# Bond valuation

A reading prepared by
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OUTLINE



## 1. Valuation of long-term debt securities

Debt securities are obligations to repay an amount borrowed, along with some compensation for the time value of money and risk. The borrowers may be corporations, the government, or governmental agencies. The lenders may be corporations, governments, pension funds, mutual funds, or individual investors.

Long-term debt securities, such as notes and bonds, are promises by the borrower to repay the principal amount. Notes and bonds may also require the borrower to pay interest periodically, typically semi-annually or annually, and generally stated as a percentage of the face value of the bond or note. We refer to the interest payments as coupon payments or ***coupons*** and the percentage rate as the ***coupon rate***. If these coupons are a constant amount, paid at regular intervals, we refer to the security paying them as having a ***straight coupon***. A debt security that does not have a promise to pay interest we refer to as a ***zero-coupon*** note or bond.

The value of a debt security today is the present value of the promised future cash flows -- the interest and the ***maturity value***.[1] Therefore, the present value of a debt is the sum of the present value of the interest payments and the present value of the maturity value:

Present value of a bond = present value of interest payments + present value of maturity value

To calculate the value of a debt security, we discount the future cash flows -- the interest and maturity value -- at some rate that reflects both the time value of money and the uncertainty of receiving these future cash flows. We refer to this discount rate as the ***yield***. The more uncertain the future cash flows, the greater the yield. It follows that the greater the yield, the lower the present value of the future cash flows -- hence, the lower the value of the debt security.

Most U.S. bonds pay interest semi-annually.[2] In Wall Street parlance, the term ***yield-to-maturity*** (***YTM***) is used to describe an annualized yield on a security if the security is held to maturity. For example, if a bond has a return of 5 per cent over a six-month period, the annualized yield-to-maturity for a year is 2 times 5 per cent or 10 per cent.[3] The yield-to-maturity, as commonly used on Wall Street, is the annualized yield-to-maturity:

Annualized yield-to-maturity = six-month yield x 2

[1] The maturity value is also referred to as the ***face value*** of the bond.
[2] You should assume all bonds pay interest semi-annually unless specified otherwise.
[3] But is this the effective yield-to-maturity? Not quite. This annualized yield does not take into consideration the compounding within the year if the bond pays interest more than once per year.

When the term “yield” is used in the context of bond valuation without any qualification, the intent is that this is the yield to maturity.

**A note about rates**

- The interest cash flows associated with a bond are determined by the coupon rate.
- The discount rate is associated with the yield to maturity.

The present value of the maturity value is the present value of a lump-sum, a future amount. In the case of a straight coupon security, the present value of the interest payments is the present value of an annuity. In the case of a zero-coupon security, the present value of the interest payments is zero, so the present value of the debt is the present value of the maturity value.

We can rewrite the formula for the present value of a debt security using some new notation and some familiar notation. Since there are two different cash flows -- interest and maturity value -- let C represent the coupon payment promised each period and M represent the maturity value. Also, let N indicate the number of periods until maturity, t indicate a specific period, and $r_d$ indicate the six-month yield. The present value of a debt security, V, is:

$$V = \left[ \sum_{t=1}^{N} \frac{C_t}{(1+r_d)^t} \right] + \frac{M}{(1+r_d)^N}$$

To see how the valuation of future cash flows from debt securities works, let's look at the valuation of a straight coupon bond and a zero-coupon bond.

## A. Valuing a straight-coupon bond

Suppose you are considering investing in a straight coupon bond that:

- promises interest 10 percent each year;
- promises to pay the principal amount of $1,000 at the end of twelve years; and
- has a yield of 5 percent per year.

What is this bond worth today? We are given the following:

Interest = $100 every year
Number of years to maturity = 12
Maturity value = $1,000
Yield to maturity = 5% per year

Most U.S. bonds pay interest twice a year. Therefore, we adjust the given information for the fact that interest is paid semi-annually, producing the following:

C = $100 / 2 = $50
N = 12 x 2 = 24
M = $1,000
$r_d$ = 5% / 2 = 2.5%

$$V = \left[ \sum_{t=1}^{24} \frac{\$50}{(1+0.025)^t} \right] + \frac{\$1{,}000}{(1+0.025)^{24}} = \$1{,}447.1246$$

This value is the sum of the value of the interest payments (an ordinary annuity consisting of 24 $50 payments, discounted at 2.5 percent) and the value of the maturity value (a lump-sum of $1,000, discounted 24 periods at 2.5 percent).

