

MORTGAGE-EQUITY AND RESIDUAL VALUATION TECHNIQUES

9

Introduction

In this chapter we move from the stable income, all-cash world depicted in the previous chapter, to the reality that property is typically purchased with financing, creating debt leverage. Our focus is the mortgage-equity or M-E technique. This valuation technique is founded on the premise that the overall rate should reflect the importance of separate yields attributable to the equity position and to the debt position.

The M-E technique is not as commonly used as capitalization or discounted cash flow in everyday real estate practice. However, the M-E technique is helpful in certain situations to:

- determine overall rates;
- derive building and land capitalization rates for residual techniques;
- analyze the capitalization rates derived through other techniques;
- test separately determined value estimates; or
- graphically analyze financial components of an overall rate.¹

This chapter begins with some history on the evolution of the M-E method. Early valuation methods did not consider the impact of mortgage financing. To explore this impact, economists in the 1950s developed new valuation techniques, the first of which was *Band of Investment* or BOI. Roughly speaking, BOI can be considered a forerunner to the mortgage-equity method, so in order to better understand M-E, it is useful to have a quick primer on its BOI predecessor.

The goal of BOI is to determine an overall capitalization rate by building up the rate from the key factors that investors consider when making an investment decision. These factors include:

- The initial loan-to-value ratio;
- Interest rate on the loan;
- The entrepreneur's equity investment; and
- The rate of return on investment expected.

¹ Appraisal Institute of Canada. 2010. *The Appraisal of Real Estate. Third Canadian Edition*. UBC Real Estate Division: P. 22.6-7.

In its simplest form, the BOI can be expressed as follows:

$$R_o = (L/V) i + ER \quad \text{where}$$

R_o is the capitalization rate

L/V is the initial loan-to-value ratio

i is the interest rate

E is the entrepreneur's investment or equity

R is the expected return on investment

The BOI formula has continued to evolve and is now commonly expressed as follows:

$$R_o = MR_m + ER_e \quad \text{where}$$

M is the initial permanent long-term, debt to price ratio

E is the initial equity down payment to price ratio (i.e., $M + E = 100\%$)

R_m is the mortgage constant, or annual payment to amortize \$1

R_e is the equity cap rate, or the first year cash flow return required by the typical investor on the initial equity investment .

Band of Investment Formula

The overall rate is the blended proportion of the mortgage constant and the equity capitalization rate.

$$R_o = MR_m + ER_e$$

The key argument for using the BOI method is that it is market-based since the inputs can be extracted from the market or the behaviour of typical investors. For example, there are typical norms for debt-to-value or price ratio associated with certain property types. Well-established office properties with a good leasing history may achieve a debt-to-price ratio of up to 75%, while hotels, with much higher risk, may only achieve a 50% debt-to-price ratio. While these ratios constantly change as risk parameters change in the market, the relationship between one property type and another tends to be somewhat consistent.

In the post-WWII era, or pre-computer days of real estate appraisal, an economist, L. W. Ellwood,² produced a series of tables that provided short-cuts for intensive manual calculations required for the BOI method. C. Akerson³ offered further simplification of the sophisticated calculations required, evolving towards the mortgage-equity concept that is the main focus in this chapter.

Mortgage-equity capitalization, like all other valuation methods, offers both benefits and pitfalls. On the plus side, mortgage-equity allows appraisers to either synthesize an overall rate, or analyze components of property value (e.g., financial, physical, legal) through residual techniques. The M-E method also provides a mechanism for dealing with property risk quantification. On the negative side, the M-E method in its basic form doesn't account for variation in annual cash flow and the concept of reversion of the investment. M-E is also conceptually difficult to understand for both appraisers and clients. Ellwood and Akerson's relative improvements to mathematical complexity in the pre-computer era are now all but erased by the power of computers. Perhaps for these reasons, the mortgage-equity method has lost much of its prominence in the appraisal world.

Nevertheless, we will offer a brief coverage of these methods in this chapter. First, understanding the past is important to fully understanding the present and the future. Second, there are some situations today where M-E methods can be applied effectively, and practitioners must have at least a familiarity with these methods, if not a working knowledge.

² L. W. Ellwood, *Ellwood Tables for Real Estate Appraising and Financing*, American Institute of Real Estate Appraisers, 1957 (Chicago).

³ Akerson, C. B. "Ellwood Without Algebra". *The Appraisal Journal*. Vol. 38, July 1970, p. 327.

This chapter is broken down into two parts. The first part will examine mortgage-equity capitalization – its tradition and potential application through discounted cash flow analysis. The second part deals with the application of residual techniques to determine one of the many component values into which property may need to be divided (e.g., financial, physical, legal). Keep in mind that the mortgage-equity method is only one of the concepts that may be applied in segregating elements of value.

PART I – MORTGAGE-EQUITY VALUATION TECHNIQUES

In the mortgage-equity section of this chapter, we will review the conceptual basis of the mortgage-equity techniques, i.e., the financial “splitting” of value. We will first touch briefly on traditional methods of determining overall capitalization rates through algebraic formulation, and through Akerson’s modified band of investment technique. We will then focus on applying the mortgage-equity concept through discounted cash flow analysis to directly estimate value.

Most of the material covered in this chapter could (and should) be linked directly with the content of previous chapters since we are using, simultaneously, the discounted cash flow and the capitalization of income concepts of value. The difference is that, in this chapter, we are discounting before-tax cash flows and we are using a composite-adjusted rate of capitalization which accounts for financing conditions, property appreciation or depreciation, and annual cash flows.

Before examining the mortgage-equity approach in detail, we will review two basic examples of the equity and mortgage residual techniques.

Equity Residual Method – Basic Application

The following example illustrates the basic application of the equity residual method. Deducting the annual debt service from the net operating income results in the residual income attributed to the equity. This residual equity income can be converted to an indication of the equity value by applying the equity capitalization rate.

Mortgage Value		\$375,000
Net operating income (annual)	\$60,000	
Less mortgage debt service (annual)	<u>- 31,519</u>	
Residual income to equity	\$28,481	
Equity Value (capitalized at equity capitalization rate of 13.0% → \$28,481 / 13.0%)		+ <u>\$219,085</u>
Indicated Property Value		\$594,085

Source of the various inputs above:

1. Mortgage value – provided by mortgage company.
2. Net Operating Income – income/expense analysis and stabilization for the subject property.
3. Mortgage debt service (annual) – mortgage amount multiplied by the mortgage constant.
4. Equity capitalization rate – derived from analyses of investments in real property from the formula equity capitalization rate = income to equity / equity investment and other market sources.

Example derived from *The Appraisal of Real Estate, 3rd Cdn. Ed.*, Ch. 22.

Mortgage Residual Method – Basic Application

The following example illustrates the basic application of the mortgage residual method. Deducting the annual return on equity from the net operating income results in the residual income attributed to the mortgage. This residual mortgage income can be converted to an indication of the equity value by applying the mortgage capitalization rate (the mortgage constant).

Equity Value		\$219,085
Net operating income (annual)	\$60,000	
Less mortgage debt service (annual)	<u>- 28,481</u>	
Residual income to equity	\$31,519	
Equity Value (capitalized using the mortgage equity capitalization rate)		+ <u>\$375,000</u>
Indicated Property Value		\$594,085

Example derived from *The Appraisal of Real Estate, 3rd Cdn. Ed.*, Ch. 22

Background

L.W. Ellwood, an innovative chief appraiser of the New York Life Insurance Company, introduced an algebraic formula for determining capitalization rates in 1957 that has had considerable influence on the appraisal profession. Ellwood's approach represented an important addition to prevailing financial theory. The basis of Ellwood's methodology was to apply adjustments to overall capitalization rates to account for investor's equity and debt positions. As noted earlier, he is perhaps best known for developing the financial factors required for analyzing properties with stable or stabilized income streams.

Prior to Ellwood's theories on financing adjustments and his Ellwood tables,⁴ appraisal theory and practice was based on a very simple premise: all real estate transactions were assumed to have occurred on a cash basis. The economic concepts of *return on capital* and *return of capital* did not yet play a role in appraisal thinking. The only opportunities to identify separate components of value was limited to land, buildings, or natural resources, or legal interests, such as leases. Although the basic analytical framework for mortgage-equity analysis had existed for several decades before Ellwood published his tables, its use was limited to applications by financial investors, primarily in working with financial instruments. Increased inflation in the post-Second World War period and commoditization of more highly leveraged investments (including real estate) demanded a change in appraisal thinking.

The impact of financial leverage – separating an investment into its financial components, or mortgage and equity positions – was noted in Chapter 1. This separation reflects investor behaviour in the marketplace. As Kinnard states, “the most probable purchaser-investor is presumed to seek to maximize or optimize his *cash* position: both income flows and cash equity”.⁵ In other words, “why tie up my money when I can use your cash!”

Ellwood made clear how the equity investor in real estate was motivated by opportunities for financial leverage, and how those opportunities presented greater risk to that same equity investor. That is, just as the investor sought to maximize the benefits of positive financial leverage, they also risked accelerated losses due to higher risk and less certain returns.

⁴ Ellwood, L.W. 1957. *Ellwood Tables for Real Estate Appraising and Financing, first edition*. Ellwood L.W., Ridgewood, N.J.

⁵ Kinnard, W.N., Jr. 1971. *Income Property Valuation*. Lexington, Massachusetts. Heath Lexington Books. Page 256.

The next sections of this chapter outline the basic premise of the mortgage-equity method and its application in a series of techniques.

Rationale of Mortgage-Equity Concept

Mortgage-equity capitalization presumes that mortgage terms and equity yields influence the overall rate. As Akerson explains, “the overall rate is the fraction of the total investment that must be collected each year, on the average, to service the debt (principal as well as interest payments), yield the required benefits (cash flow and/or equity build-up), and compensate for depreciation or appreciation”.⁶

Thus, mortgage-equity is a band of investment approach whereby the overall rate is calculated as the sum of capitalization rates for the mortgage component and the equity investment component – adjusted for changes in the equity position.

To calculate debt service, the analyst identifies the most probable mortgage loan terms available to the typical investor for the property type being analyzed, and then determines the mortgage constant. All that remains for the appraiser is to determine the equity yield rate. This requires an estimate of the proportionate allowance for cash throw-off to equity and provision for capital recovery. An increase in equity will occur through mortgage amortization and appreciation (or depreciation) in the residual captured at the conclusion of the holding period, when the property is sold.



Going beyond the simplistic premise of regular or stable annual cash-flow, mortgage-equity first accounted for income streams that were expected to grow or decline systematically. A series of tables containing Ellwood “J” and “K” factors have been developed to address this short-coming.

In the *Appraisal of Real Estate, 3rd Canadian edition*, these factors are defined as follows.

J = an income stabilization factor used to convert an income stream changing on a curvilinear basis into its level equivalent.

K = an income stabilization factor used to convert an income stream changing at a constant ratio into its stable or level equivalent.

The “J” factor addresses the accelerating or decelerating income changes based on compounding or discounting, while the “K” factor deals with the straight line growth premise.

The difficulty with both of these adjustments is the complexity they add to a mortgage-equity analysis. Mortgage-equity formulae are much more difficult to grasp than the intuitive and straightforward discounted cash-flow (DCF) methods. Advantages of DCF over Ellwood include:

⁶ Akerson, C.B. 2000. *Capitalization Theory and Techniques. Second Edition*. Chicago, Illinois. Appraisal Institute.

- a more direct and immediate connection between changes in the periodic income stream and the value of the asset versus the indirect impact of mortgage-equity adjustments on the overall rate; and
- greater accuracy when dealing with predictable annual variation in the net income as a result of lease set-ups and re-leasing.

However, for illustration purposes, and for the sake of comprehensiveness, we will now examine the basic and more advanced application of mortgage-equity techniques.

The Mortgage-Equity Techniques

Traditional Techniques – Elwood and Akerson

Traditional mortgage-equity techniques involve converting income into value estimates, typically through direct capitalization. Both Ellwood's algebraic formula and Akerson's modified band of investment contemplate an overall rate comprising two components:

- A basic capitalization rate that does not reflect changes in equity position; and
- Adjustments to the basic overall rate for changes in equity position due to periodic mortgage paydown and property appreciation/depreciation.

As mentioned earlier, Ellwood's pre-computed tables and his mortgage-equity capitalization rate were a practical breakthrough in the pre-computer and pre-calculator days. However, requirements for sophisticated calculations and the advent of computer spreadsheet analytical tools have made the technique less appealing today. Appendix 9.1 at the end of this chapter briefly outlines Elwood's technique.

In an effort to reduce mathematical sophistication and enhance practical understanding, C. B. Akerson offered mortgage-equity practitioners an intuitive sense of the Ellwood formula in his article *Ellwood Without Algebra*. Building on a weighted average (i.e., debt and equity) basic overall rate, Akerson converts the Ellwood algebraic formula into a modified band of investment technique that is adjusted for changes in return levels through equity build-up and appreciation/depreciation. It is a less intimidating technique, and offers its practitioners the added benefit of application that can be aided by financial calculators or computer spreadsheets.

In modern markets that are impacted by a host of factors, both methods might be criticized as rather inflexible, pre-tax present value methods. In the pre-computer era, the Ellwood system (and its Akerson version) enabled practitioners to conduct sophisticated analyses. As we shall see in the following sections, computers can now easily provide similar results and deal with a variety of cash flows and financing packages. It should be noted that Ellwood and Akerson formulas and more modern DCF approaches are all based on similar assumptions, require similar information, and produce the same results.

Splitting the Value “Financially”: The Basis of the Mortgage-Equity Approach

In previous chapters we have discussed the “slicing of the pie” analogy and concluded that the concept of market value can be analyzed in terms of:

- the asset's capital structure: the relative shares of debt and equity financing used in the acquisition and holding of real property

$$\text{Value} = \text{Debt} + \text{Equity}$$

$$\text{Value} = \text{Debt} + \text{Equity}$$

$$V = D + E$$

$$\text{Value} = \text{Present Value of NOIs} + \text{Present Value of Reversion}$$

- the relative contribution of the operating flows and the residual flows.

$$V = \text{Present Value of NOI Flows} + \text{Present Value of Residual Flows}$$

$$V = \frac{\text{NOI}_1}{(1+k_a)^1} + \frac{\text{NOI}_2}{(1+k_a)^2} + \frac{\text{NOI}_3}{(1+k_a)^3} + \dots + \frac{\text{NOI}_n}{(1+k_a)^n} + \frac{\text{REV}_n}{(1+k_a)^n}$$

where NOI_i = net operating income in period i ($i=1,2,3,\dots,n$)
 REV_n = reversion value in period n
 k_a = overall internal rate of return based on the total value

The equation above can be summarized as:

$$V = \sum_{t=1}^n \frac{\text{NOI}_t}{(1+k_a)^t} + \frac{\text{REV}_n}{(1+k_a)^n} \tag{Equation 9.1}$$

In the last identity, k_a is the internal rate of return on the full value of the investment.

Furthermore, since we also know how to allocate the periodic net operating income flows and the residual flows between debt and equity, we can proceed with our “financial split” based on the example described in Table 9.1 (The Dixsept Building).

