The Beta Anomaly and Mutual Fund Performance*

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November 14, 2018

Abstract

We contend that mutual fund performance cannot be properly measured using the alpha from standard asset pricing models if passive portfolios have nonzero alphas. We show how controlling for the passive component of alpha produces an alternative measure of managerial skill that we call "active alpha." Active alpha is persistent and associated with superior portfolio performance. Therefore, it would be sensible for sophisticated investors to reward managers with high active alpha. In addition to allocating their money based on standard alpha, we find that a subset of investors allocate their assets to funds with high active alpha performance.

*We would like to thank Jeffrey Busse, Kevin Crotty, Jon Fulkerson, Tong Xu, James Yae, Virgilio Zurita, seminar participants at Emory University and Fudan University, and participants at the KAEA-ASSA 2018 Workshop, the MFA 2018 Annual Meeting, the 2018 EFA Annual Meeting, the GCFC2018, and the 2018 Lone Star Finance Conference for their comments. We apologize for any errors remaining in the paper.
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1 Introduction

The empirical asset pricing literature supplies convincing evidence that high-beta assets often deliver lower expected returns than predicted by the CAPM, and that lower beta assets deliver returns higher than expected according to the CAPM (Black, Jensen, and Scholes (1972), Gibbons, Ross, and Shanken (1989), Baker, Bradley, and Wurgler (2011)). Recently, Frazzini and Pedersen (2014) reinvigorate interest in this so-called beta anomaly with a compelling theoretical argument. They propose a betting-against-beta (BAB) factor that captures the return spread from the beta anomaly.

Given the evidence for the beta anomaly, it has long been noted that actively managed funds can show significant performance by passively investing in low-beta stocks. The standard approach to measuring mutual fund performance today is to use the Carhart (1997) 4-factor model. According to this model, in the absence of active management, the expected excess return for a fund is the sum of the products of the betas with four factor risk premia. The expected difference between the portfolio return and its benchmark return is the Carhart measure of abnormal performance, or the alpha. The Carhart approach in effect assumes that a matching passive portfolio alpha is zero. However, in the context of asset pricing anomalies such as the beta anomaly, this assumption is not innocuous. More importantly, whether any asset pricing model effectively controls for the beta anomaly is unclear.

This paper examines whether accounting for the beta anomaly can systematically affect inferences about mutual fund performance. According to the CAPM, higher mutual fund alpha indicates skill. However, given the existence of the beta anomaly, higher alpha could also reflect a low beta tilt. That is, if fund A tends to hold high-beta assets relative to fund B, we ought to expect that, given equal skill, A has a lower alpha than B. In the standard attribution framework we might spuriously attribute this result to differences in skill.

It is not immediately clear how to account for the beta anomaly in mutual fund performance evaluation. More generally, there is no existing method to estimate the value-added of a fund when factor sensitivities are associated with a consistent pattern of alphas. We address the accounting issue by introducing a new performance measure that we call "active alpha." Active alpha subtracts from the fund's standard alpha its passive component, which is measured as the value-weighted alpha of those individual stocks whose betas are similar to the fund's estimated beta. If the active alpha is positive, investors seeking that particular level of risk would benefit from owning these funds.

In our sample of actively managed U.S. domestic equity funds, we find that fund alphas are almost monotonically declining in beta, just as they do for equities. In contrast, we find that active alpha tends to improve with beta. These results suggest that the alpha from standard asset pricing models cannot effectively control for the beta anomaly, and that inference based on our active alpha measure, which accounts for cross-sectional return differences due to the beta anomaly, differs considerably from that based on standard alpha measures. Notably, introducing the Frazzini-Pedersen (2014) BAB factor to the commonly used Carhart (1997) four-factor model does not suffice to control for the beta anomaly in fund performance. Although the magnitude of the alpha-beta relation is smaller based on more sophisticated multifactor return models, standard alphas continue to be significantly negatively related to fund beta.

In our main analysis, we show that active alpha is persistent, indicating that it captures the existence of investment skill over and above allocating capital to low-beta stocks. We also show that active alpha can be used to identify funds with desirable portfolio characteristics, including market-adjusted return and the Sharpe ratio.[1] These findings raise the question of whether investors recognize and respond to active alpha when allocating their capital to funds. To answer this question, we analyze mutual fund flows as a function of standard

[1] There are other benefits of using active alpha to measure managerial skill. By controlling for passive beta outperformance or underperformance, active alpha controls for any time-variation in average mutual fund beta documented by Boguth and Simutin (2018).

alpha and active alpha. Consistent with the literature, we find that standard alpha generates future fund flows. On the other hand, we also find that fund flows respond to our active alpha measure in and beyond standard alpha. These findings suggest that while most mutual fund investors allocate their capital based on the standard alpha, some investors are sufficiently sophisticated to account for the beta anomaly, allocating their fund flows based on active alpha. We provide supporting evidence for this investor heterogeneity comparing fund flows from institutional and retail share classes. we find that fund flows from institutional share classes, where presumably a higher proportion of fund investors are sophisticated, are more responsive to active alpha.

To provide an economic explanation for the empirical sensitivity of fund flows to active alpha, we develop a simple model of fund flows with the presence of both sophisticated and naive investors. In our model, some investors are sophisticated and are able to invest in a passive benchmark with the same risk as the fund. Other investors are naive and only make risky investments via the fund. Both types of investors update the fund's managerial skill as Bayesians. This framework shows how sophisticated investors' demand for the fund can be positively related to posterior expectations of active alpha, whereas naive investors' demand for the fund is positively related only to their posterior expectations of the standard alpha. Intuitively, sophisticated investors concern themselves with active alpha since they can identify (and short) the passive benchmark portfolio, in turn extracting only the performance truly attributable to managerial ability. On the other hand, naive investors care equally about all sources of alpha, since they are comfortable making risky investments only with the fund manager.