Another way of representing the bond valuation is to state all the monetary inputs in terms of a percentage of the face value. Continuing this example, this would require the following:

| TI-83/84 Using TVM Solver | HP10B |
|---|---|
| N = 24 | 1000 FV |
| I = 2.5 | 24 n |
| PMT = 50 | 2.5 i/YR |
| FV = 1000 | 50 PMT |
| *Solve for PV* | PV |

C = 10 / 2 = 5
N = 12 x 2 = 24
M = 100
$r_d$ = 5% / 2 = 2.5%

$$V = \left[\sum_{t=1}^{24} \frac{5}{(1+0.025)^t}\right] + \frac{100}{(1+0.025)^{24}} = 144.71246$$

This produces a value that is in terms of a bond quote, which is a percentage of face values. For a $1,000 face value bond, this means that the present value is 144.71246 percent of the face value, or $1,447.1246.

**Try it. Bond quotes**

| Bond | Quote | Face value | Value of bond |
|---|---|---|---|
| A | 103.45 | $1,000 | |
| B | 98.00 | $1,000 | |
| C | 89.50 | $500 | |
| D | 110.00 | $100,000 | |
| E | 90.00 | €1000 | |
| F | 120.25 | ¥10000 | |
| G | 65.45 | $10,000 | |

*Solutions are provided at the end of the reading.*

Why bother with bond quotes? For two reasons: First, this is how you will see a bond's value quoted on any financial publication or site; second, this is a more general approach to communicating a bond's value and can be used irregardless of the bond's face value. For example, if the bond has a face value of $500 (i.e., it's a baby bond), a bond quote of 101 translates into a bond value of $500 x 101% = $505.

| TI-83/84 Using TVM Solver | HP10B |
|---|---|
| N = 24 | 100 FV |
| I = 2.5 | 24 n |
| PMT = 5 | 2.5 i/YR |
| FV = 100 | PV |
| *Solve for PV* | |

This bond has a present value greater than its maturity value, so we say that the bond is selling at a ***premium*** from its maturity value. Does this make sense? Yes: The bond pays interest of 10 percent of its face value every year. But what investors require on their investment -- the capitalization rate considering the time value of money and the uncertainty of the future cash flows -- is 5 percent. So what happens? The bond paying 10 percent is attractive -- so attractive that its price is bid upward to a price that gives investors the going rate, the 5 percent. In other words, an investor who buys the bond for $1,447.1246 will get a 5 percent return on it if it is held until maturity. We say that at $1,447.1246, the bond is *priced to yield* 5 percent per year.

Suppose that instead of being priced to yield 5 percent, this bond is priced to yield 10 percent. What is the value of this bond?

C = $100 / 2 = $50
N = 12 x 2 = 24
M = $1,000
$r_d$ = 10% / 2 = 5%

$$V = \left[\sum_{t=1}^{24} \frac{\$50}{(1+0.05)^t}\right] + \frac{\$1{,}000}{(1+0.05)^{24}} = \$1{,}000$$

| TI-83/84 Using TVM Solver | HP10B |
|---|---|
| N = 24 | 1000 FV |
| I = 5 | 24 n |
| PMT = 50 | 5 i/YR |
| FV = 1000 | PV |
| *Solve for PV* | |

The bond's present value is equal to its face value and we say that the bond is selling "at par". Investors will pay face value for a bond that pays the going rate for bonds of similar risk. In other words, if you buy the 10 percent bond for $1,000.00, you will earn a 10 percent annual return on your investment if you hold it until maturity.

Suppose, instead, the interest on the bond is $20 every year -- a 2 percent coupon rate. Then,

C = $20 / 2 = $10
N = 12 x 2 = 24
M = $1,000
$r_d$ = 10% / 2 = 5%

$$V = \left[ \sum_{t=1}^{24} \frac{\$10}{(1+0.05)^t} \right] + \frac{\$1,000}{(1+0.05)^{24}} = \$448.0543$$

The bond sells at a ***discount*** from its face value. Why? Because investors are not going to pay face value for a bond that pays less than the going rate for bonds of similar risk. If an investor can buy other bonds that yield 5 percent, why pay the face value -- $1,000 in this case -- for a bond that pays only 2 percent? They wouldn't. Instead, the price of this bond would fall to a price that provides an investor earn a yield-to-maturity of 5 percent. In other words, if you buy the 2 percent bond for $448.0543, you will earn a 5 percent annual return on your investment if you hold it until maturity.

So when we look at the value of a bond, we see that its present value is dependent on the relation between the coupon rate and the yield. We can see this relation in our example: if the yield exceeds the bond's coupon rate, the bond sells at a discount from its maturity value and if the yield is less than the bond's coupon rate, the bond sells at a premium.