Table 9.1 The Dixsept Building		
• Total initial value	V	\$40,000
• Initial Mortgage	D	\$32,000
· Annual nominal rate	k_d	$j_i = 15\%$
· Frequency of Compounding		Annual
· Payments are made annually		
· Amortization Period	m	25 years
• Disposition Price after 8 years	V_n	\$44,000
• Expected overall return on Equity	k_e	18%
• Expected equity dividend return	y	13.12%
• Net operating Income (assumed constant)	NOI	\$6,000
• Holding period	n	8 years

The Value of the Debt Component of Total Value

The value of the debt (D) can be written:

$$D = \frac{PMT_1}{(1+k_d)^1} + \frac{PMT_2}{(1+k_d)^2} + \dots + \frac{PMT_n}{(1+k_d)^n} + \frac{OSB_n}{(1+k_d)^n} \quad \text{Equation 9.2}$$

where

- PMT = debt (mortgage) payments
- n = number of payments in the loan term
- OSB_n = outstanding balance on the loan at the term
- k_d = cost of debt (mortgage rate) per payment period

Since the mortgage payments are constant, we can also write:

$$D = PMT \times a_{\overline{n}|k_d} + \frac{OSB_n}{(1+k_d)^n} \quad \text{Equation 9.3}$$

The value of the mortgage contract (to the Mortgagee) is the present value of the flow of payments and of the outstanding balance if the loan is not fully amortized. The discount rate to be used here is k_d, the mortgage rate. The mortgage rate is the cost of capital for the mortgagor and the return on the mortgagee's capital, i.e., the internal rate of return to the lender on this loan.

Let us verify this identity with the "Dixsept" Building:

$$\begin{aligned} D &= \$4,950.39 \times a_{\overline{8}|0.15} + \frac{\$29,935.62}{(1+0.15)^8} \\ D &= [\$4,950.39 \times 4.4873215] + \left[\frac{\$29,935.62}{3.0590229} \right] \\ D &= \$22,213.99 + \$9,786.01 \\ D &= \$32,000.00 \end{aligned}$$

The calculation of the annuity term, $a_{\overline{8}|0.15}$ or the present value of \$1 per year for 8 years at 15%, is illustrated in Appendix 9.3 at the end of this chapter.

The Value of the Equity Component of Total Value

Let E represent the value of the equity portion (before income tax). This can be written:

$$E = \frac{BTCF_1}{(1+k_e)^1} + \frac{BTCF_2}{(1+k_e)^2} + \dots + \frac{BTCF_n}{(1+k_e)^n} + \frac{BTER_n}{(1+k_e)^n} \quad \text{Equation 9.4}$$

Where we define k_e as the (internal rate of) return on equity.

As before, we can simplify the previous identity since we deal here with constant before-tax cash flows:

$$E = \text{BTCF} \times a_{[n, k_e]} + \frac{\text{BTER}_n}{(1 + k_e)^n}$$

In our example,

$$E = (\text{NOI} - \text{PMT}) \times a_{[8, 0.18]} + \frac{(\text{REV}_n - \text{OSB}_n)}{(1 + 0.18)^8}$$

$$E = (\$6,000 - \$4,950.39) \times 4.0775658 + \frac{(\$44,000 - \$29,935.62)}{3.7588592}$$

$$E = \$4,279.85 + \$3,741.66$$

$$E = \$8,021.51 \text{ rounded to } \$8,000.00^7$$

Now we can put this together again and write, once more, the full equation of market value as:

$$V = \frac{\text{NOI}_1}{(1 + k_a)^1} + \frac{\text{NOI}_2}{(1 + k_a)^2} + \dots + \frac{\text{REV}_n}{(1 + k_a)^n}$$

or (since NOI is constant)

$$V = \text{NOI} a_{[n, k_a]} + \frac{\text{REV}_n}{(1 + k_a)^n}$$

Here k_a is the internal overall rate of return, i.e., the rate of return which satisfies the above equation for the total value.

Although this may appear to be some form of pedagogical overkill, it should be emphasized that the splitting process can be described as follows:

⁷ Here, as in all further computations, the results are approximate because of rounding errors. The readers should understand that any "real-life" appraisal situation numbers and coefficients may not be as accurate. The degree of precision for the rates and adjustment factors described in this chapter is required to prove the theoretical validity of the formulae. This accuracy should not lull the reader into believing that he or she deals with exact sciences. Appraisal is not physics and fourth-digit precision numbers should not be taken too seriously.

Figure 9.1
Splitting the Value of the Dixsept Building

$V = \$40,000$	
$V = [\text{NOI} \times a_{\overline{n} k_a}] + [\text{REV}_n(1+k_a)^{-n}]$	
$V = [(\$6,000) \times a_{\overline{n} k_a}] + [(\$44,000)(1+k_a)^{-n}]$	
$E = \$8,000$	$D = \$32,000$
$E = \text{BTCF} \times a_{\overline{n} k_e} + \text{BTER}(1+k_e)^{-n}$	$D = \text{PMT} \times a_{\overline{n} k_d} + \text{OSB}_n(1+k_d)^{-n}$

The same analysis can be conducted in a tabular format.

Table 9.2
The DCF Mortgage-Equity Identities

		<u>Years 1-8</u>		<u>Year 8</u>		
Debt	(D)	\$32,000	PMT	\$4,950.39	OSB	\$29,935.62
Equity	(E)	<u>8,000</u>	BTCF	<u>1,049.61</u>	BTER	<u>14,064.38</u>
Value	(V)	\$40,000	NOI	\$6,000.00	REV	\$44,000.00

The Discounted Cash Flow Mortgage-Equity Valuation: A Direct Estimation of Value

Thus far, the analysis may have seemed to be quite tautological since we started from the value ($V = \$40,000$) to reconstruct the same market value (V). But we can show now that the discounted cash flow mortgage-equity method is in fact a valuation instrument (i.e., we can find V directly) as long as the debt-to-value ratio (D/V) is either known or simply assumed. When, as in a typical appraisal context, Debt (D) and Value (V) are unknown, we can still transform the Value identity as shown below.

Let $V_n =$ Net Property Value in period n

$V =$ total property value today (Unknown)

Let's take the development one step at a time.

1. $V = D + E$

2. $E = \text{BTCF} \times a_{\overline{n}|k_e} + \text{BTER}(1+k_e)^{-n}$

but

3. $\text{BTCF} = \text{NOI} - \text{PMT}$

Therefore,

$$4. E = [(NOI - PMT) \times a[n, k_e]] + [BTER(1 + k_e)^{-n}]$$

and

$$5. PMT = \frac{D}{a[m, k_d]}$$

Therefore,

$$6. E = \left[\left(NOI - \frac{D}{a[m, k_d]} \right) \times a[n, k_e] \right] + [BTER(1 + k_e)^{-n}]$$

and we know

$$7. BTER = V_n - (\%OSB_n \times D)$$

where V_n = Value in the n^{th} year (end of the holding period)
 $\%OSB_n$ = the percent outstanding balance after 'n' periods

Therefore,

$$8. E = \left[\left(NOI - \frac{D}{a[m, k_d]} \right) \times a[n, k_e] \right] + [(V_n - \%OSB_n \times D)(1 + k_e)^{-n}]$$

Finally, since we do not always know the amount of debt, we can substitute the loan/value ratio and property value for D.

$$9. D = L/V \times V$$

Therefore

$$10. E = \left[\left(NOI - \frac{L/V \times V}{a[m, k_d]} \right) \times a[n, k_e] \right] + (V_n - \%OSB \times L/V \times V)(1 + k_e)^{-n}$$

Note that the equity value (E) in step 10 is expressed with only one unknown (V) and can be solved.

Let us solve this equation on the example shown in Table 9.3:⁸



⁸ Calculation steps for certain parts of these equations can be found in Appendix 9.2.

Table 9.3
The Swift Building

Loan-to-value ratio ⁹	L/V	0.8
Rate on loan	k _d	15% p.a.
Frequency of compounding (k _d)		1
Loan payments per year		1
Amortization period	m	25
Net operating income (constant)	NOI	\$6,000
Holding Period	n	8 (years)
Expected growth in property value in n years (n=8)	g	10%
Expected return on equity	k _e	18% p.a.

$$V = D + E$$

$$V = (L/V \times V) + \left[\left(\text{NOI} - \frac{L/V \times V}{a[m, k_d]} \right) \times a[n, k_e] \right] + (V_n - \% \text{OSB} \times L/V \times V)(1 + k_e)^{-n}$$

$$V = .8(V) + \left[\$6,000 - .8(V) \times 1 / a[25, .15] \right] \times a[8, .18] + \left[[1.1(V) - .8(V) \times \% \text{OSB}_n] / (1 + .18)^8 \right]$$

$$V = 0.8(V) + [\$6,000 - .8(V) \times 1 / 6.464] \times 4.078 + [(1.1(V) - .8(V) \times .9355) / 3.7589]$$

$$V = 0.8(V) + \$24,468 - 0.5047(V) + [.3516(V)/3.7589]$$

$$V = 0.8(V) + \$24,468 - 0.5047(V) + 0.0935(V)$$

$$V = 0.3888(V) + \$24,468$$

$$V - 0.3888(V) = \$24,468$$

$$.6112(V) = \$24,468$$

$$V = \frac{\$24,468}{0.6112} = \$40,032.72, \text{ say, } \$40,000$$

Just to make sure, we can always check:

$$V = [0.80 \times 40,000] + [6,000 - (0.80 \times 40,000 \times 0.1546994)](4.0775658) + [1.1(40,000) - 0.80(40,000)(0.9354881)]/3.7588592$$

$$V = \$32,000 + (\$6,000 - 4,950.39)(4.0775658) + (44,000 - 29,935.62)/3.7588592$$

$$V = \$32,000 + \$4,279.89 + \$3,741.66$$

$$V = \$40,021.55, \text{ say, } \$40,000$$

⁹ The terms loan to value ratio (L/V) and debt to value (D/V) ratio are used interchangeably throughout this chapter.

Thus the value identity can be written in terms of different factors and percentages using the present value concept. The value of the property is the sum of the present value of the debt and the present value of the equity. All we need to know is the stream of the net operating income, the capital structure (D/V), and the reversion hypothesis (appreciation or depreciation of the property). This technique is direct and very similar to the general approach developed in Chapter 8.

Alternative techniques of valuation are less direct, i.e., find a capitalization rate which reflects the capital structure and reversion hypothesis, and then capitalize the stream of net operating income. This alternative path is considered in Appendix 9.3.

Summary of Mortgage-Equity Methods

Application of either traditional or computerized DCF mortgage-equity methods will turn our seemingly naive exercise of splitting the value “financially” into a fairly intimidating set of formulae. (See the summary in Table 9.4, which also summarizes the approaches covered in Appendix 9.3.)

Contrasted to traditional mortgage-equity techniques, the discounted cash flow approach is intuitively more appealing and computerized calculations reduce the level of intimidation and facilitate application. Furthermore, the DCF mortgage-equity valuation formula presents the distinct advantage of being structurally analogous to the DCF equity valuation model, which was previously described as a cornerstone in real estate analysis.

Table 9.4 Summary of Mortgage-Equity Formulae		
<i>Technique Name</i>	<i>Technique</i>	<i>Formula</i>
Discounted Cash Flow	Direct Computation of V	$V = D + E$ $V = L/V \times V + (NOI - L/V \times V \times 1/a[m,ka]) \times a[n,ke] + [(1 + g)V - L/V \times V \times \%OSB_n]/(1 + ke)^n$
Band of Investment	Indirect computation of V through NOI/R.	$R = [D/V \times f] + [E/V \times y]$
Adjusted Band of Investment: Akerson	Indirect computation of V, through NOI/R.	$R = [D/V \times f] + [E/V \times ke] - [D/V(1 - \%OSB_n) \times 1/s[n,ke]] - [g \times 1/s[n,ke]]$
<p>General Assumptions</p> <ul style="list-style-type: none"> • Limited holding period • Constant net operating income • Amortized mortgages (thus equity build-up) • y and ke are known or assumed • Appreciation or depreciation of the property • Mortgage conditions affect values 		

Further, we see that the DCF mortgage-equity formula can be directly adapted to deal with variable net operating income, with more complex financing schemes and to the final outrage: the introduction of taxation. Clearly, if you move from constant discounted before-tax cash flows to variable discounted after-tax cash flows, you graduate from mortgage-equity appraisal to investment analysis and you will have to deal with a (presumably) more realistic methodology. As a matter of fact, similar attempts of emancipating the capitalization techniques (Akerson) from their constraining hypotheses have been suggested.

It may be reassuring to realize that traditional capitalization techniques can be adapted to deal with more realistic hypotheses, but we do not see much usefulness. Not only do discounted cash flow mortgage-equity models provide the same results as any adapted version of Akerson's formula, but they can also deal with any form or shape of cash flows and can explicitly account for any type of financing package.

The *raison d'être* of traditional mortgage-equity capitalization techniques was that, in pre-computer times, they made computations feasible thanks to the publication of pre-computed tables. This justification is not sufficient anymore. Appraisers and analysts today concentrate more on the direct valuation techniques.

Recall that one of the uses for the mortgage-equity approach was to derive building and land capitalization rates for the residual techniques. And, because mortgage-equity techniques separately consider equity, some appraisers refer to this approach as an equity residual technique. Students with further interest in this topic may wish to review Akerson's *Capitalization Theory and Techniques*.

Conclusion: Mortgage-Equity Method

Mortgage-equity techniques have been criticized as being overly complex, requiring many awkward calculations which are difficult to understand and explain, demanding of data, and limited to a few income patterns. The validity in these complaints regarding its application does not diminish the historical contribution of the concept or its instructiveness in introducing factors that need to be considered in discounted cash flow analysis. As discussed earlier, mortgage-equity "tools" (especially the Ellwood variant or Akerson modification) brought critical elements of financial theory to appraisal practice, recognizing the importance of financial and operating leverage in analyzing real estate investments.

In the absence of computerized DCF applications that allow for income fluctuations, Ellwood and its variations introduced tables that adjusted the *yield* rate for a host of factors including:

- explaining and measuring capital recovery and the value of a wasting asset through both debt and equity claims; and
- accommodating varying income patterns.

Accommodating many market factors through adjustments to the yield rate (rather than within the income stream, as can be readily accomplished within DCF analysis)¹⁰ resulted in extremely complex mortgage-equity formulae which notably reduced understandability by real estate practitioners. This lack of understandability and the rigour required in properly supporting mortgage-equity conclusions greatly diminished appraisers' enthusiasm for popular application.

The advent of computerized technology enabled DCF analysis, permitting painless adjustments for factors that impacted income to be made directly to income, rather than indirectly applied by manipulating yield rate calculations. This development has greatly enhanced the understandability and explainability of both

¹⁰ Note that DCF valuation and Ellwood technique applications produce the same estimate of value when the same assumptions are used.

methodology and conclusions, and has increased the palatability of these previously complex techniques for both appraisers and their customers. To the extent that income forecasts are correct, DCF analysis can also provide greater accuracy where the analyst anticipates an irregular income stream over the holding period.



Before casting mortgage-equity on the pile of interesting but outdated antiquities, and happily taking up such conveniences as DCF analyses, analysts might note that the awkward rigour required in applying mortgage-equity techniques helped ensure a degree of competency on the part of the practitioner. In most commercial real estate investment, mortgage financing and equity yields substantially influence the overall rate. As noted in the Webb-McIntosh survey in the following text box, many of those real estate investors use an increasing array of investment criteria with the equity dividend rate being their most popular before-tax measure.

