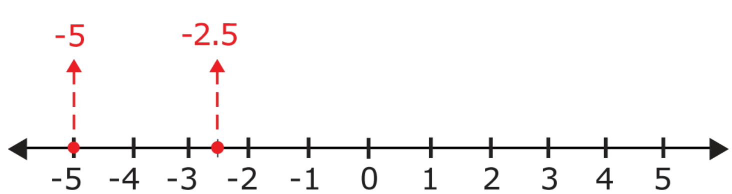
Example

Choose the inequality symbol that goes in the blank to make this statement true.

-2.5 ___ -5

First, Draw a number line from -5 to 5.

Place the numbers -2.5 and -5 on that number line. Since $0.5 = \frac{1}{2}$, -2.5 will be halfway between -2 and -3 on the number line.



Since -2.5 is further to the right on the number line than -5 is, -2.5 is greater than -5

The symbol >goes in the blank because $-2.5 > -5$.

Example

Order these rational numbers from least to greatest.

$$\frac{4}{5} \qquad 0.6 \qquad 1 \qquad 0.\bar{6}$$

It is often fairly easy to place decimals on a number line that is divided into tenths.

So, we can draw a number line from 0 to 1 and divide it into tenths. Then we can place all four numbers on the number line.

First, we should change $\frac{4}{5}$ to a fraction with a denominator of 10:

$\frac{4}{5} = \frac{4 \times 2}{5 \times 2} = \frac{8}{10}$

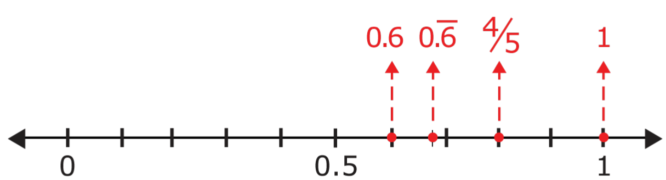
Since eight tenths is equivalent to $\frac{4}{5}$, we can find eight tenths on the number line and place $\frac{4}{5}$ there.

0.6 means six tenths. So, we can find six tenths on the number line and place 0.6 there.

1 is shown on the number line, so we can add a dashed line to show that number also.

$0.\bar{6}$ means 0.666... So, $0.\bar{6}$ is a little greater than six tenths, but less than seven tenths. We can place $0.\bar{6}$ roughly where it belongs on the number line.

The number line will look like this when we are finished.



From the number line, we can see that $0.6 < 0.\bar{6} < \frac{4}{5} < 1$.

So, ordered from least to greatest, the numbers are $0.6,\ 0.\bar{6},\ \frac{4}{5},\ 1$.

Yes. Thinking about the relationships between the numbers (in this case, how is each one bigger or smaller in relation to the other numbers) will help you. That is how you can be sure that the numbers are in the correct order. Remember, they are all rational numbers!

4O. Lesson Exercises

Compare the following rational numbers.

1. $-.7$ ____ $-\frac{7}{10}$
2. $.34$ ____ $\frac{1}{2}$
3. 67 ____ -10

Take a few minutes to check your answers with a partner.

III. **Identifying and Applying Properties with Rational Numbers**

Next, let's review some properties of numbers. You may recall these properties from the work you have done with whole numbers. In this section, we will see how these properties can help us compute with rational numbers, too.

Here are the properties that we will be using in this section.

- ***The Commutative Property of Addition*** states that numbers being added can be added in any order. ***The Commutative Property of Multiplication*** states that numbers being multiplied can be multiplied in any order.

Examples

$$0.3+7.5=7.5+0.3$$
$$\frac{1}{2}\times(-3)=-3\times\frac{1}{2}$$

- ***The Associative Property of Addition*** states that the grouping of numbers that are being added does not matter. ***The Associative Property of Multiplication*** states that the grouping of numbers being multiplied does not matter.

Examples

$$\left(\frac{3}{10}+\frac{11}{5}\right)+\frac{1}{5}=\frac{3}{10}+\left(\frac{11}{5}+\frac{1}{5}\right)$$
$$(-3\times 4)\times 10=-3\times(4\times 10)$$

- ***The Inverse Property of Addition*** states that when a number is added to its opposite (or ***additive inverse***), the sum is zero.

