

# Why do we Need a Mathematical Model?

*SAMSI/CRSC*

*Undergraduate Workshop 2006*

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# 1. The Reality, the experiment and the model.

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Question:

- Is it always possible to find a mathematical formula to express  $y_{\text{true}}$  as function of time?
- How can we compare  $y_{\text{true}}$  and  $y_{\text{data}}$  ?
- How can we compare  $y_{\text{true}}$  and  $y$  ?
- How can we compare  $y_{\text{data}}$  and  $y$  ?

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## 1. Answer:

- It is not always possible to find a mathematical expression of the reality.
- Due to measurement errors the data collected always differ from the reality.
- In a mathematical model it is impossible to take into account all parameters of the experiment, for example in the beam model we do not take into account temperature of lab and any other gravitational forces that exist in the lab. This is why  $y_{\text{true}}$  differs from  $y$ .
- As we have seen throughout this week,  $y_{\text{data}}$  differs from  $y$ . This leads to the errors analysis in the model.

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- Assume we do not have a model. Can we just with data collected in the CRSC lab derive a good understanding of the system?
- Yes, just with data collected in CRSC lab we can interpolate the data using the least square approach or any other interpolation technique to derive functional relationship between the displacements of the beam and the times.



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- Assume we do not have a model, and that we have interpolate the data from the Beam in the CRSC lab and obtain a functional relationship between the displacements and the times.
- Can we use this functional relationship to study a larger beam?
- No, without a model we cannot use the functional relationship of a smaller beam to study a larger beam.

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- Is it possible, just with experiments to obtain all necessary or desirable data of a physical system?
- No, there are certain data that are not observable, we cannot obtain them just with experiments. In fact, we need experiments and models in order to filter out unobservable data.

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- No, in order to calibrate and validate the model we need experiments

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Why do scientists use both models and observed data?

- Control and design
- Navigation (Space shuttles, Satellites, Rockets)
- Predictions and Forecasting etc...

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- In Biomedical sciences, certain measurements in vivo can be destructive
- In the design and the development of jet engines and big airplanes like the Airbus A380 etc...