While appraisers may properly de-emphasize reliance on mortgage-equity capitalization due to complex mathematical processes, any alternate analysis needs to recognize that investors do make decisions based on such factors as financial and operating leverage, potential for appreciation (or risk of depreciation), and tax shield implications. The convenient sophistication of computer-enabled DCF and its implied assumptions can increase risk by obscuring understanding. Life becomes so much more complicated in the absence of four good comparables!

As a closing word on mortgage-equity methods, readers may find the text box below of interest, “Do investors actually use mortgage-equity techniques?” Following this text box, the remainder of this chapter will examine residual valuation techniques. Residual methods are a sibling of mortgage-equity techniques in that they also split property value into components: e.g., the financial, physical, and legal components that encompass overall value.

As a closing word on mortgage-equity methods, readers may find the text box below of interest, “Do investors actually use mortgage-equity techniques?” Following this text box, the remainder of this chapter will examine residual valuation techniques. Residual methods are a sibling of mortgage-equity techniques in that they also split property value into components: e.g., the financial, physical, and legal components that encompass overall value.

Application: Do Investors Use Mortgage-Equity Techniques?

An investor’s goal is not complicated: to acquire a property today for a price less than it can be resold now or in future, thereby earning a reasonable time- and risk-adjusted return on capital. As the appraiser is generally striving to emulate the investor/purchaser’s actions, it is important to consider mortgage-equity techniques as an investor might rely upon them.

Poorvu and Cruikshank explain the private investor’s perspective: “Seasoned real estate people know that on the negotiation end of this business, you have to make quick decisions, and in many cases you have to make them more on gut feel than on the basis of exhaustive number-crunching. Most professionals realize, too, that the average real estate investment only pays for itself in the long-term, and that assumptions that are projected out ten years aren’t much grounded in reality.” He argues that investors can find greater validity in a back-of-envelope analysis that considers the “four corners” of the real estate industry comprising properties, capital markets, market players, and the external environment. However, “sophisticated quantitative analyses also have their place in certain kinds of real-world transactions. For example, computerized spreadsheets have made it easier to analyze multiple scenarios, to calculate potential returns based on each of those scenarios, and to keep track of what has happened over the longer-term.” (Poorvu, William J. and Cruikshank, Jeffrey L. 1999. *The Real Estate Game: The Intelligent Guide To Decision-Making And Investment.*)

continued

In a mid-1990s literature review and survey of REITs on all facets of their real estate investments, Webb and McIntosh document trends in investors' preferred criteria (measures) for before-tax and after-tax investments. This comprehensive survey covered real estate portfolio size and type, portfolio composition, investment by property type, international investments, before-tax analysis, after-tax analysis, diversification strategies, computer usage, holding period assumptions and criteria for obtaining mortgages, equity positions and construction loans. Before- and after-tax investment criteria were compared with previous studies as shown in Tables 1 and 2 below.

Table 1
Comparison of Before-Tax Investment Criteria

Before-Tax Measure	Wiley (1972)	Farragher (1982)	Page (1982-1983)	Webb (1982-1983)	Webb & McIntosh (1984-1985)
Gross income/Purchase price	10%	6%	5%	35%	30%
Net income/Initial equity	36%	16%	14%	80%	68%
Equity dividend rate	58%	61%	33%	72%	94%
Payback period	11%	17%	12%	38%	38%
Net present value	32%	27%	NA	65%	57%
Internal rate of return	NA	57%	57%	77%	66%
Overall rate	NA	21%	45%	NA	NA
None used	9%	NA	5%	11%	0%

Webb & McIntosh found that *all* firms used before-tax analysis. The most used before-tax investment criteria was clearly the equity dividend rate. Its popularity increased with later studies, showing the influence of increased computer usage.

Table 2
Comparison of After-Tax Investment Criteria

Before-Tax Measure	Wiley (1972)	Farragher (1982)	Page (1982-1983)	Webb (1982-1983)	Webb & McIntosh (1984-1985)
Cash-on-cash	25%	39%	17%	61%	62%
Brokers rate	12%	45%	NA	21%	19%
Payback period	8%	NA	11%	26%	26%
Net present value	7%	NA	20%	48%	28%
Internal rate of return	18%	NA	50%	65%	42%
Tax shelter benefits	18%	NA	9%	46%	28%
Financial management rate	NA	18%	11%	NA	NA
None used	46%	NA	26%	29%	19%

Webb & McIntosh found more firms using after-tax investment criteria, and greater use of more sophisticated methods adapted from current financial theory – particularly evident in the increased use of cash-on-cash analysis. Investors' use of more sophisticated analysis like net present value and internal rate of return has also increased over time.

These results show that certain of the mortgage-equity elements remain critical in investors' decision-making. The techniques applied by investors today stand on the shoulders of the mortgage-equity concept, but reflect more efficient use of modern technology and current financial theory. Based on this research and the observations of seasoned real estate practitioners, it can be concluded that mortgage-equity techniques continue to be useful only to the extent that their careful application reflects the decisions of investor-purchasers in the marketplace.

PART II – RESIDUAL VALUATION TECHNIQUES

The Capitalization of a Dual Stream of Income

“Componentizing” or Splitting Value

Residual techniques enable the appraiser to analyze income properties as they might be “split” in three different ways:

- *Legal* components reflected under the terms of a lease contract, where the lessor and lessee have separate interests;
- *Financial* components as discussed above using mortgage-equity analysis, where the mortgagees (lenders) and equity investors hold separate interests; and
- *Physical* components - Traditional appraisal theory recognizes the separate contributions to total property value of land and buildings (or other improvements). According to this tradition, each component contributes to net operating income. An important difference is that improvements are considered a depreciable component that necessitate an allowance for capital recovery (return of investment in addition to the return *on* investment), while land is viewed as imperishable and thus requires only a return *on* investment.¹¹

As leases are considered separately, the following comments touch only on physical and financial component “residuals”. Residual analysis can be useful where the appraiser has supportable evidence of value for one component of value (e.g., land, building, or total property), but needs to extrapolate that available information to estimate value of other property elements. Selection of the appropriate residual technique to apply depends upon the nature of the property, the information available, and the requirements of the appraisal assignment.

The following excerpt from the *Appraisal of Real Estate* (3rd Canadian Edition) introduces the methodology for applying residual techniques:

Regardless of which known and unknown (residual) components of the property are being analyzed, the appraiser starts with the value of the known items and the net operating income, as shown in Table 22.3 (*reproduced below as Table 9.5*). The appraiser does the following:

- applies an appropriate capitalization rate to the value of the known component to derive the annual income needed to support the investment in that component;
- deducts the annual income needed to support the investment in the known component from the net operating income to derive the residual income available to support the investment in the unknown component;
- capitalizes the residual income at a capitalization rate appropriate to the investment in the residual component to derive the present value of this component; and
- adds the values of the known component and the residual component to derive a value indication for the total property.

¹¹ Of course, appraisers now recognize that land can and does decline in value, thus this simplistic assumption must be challenged according to the nature of any specific appraisal assignment.

Table 9.5
Known and Unknown Variables in Residual Calculations

Residual Technique	Known	Unknown
Land residual	Net operating income (NOI) Building value (V_B) Building capitalization rate (R_B) Land capitalization rate (R_L)	Land or site value (V_L)
Building residual	Net operating income (NOI) Land or site value (V_L) Land capitalization rate (R_L) Building capitalization rate (R_B)	Building value (V_B)
Equity residual	Net operating income (NOI) Mortgage amount (V_M) Mortgage capitalization rate (R_M) Equity capitalization rate (R_E)	Amount of equity (V_E)

Splitting the Value “Physically”

Thus far, we have conveniently looked at a very abstract and unified form of property in which value is simply the present value of a stream of income. But in some particular appraisal situations, we may want to “split” the total value of a property among some of its many component parts: improvements and land, mortgage and equity, or various legal interests. It is possible for an appraiser to estimate the value of an unknown component if the known components of the equation have been satisfied, by capitalizing the income allocated to the unknown component.

To simplify matters in this section we will deal with only two components of value: the value of the buildings and the value of the land. We can write the equation:

$$V = V_B + V_L$$

where,

$$\begin{aligned} V &= \text{Total property value} \\ V_B &= \text{Value of improvements} \\ V_L &= \text{Value of land} \end{aligned}$$

Property Value = Building Value + Land Value

Why should the two components of value be treated differently? Because buildings have a finite life, they depreciate in value and the investor may require a higher rate of return to compensate for this depreciation. This higher rate of return should include a *recovery of the capital* over and above the *return on the capital*. On the other hand, the land component is not considered to be a wasting asset and, therefore, not depreciable¹² and the rate of return should only reflect the opportunity cost of the capital invested: the return on the land value.

¹² We simply say that no capital cost allowance can be taken on the land value. Obviously land values can decline (or increase) over time, as can building value (due to market conditions as opposed to depreciation) and thus a compensating return for this drop in value should also be included. Generally, land is not seen to depreciate, except perhaps in some circumstances such as waterfront land being eroded by the river, lake, or ocean on which it fronts. This fact seems to be rarely considered in the appraisal literature dealing with residual techniques.

This concept of providing for a return *of* capital extends beyond the valuation of the building component of real estate and will apply to any “wasting asset” (an asset that declines in value as it wears out or is depleted). This principle will apply to an oil well, gravel pit, coal mine, or a leasehold estate.

The general principle can be illustrated as follows. Assume you purchase (prepay rent) a leasehold estate having 20 years remaining. The purchase price is \$100,000. The property is sub-let and produces an income of \$20,000 per year. A simple yield calculation would show a return of 19.42%.

$$\begin{aligned} \text{Value} &= \text{Income} \times a_{\overline{n}|i\%} \\ \$100,000 &= \$20,000 \times a_{\overline{20}|i\%} \\ i &= \text{internal rate of return} \\ &= 19.425795\% \end{aligned}$$

However, if the investor spends all of the \$20,000 each year, at the end of 20 years they would have nothing, not even the original \$100,000. Hence the \$20,000 per annum includes both a return *on* capital and a recovery *of* (the \$100,000) capital. Therefore it is necessary to find some way to explicitly handle the recovery *of* capital. In fact, as we will see later, this calculation for this simple leasehold example does include provision for the recovery *of* capital; it is just hidden in the arithmetic and we will return to this problem later.

From this conceptual physical splitting of the property value stems the residual technique of capitalization. The general principle is that the value of one of the physical components can be measured separately and then the residual portion of the total value measures the contribution of the other component to total value. As we shall see shortly, these techniques should be used only under fairly specific conditions and they are designed to address particular appraisal problems. Land and building residual techniques are most appropriately applied when:

- The current use either represents the highest and best use of the land or the property is assumed to be ready for redevelopment to its highest and best use.
- The amount of depreciation attributable to the improvements is minimal.

Such specific problems are more appropriately illustrated with case analysis and we will limit the presentation here to a technical and brief presentation of two generally accepted residual techniques: the land residual and the building residual. Since any of the residual techniques (land, building, and property residual techniques) may rely on two different depreciation models, we are faced with six alternative computational choices. Let us first clear the way by reviewing the different depreciation alternatives generally adopted in the appraisal literature.

Quite aside from the conceptual reasons for valuing land and improvements separately, appraisers may find a number of occasions where they are asked to produce separate valuations for the component parts. For example, in some areas the property tax on land is much higher than that applied for improvements, and in these instances the split in value between land/improvements becomes critical (and hotly contested in appeals).

Depreciation Hypotheses and the Value of the Building

Two primary methods of handling depreciation are described in the appraisal literature.¹³ These include:

1. The Hoskold method
2. The Inwood method

To illustrate the two methods of estimating depreciation, an improvement costing \$100,000 which has an expected life of 15 years will be used.

The Hoskold Premise

The recovery of capital can be obtained through the accumulation of a sinking fund: the annuity required to accumulate the initial capital value of the building. If this accumulation was safely done (for example through saving accounts or Guaranteed Investment Certificates) at the safe rate of return ($r\%$) over the n years of the building's life, the annual recovery would be:

$$\text{Annual Recovery} = V_B \times (1/s[n, r])$$

where $s[n, r]$, which is the future value of an annuity for n periods at an interest rate r . The calculation for this is illustrated in Appendix 9.2 at the end of this chapter.

The above equation can be viewed as simply a calculation for the future value of a series of payments such as:

$$\begin{aligned} \text{FV} &= \text{PMT} \times s[n, r] \\ \text{or} \\ \text{PMT} &= \text{FV} \times \frac{1}{s[n, r]} \end{aligned}$$

However, in the Hoskold model the future value is equal to the initial capital cost (the depreciation base) of the improvement and the payment is the annual amount of depreciation.

Using our example of a \$100,000 asset with a 15 year life, if we assume a *safe rate* of 6%, the annual recovery will be \$4,296.28 found as follows:

$$\begin{aligned} \text{Annual Recovery} &= \$100,000 \times \frac{1}{s[15, 6\%]} \\ &= \$4,296.28 \end{aligned}$$

Since the safe rate $r\%$ is probably different from the expected rate of return on the unlevered investment (k), the expression “dual rate of return” model is sometimes used to describe this concept.

¹³ A third depreciation method is the Babcock premise (Babcock, F.M., *Appraisal Principles and Procedures*, R. Irwin, 1968). This method is outdated and will not be covered in depth in this chapter. In brief, the Babcock method proposes that a building may be assumed to depreciate linearly at the rate $1/n$ over the n years of the economic life of the asset. This straight line method is a special case of the more general problem of accounting for the recovery or “return of capital” (as opposed to the “return on capital”). This straight line method is not preferred because it basically assumes an investor will set aside a return of capital each year, equal to $1/n$ th the value, and these annual deposits will bear no interest (the old “under the mattress” approach).

The Inwood Premise

In this method the recovery of capital takes the form of an annuity which accumulates at the *same rate* as the return on the capital (k). The return *on* the capital and the return *of* the capital are blended in an annuity payment of:

$$\text{Annual Recovery} = V_B \times \frac{1}{s\llbracket n, k \rrbracket}$$

Once again, using our example and an assumed rate of 9%, the annual recovery of \$3,405.89 is found as:

$$\begin{aligned} \text{Annual Recovery} &= \$100,000 \times \frac{1}{s\llbracket 15, 9\% \rrbracket} \\ &= \$3,405.89 \end{aligned}$$

Note that in the Inwood approach the rate used for the return *of* capital is $k\%$, the same rate as we use for the return *on* capital.

Is the Inwood or Hoskold method the best? On theoretical grounds one can defend the Hoskold method but the Inwood method is much more commonly used and easier (which may explain why it is more commonly used). Perhaps if we extend the analysis into the next step in the valuation process some further light will be shed on the issue. The next step is to incorporate each depreciation method into the valuation model itself and in doing so, we can observe how the capitalized value of the assets should be affected by the choice of a particular recovery scenario (Hoskold's or Inwood's). If the share of net operating income attributable to the building component is noted NOI_B ,¹⁴ we derive the capitalized value of this building by the direct capitalization technique where the capitalization rate should account for the expected return *on* the capital (k) and the return *of* the capital, i.e., the depreciation allowance.