**Example: Bond valuation**

*Problem*

Suppose a bond has a $1,000 face value, a 10 percent coupon (paid semiannually), five years remaining to maturity, and is priced to yield 8 percent. What is its value?

*Solution*

PV of interest = $405.54
PV of face value = $1,000 (0.6756) = $675.60
Value = $405.54 + 675.60 = **$1,081.14**

Using a calculator,

4 I/YR
10 N
50 PMT
1000 FV
PV

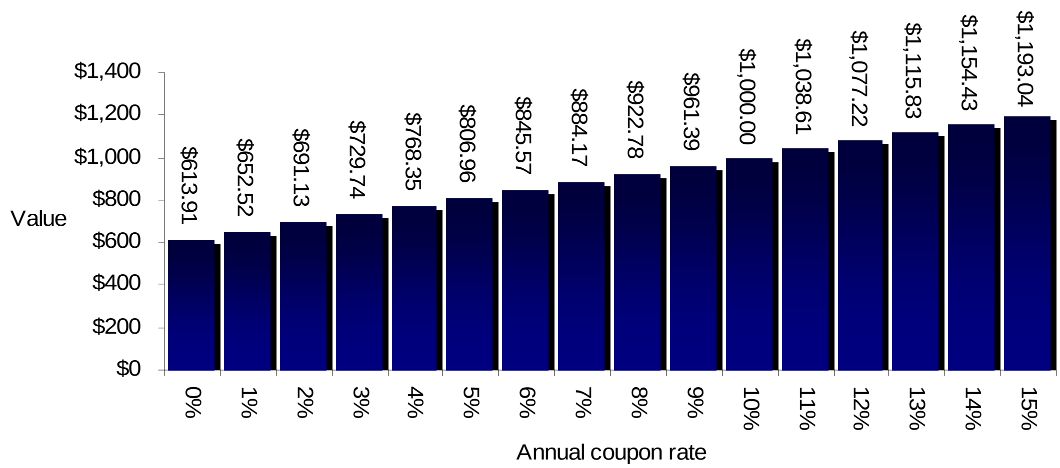
Using Microsoft's Excel® spreadsheet function,

=PV(rate,nper,pmt,fv,type)*-1

=PV(.04,10,50,1000,0)*-1

As another example, consider a bond with five years remaining to maturity and is priced to yield 10 percent. If the coupon on this bond is 6 percent per year, the bond is priced at $845.57 (bond quote: 84.557). If the coupon on this bond is 14 percent per year, the bond is a premium bond, priced at $1,154.43 (bond quote: 115.443). The relation between this bond's value and its coupon is illustrated in Exhibit 1.

**Exhibit 1: Value of a $1,000 face-value bond that has five years remaining to maturity and is priced to yield 10 percent for different coupon rates**




## Different value, different coupon rate, but same yield?

The yield to maturity on a bond is the market's assessment of the time value and risk of the bond's cash flows. This yield will change constantly to reflect changes in interest rates in general, and it will also change as the market's perception of the debt issuer's risk changes.

At any point in time, a company may have several different bonds outstanding, each with a different coupon rate and bond quote. However, the yield on these bonds – at least those with similar other characteristics (e.g., seniority, security, indentures) – is usually the same or very close. This occurs because the bonds are likely issued at different times and with different coupons and maturity, but the yield on the bonds reflects the market's perception of the risk of the bond and its time value.

Consider two bonds:

Bond A: A maturity value of $1,000, a coupon rate of 6 percent, ten years remaining to maturity, and priced to yield 8 percent. Value = $864.0967

Bond B: A maturity value of $1,000, a coupon rate of 12 percent, ten years remaining to maturity, and priced to yield 8 percent. Value = $1,271.8065.

How can one bond costing $864.0967 and another costing $1,271.8065 both give an investor a return of 8 percent per year if held to maturity? Bond B has a higher coupon rate than Bond A (12 percent versus 6 percent), yet it is possible for the bonds to provide the same return. Bond B you pay more now, but also get more each year ($120 versus $60). The extra $60 a year for 10 years makes up for the extra you pay now to buy the bond, considering the time value of money.

## Same bond, different yields, hence different values

As interest rates change, the value of bonds change in the opposite direction; that is, there is an inverse relation between bond prices and bond yields.