Example

$4+(-4)=0$

- ***The Inverse Property of Multiplication*** states that when a number is multiplied by its ***reciprocal*** (or ***multiplicative inverse***), the product is 1. You can find the reciprocal of a fraction by flipping it. For example, the reciprocal of $\frac{7}{5}$ can be found by flipping the fraction to get its reciprocal, $\frac{5}{7}$.

Example

$\frac{7}{5} \cdot \frac{5}{7} = 1$

- ***The Identity Property of Addition*** states that when zero is added to any number, the sum is that number.

Example

$3\frac{1}{25} + 0 = 3\frac{1}{25}$

- ***The Identity Property of Multiplication*** states that when a number is multiplied by 1, the product is that number.

Example

$0.16 \times 1 = 0.16$

Examples

Identify the number property that each equation illustrates.

a. $-159 + 0 = -159$

b. $(0.3 + 1.2) + 0.8 = 0.3 + (1.2 + 0.8)$

c. $8 \times \frac{3}{4} = \frac{3}{4} \times 8$

d. $6 \cdot \frac{1}{6} = 1$

Consider the equation in *a*.

In $-159 + 0 = -159$, a negative integer is being added to zero and the sum is equal to the negative integer.

This is an example of the Identity Property of Addition.

Consider the equation in *b*.

In $(0.3 + 1.2) + 0.8 = 0.3 + (1.2 + 0.8)$, the parentheses show that the sums remain equal even when the numbers are grouped in different ways.

This is an example of the Associative Property of Addition.

Consider the equation in *c*.

In $8 \times \frac{3}{4} = \frac{3}{4} \times 8$, the order of the numbers being multiplied has been changed but they remain equal.

This is an example of the Commutative Property of Multiplication.

Consider the equation in *d*.

In $6 \cdot \frac{1}{6} = 1$, the integer 6 is multiplied by its reciprocal, $\frac{1}{6}$. (Since $6 = \frac{6}{1}$, its reciprocal is $\frac{1}{6}$.)

This is an example of the Inverse Property of Multiplication.

It is not enough to be able to identify the different number properties. You also need to consider how those properties can be applied. The next section will show how these number properties can make some computations easier.

IV. **Apply Properties and Use Order of Operations to Evaluate Numerical and Variable Expressions**

Properties can help you to evaluate numerical expressions. Do you remember what a numerical expression is? **A *numerical expression* is a phrase that contains numbers and operations.** Now that you know about rational numbers, you may see them in numerical expressions as well.

Let's look at applying properties to an example that is a numerical expression.

Example

Use one or more number properties to help you find the value of this expression.

$(0.3892 \times 7) \times \frac{1}{7}$

We should consider which of these rational numbers can be multiplied easily.

Multiplying by a decimal with four digits, such as 0.3892, will be time-consuming.

So, use the **Associative Property** to group the numbers differently.

$(0.3892 \times 7) \times \frac{1}{7} = 0.3892 \times \left(7 \times \frac{1}{7}\right)$

You will multiply the expression inside the parentheses, $\left(7 \times \frac{1}{7}\right)$, first.

7 is the reciprocal of $\frac{1}{7}$. So, according to the **Inverse Property of Multiplication**, the product of those two numbers will be 1.

$0.3892 \times \left(7 \times \frac{1}{7}\right) = 0.3892 \times 1$

Now, you need to multiply the decimal by 1. The **Identity Property of Multiplication** states that any number multiplied by 1 is equal to itself.

$0.3892 \times 1 = 0.3892$

The value of the expression is 0.3892.

You could have multiplied that decimal by 7 and then multiplied that product by $\frac{1}{7}$ to find that answer.

Yes. You can solve it without applying what you know about the properties, but using properties is definitely simpler in this example.

When evaluating expressions, it is also important to keep in mind the ***order of operations***. Let's review this order below.

- First, complete computations that are inside grouping symbols, such as parentheses.
- Second, evaluate any exponents.

- Third, multiply and divide in order from left to right.
- Finally, add and subtract in order from left to right.

We can also apply properties when we evaluate variable expressions. Remember that a variable expression is an expression with numbers, variables and operations.

Example

Find the value of this expression. Be sure to use the correct order of operations.

$-12 \div (8-6) \times p$

According to the order of operations, you should do the computation inside parentheses first. So, subtract.

$-12 \div (8-6) \times p = -12 \div 2 \times p$

There are no exponents to evaluate. So, the next step is to multiply and divide in order from left to right.

$-12 \div 2 \times p = -6 \times p = -6p$

The value of the expression is $-6p$.