Generally, the value of the building V_B can be written as:

$$V_B = \frac{\text{NOI}_B}{R} \quad \text{or} \quad \frac{\text{NOI}_B}{k + \text{depreciation}}$$

Under the alternative models, the recovery adjustment to the capitalization rate will be:

$$= 1/s\llbracket n, r \rrbracket \quad \text{in the Hoskold method}$$

$$= 1/s\llbracket n, k \rrbracket \quad \text{in the Inwood method}$$

Substituting these into the above equation we get the following two valuation formulae:

$$(1) \quad V_B = \frac{\text{NOI}_B}{k + \frac{1}{s\llbracket n, r \rrbracket}} \quad (\text{Hoskold})$$

¹⁴ Just how to split net operating income between land and building is another matter to be discussed later in this chapter.

$$(2) \quad V_B = \frac{NOI_B}{k + \frac{1}{s[n, k]}} \quad (\text{Inwood})$$

Before moving forward, let's take a closer look at what is happening. Some numbers in an example will help.

Example 9.1

Assume we have an asset which has an unknown building value of V_B . The net operating income attributed to the building is \$60,000 per annum. The building has an expected life of 20 years. The required rate of return is 12% and a *safe rate* for recovery of capital is considered to be 10%.

Using the two depreciation models, find V_B , the value of the building.

$$\begin{aligned} NOI_B &= \$60,000 \\ n &= 20 \text{ years} \\ k &= .12 \text{ or } 12\% \\ r &= .10 \text{ or } 10\% \end{aligned}$$

The Hoskold Sinking Fund Method

Let us first consider the Hoskold method. We can find V_B (the value of the building), then re-examine it.

$$V_B = \frac{NOI_B}{k + \frac{1}{s[n, r]}} = \frac{\$60,000}{.12 + \frac{1}{s[20, 0.10]}}$$

Readers may use a financial calculator to find the value of $s[20, 10\%]$ (the future value of an annuity of \$1 for 20 periods, compounded at 10%; see Appendix 9.2).

$$V_B = \frac{\$60,000}{0.12 + \frac{1}{57.27}}$$

$$V_B = \frac{\$60,000}{0.12 + 0.01746} = \frac{\$60,000}{0.13746}$$

$$V_B = \$436,490.62$$

Let's now work backwards to check out the model. If we pay \$436,490.62 for the building and we want to recover this amount over a 20 year expected life, how much must we set aside each year if our recovery fund will grow at 10%?

We want to make 20 annual deposits to an account which grows at 10% compounded per annum and will grow to \$436,490.62.

$$FV = \text{Deposit} \times s_{\overline{n}|i\%}$$

$$\$436,490.62 = \text{Deposit} \times s_{\overline{20}|10\%}$$

$$\text{Deposit} = \$436,490.62 \times \frac{1}{s_{\overline{20}|0.10}}$$

$$\text{Deposit} = \$436,490.62 \times \frac{1}{57.277}$$

$$\text{Deposit} = \$436,490.62 \times 0.017459$$

$$\text{Deposit} = \$7,620.69$$

Hence if we deposit \$7,620.69 each year for 20 years and it grows at 10% per annum, we will have a future value of \$436,490.62.

Will we still earn 12%? Let's check. We pay \$436,490.62; receive \$6,000 each year for 20 years but must set aside \$7,620.69 each year. However, after 20 years we get our recapture account which by then has \$436,490.62. Therefore, our yield is:

$$\$436,490.62 = (\$6,000 - \$7,620.69)a_{\overline{20}|i\%} + 436,490.62(1 + i)^{-20}$$

and $i = \text{internal rate of return} = 12\%$ (our required rate $k\%$).

This method is summarized in Table 9.6.

Table 9.6 The Hoskold Sinking Fund Hypothesis					
Years	V_B	Return <i>on</i> the Capital	Return <i>of</i> the Capital	Accumulated Return of Capital	NOI _B
1	\$436,490.62	\$52,379.31	\$7,620.69	\$ 7,620.69	\$60,000
·	·	52,379.31	7,620.69	16,003.45	60,000
·	·	52,379.31	7,620.69	25,224.48	60,000
·	·	·	·	·	·
20	0	\$52,379.31	\$7,620.69	\$436,490.62	\$60,000
\$152,413.80					

Note that the accumulated return of capital increases each year by the amount of the deposit (\$7,620.69) plus one year's interest on the previous deposits. Hence, after two years the accumulated total is:

$$\$7,620.69 + \$7,620.69 (1 + .10)^1 = \$16,003.45$$

Over the 20 years you deposit \$152,413.80 (20 x \$7,620.69), but the compound interest increases this to \$436,490.62.

We can now examine the third method of handling recovery of capital, and then try to generalize this issue.

The Inwood Sinking Fund Method

The Inwood approach to the return of capital is to assume that the return of capital can be recovered at the same rate as the required return on capital. Therefore, in our example, the value of the building is:

$$V_B = \frac{NOI_B}{k + \frac{1}{s[n, k]}} = \frac{\$60,000}{.12 + \frac{1}{s[20, 0.12]}}$$

$$V_B = \frac{\$60,000}{.12 + \frac{1}{72.0524}} = \frac{\$60,000}{.12 + 0.01387}$$

$$V_B = \frac{\$60,000}{0.13387}$$

$$V_B = \$448,196.01$$

Let's rework this method backwards to see how it compares with the Hoskold method.

If we pay \$448,196.01 and expect our recovery of capital to grow at 12% per annum, how much must we set aside each year for 20 years?

$$FV = \text{Deposit} \times s[n, i\%]$$

$$\$448,196.01 = \text{Deposit} \times s[20, 12\%]$$

$$\text{Deposit} = \$448,196.01 \times \frac{1}{s[20, 12\%]}$$

$$\text{Deposit} = \$448,196.01 \times 0.013878$$

$$\text{Deposit} = \$6,220.06$$

And if we deposit \$6,220.06 into our recovery account, which grows at 12%, after 20 years we will have \$448,196.01. In the meantime our "return on capital" each year is \$5,377.96 (\$6,000 - \$622.04).

Will we earn our required 12%? Let's check.

$$\$448,196.01 = (\$60,000 - \$6,220.06)a[20, i\%] + \$448,196.01(1 + i)^{-20}$$

$$\$448,196.01 = \$53,779.94a[20, i\%] + \$448,196.01(1 + i)^{-20}$$

and $i = \text{the internal rate of return} = 12\%$

The pattern of cash flows for this method is summarized in Table 9.7.

Table 9.7
The Inwood Sinking Fund Hypothesis

Years	V_B	Return <i>on</i> the Capital	Return <i>of</i> the Capital	Accumulated Return of Capital	NOI _B
1	\$448,196.01	\$53,779.94	6,220.06	\$6,220.06	\$60,000
2	441,975.95	53,779.94	6,220.06	13,186.53	60,000
.	.	53,779.94	6,220.06	20,988.97	.
.
.
20	0	\$53,779.94	\$6,220.06	\$448,196.01	\$60,000

Now we are in a position to summarize these two methods of handling return of capital.

The *only* difference between the Hoskold and the Inwood method is the choice of the rate of earnings to be applied to the annual deposits of the return of capital:

- the Hoskold method uses a *safe rate* to accumulate the return of capital, then $k\%$ to find the value;
- the Inwood method uses the required rate of return ($k\%$) for both the accumulation of the return of capital and to find the value.

It is for these reasons that the Hoskold method is frequently called the *dual rate method* (for the two rates in the analysis) and the Inwood method is called the single rate method. Readers are perhaps more accustomed to seeing the Inwood method in a slightly different form where:

$$V_B = \text{NOI}_B \times a[\overline{n}, k]$$

In other words, V_B is equal to the present value of an annuity of NOI_B for “ n ” periods, discounted at $k\%$ (a simple annuity problem). However, the equation above can be rewritten as:

$$V_B = \text{NOI}_B \times \left[\frac{1 - (1 + k)^{-n}}{k} \right]$$

We can return to our original example involving the leasehold interest and now relate it to the Inwood method. The original leasehold example involved a prepaid lease, costing \$100,000, which produced an income of \$20,000 per annum for 20 years on a sublet basis. The yield was found to be 19.42% calculated as above:

$$\$100,000 = \$20,000 s[\overline{20}, i\%]$$

$$i\% = 19.42\%$$

Where is the recovery of capital? This Inwood approach implicitly assumes the investor can recover capital at the same rate they earn on the investment (in this case 19.42%). Let’s check. We want to find an annual deposit which, made for 20 years and growing at 19.42%, will accumulate to \$100,000.

$$\$100,000 = \text{Deposit} \times s_{\overline{20}|19.42\%}$$

$$\text{Deposit} = \frac{\$100,000}{s_{\overline{20}|19.42\%}}$$

$$\text{Deposit} = \$574.61$$

If the investor sets aside \$574.61 per year for 20 years, it will grow to \$100,000 (at 19.42%). Will this still leave the investor with a yield of 19.42% on the original investment?

$$\$100,000 = (\$20,000 - \$574.61) a_{\overline{20}|i} + \$100,000(1 + i)^{-20}$$

$$i = \text{internal rate of return} = 19.42\%$$

Therefore the investor earns 19.42% (and accumulates capital at 19.42%) with the Inwood annuity factor.

Which is Best? Hoskold or Inwood

We earlier posed the question as to which method is best? Inwood is used more often, and simpler, but is it best? Note that the periodic recovery of capital is assumed to be reinvested in either method: Inwood reinvests at the required rate of return while Hoskold uses a (lower) safe rate.¹⁵ Keep in mind that the recovery rate is the rate at which the fund will grow; if you pay income tax, this will be lower than the rate you earn on the recovery fund [$g = (1-t)i$] where i is the rate you earn, t is the tax rate, and g is the rate the fund grows. Moreover, this accumulation rate for the recovery fund must be available each year and for sums of money which are considerably smaller than the original investments. For these reasons, using a rate of growth on the recovery fund which is less than the expected return on the original investment is at least conceptually superior.

Applying Residual Techniques

Freshly equipped with these new little methodological gadgets, we may now come back to our main goal in this chapter, to split the value “physically” and then to value each slice of value separately. Depending on whether we have information on NOI_B (the net operating income from the building), on NOI_L (the net operating income from the land), or only on the total net operating income and the probable reversion value of land, we may choose among three appropriate valuation procedures: the land residual method, the building residual method, and the property residual method.

No matter which residual technique is being applied, an appraiser begins the valuation process with the value of the known element and the net operating income. The generic residual process is represented in the following steps:

¹⁵ It would not be logical to assume reinvestment at a rate in excess of the required rate, otherwise you would skip the original investment and put all your money in the recovery fund.

1. Apply a reliable capitalization rate to the value of the known component to determine the proportionate income required to support that component.
2. Subtract the income derived in step 1 from the NOI to determine the residual income attributable to support investment in the unknown component.
3. Calculate the present value of the residual component by capitalizing that residual income (step 2) using a capitalization rate appropriate to the unknown component.
4. Add the known component value to the residual component value to arrive at an estimated value for the entire property.

The land residual method, the building residual method, and the property residual method will each be briefly illustrated.

The Land Residual Method

The procedure for the land residual method is as follows:

- 1st Step: Estimate the value of the building V_B (any approach will do: income, cost, or market).
- 2nd Step: Allocate the total NOI for the property to the building (NOI_B) and to the land (NOI_L).
- 3rd Step: Estimate the value of the land (V_L) through the direct capitalization of NOI_L . The value of the land is the capitalized value of the residual NOI_L .
- 4th Step: Finally, you obtain the full value through the identity $V = V_B + V_L$

Land Residual Method – Basic Application

The following example illustrates the basic application of the land residual method, before considering the Hoskold or Inwood formula. We need to estimate the contributory value of the land in order to estimate the value of the property; therefore, we must calculate the residual income to the land and capitalize it to provide an indication of the land value.

Estimated building value		\$545,000
Net Operating Income (Property)	\$67,500	
Income attributable to building (at a building capitalization rate of 10.0% → \$545,000 x 10.0%)	- \$54,500	
Residual Income to Land	\$13,000	
Capitalized land value (at a land capitalization rate of 6.5% - \$13,000 / 6.5%)		+ \$200,000
Indicated property value		\$745,000

Example adopted from *The Appraisal of Real Estate, 3rd Cdn. Ed., Ch. 22.*

Source of the various inputs above:

1. Estimated building value – depreciated cost estimate of the building (assumed to represent the contributory of the building to the market value of the property). This is most reliable for newer buildings with minimal depreciation.
2. Net Operating Income – income/expense analysis and stabilization for the subject property.
3. Building capitalization rate – return *on* the investment in the building (normally, the same as the safe rate of interest, the rate applied to the land) plus return *of* the investment in the depreciating asset. If the building has a remaining economic life of, say, 28.5 years in our example, then the building depreciates $1/28.5$ per year, or 3.5% per year. This is the return *of*.
4. Land capitalization rate – the safe rate of return on monies invested in real estate. Normally, the rate indicated by analyzing land (or ground) rents, or other appropriate market indicators. This is the return *on*.

What is the relationship between the building capitalization rate, the land capitalization rate, and the overall capitalization rate?

In our example, the land capitalization rate (or land interest rate) is 6.5%. The building capitalization rate equals the land capitalization rate plus the rate of recapture. In this case, $6.5\% + 3.5\% = 10.0\%$. If the market indicates a land value to building value ratio of 1:3 (land to property value ratio would be 1:4, and the building to property value ratio would be 3:4) then the land is worth 25% of the property value and the building is worth 75% of the property value. So, the overall rate is the result of weighting the land and building capitalization rates as follows: $25\% \times 6.5\% + 75\% \times 10.0\% = 9.125\%$.

Example 9.2

V_B	=	\$400,000
NOI	=	\$70,000
k	=	0.12
r	=	0.10
n	=	20 years

1st Step: We assume here that the value of the building is \$400,000 obtained from a market analysis of comparable units.

2nd Step: Allocate NOI between NOI_B and NOI_L . We must choose one of the two depreciation methods and invert the computations presented previously. Thus alternatively we have:

$$\begin{aligned}
 NOI_B &= V_B(k + 1/s[n, r]) \\
 &= \$400,000(0.12 + 0.0174596) \\
 &= \$54,984 \text{ (sinking fund Hoskold form)}
 \end{aligned}$$

$$\begin{aligned}
 NOI_B &= V_B(k + 1/s[n, k]) \\
 &= \$400,000(0.12 + 0.0138788) \\
 &= \$53,552 \text{ (sinking fund Inwood form)}
 \end{aligned}$$

Now we can derive the net operating income for the land: $NOI_L = NOI - NOI_B$

Note that we have a choice of two values:

$$\text{NOI}_L = \$70,000 - \$54,984 = \$15,016$$

$$\text{NOI}_L = \$70,000 - \$53,552 = \$16,448$$

3rd Step: We capitalize one of the previous net operating incomes for the land (NOI_L) at the rate of return (k) to find the capital value of the required lot (V_L).

$$V_L = \frac{\$15,016}{0.12} = \$125,133$$

$$V_L = \frac{\$16,448}{0.12} = \$137,067$$

4th Step: Finally, we have two alternative total values from the identity:

$$V = V_B + V_L$$

$$V = \$400,000 + \$125,133 = \$525,133$$

$$V = \$400,000 + \$137,067 = \$537,067$$

The same answers could be obtained directly from the following formulae (see Table 9.8 also):

(a) In the Hoskold sinking fund case:

$$V = V_B + \frac{\text{NOI} - V_B(k + 1/s \llbracket n, r \rrbracket)}{k}$$

$$V = \$400,000 + \frac{\$70,000 - \$54,984}{0.12}$$

$$V = \$400,000 + \$125,133 = \$525,133$$

(b) In the Inwood sinking fund case:

$$V = V_B + \frac{\text{NOI} - V_B(k + 1/s \llbracket n, k \rrbracket)}{k}$$

$$V = \$400,000 + \frac{\$70,000 - \$400,000(.12 + .0138788)}{0.12}$$

$$V = \$400,000 + \$137,067 = \$537,067$$

Table 9.8
The Land Residual Procedure

Depreciation Hypothesis	Hoskold	Inwood
V_B	\$400,000	\$400,000
k	.12	.12
Depreciation factors	$1/s[20,.10] = 0.0174596$	$1/s[20,.12] = 0.0138788$
NOI	\$70,000	\$70,000
NOI_B	<u>54,984</u>	<u>53,552</u>
NOI_L	\$15,016	\$16,448
V_L	\$125,133	\$137,067
$V = V_L + V_B$	\$525,133	\$537,067

Applicability of the Land Residual Procedure

Two obvious and immediate problems in this method come to mind: first, we must rely on some estimation of the value of the building (V_B) and second, why not find the overall value (V) using the income method and simply deduct the value of the building to get the land residual?