Let's look at another example, this time keeping the coupon rate the same, but varying the yield. Suppose we have a $1,000 face value bond with a 10 percent coupon rate that pays interest at the end of each year and matures in five years. If the yield is 5 percent, the value of the bond is:

$$V = \$432.95 + \$783.53 = \$1,216.48$$

If the yield is 10 percent, the same as the coupon rate, 10 percent, the bond sells at face value:

$$V = \$379.08 + \$620.92 = \$1,000.00$$

If the yield is 15 percent, the bond's value is less than its face value:

$$V = \$335.21 + \$497.18 = \$832.39$$

When we hold the coupon rate constant and vary the yield, we see that there is a negative relation between a bond's yield and its value.

We see a relation developing between the coupon rate, the yield, and the value of a debt security:

- if the coupon rate is more than the yield, the security is worth more than its face value -- it sells at a premium;
- if the coupon rate is less than the yield, the security is less that its face value -- it sells at a discount.
- if the coupon rate is equal to the yield, the security is valued at its face value.

**Try it! Bond values and yields**

Consider a bond that pays interest at the rate of 6 percent per year and has ten years remaining to maturity. Calculate the value of the bond if its face value is $1,000 and the bond quote for the specific yields to maturity:

| Yield to maturity | Value of bond | Bond quote |
|---|---|---|
| 5% | | |
| 6% | | |
| 7% | | |
| 8% | | |

*Solutions are provided at the end of the reading.*

**Example: Bond valuation**

*Problem*

Suppose we are interested in valuing a $1,000 face value bond that matures in five years and promises a coupon of 4 percent per year, with interest paid semi-annually. This 4 percent coupon rate tells us that 2 percent, or $20, is paid every six months. What is the bond's value today if the annualized yield-to-maturity is 6 percent? 8 percent

*Solution*

If the yield-to-maturity is 6 percent:

Interest, C = $20 every six months
Number of periods, N = 5 times 2 = 10 six-month periods
Maturity value, M = $1,000
Yield, $r_d$ = 6%/2 = 3% for six-month period

The value of the bond is **$914.69797**

If the yield-to-maturity is 8 percent, then:

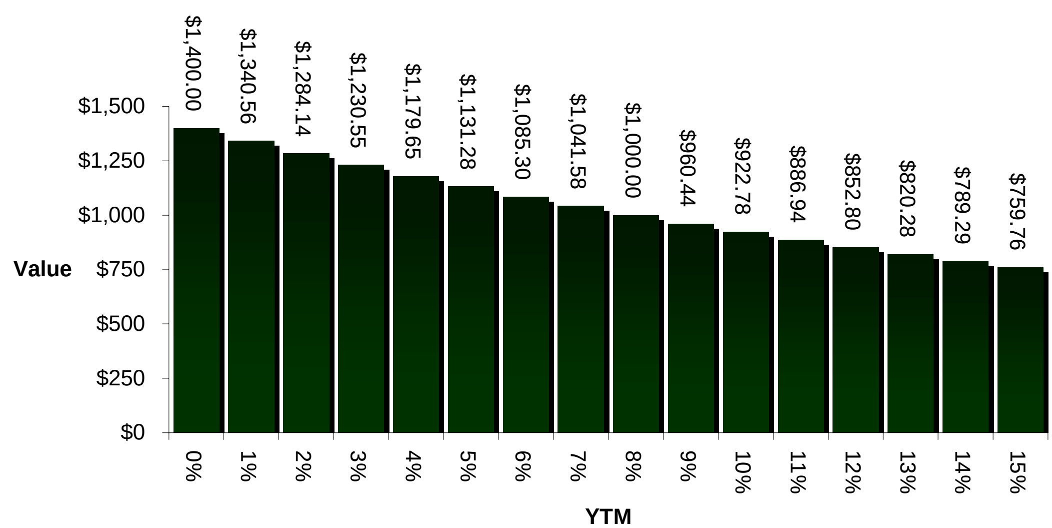
Interest, C = $20 every six months
Number of periods, N = 5 times 2 = 10 six-month periods
Maturity value, M = $1,000
Yield, $r_d$ = 8% / 2= 4% for six-month period

The value of the bond is **$837.7821**

We can see the relation between the annualized yield-to-maturity and the value of the 8 percent coupon bond in Exhibit 2. The greater the yield, the lower the present value of the bond. This makes sense since an increasing yield means that we are discounting the future cash flows at higher rates.

For a given bond, if interest rates go up, its price goes down; if interest rates go down, its price goes up.

**Exhibit 2:** **Value of a five-year, $1,000 face value bond with an 8 percent coupon for different yields to maturity**




## B. Valuing a zero-coupon bond

A ***zero-coupon bond*** is a debt security that is issued without a coupon. The value of a zero-coupon bond is easier to figure out than the value of a coupon bond. Let's see why. Suppose we are considering investing in a zero-coupon bond that matures in five years and has a face value of $1,000. If this bond does not pay interest -- explicitly at least -- no one will buy it at its face value. Instead, investors pay some amount less than the face value, with its return based on the difference between what they pay for it and -- assuming they hold it to maturity -- its maturity value.