Real-Life Example Completed

Comparing Distances

Here is the original problem once again. Reread it and underline any important information.

On the dive boat one morning, Cameron began talking with another boy named Chet. Chet and his family were from Colorado and Chet was just two years older than Cameron. The boys struck up a great conversation about diving and fish and the things that they had seen on their dives.

After a little while, they spotted some dolphins swimming with the boat. This is something that often happens as dolphins love the rushing water generated by the motor on the back of the boat.

"Did you know that they can swim .83 miles in one minute?" Chet asked Cameron.

"Really, no I didn't know that. I do know that a swordfish can swim almost one-half mile in a minute. I think the exact number is $\frac{9}{20}$ of a mile."

"Wow, which one can swim the farthest in one minute?" Chet asked, thinking carefully through the math.

By the time they reached the dive site, Cameron had figured out which one can swim farthest in one minute.

To figure out which one can swim the farthest in one minute, we will need to compare these two rational numbers.

A dolphin = .83 of a mile in one minute

A swordfish $= \frac{9}{20}$ **of a mile in one minute**

To figure this out, we first need to change the fraction into a decimal so that both numbers are in the same form.

$\frac{9}{20} = \frac{45}{100} = .45$

Next, we compare .83 to .45.

.83 >.45

A dolphin can swim farther than a swordfish in one minute.

Vocabulary

Here are the vocabulary words that are found in this lesson.

Rational Number
: any number positive or negative that can be written as a ratio.

Ratio
: a comparison between two quantities. Can be written using the word "to", using a colon, or using a fraction bar

Terminating Decimal
: a decimal that has a definite ending

Repeating Decimal
: a decimal where some of the digits repeat themselves.

Irrational Number
: a decimal that does not terminate or repeat but continues indefinitely.

Commutative Property of Addition
: states that the order that you add numbers does not change its sum.

Commutative Property of Multiplication
: states that the order that you multiply values does not change the product

Associative Property of Addition
: the groupings of the numbers being added does not change the sum

Associative Property of Multiplication
: the groupings of the numbers being multiplied does not change the product

Inverse Property of Addition
: any number added with its opposite is zero.

Inverse Property of Multiplication
: any number multiplied by its reciprocal is one.

Reciprocal
a flipped or inverted number

Identity Property of Addition
any number plus zero is that number

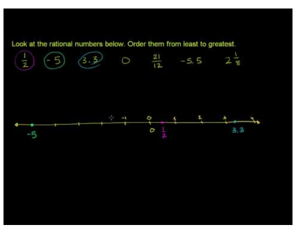
Identity Property of Multiplication
any number times one is that number

Numerical Expression
a phrase that contains numbers and operations

Variable Expression
a phrase that contains numbers, variables and operations

Technology Integration

This video shows how to compare rational numbers on a number line.



MEDIA
Click image to the left for more content.

KhanAcademy, Comparing RationalNumbers

Time to Practice

Directions: Rewrite each number as the ratio of two integers (a fraction) to prove that each number is rational.

1. -11
2. $3\frac{1}{6}$
3. 9
4. .08
5. -.34
6. .678
7. $\frac{4}{5}$
8. -19
9. 25
10. -7

Directions: Choose the inequality symbol (>, <, or =) that goes in the blank to make each statement true.

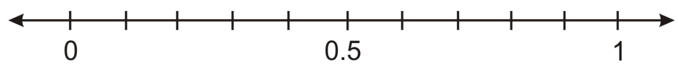
11. 1.1 ___ $1\frac{1}{10}$

12. -2 ___ $1\frac{1}{3}$

13. $\frac{2}{5}$ ___ 0.3

Directions: Place each rational number on the number line below. Then list these rational numbers in order from greatest to least.

14. $\frac{1}{2}$ 0.9 0 $0.\bar{9}$



Directions: For each equation, identify the number property that is demonstrated.

15. $\left(-3\frac{1}{2}\right) \times 1 = -3\frac{1}{2}$

16. $\frac{8}{5} \times \frac{5}{8} = 1$

17. $(22+4)+6 = 22+(4+6)$

18. $9.5+5.5 = 5.5+9.5$

19. $17+(-17) = 0$

Directions: Simplify each expression. Consider how number properties and the order of operations help you with this task.

20. $-6a+(8-4)$

21. $-12a \div (3a+a)$