In principle, we could trust the cost approach to help us determine the value of the building. But, unless we deal with a brand new construction on which we have good information we know how carefully we should handle values obtained via the cost approach. In practice, this technique could nevertheless be vaguely justified when the value of the improvements is small compared to the value of the land or when the improvements are new; hence, depreciation is not a major consideration (e.g., golf courses or cemeteries.) In such cases, any error on the building value (V_B) will be harmless and almost the entire NOI will be allocated to land values in order to derive V_L . Here V_L would be a residual, but nevertheless the most important component of value.

This explains in part, why this method of finding a land residual is generally limited to situations where the improvements have relatively little value (which may include properties ripe for development) or where an assumed or actual redevelopment is about to occur and reliable cost estimates are available. These limiting conditions suggest when the land residual is best applied and explain why it is often called the “development method” of appraisal. This is illustrated in detail later in this chapter.

Our second problem is of greater concern. If we know or can estimate the full net operating income and the value of the improvements, why not simply capitalize the full net operating income, using a familiar income method of valuation, and deduct the value of the improvements to arrive at a land residual?

$$V = \frac{NOI}{R} = \frac{\$700,000}{0.13} = \$538,462$$

where $R = 13\%$ is the market determined overall discount rate and

$$V_L = V - V_B$$

$$V_L = \$538,462 - \$400,000$$

$$V_L = \$138,462$$

This simplified approach is, in fact, the more common direct approach for finding either a land or building residual.

The Building Residual Method

The procedure to be followed on the building residual is identical to those for the land residual, but with a different unknown.

- 1st Step: The value of the site (V_L) is given, assumed, or presumably obtained from market valuation.
- 2nd Step: Allocate the total NOI between the land (NOI_L) and to the building (NOI_B).
- 3rd Step: Derive the value of the building (V_B) through the direct capitalization of NOI_B . (The value of the building is the capitalization of the residual NOI_B .)
- 4th Step: Finally, the full value of the property is $V = V_B + V_L$.

Building Residual Method – Basic Application

The following example illustrates the basic application of the building residual method, before considering the Hoskold or Inwood formula. We need to estimate the contributory value of the building in order to estimate the value of the property; therefore, we must calculate the residual income to the building and capitalize it to provide an indication of the building value.

Estimated land value		\$200,000
Net Operating Income (Property)	\$67,500	
Income attributable to land (at a land capitalization rate of 6.5% → \$200,000 x 6.5%)	- \$13,000	
Residual Income to Land	\$54,500	
Capitalized land value (at a building capitalization rate of 10.0% → \$54,500 / 10.0%)		+ \$545,000
Indicated property value		\$745,000

Example adopted from *The Appraisal of Real Estate, 3rd Cdn. Ed.*, Ch. 22.

Example

V_L	=	\$20,000
NOI	=	\$70,000
k	=	0.12
r	=	0.10
n	=	20 years

This example is summarized in Table 9.9 where we observe again that the choice between depreciation hypotheses gives us a choice of two possible valuations for the property. We do not need to lead the reader through each step of the example so we can present the direct solutions formally.

(a) The Hoskold sinking fund case:

$$V = V_L + \frac{NOI - (V_L \times k)}{k + (1/s[[n, r]])}$$

$$V = \$20,000 + \frac{\$70,000 - (20,000 \times .12)}{0.12 + 0.0174596}$$

$$V = \$20,000 + \$491,780.86 = \$511,780.86$$

(b) The Inwood sinking fund case:

$$V = V_L + \frac{NOI - (V_L \times k)}{k + (1/s[[n, k]])}$$

$$V = \$20,000 + \frac{\$70,000 - (\$20,000 \times .12)}{0.1338788}$$

$$V = \$20,000 + \$504,934.31 = \$524,934.31$$

Table 9.9		
The Building Residual Procedure		
Depreciation Hypothesis	Hoskold	Inwood
V_L	\$20,000	\$20,000
k	0.12	0.12
Depreciation factors	$1/s[[20, .10]] = 0.0174596$	$1/s[[20, .12]] = 0.0138788$
NOI	\$70,000	\$70,000
$\underline{NOI_L}$	<u>2,400</u>	<u>2,400</u>
$\underline{NOI_B}$	\$67,600	\$67,600
V_B	\$491,780.86	\$504,934.31
$V = V_L + V_B$	\$511,780.86	\$524,934.31

Applicability of the Building Residual Method

Since “pure” market land values may be easier to obtain than pure building values, this technique may seem more appealing than the previous one. But, in fact, the same awkward problem remains: on what basis can we seriously justify the allocation of the income between land and building?

The Property Residual Technique

This technique would typically apply to the appraisal of the development value of a site not used at its “highest and best” potential. We have the existing NOI and we assume an expected reversion value for the site:

- 1st Step: NOI is known for the existing property.
- 2nd Step: The total value of the property based on its current use is obtained through the capitalization of NOI.
- 3rd Step: The reversion value of the site is assumed and the present value is computed.
- 4th Step: The formally computed value is now adjusted to account for the reversion value of the site.

Example 9.3

NOI = \$50,000 for 3 years
 k = 0.12
 r = 0.10
 n = 3 years
 Resale V_L = \$500,000 in 3 years.

This example is summarized in Table 9.10.

The direct solutions are now:

- (a) The Hoskold sinking fund case:

$$V = \frac{\text{NOI}}{k + 1/s[[n, r]]} + \frac{V_L}{(1+k)^n}$$

$$V = \frac{50,000}{0.12 + .3021148} + \frac{500,000}{1.404928}$$

$$V = 118,451.19 + 355,890.12 = \$474,341.31$$

- (b) The Inwood sinking fund case:

$$V = \frac{\text{NOI}}{(k + 1/s[[n, k]])} + \frac{V_L}{(1+k)^n}$$

$$V = \frac{\$50,000}{0.416349} + \frac{\$500,000}{1.404928}$$

$$V = \$120,091.56 + \$355,890.12 = \$475,981.68$$

Table 9.10
The Property Residual Procedure

Depreciation Hypothesis	Hoskold	Inwood
r	0.1	0.1
k	0.12	0.12
Depreciation factors	$1/s[3, 0.10] = 0.3021148$	$1/s[3, 0.12] = 0.2963490$
NOI	\$50,000	\$50,000
V _L (Resale)	\$500,000	\$500,000
V	\$474,341.31	\$475,981.68

Applicability of the Property Residual Method

As we said above, the only justifiable use of this method is when one deals with a redevelopment project.

The existing building generates a short term (and sub-optimal) stream of income and you have some reliable information on the full potential value of the site and on the timing of the project. The earlier the redevelopment, the more reliable the appraisal will be.

Conclusion: Residual Methods

The general income capitalization approach may look like a naive way to handle realistic valuation problems. We have nevertheless demonstrated that there is more than meets the eyes behind the very simple $V = NOI/R$ model. We have reviewed the principal implicit assumptions and concluded that, when properly handled, this model should give reliable answers. Table 9.11 provides a synopsis of the income approach, with residual approaches considered.

While the residual techniques may give an illusion of sophistication, in fact they raise difficult empirical problems and they should be used parsimoniously under quite specific conditions. The empirical problem of “properly handling” the computation of the capitalization rate is not a trivial one and unfortunately not much guidance can be offered here. The brief review in this chapter of the “physical split” family of techniques should have been sufficient to drive the message home: user beware!

Application: Developer's Residual Method of Appraisal

So far, this chapter has described the theory underlying the residual method in great detail, but has not provided any examples which illustrate its practical applications. The method which is often used by real estate developers to determine the value of land is an application of residual method called developer's residual analysis, or the development method of appraisal.

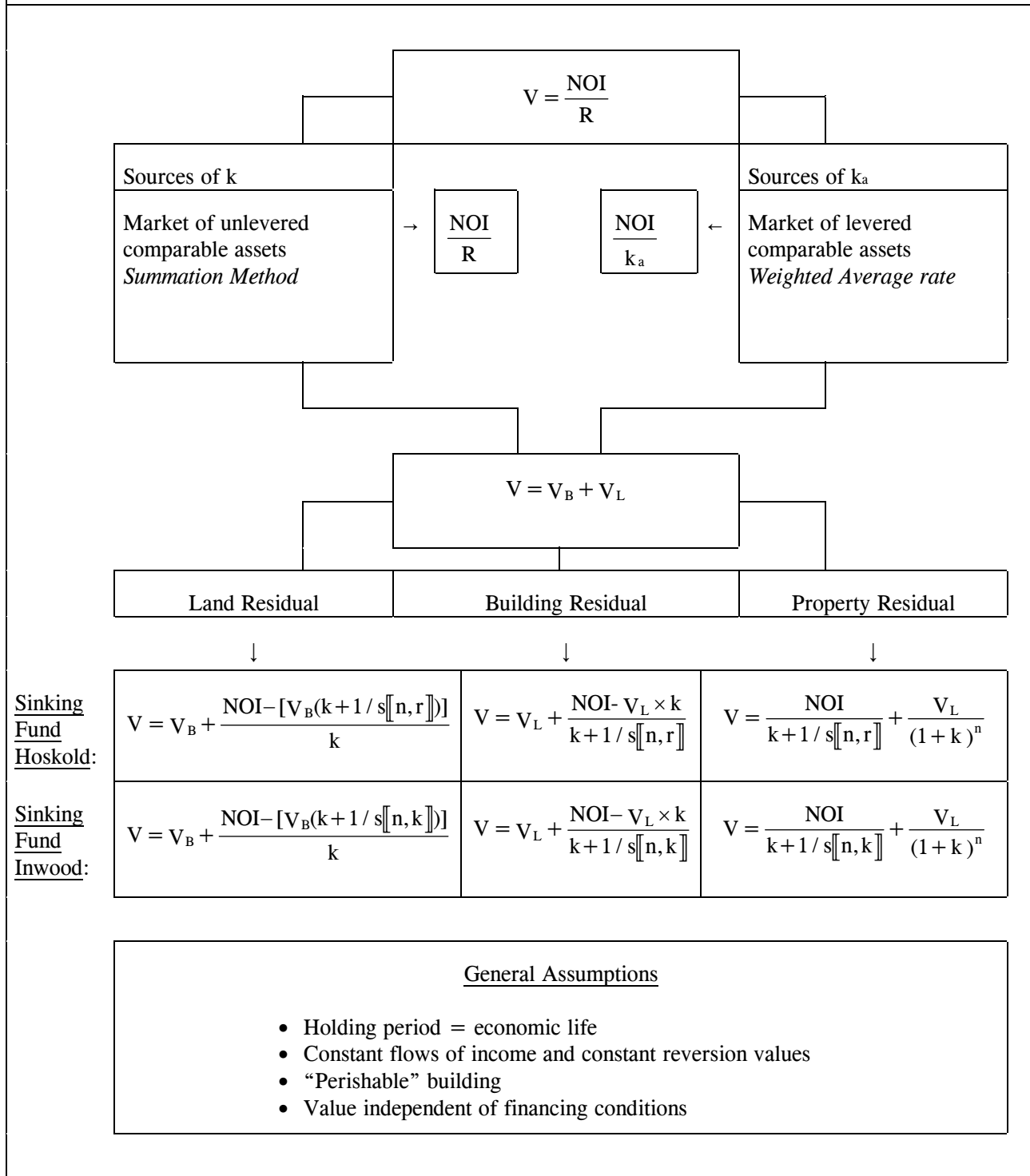
Conceptual Basis

Land is valuable because of the utility it provides people. In its natural unimproved state, land has little value. For example, millions of square miles of land in Northern Canada have virtually no economic value because they are of no economic use. However, the parcels that are demanded for some useful purpose, such as hydroelectric dams, access to timber, oil, or mineral resources, or fishing lodges do have value. The problem facing urban land economists and appraisers is how to determine the value of these developable parcels of land.

There are several methods of determining the value of developable land. Perhaps the simplest method is to compare the plot of land to other similar plots which have recently sold, and assume that the plot in question should sell for approximately the same price. For example, if you are valuing an undeveloped recreational property and there are many recent sales of properties which are similar, these sale prices could represent an accurate estimate of value. This is an application of the direct sales comparison approach and can operate very efficiently, as long as there are sales of similar properties for comparison. However, it can be difficult to apply this approach if the property in question is unique or if there are no similar sales. For example, if the property above was very large, or had a unique view, or was in an isolated area, it is possible that no similar properties could be found to compare to, and it would be difficult to find a justifiable value estimate.

This problem of unique parcels of land is particularly relevant for valuing potential commercial developments in urban areas. These properties are often unique in size and attributes, and each alternative use has its own subset of desirable features. Two lots which are in the same neighbourhood and are identical in size may have very different values if one has the attributes required for a shopping centre, while the other is best suited for a warehouse. In these cases, it is often difficult to find sales which are similar enough to the property in question to be able to infer value. The direct sales comparison method cannot be used to value many of these properties.

Table 9.11
The Income Approach: A Synopsis



Rather than examining the sales of other lots to determine the value of a particular piece of land, the development method focuses the appraisal on the property in question. Try to view the situation from the perspective of a potential developer. If you were planning to buy this piece of land in order to build something and then sell the property for a profit, how much would you be willing to pay for the land? In this sense, the maximum you would pay for this land would be just enough so that the land cost plus the cost of improving the land exactly equals the expected proceeds of selling the property (of course, the cost of improving the property would have to include your required profit as the developer). Your maximum payment for the land is therefore the amount left over after paying all other costs associated with the development. This is the basis of the development method of appraisal; the price of land is determined by what developers of end-use products can afford to pay after accounting for all costs of development.

The value of land under this appraisal method is therefore a residual amount resulting from the improvement of land. Any improvement that increases the value of the land's final use increases the land residual. For example, if house prices are increasing, with other costs remaining constant, land prices should rise. Similarly, anything that raises the cost of development lowers the land residual, and lowers the value of land. The cost of construction is therefore directly related to the value of land. Interest rates are also directly related, since financing charges form part of the cost of development.