If these bonds are priced to yield 10 percent, their present value is the present value of $1,000, discounted five years at 10 percent. We are given:[4]

M = $1,000
N = 10
$r_d$ = 5%

The value of the debt security is:

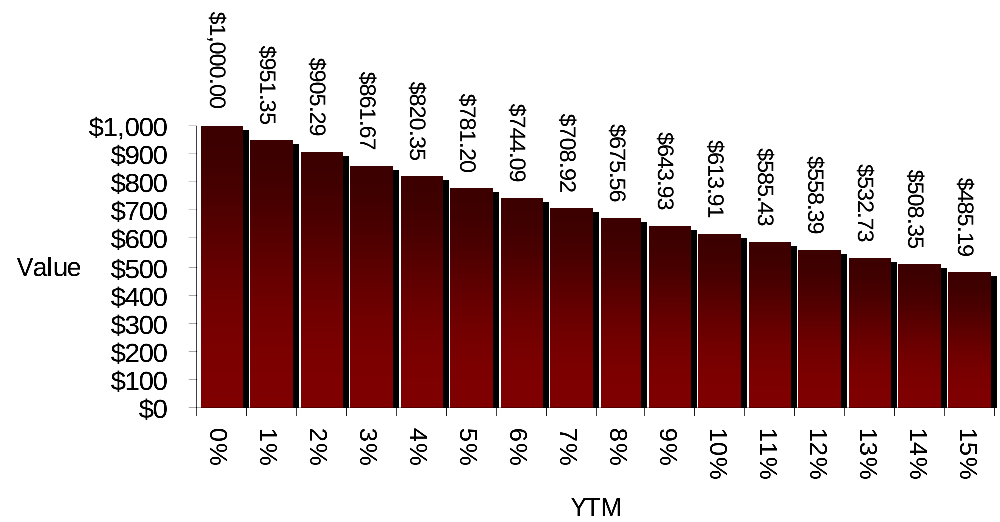
$$V = \frac{\$1{,}000}{(1+0.05)^{10}} = \$613.91325$$

| TI-83/84 Using TVM Solver | HP10B |
|---|---|
| N = 10<br>I = 5<br>PMT = 0<br>FV = 1000<br>*Solve for PV* | 1000 FV<br>10 n<br>5 i/YR<br>PV |

The price of the zero-coupon bond is sensitive to the yield: if the yield changes from 10 percent to 5 percent, the value of the bond increases from $613.91325 to $781.19840. We can see the sensitivity of the value of the bond's price over yields ranging from 1 percent to 15 percent in Exhibit 3.

[4] You will notice that we still convert the number of years into the number of six-month periods and we convert the yield to maturity to a six-month yield. This is because the convention for reporting yields on bonds, whether coupon or zero-coupon, is to assume an annualized yield that is the six-month yield multiplied by two.

**Exhibit 3: Value of a five-year maturity zero coupon bond for different yields to maturity**




# 2. Calculating the yield to maturity

In the previous section, we valued a bond, given a specific yield-to-maturity. But we are often concerned the yield that is implied in a given bond's price. For example, what is the yield-to-maturity on a bond that has a current price of $900, has five years remaining to maturity, an 8 percent coupon rate, and a face value of $1,000? We are given the following:

N = 10
C = $40
M = $1,000
V = $900

The six-month yield, $r_d$, is the discount rate that solves the following:

$$\$900 = \left[\sum_{t=1}^{10} \frac{\$40}{(1+r_d)^t}\right] + \frac{\$1{,}000}{(1+r_d)^{10}}$$

| TI-83/84 Using TVM Solver | HP10B |
|---|---|
| N = 10 | 1000 FV |
| PV = -900 | 900 +/- PV |
| PMT = 40 | 10 n |
| FV = 1000 | 40 PMT |
| *Solve for i* | i |

There is no direct solution, so we must use iteration.[5] In other words, without the help of a financial calculator or a spreadsheet, we would have to try different values of $r_d$ until we cause the left and right hand sides of this equation to be equal. Fortunately, calculators and spreadsheets make calculations much easier. Once we arrive at $r_d$, we multiply this by two to arrive at the yield-to-maturity. We can use

[5] That is, we cannot algebraically manipulate this equation to produce rd on the left-hand side and the remainder on the right-hand side of the equation and solve.

either the dollar amounts (that is, $40, $1000, and $900 for PMT, FV, PV, respectively) or in bond quote terms (that is, 4, 100, and 90 for PMT, FV, and PV, respectively).

| TI-83/84 Using TVM Solver | HP10B |
|---|---|
| N = 10 | 100 FV |
| PV = -90 | 90 +/- PV |
| PMT = 4 | 10 n |
| FV = 100 | 4 PMT |
| *Solve for i* | I |
| *Then multiply by 2* | X 2 |

Using a calculator, the six-month yield is 5.315% and, therefore, the yield to maturity is 10.63%. Using Microsoft Excel®, we use the RATE function and multiply this rate by two, as follows:

=RATE(10,40,-900,1000,0) * 2

The yield-to-maturity calculation is similar for a zero-coupon bond, with the exception that there is no interest: there is simply a present value, a future value, and a number of six-month periods. Again, the rate from this calculation must then be multiplied by two to produce the yield-to maturity.

**Example: Yield-to-maturity**

*Problem*

B, Inc. has a bond outstanding with 8 years remaining to maturity, a $1,000 face value, and a coupon rate of 8% paid semi-annually. If the current market price is $880, what is the yield to maturity (YTM) on the BD bonds?

*Solution*

Given:

FV = $1,000
N = 16
PV = $880
PMT = $40

Solve for i

i = 5.116434%
YTM = 5.116434 x 2 = **10.232868%**

**Example: YTM for a zero-coupon bond**

*Problem*

Suppose a zero-coupon with five years remaining to maturity and a face value of $1,000 has a price of $800. What is the yield to maturity on this bond?

*Solution*

Given:

FV = $1,000
N = 10
PV = $800
PMT = $0

Solve for i

i = 2.2565%
YTM = 2.2565% x 2 = **4.5130%**

**Try it!** **Yields to maturity**

Consider a bond that pays interest at the rate of 6 percent per year, a face value of $1,000, and has ten years remaining to maturity. Calculate the yield to maturity of the bond for the various bond values:

| Bond value | Yield to maturity |
|---|---|
| $1,100 | 4.733% |
| $1,000 | 6.000% |
| $900 | 7.435% |
| $800 | 9.087% |

*Solutions are provided at the end of the reading.*

# 3. Issues

## A. Changes in interest rates

We have already seen that value of a bond changes as the yield changes: if the yield increases, the bond's price decreases; if the yield decreases, the bond's price increases. Just how much a bond's value changes for a given yield change depends on the cash flows of the bond and the starting point, in terms of yield.

Consider the 8 percent coupon bond with five years to maturity that we saw earlier. If the yield changes from 5 percent to 6 percent, the price of the bond goes from $1,131.28 to $1,085.30; in percentage terms, the price declines 4.064 percent. But if the yield changes from 10 percent to 11 percent, the price changes from $922.78 to $886.94, a decline of 3.884 percent. In other words, this bond's price is more sensitive to yield changes for lower yields.

We can also compare bonds and their price sensitivities. Consider two bonds with the following characteristics:

Bond C: A 5 percent coupon bond with six years remaining to maturity and a face value of $1,000.

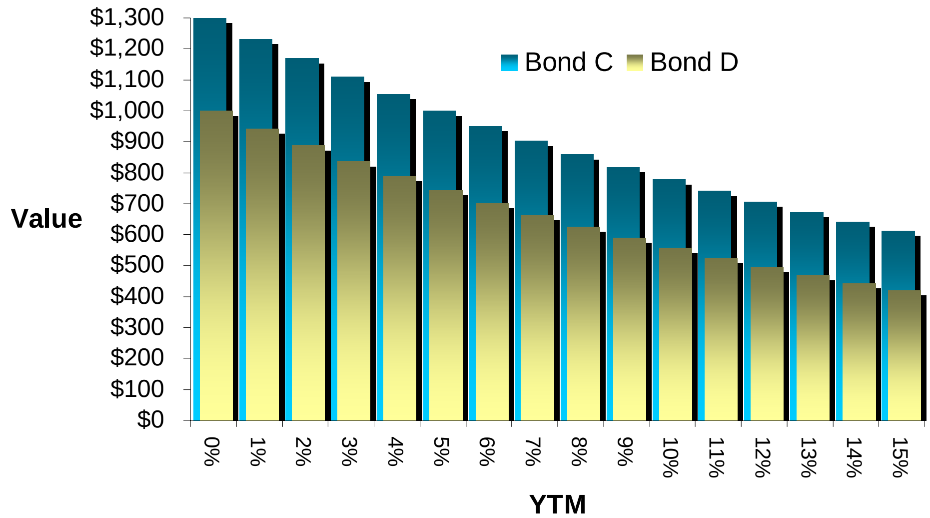
Bond D: Zero-coupon bond with six years remaining to maturity and a face value of $1,000.