An underlying assumption of urban land markets is that land should be used for the activity which yields the greatest utility, that is, the use which generates the highest rent. The residual method operates under this assumption as well. For any particular parcel of land, the better suited the improvements, the higher the potential residual should be. The developer who is proposing the project with the highest residual should be able to bid the highest for the land and be able to implement this end use. Thus, the land will be used for the best suited project, with the highest possible utility, meeting the requirement for the land to be in its highest and best use.

Development Process

The calculation of residual values depends on estimates of future revenues, expenses, and risk. The development residual method entails forecasting cash flows into the future, and can involve considerable complexity. The strength of the end-use market has a great effect on value. Consumer attitudes, interest rates, and the general state of the economy affect what people will pay, which affects sale prices and filters down to the land residual. A large part of this appraisal process is to assess the trends in end-use markets in order to determine the maximum future sale proceeds.

The first step in examining a potential development is to determine what type of end-use would be most suitable for the land in question. The developer should examine the site, check the zoning, and try to determine what is the highest and best use of the property. Next, the developer must decide roughly what the improvements will consist of. Conceptual plans should be drawn and estimates made of the saleable space which will be created.

Once the form of improvements has been decided on, with rough estimates of the general size, the value of the property upon completion must be calculated. This value can be found by capitalizing the expected future income stream at the market capitalization rate (income method of appraisal). This amount represents the maximum cash flow which will be received from this project. If the developer purchases the land and undertakes this project, the profit made would depend on how much of this amount is remaining when the project is complete.

The next step is to determine the expenses necessary to develop the land into a new end use. Some examples of these costs include construction, real estate commissions, architect fees, financing charges, and developer's profit. After computing all of these expenses, and subtracting these from the expected income from the property, the remainder is the land residual.

Framework for Development Residual Method

The land residual can be calculated under several different frameworks, but all of these have a similar theme: calculate the income expectations for the developed land, subtract all expenses associated with this development, and the remainder is the land residual. The following is one common framework, with its components explained below.

Development Method Framework

- Gross value upon completion (1)
 - costs of sale (2)
- Net value upon completion
 - hard costs (3)
 - soft costs (4)
 - development financing costs (5)
 - developer's profit (6)
- Residual to land and land financing cost (7)
 - Land financing cost (8)
- Land residual (9)



Gross Value Upon Completion:

This is the forecast for the maximum cash flow that the property will generate when completed. This figure could be as simple as a rough guess at future sale prices, or it could involve complex discounted cash flow analysis. Some terminology used in this type of analysis includes:

- gross buildable area – the total area of the anticipated improvements;
- net saleable area – the portion of the gross buildable area which can be sold to generate revenue; and
- building efficiency – a measure of how much of the gross building area is saleable. The more efficient the building, the closer to 100% of the gross buildable area can be sold.

Some calculations of gross value upon completion include the net saleable area multiplied by an estimate of the value per square foot when sold, or net rentable area multiplied by the estimated rent per square foot (an estimate of future gross potential rent) and capitalized at a market capitalization rate.

Costs of Sale:

Closing costs are expenses required when selling real estate, and include items such as real estate commissions and legal fees. Marketing costs are also often included in costs of sale, as large development projects often require considerable promotional expense to facilitate their sale.

Hard Costs:

Hard costs represent the expenses directly associated with construction. The simplest method to calculate hard costs would be to estimate the expected construction cost per square foot and then apply this figure to the gross buildable area. More complex methods would involve estimating the cost of each component

separately (such as the cost per square foot to build the roof, exterior walls, foundation, and so on) or pricing the materials and labour needed for every portion of the planned development.

Soft Costs:

Soft costs are the overhead associated with the development process. Some examples of these include consultant's fees (such as architects and engineers), property taxes during construction, and interim financing costs. Soft costs are often expressed as a percentage of hard costs, although they can be forecasted and itemized in detail if required. In particular, the interim financing costs during construction generally require fairly detailed analysis, and will be dealt with below in a separate category.

Development Financing Costs:

Most development projects are undertaken using debt financing for a large portion of the development cost. The typical scenario is for a developer to pay a down payment on a piece of land, with a mortgage loan for the remainder of the purchase price, and then arrange a construction loan to supply funds for all expenses incurred between the time of purchasing the land and selling the completed development. The funds for this loan would be advanced periodically as cash flow is needed for the development. These advances could be based on a pre-set schedule of cash flow requirements or could be an open line of credit to a certain limit, or any other arrangement which is made with a lender. The loan would generally be interest accruing, requiring payment of principal plus interest at the end of the loan term. Many of these construction loans also have arranged long-term take out commitments for the end of the term to add security for the construction lender.

The calculations for this construction loan will generally involve fairly detailed present value analysis. The cash flow advances for each period will have to be accumulated and accrued interest calculated.

Developer's Profit:

The amount remaining after deducting all of the expenses above must be enough to cover the cost of purchasing land plus a profit for the developer. The required developer's profit could be stated in many ways. Some of the possibilities include a lump sum amount, a percentage of gross or net value on completion, or a percentage of total project cost. The latter method can involve a complicated calculation, as the developer's profit is based on total cost, which also includes land and land financing, both of which are unknown at this point.

Residual to Land and Land Financing Cost:

A developer must purchase a plot of land before a development project can be undertaken. In most situations, a developer will want to finance at least part of the price of the land, and will pay a portion as a down payment (the developer either does not have enough money today to pay for the entire land purchase price, or wants to take advantage of financial leverage). This loan will accrue interest over the development period until the project is built and sold, when there will be proceeds to repay the loan amount plus interest.

The residual to land and land financing cost is the amount of proceeds remaining from the sale of the completed project once all expenses associated with developing the land (including profit) have been deducted. This residual amount will consist of accumulated interest on the land loan (if financing was used), plus the future value of the land cost.

Land Financing Cost:

The interest on the land loan should be a simple calculation, as it is just an interest accruing loan. However, the calculation is complicated because the land cost is still unknown (and is in fact what we are trying to find). The interest on this loan has to be expressed algebraically, in terms of the unknown land variable. If

you are working under the assumption that no down payment is provided by the developer and the purchase price of the land is provided 100% by debt financing, this makes the calculation slightly easier. However, it is not a very realistic assumption, as most development projects will require at least some equity.

The following illustrates how this financing cost could be expressed using formulas:

$$\begin{aligned}\text{Loan balance at end of term} &= \text{original loan amount} + \text{interest} \\ &= \text{original loan amount} \times (1+i)^n \\ &= (\text{total land cost} - \text{equity}) \times (1+i)^n \\ \\ \text{Interest on land loan} &= \text{loan balance at end of term} - \text{original loan amount} \\ &= [(\text{total land cost} - \text{equity}) \times (1+i)^n] - (\text{total land cost} - \text{equity}) \\ &= (\text{total land cost} - \text{equity}) \times [(1+i)^n - 1]\end{aligned}$$

Land Residual:

The land residual is the amount remaining when all expenses of the development project other than land have been covered. This residual would be used to repay the principal of any financing used to purchase the land, together with a repayment of the initial equity invested (the interest on the land loan and the return on the developer's equity have already been accounted for above). To determine the maximum amount that should be bid for a plot of land today by the developer, the present value of this land residual should be calculated. However, what makes this calculation difficult is that the land residual figure is expressed as a formula, not as a dollar figure. To calculate the present value of this requires solving for the unknown in the discounting formula.

- *Timing of Cash Flows:* In developing a property, there are two components to the time frame from purchase of the property to completion of the sale of (all) the property. The time frame establishes the investment horizon, or holding period to be used as the basis for discounted cash flow calculations.
- *Development Time:* The development of time is the period from purchase to completion of all the work necessary to sell all the completed project.
- *Absorption Period:* The absorption period is the time required for the market to absorb the entire property, whether leasing out the project or selling all developed lots. Overlapping the development time and the absorption period will be the *marketing period* – the time frame from when the developer begins to market the finished product to the time when the last product has been sold.

Example: Development Residual Method

You have found a block of vacant land for sale which you believe would be a good site for an apartment building. You want to put in a bid, and as you believe that this is a highly desirable site for other developers, you want to ensure that your bid is the maximum possible. The following are your projections for this development:

- can build one hundred units, 1,500 square feet each (net saleable area of 150,000 square feet)
- the building efficiency will be 80%, with a gross buildable area of 187,500 square feet
- development will take 6 months to build and sell
- estimated sale price of each unit will be \$300,000
- closing costs will be 3% of gross value upon completion
- hard costs will be \$100 per square foot of gross buildable area
- soft costs (including property tax, consulting fees, and legal fees) will be 20% of hard costs

- cash for hard and soft costs will be advanced in 6 equal advances at the beginning of each month in the development process. Interest will accrue at a rate of $j_{12} = 9\%$ with all principal and interest due at the end of the 6 month construction phase.
- developer's profit will be 15% of the net value upon completion
- land purchase will be 70% financed, with an interest accruing loan at a rate of $j_{12} = 12\%$, with all interest and principal due at the end of 6 months.
- cash flows received in the future will be discounted at $j_{12} = 15\%$

Solution:

Gross value upon completion: 100 units @ \$300,000 per unit	\$30,000,000
- costs of sale (3%)	<u>900,000</u>
Net value upon completion	\$29,100,000
- hard costs: \$100 × 187,500 sq.ft.	18,750,000
- soft costs: 20% × \$18,750,000	3,750,000
- development financing cost*	598,063
- developer's profit: 15% of \$29,100,000	<u>4,365,000</u>
Residual to land and land financing cost	\$ 1,636,937
- land financing cost**	<u> ?</u>
Land residual ***	<u><u> ?</u></u>

To find the land financing cost and the land residual, the formulas shown in the following supporting calculations must be solved algebraically. Once the land residual is found, the maximum bid price for the land today can be calculated using present value analysis (based on a discount rate of $j_{12} = 15\%$).

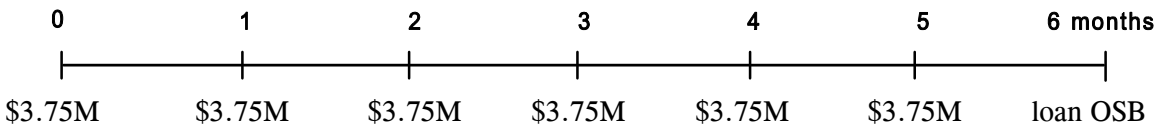
Supporting calculations:

***Development Financing Cost:**

Total development cost is \$18,750,000 hard costs plus \$3,750,000 soft costs. This amount will be paid out in 6 equal instalments at the beginning of each month.

$$= (\$18,750,000 + \$3,750,000) \div 6$$

$$= \$3,750,000$$



Interest owing at the end of 6 months:

Interest rate: $j_{12}=9\%$ (or $i_{mo}=0.75\%$)

$$FV = \$3,750,000 (1+0.0075)^6 + \$3,750,000 (1+0.0075)^5 + \$3,750,000 (1+0.0075)^4$$

$$+ \$3,750,000 (1+0.0075)^3 + \$3,750,000 (1+0.0075)^2 + \$3,750,000 (1+0.0075)^1$$

$$FV = \$23,098,063$$

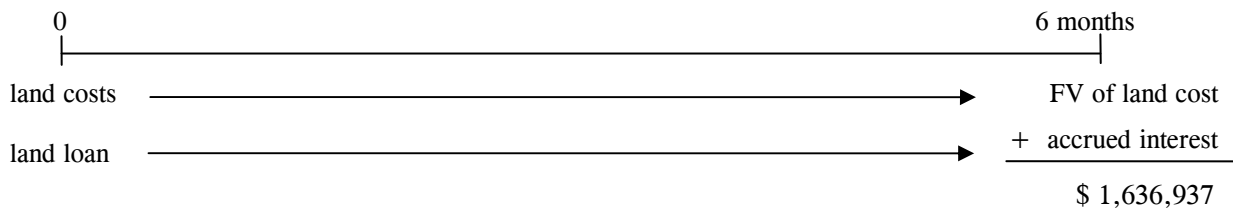
$$\text{Interest} = FV - PV = \$23,098,063 - 22,500,000 = \$598,063$$

****Land Financing Cost:**

Amount available for land and land financing = \$1,636,937

Purchase of land was 70% financed, therefore original land loan can be shown as 0.7 L
(where L = present value of total land cost)

Therefore, future value of land cost (equity + debt) plus accrued interest on land loan must equal \$1,636,937.



$$\begin{aligned}
 \text{Interest on land loan} &= (\text{total land cost} - \text{equity}) \times [(1+i)^n - 1] \\
 &= 0.7L \times [(1+0.01)^6 - 1] \\
 &= 0.0430641L
 \end{aligned}$$

The dollar amount of the land financing cost can be calculated once the land residual (and therefore the amount of the land loan) are calculated.

*****Land Residual:**

Residual to land and land financing cost = FV of land cost + accrued interest

$$1,636,937 = L \times (1+0.0125)^6 + 0.0430641L$$

$$1,636,937 = 1.0773832L + 0.0430641L$$

$$1.120447L = 1,636,937$$

$$L = 1,460,968$$

Since L is equal to the present value of the total land cost, the maximum bid price for the land today will be \$1,460,968.

Summary and reconciliation:

If the developer pays \$1,460,968 for the land today and the purchase is 70% financed, the financing will account for \$1,022,678. At an interest rate of $j_{12}=12\%$, this loan will accrue \$62,915 interest over 6 months.

$$\begin{aligned}
 j_{12} &= 12\%, \quad i_{mo} = 1.00\% \\
 \text{Interest} &= 1,022,678 [(1.01)^6 - 1] \\
 &= \$62,915
 \end{aligned}$$

Therefore, the funds available for land at the end of 6 months would be $\$1,636,937 - 62,915 = \$1,574,022$. The present value of this at a discount rate of $j_{12} = 15\%$ is equal to \$1,460,968.

Conclusion: Development Residual Method

The development residual method is a model which abstracts and simplifies reality. However, it is a fairly good approximation of investor behaviour, combining discounted cash flow techniques, and the income and the cost methods of appraisal. This development method is best used in situations where no applicable direct sales evidence can be found. This method requires extensive forecasting, with many assumptions required, such as absorption rates, discount rates, and market trends. Considerable research must be done in order to ensure that these estimates are as accurate as possible. In this model small changes in assumptions could have a large effect on value, so a sensitivity analysis may be useful to test the impact of the assumptions.

Summary of Chapter 9

This chapter built upon the income capitalization and discounted cash flow concepts illustrated in previous chapters, covering mortgage-equity and residual valuation techniques. These techniques isolate and separately consider the components of real property investments, either equity and financing or land and building. These techniques are not as commonly applied in day-to-day real estate practice as direct capitalization or DCF, but they are helpful in certain situations and are therefore another tool that may be applied by real estate practitioners in analyzing complex investment scenarios.

With this chapter concluded, this ends our coverage in this course of the techniques that specifically relate to the valuation of single properties. The final three chapters will examine other related real estate investment considerations, including risk analysis, portfolio issues, and the different ownership vehicles that can be used real estate investments.

APPENDIX 9.1

The Modified Band of Investment à la Ellwood

This technique was introduced in 1959 by L.W. Ellwood: a very innovative chief appraiser of the New York Life Insurance Company. His contribution has had a considerable influence on the appraising profession and was remarkably modern on some aspects of financial theory.

Ellwood's pre-computed tables and his mortgage-equity capitalization rate were a practical breakthrough in the pre-computer and pre-calculator days. Now the technique is less appealing but it still retains its aficionados among the more traditional appraisers.

We have been through Akerson's adaptation of Ellwood's method, thus the conceptual roadblocks have all been removed and we only have to deal with different notations and a slightly different equation.

Let us first tackle Ellwood's notational jungle with the help of Table 9.12.

Table 9.12		
Notational Equivalence		
<u>Ellwood</u>		<u>Meaning</u>
Y	k_e or EYR or IRR_e	The equity yield rate: an internal rate of return on the equity
I	k_d	The annual mortgage rate
f	f or $1/a[m, k_d]$	The mortgage constant
M	D/V	The mortgage to value ratio
P	$(1 - \%OSB)$	The % of mortgage principal paid off at the end of the holding price
app	$g \times V$	Value appreciation (g is positive)
dep	$g \times V$	Value depreciation (g is negative)
d	NOI	The expected net operating income
R	R	The capitalization rate

Using these notations Ellwood also defines a mortgage coefficient:

$$c = Y + P \times 1/s[n, Y] - f$$

This coefficient is therefore the equity yield adjusted for the equity build-up and the mortgage payment. In its most compact form the Ellwood's formula reads:

$$R = Y - MC \left[\begin{array}{c} + \text{dep} \\ - \text{app} \end{array} \right] (1 / s[n, Y])$$

In our previous notation it would be translated into:

$$R = k_e - \frac{D}{V} [k_e + (1 - \%OSB)1/s[[n,k_e]] - f] - [g \times 1/s[[n,k_e]]]$$

and, with a minor re-shuffling:

$$R = \left[\frac{D}{V} \times f\right] + \left(1 - \frac{D}{V}\right)k_e - \frac{D}{V} (1 - \%OSB)(1/s[[n,k_e]]) - [g \times 1/s[[n,k_e]]]$$

We are back to our previous Akerson formula where an eventual depreciation is also considered (g can be a positive or negative variable; if the property depreciates, g is negative, thus making this adjustment factor a positive addition to R). Note carefully (it may still be counter intuitive) that you subtract the appreciation adjustment factor (a smaller R implies a larger value) and you add the depreciation adjustment factor (a larger R implies a smaller value).

With his formula Ellwood also provided a special set of pre-calculated tables for all the required factors within a certain range of rates of returns and appreciation (depreciation) rates.

The following example will illustrate the step wise procedure for the use of Ellwood's Tables:

Hypotheses:

$$\begin{aligned} \text{NOI} &= \$32,250 \\ \text{D/V} &= 0.75 \\ k_d &= 5.5\% \text{ per annum, monthly compounding} \\ m &= 300 \text{ months} \\ g &= 0.15 \text{ (15\% depreciation of the property)} \\ k_e &= 0.11 \end{aligned}$$

You are required to find R , and then V from the Ellwood tables.

The answer can be quickly calculated.

First, obtain from the Table the overall discount rate before depreciation or appreciation (circled). (See sample page in Table 9.13).

- Find the Table for a 75%, 25-year mortgage.
- Find the column headed "5 1/2%", the mortgage rate.
- Find the 10-year holding period (a forecast by the appraiser).
- Find the basic rate before depreciation or appreciation = 0.0717.
- Find the appreciation or depreciation factor, in this case, 0.0598, and multiply it by the total appreciation or depreciation.

$$0.0598 \times 0.15 = 0.00897$$

- Add (or subtract) the appreciation or depreciation factor calculated in step (5) to (from) the basic rate to find the overall capitalization rate.

$$\begin{aligned}\text{Overall capitalization rate} &= 0.0717 \\ &+ 0.00897 \\ &= 0.08067\end{aligned}$$

- Finally, find the indicated market value.

$$\begin{aligned}\text{Market Value} &= \frac{\$32,250}{0.08067} \\ &= \$399,776.87\end{aligned}$$

$$\begin{aligned}\text{or Market Value} &= \$32,250 \times a[999, 8.067\%] \\ &= \$399,776.87\end{aligned}$$

On the basis of computational convenience, the use of Ellwood's tables can be ruled out despite their strong nostalgic appeal. The tables are not always available when you need them, they are awkward to use, they require interpolation for intermediate values of the variables, and may not cover the full range of variations. Ellwood's Tables also do not apply directly to Canadian mortgages with semi-annual compounding.

The choice of the Ellwood or Akerson formula is a matter of personal taste. The two formulas require the same calculations and they produce identical results. While Ellwood's formula has seniority, Akerson's formula has a pedagogical advantage.

Table 9.13
Ellwood's Table

CAP. RATES

75% Mtg. 25 Years

CAPITALIZATION RATES
Assuming 75% of Purchase Price Financed by Mortgage
25 YEARS AMORTIZATION: 4¼% TO 6%

Step 1 → Step 2

Interest Rate	4¼%	5%	5¼%	5½%	5¾%	6%	$\frac{1}{s[n]}$	
Annual Requirement (f)	.06852	.07020	.07200	.07380	.07560	.07740		
Coverage Min. Rate	.05139	.05265	.05400	.05535	.05670	.05805		
Projection	Balance (b)	.881633	.885523	.888782	.892087	.895437	.898833	+ Dep
	Equity Yield	Basic Rate before Depreciation or Appreciation						- App
5 Years n = 5	5% .05	.0479	.0497	.0515	.0533	.0551	.0569	.1810
	6% .06	.0507	.0525	.0543	.0561	.0579	.0597	.1774
	7% .07	.0535	.0553	.0571	.0589	.0606	.0624	.1739
	8% .08	.0563	.0581	.0599	.0616	.0634	.0652	.1705
	9% .09	.0591	.0609	.0626	.0644	.0662	.0679	.1671
	10% .10	.0619	.0637	.0654	.0671	.0689	.0707	.1638
	11% .11	.0647	.0664	.0683	.0699	.0716	.0734	.1606
	12% .12	.0675	.0692	.0710	.0726	.0744	.0762	.1574
	13% .13	.0703	.0720	.0738	.0754	.0772	.0789	.1543
	14% .14	.0730	.0747	.0765	.0782	.0799	.0816	.1513
	15% .15	.0758	.0774	.0792	.0809	.0826	.0843	.1483
	16% .16	.0785	.0802	.0820	.0836	.0853	.0871	.1454
	17% .17	.0813	.0830	.0847	.0864	.0881	.0898	.1426
18% .18	.0840	.0857	.0874	.0891	.0908	.0925	.1398	
19% .19	.0968	.0885	.0901	.0919	.0935	.0952	.1371	
20% .20	.0896	.0912	.0929	.0945	.0962	.0979	.1344	
	Balance (b)	.731604	.738608	.744262	.750105	.756141	.762375	+ Dep
	Equity Yield	Basic Rate before Depreciation or Appreciation						- App
10 Years n = 10	5% .05	.0479	.0496	.0513	.0530	.0547	.0564	.0795
	6% .06	.0512	.0528	.0545	.0562	.0579	.0596	.0759
	7% .07	.0544	.0560	.0577	.0593	.0610	.0627	.0724
	8% .08	.0575	.0592	.0608	.0625	.0641	.0658	.0690
	9% .09	.0607	.0623	.0639	.0656	.0672	.0689	.0658
	10% .10	.0638	.0654	.0670	.0686	.0703	.0719	.0628
	11% .11	.0669	.0685	.0701	.0717	.0733	.0749	.0598
	12% .12	.0700	.0715	.0732	.0747	.0763	.0780	.0570
	13% .13	.0730	.0745	.0761	.0778	.0793	.0809	.0543
	14% .14	.0761	.0776	.0791	.0807	.0823	.0839	.0517
	15% .15	.0790	.0806	.0821	.0837	.0852	.0868	.0493
	16% .16	.0820	.0835	.0850	.0866	.0882	.0898	.0469
	17% .17	.0850	.0865	.0880	.0896	.0911	.0926	.0447
18% .18	.0879	.0894	.0909	.0924	.0940	.0955	.0425	
19% .19	.0908	.0923	.0938	.0953	.0969	.0984	.0405	
20% .20	.0937	.0952	.0967	.0982	.0997	.1013	.0385	

Step 3 → Step 4 Step 5

Source: Ellwood Tables for Real Estate Appraising and Financing. American Institute of Real Estate Appraisers, Chicago, Ill., 1968.

APPENDIX 9.2

Discounted Cash Flow Mortgage-Equity Valuation: Calculation of Variables

The Mortgage Constant – A Two Step Calculation

Equation: $1/a[25, 0.15]$

This calculation represents the calculation of the mortgage constant. The mortgage constant is calculated in two steps. First the present value of a \$1 annuity is calculated (6.464). Then, the inverse of the present value of a \$1 annuity is taken in order to calculate the “mortgage constant”. In order to calculate the value of the mortgage constant with this two step process, one would follow the calculator steps shown below:

<u>Press</u>	<u>Display</u>	<u>Comments</u>
1 ■ P/YR	1	
25 N	25	Amortization Period
15 I/YR	15	Rate on the Loan
1 +/- PMT	-1	\$1 Annuity
0 FV	0	
PV	6.46414908527	PV of \$1 Annuity
■ 1/x	0.15469940232	The “Mortgage Constant”

The inverse of the present value of a \$1 annuity results in the mortgage constant, 0.15469940232.

The Mortgage Constant – A One Step Calculation

Equation: $1/a [25, 0.15]$

Again, this equation is used to calculate the mortgage constant. Using a rate of $j_1 = 15\%$ and annual payments over 25 years, the mortgage constant, that is, the debt service (principal and interest) per period on a loan of one dollar is 0.15469940232. In other words, a loan of \$1 amortized over 25 years would require annual payments of just over \$0.15.

<u>Press</u>	<u>Display</u>	<u>Comments</u>
1 ■ P/YR	1	Annual Payments
25 N	25	Amortization Period
15 I/YR	15	Rate on the Loan
1 +/- PV	-1	\$1 Loan
0 FV	0	
PMT	0.15469940232	The “Mortgage Constant”

Therefore, the mortgage constant can be calculated in two different ways, both arriving at the same answer.

Calculation of Other Variables

Equation: a[8, 0.18]

The calculation presented is the present value of a \$1 annuity given the holding period and the expected return on equity.

<u>Press</u>	<u>Display</u>	<u>Comments</u>
1 ■ P/YR	1	
8 N	8	Holding Period
18 I/YR	18	Expected Return on Equity
1 +/- PMT	-1	\$1 Annuity
0 FV	0	
PV	4.07756575705	PV of \$1 Annuity

The present value of a \$1 annuity given the holding period and the expected return on equity is \$4.08.

Equation: s[15, 0.06]

The calculation presented is the future value of a \$1 annuity given the holding period and the expected return on equity.

<u>Press</u>	<u>Display</u>	<u>Comments</u>
1 ■ P/YR	1	
15 N	15	Holding Period
6 I/YR	6	Expected Return on Equity
1 +/- PMT	-1	\$1 Annuity
0 PV	0	
FV	23.275969885	FV of \$1 Annuity

The future value of a \$1 annuity given the holding period and the expected return on equity is \$23.28.

Equation: %OSB_n

This equation requires the calculation of the mortgage constant, that is, the debt service (principal and interest) on a loan of one dollar per period. Next, the holding period must be used in order to calculate the %OSB_n.

<u>Press</u>	<u>Display</u>	<u>Comments</u>
1 ■ P/YR	1	Annual Payments
25 N	25	Amortization Period
15 I/YR	15	Rate on the Loan

1	$\boxed{+/-}$	\boxed{PV}	-1	\$1 Loan
0	\boxed{FV}		0	
	\boxed{PMT}		0.15469940232	The "Mortgage Constant"
8	\boxed{N}		8	Holding Period
	\boxed{FV}		9.35492155 E-1	OSB of \$1 loan after 8 years

The %OSB_n is 0.9355.

APPENDIX 9.3

The Mortgage-Equity Capitalization Techniques: An Indirect Estimation of Value

Overall Capitalization Rate (R) Through the “Band of Investment” Technique

This ground is not totally uncharted: we were first exposed to a similar technique when we computed a weighted average capitalization rate.

Table 9.14			
The Weighted Average Capitalization Rate			
	<u>Relative Weight</u>	<u>Expected Return</u>	<u>Factor</u>
Debt	D/V	k_d	Mortgage component of R
Equity	E/V	y	Equity component of R
			Overall R

As we mentioned previously, this procedure applies only (as it does in corporate finance) to perpetual investments. Specifically, it would apply to non-amortized debt (e.g., bonds) and to constant equity (i.e., the case where the value of the property remains constant).

A look-alike approach has been developed to apply to more realistic real estate situations. In the so called band of investment technique we explicitly recognize that mortgages are amortized but, in this simple form, we still assume that the disposition value is equal to the initial value.

Let us again use our previous example for which we are only given the following information:

$$\begin{aligned}
 \text{NOI} &= \$6,000 \\
 \text{D/V} &= 0.80 \\
 \text{E/V} &= 0.20 \\
 k_d &= 0.15 \\
 m &= 25 \text{ years} \\
 y &= 13.12013\% \\
 f &= 1/a[25, 0.15] = 0.1546994 \text{ “The mortgage constant”}
 \end{aligned}$$

A note of caution is warranted here. The rate of return (y) described here is the Equity Dividend Rate. In this example, y represents the current before-tax cash flow dividend on the initial equity.

$$y = \frac{\text{BTCF}}{E} = \frac{\$1,049.61}{\$8,000} = 0.1312013$$

We want first to derive R, the overall capitalization rate, and then V, the appraised value, through the familiar model:

$$V = \frac{\text{NOI}}{R}$$

The “Band of Investment” formula for R is a form of weighted average between the mortgage constant (f) and the equity dividend yield (y):

$$\begin{aligned}
 R &= [D/V \times f] + [E/V \times y] \\
 R &= 0.80 \times 0.1546994 + 0.20 \times 0.1312013 \\
 &\quad \underbrace{\hspace{10em}} \\
 &\quad \text{Mortgage} \qquad \qquad \text{Equity} \\
 &\quad \text{Component of R} \qquad \text{Component of R} \\
 &= 0.1499998 \text{ say, } 0.15 \\
 &= 0.1499998 \text{ say, } 0.15
 \end{aligned}$$

$$\text{Thus } V = \frac{\text{NOI}}{R} = \frac{\$6,000}{.15} = \$40,000.00$$

In a tabular form we would write:

Table 9.15			
The Band of Investment R for the Swift Building			
	(1)	(2)	(1) × (2) = (3)
	Relative Weight	Rate	Factor
Debt (D)	D/V = 0.80	f = 0.1546994	0.1237595
Equity (E)	E/V = 0.20	y = 0.1312013	0.0262403
Overall R			0.1500
Value (V)			\$40,000.00

The reader should carefully compare Tables 9.14 and 9.15 and note that in Table 9.15 we use f, the mortgage constant $1/a[m, k_d]$, instead of k_d , the mortgage rate. The use of the mortgage constant is required here because the loan is amortized, i.e., some principal is repaid annually.