Bond C is more valuable because it has the additional cash flows from interest, relative to Bond D. But which bond's value is more sensitive to changes in yields? The value of each bond is graphed in Exhibit 4 for yields from 0 percent to 15 percent. The change in price for a given yield change is more for the zero-coupon bond, Bond D, than the coupon bond; for example:

| | Percentage change in the bond's value | |
|---|---|---|
| Change in yield | Bond C | Bond D |
| From 5 percent to 6 percent | -4.98% | -5.67% |
| From 8 percent to 9 percent | -4.84% | -5.59% |
| From 14 percent to 15 percent | -4.57% | -5.44% |

This is because the entire cash flow for the zero-coupon bond is twelve periods into the future, whereas the coupon bond has cash flows in the near periods as well, which are not as affected by the yield change as the maturity value.

**Exhibit 4:** **Value of bonds with six years to maturity: Bond C has a 5 percent coupon and Bond D is a zero-coupon bond**




## B. Time passage

We have seen examples of bonds that trade at either a premium or a discount from their face values. This is usually the case: bonds are often issued at or near their face value, but as time passes, yields change and hence the value of the bond changes. But eventually, the value of the bond must be equal to the maturity value.[6] If the yield is held constant, the value of a bond approaches the maturity value as time passes. If the yield changes during the life of the bond, the value still approaches the maturity value as time passes, but perhaps not in a smooth path.

Consider a bond that has a 10 percent coupon, a maturity value of $1,000, ten years (i.e., 20 periods) remaining to maturity, and is priced to yield 6 percent. If the yield does not change until the bond matures, the price of the bond will decline until it reaches $1,000, the maturity value, as shown in Exhibit 5. If this bond's yield changes, say to 4 percent with 10 periods remaining, the value adjusts appropriately (i.e., increasing) and the bond's value will decline towards $1,000 at maturity, as shown in Exhibit 6.

[6] Otherwise there would be a windfall gain or a large loss to someone owning the bond just prior to maturity.

**Exhibit 5: Value of a 10 percent coupon, $1,000 maturity value bond, with ten years to maturity, and priced to yield 6 percent**

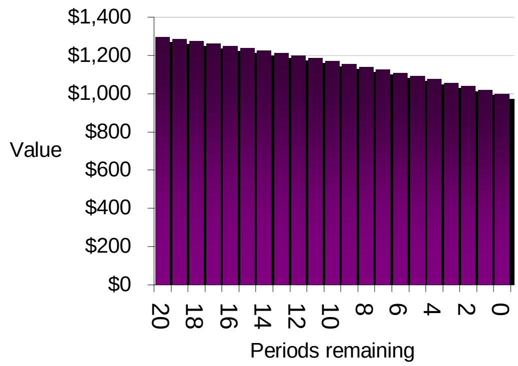
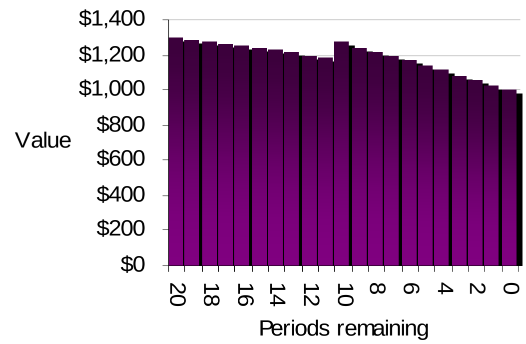
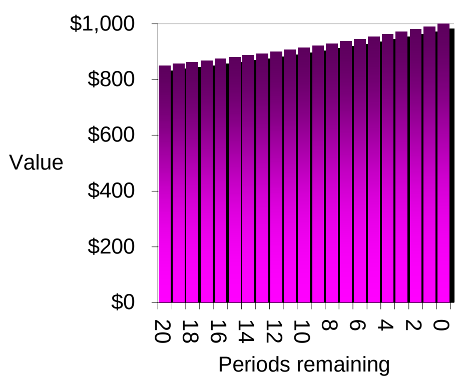



**Exhibit 6: Value of a 10 percent coupon, $1,000 maturity value bond, with ten years to maturity, and priced to yield 6 percent for the first five years and 4 percent for the last five years**




In a similar manner, a discount bond's value will increase over time, approaching the maturity value, as shown in Exhibit 7.