Surprisingly we found the same value of \$40,000 from the “Band of Investment” as the one we obtained previously from the DCF mortgage-equity despite the fact that we neglected the *equity appreciation* in the Band of Investment computation of R. Is it so surprising? No, because in the computation of R we used y, the equity dividend rate (13.12013%) whereas we used k_e (the investment yield of 18%) in the DCF method.¹⁶ This is worth repeating ... and repeating again:

¹⁶ In fact you should be surprised: we must admit that the estimate of y is *fudged*. The rate of 13.12013% was not picked up from the thin air; it was computed backward from the knowledge of k_e . Clearly (try it) no other value of y would lead to the same result for V. The reader is asked to perform the same technique in the assignments. Through this devious choice of hypotheses we hope to clarify the relationship between y and k_e . This is crucial for the understanding of the rest of this appendix.

y: the *equity dividend rate* (EDR) is the “short-term” annual return on the equity: = BTCF/ E_0

k_e : the *equity yield rate* (EYR) is the “long-term” overall return on the equity; it includes the annual return and the return from the increased equity. k_e is the (internal) equity yield rate of return on the equity:

$$E_0 = \sum \frac{\text{BTCF}_t}{(1 + k_e)^t} + \frac{\text{BTER}}{(1 + k_e)^t}$$

In our example the k_e of 18% reflects the increased value of the equity whereas y (of 13.12013%) does not. Indeed we could write that:

$$\left(\frac{E}{V}\right)y = \left(\frac{E}{V}\right)k_e - \text{(the adjustment for changes in equity)}$$

and we would infer that $y = k_e$ only when the equity value remains constant.

In this ongoing argument we have been prudently vague about what we meant by changes in “Equity”. Did we say how the adjustment occurs? No, we did not, because we still need some more time to develop some other points.

The Equity Adjustment Factors: From k_e to y

What does happen to the investor’s initial equity? In our case it went from E_0 (the downpayment of \$8,000) to BTER (the before tax equity reversion: \$14,064.38) and we can further observe that this final equity comes from two different sources.

Equity Build-up

From the *equity build-up* through the process of principal amortization. The outstanding balance on the mortgage loan is reduced, yearly, by the annual principal repayment. In this example this equity build-up, after eight years, is:

$$\begin{aligned} \text{Principal Paid} &= D - \text{OSB}_n \\ &= \$32,000 - \$29,935.62 \\ &= \$2,064.38 \end{aligned}$$

Since we want an annual measure of the adjustment factor we can transform this future value of the change in equity (a value to be recovered in 8 years) into an annuity: a *sinking fund annuity* accumulated at the investor’s expected return on equity k_e . Thus, in dollar terms the annuity equivalent to the total principal paid would be:

$$\text{Equity Build-up Annuity} = (D - \text{OSB}_n) \times 1/s\llbracket n, k_e \rrbracket$$

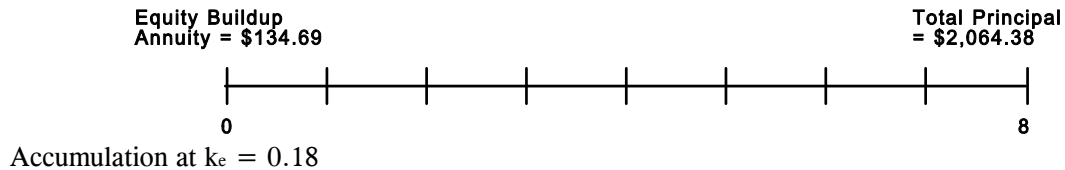
$$\text{Equity Build-up Annuity} = \$2,064.38 \times 1/s\llbracket 8, 0.18 \rrbracket$$

The calculator steps for the depreciation factor are as follows:

Calculations

<u>Press</u>	<u>Display</u>	<u>Comments</u>
1 ■ P/YR	1	
8 N	8	Amortization Period
18 I/YR	18	Expected Return on Equity
1 +/- FV	-1	\$1 Future Value of Sinking Fund
0 PV	0	
PMT	6.52443589E-2	Depreciation Factor
Equity Build-up Annuity =	$\$2,064.38 \times (0.0652444)$	
Equity Build-up Annuity =	$\$134.69$	

Graphically, the equity build-up due to principal repayment could be represented, in dollars terms as:



In percentage terms the same expression can now be written:

$$\% \text{ Equity Build-up Annuity} = (1 - \% \text{OSB}_n) \times 1/s[n, k_e]$$

$$\% \text{ Equity Build-up Annuity} = (1 - 0.9354881) \times (0.0652444)$$

$$\% \text{ Equity Build-up Annuity} = 0.004209$$

Since we do not know the absolute values of V and D , but only their ratio D/V , we can finally write the required annuity adjustment factor as:

$$\text{Equity Build-up Adjustment Factor} = D/V \times (1 - \% \text{OSB}_n) \times 1/s[n, k_e]$$

Appreciation

The second reason why the equity has increased is that the property has appreciated over the holding period. This appreciation is \$4,000 in our example.

$$\begin{aligned}
 \text{Appreciation} &= \text{REV}_n - V_0 \\
 &= \$44,000 - \$40,000 \\
 &= \$4,000
 \end{aligned}$$

or, if we note g the percentage of growth over the n years:

$$\begin{aligned}
 \text{Appreciation} &= g \times V_0 \\
 &= 0.10 \times \$40,000 \\
 &= \$4,000
 \end{aligned}$$

Here again we can turn this future value of appreciation (to be recovered in 8 years) into another sinking fund annuity accumulated at k_e .

In dollars terms we may thus notionally consider that the investor receives annually:

$$\text{Appreciation Annuity} = \text{Appreciation} \times 1/s[n, k_e]$$

or, by substituting $g \times V$ for Appreciation:

$$\text{Appreciation Annuity} = g \times V \times 1/s[n, k_e]$$

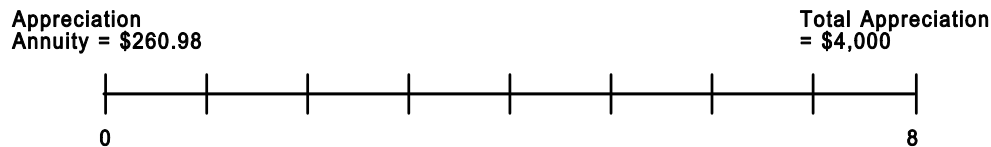
$$\text{Appreciation Annuity} = 0.10 \times \$40,000 \times 1/s[8, 0.18]$$

$$\text{Appreciation Annuity} = \$4,000 \times 1/s[8, 0.18]$$

$$\text{Appreciation Annuity} = \$4,000 \times 0.0652444$$

$$\text{Appreciation Annuity} = \$260.98$$

Graphically, the notional annual appreciation would be:



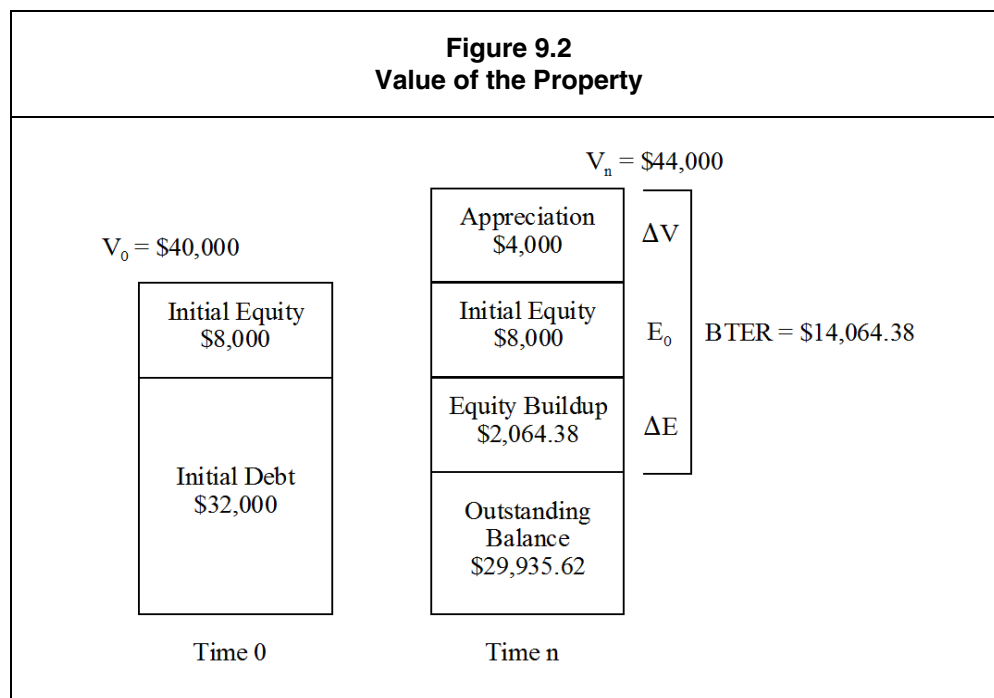
And, once more since in a typical appraisal problem we do not know V , the necessary adjustment factor will simply read:

$$\text{Appreciation Adjustment Factor} = g \times 1/s[n, k_e]$$

We can thus write, generally, that the residual equity of BTER_n is composed of the initial equity, the equity build-up and the building appreciation.

$$\text{BTER}_n = E_0 + \text{Appreciation} + \text{Equity Build-up}$$

A final graphic representation should reinforce the concept; again here, for illustration purposes, we assume that the value of the property is known. Usually this value is of course the unknown but we know how to transform our expressions into percentage terms. The reader should also mentally transform the appreciation and equity build-up curves into regular flows of annuities.



In most applications the final equity is increased because part of the loan is repaid (equity build-up) and because the property value increases (appreciation). In some cases though, the final equity may increase even when the property value drops as long as the equity build-up more than compensates for the property depreciation.

Clearly the final equity may also decline when the equity build-up is negative (a refinancing situation) and/or when the property value drops. The reader must satisfy himself that the general formulation still applies to these situations but that the adjustment factors will have different signs. Any increase in value should be reflected by a lower R (remember $V = NOI/R$) and thus by negative adjustment factors. Whereas any decrease in value is reflected by a higher R and thus by positive adjustment factors.

Back from this long detour to our adjustment problem. We had (in case you forgot) an innocuous little formula:

$$\left(\frac{E}{V}\right)y = \left(\frac{E}{V}\right)k_e - (\text{the adjustment for changes in equity variation})$$

Which transforms itself into:

$$y = k_e - \underbrace{\frac{V}{E} \left[\frac{D}{V} (1 - \%OSB_n) \times 1 / s \llbracket n, k_e \rrbracket \right]}_{\text{Adjustment for equity build-up}} + \underbrace{(g \times 1 / s \llbracket n, k_e \rrbracket)}_{\text{Adjustment for appreciation}}$$

we can verify:¹⁷

$$\frac{E}{V} \times y = \frac{E}{V} \times k_e - \left[\frac{D}{V} (1 - 0.9355)(0.0652) + g(0.0652) \right]$$

$$y = 0.18 - \frac{V}{E} \left[\frac{D}{V} (1 - 0.9355)(0.0652) + g(0.0652) \right]$$

$$y = 0.1306$$

Therefore, if we allow for some rounding errors, y is approximately equal to 0.13.

These adjustment factors will now be used to amend our capitalization rate in two different presentations of the same general technique: the modified Band of Investment. This “modification” (or adjustment) being simply the replacement of y in the Band of Investment equation by the equivalent: $(E/V)k_e$ – equity adjustment factors.

The Modified Band of Investment à la Akerson¹⁸

Readers will recall that the standard “Band of Investment” capitalization rate was a weighted average of the mortgage constant (f) and the equity dividend rate (y).

$$R = [D/V \times f] + [E/V \times y]$$

We spent the best of the last few pages to convince the reader that the same R could also be obtained using k_e instead of y. The adjustment process follows the path first suggested by C. B. Akerson who wanted to make more intuitive sense out of Ellwood’s formula which is seen in Appendix 9.1.

The Akerson’s R can be fully developed as:¹⁹

$$R = [D/V \times f] + [E/V \times k_e] - \left[\frac{D}{V} (1 - \%OSB_n) \times \frac{1}{s[[n, k_e]]} \right] - \left[\frac{g}{s[[n, k_e]]} \right]$$

or, in our example:

$$R = [0.80 \times 0.1547] + [0.20 \times 0.18] - [0.80(1 - 0.9355) \times (0.065)] - [0.10 \times (0.065)]$$

$$R = 0.1499 \text{ say, } 0.15$$

¹⁷ Since the capitalization rate R has been rounded to 15% in the previous computations, the other factors are now rounded accordingly to derive the value of y. Rounding must be consistent, otherwise small rounding errors compound. In practice, extreme accuracy is neither required nor logically justified since we deal with inexact estimates of the different rates.

¹⁸ Charles B. Akerson, “Ellwood Without Algebra”, *The Appraisal Journal*, Vol. 38, July 1970, p. 327.

¹⁹ In the case of property which has depreciated, g is a negative number and the adjustment factor $(g / s[[n, k_e]])$ will be a positive addition to R.

And we still find the same estimate of value:

$$V = \frac{\text{NOI}}{R} = \frac{\$6,000}{0.15} = \$40,000$$

In a more familiar tabular form the same results can be presented.

Table 9.16 The Akerson's R for the Swift Building			
	Relative Weight	Rate (rounded)	Factor
Debt	0.80	0.1547	0.1238
Equity	0.20	0.18	0.0360
Adjustment for Equity Build-up	0.80	0.004	-0.0032
Adjustment for Appreciation	0.10	0.0652	-0.0065
Overall Cap. Rate R			0.15
Appraised Value V			\$40,000

With such error-prone type of computations, a numerical verification is highly recommended. Two possible checking procedures can be chosen.

- We can fall back on R using “y” through the simple band of investment method; or
- We can fall back on V directly using the discounted cash flow mortgage-equity model.

$$(1) \quad R = [0.80 \times (0.1547)] + [0.20 \times y]$$

$$\begin{aligned} \text{and} \quad R &= \frac{\$6,000}{\$40,000} \\ &= 15\% \end{aligned}$$

Therefore,

$$0.15 = (0.80 \times 0.1547) + (0.20 \times y)$$

$$\begin{aligned} \text{where } y &= \text{BTCF/E} \\ &= (\text{NOI} - \text{PMT})/\text{E} \\ &= \frac{6,000 - 0.80(40,000)(0.1547)}{0.20(\$40,000)} \\ &= 0.1312 \text{ or } 13.12\% \end{aligned}$$

$$\text{Therefore, } y \approx 0.13$$

$$(2) \quad V = \left[\frac{D}{V} \right] + \text{BTCF} \times a[8, 0.18] + [\text{BTER}/(1 + 0.18)^8]$$

$$V = \left[\frac{D}{V} \right] + [\text{NOI} - 0.80V \times f] \times a[8, 0.18] + \frac{[(1 + g) \times V - 0.80V(\% \text{OSB})]}{(1.18)^8}$$

$$V = \frac{0.80(40,000 + 6,000 - 0.80(40,000)(0.1546)) \times 4.077 + [1.10(40,000) - 0.80(40,000)(0.9354)]}{3.7588}$$

$$V = \$40,000$$

This “verification” should, once more, convince the reader that the three approaches (DCF, Band of Investment R, Akerson’s Modified Band of Investment R) are perfectly equivalent. A fourth approach, the Modified Band of Investment à la Ellwood, requires the same calculations and produces identical results to Akerson’s Modified Band of Investment R. This approach was further discussed in Appendix 9.1.