**Exhibit 7: Value of a 4 percent coupon, $1,000 maturity value bond, with ten years to maturity, and priced to yield 6 percent**




## C. Other valuation issues

A borrower could design a debt security with any features that are necessary for the issuer's financial situation or creditors' demand. There are endless variations in debt securities' characteristics that may affect how we value the security. Consider a few of these characteristics:

| Feature | Description | Valuation challenge |
|---|---|---|
| Convertible debt | At the discretion of the creditor, the debt security may be exchanged for another security, such as a specified number of common shares. | The value of the security is the value of the straight bond, plus the value of the option to convert. |
| Deferred interest | Interest is scheduled such that it is not paid in the first few years, but begins some time in the future. | The value of the security is the present value of the interest (a deferred annuity) and the face value. |
| Variable interest | The coupon amount depends on the interest rate of some other security (i.e, benchmarked); for example, it may be stated as prime rate + 5%. | The valuation requires a forecast of the coupon, based on forecasts of the benchmark's interest rate. |

Other features include ***security*** (i.e., collateral), a ***put option*** (the investor's option to sell the security back to the issuer), a ***sinking fund*** (i.e., putting aside funds or periodically retiring the debt). These, and many more, can be combined in a given debt security, making the valuation of the security quite challenging. Most of these features will affect the risk associated with the bond's future cash flows; for example, if the bond is secured, this reduces the risk of the bond's future cash flows because this collateral can be used to pay off the debt obligation.

## D. Risk and yields

The risk associated with the debt obligation's future cash flows affects the discount rate that investors use to value the debt: the greater the risk, the greater the discount rate applied. Consider the following two bonds as of February 2006:[7]

- Verizon Communications, 5.55% coupon, maturing February 2016
- Boise Cascade Corporation, 7.35% coupon, maturing February 2016

**The yield curve**

The **yield-curve** is the relation between maturity and yield. The *normal yield curve* is one in which the securities with longer maturities are associated with higher yields.

However, *inverted yield curves* do occur, such that shorter-term securities have higher yields. The inverted yield curve is often a pre-cursor of a recessionary economic period.

Check out Yahoo! Finance for the current U.S. Treasury yield curve.

Both bonds mature at the same time. However, the bond rating on the Verizon bond is A, which indicates that it is investment grade debt, whereas the bond rating on the Boise Cascade bond is BB, which indicate that it is speculative grade debt. The former has a yield to maturity of 5.492 percent, whereas the latter has a yield to maturity of 7.284 percent. The greater yield for the Boise Cascade bond reflects the greater relative risk of this bond. These yields are higher than those of a similar-maturity U.S. Treasury Bond, which has a yield to maturity of 4.653 percent. The U.S. Treasury bond has no default risk and hence it is priced to yield less than the similar-maturity corporate bonds.

Why do we mention maturity in the comparison of bonds' yields? Because one of the influences of a debt security's yield is the yield curve. The ***yield curve*** is the relation between the time remaining to maturity and the yield. A bond's yield is influenced by the time value of money, the riskiness of the future cash

[7] Source: Yahoo! Finance, February 13, 2006.

flows, and the yield curve.  The yield curve is affected by a number of factors, with the largest influence being the general economy.

# 4. Summary

The valuation of debt securities is an application of the time value of money mathematics.  The key is to take the bond's characteristics (i.e., coupon, maturity value) and translate them into inputs for the financial mathematics.

Bond valuation can get more complicated that what we've discussed in this reading because issuers have a great deal of flexibility in designing these securities, but any feature that an issuer includes in the debt security is usually just a simple an extension of asset valuation principles and mathematics.

# Solutions to Try it!

## Bond quotes

| Bond | Quote | Face value | Value of bond[8] |
|---|---|---|---|
| A | 103.45 | $1,000 | $1,034.50 |
| B | 98.00 | $1,000 | $980.00 |
| C | 89.50 | $500 | $447.50 |
| D | 110.00 | $100,000 | $110,000 |
| E | 90.00 | €1000 | €900,000 |
| F | 120.25 | ¥10000 | ¥12025 |
| G | 65.45 | $10,000 | $6,545 |

## Bond values and yields

| Yield to maturity | Value of bond | Bond quote |
|---|---|---|
| 5% | $1,077.95 | 107.795 |
| 6% | $1,000.00 | 100.00 |
| 7% | $928.94 | 92.894 |
| 8% | $864.10 | 86.410 |

## Yields to maturity

| Bond value | Yield to maturity |
|---|---|
| $1,100 | 4.733% |
| $1,000 | 6.000% |
| $900 | 7.435% |
| $800 | 9.087% |



[8] Note that a comma is used in European math conventions, which is different than the U.S. convention of a decimal place.