The model shows how the flow sensitivities to active alpha and to standard alpha vary as a function of the number of sophisticated investors. Importantly, the empirical fact that flows jointly respond positively to both active alpha and standard alpha measures can be consistent with our rational learning model only given the coexistence of both sophisticated and naive investors. Quantitatively comparing the simulated results from our model to the empirical magnitudes of the capital response, we find that roughly 24% of mutual fund investors are sophisticated, suggesting that sophisticated investors are a sizeable group.

Our paper contributes to the literature on mutual fund performance accounting for return anomalies from the empirical asset pricing literature. Ours is the first to account for the beta anomaly and to produce an estimate of managerial skill that does not attribute skill to a low-beta portfolio tilt. However, the factor-model regression approach is not the only popular performance attribution method. The characteristic-based benchmark approach of Daniel, Grinblatt, Titman, and Wermers (DGTW, 1997) is also prominent. Since then, the literature has recognized the importance of accounting for the stock characteristics such as size, value and momentum effects in fund returns. Busse, Jiang, and Tang (2017) propose to marry the factor-model regression approach and DGTW approach with a double-adjusted performance measure. Recently, Berk and van Binsbergen (2015) use the value added by a fund as the measure of skill, arguing that return measures of managerial skill alone do not suffice.[2]

Related to our analysis of fund flows is the fascinating question of what excess return model investors use to allocate their fund flows. Using a Bayesian framework that allows for alternative degrees of belief in different asset pricing models, Busse and Irvine (2006) show that fund flow activity varies by investor beliefs and by the time period under consideration. They report that a 3-year return history has a stronger correlation with fund flows than a single year's performance. Berk and van Binsbergen (2016) and Barber, Huang, and Odean (2016) use mutual fund flows to test which asset pricing model best fits investor behavior. They test a large number of asset pricing models and find that the CAPM best reflects

[2] Cremers, Fulkerson, and Riley (2018) review the literature on active management since Carhart (1997). They suggest that active management is more valuable than the conventional wisdom claims. In particular, they argue that it is still not clear what is the appropriate model for evaluating fund performance. Our search for a better measure of skill contributes to answering this important question.

investor behavior.[3]

Finally, as we propose fund beta as a predictor of fund performance, the existing literature has proposed other fund characteristics that predict performance, including but not limited to the return gap in Kacperczyk, Sialm, and Zheng (2008), active share in Cremers and Petajisto (2009), and mutual fund's R^2 in Amihud and Goyenko (2013).[4]

2 Data and Methods

2.1 Mutual fund sample

The Morningstar and CRSP merged dataset provides information about fund names, returns, total assets under management (AUM), inception dates, expense ratios, investment strategies classified into Morningstar Categories, and other fund characteristics. From this data set we collect monthly return and flow data on over 2,838 U.S. diversified equity mutual funds actively managed for the period 1983-2014. Panel A of Table 1 presents summary information about the sample. There are 298,055 fund-month observations. Funds have average total net assets (TNA) of $1.277 million, with a standard deviation of $2.838 billion. For the usual reasons related to scaling, we use the log of a fund's TNA as the proxy of fund size. We report the summary statistics for this variable in the row below that of fund size. We compute the fund age from the fund's inception date and find the typical fund has a life of 199 months. Funds earn an average gross return of 0.78% per month and collect fees of 9.8 basis points per month. Monthly firm volatility is 4.64% and average fund beta is 0.99. This

[3] Agarwal, Green, and Ren (2017) examine hedge fund flows and they also find that CAPM alpha consistently wins a model horse race in predicting hedge fund flows.

[4] Hunter, Kandel, Kandel, and Wermers (2014) and Hoberg, Kumar, and Prabhala (2018) generate measures of skill as a fund's outperformance relative to its peers instead of passive benchmarks. Hunter, Kandel, Kandel, and Wermers (2014) find that this approach significantly improves the selection of funds with future outperformance. Hoberg, Kumar, and Prabhala (2018) show that these funds generate future alpha when they face less competition, highlighting the importance of competition in limiting fund managers' ability to earn persistent alpha.

beta average suggests that in the fund beta sort results presented below, one can consider the middle decile portfolios to roughly bracket the market beta.

2.1.1 Estimating mutual fund alphas

We estimate the abnormal return (alpha) for each fund using five performance evaluation models: i) the CAPM, ii) the Fama-French (1993) three factor model (FF3), iii) the Carhart (1997) four factor model (Carhart4), iv) a five factor model we call PS5 augmenting the Carhart (1997) four-factor model with the Pastor and Stambaugh (2003) liquidity factor as in Boguth and Simutin (2018), and v) the Carhart (1997) four factor model augmented with the Pastor and Stambaugh (2003) liquidity factor and the Frazzini and Pedersen (2014) betting against beta factor (FP6). Alpha estimates are updated monthly based on 36-month rolling estimation window for each model. For example, in the case of the four-factor model for each fund in month t, we estimate the following time-series regression using thirty-six months of returns data from months $\tau = t-1, \ldots t-36$:

$$(R_{p\tau} - R_{f\tau}) = \alpha_{pt} + \beta_{pt}\left(R_{m\tau} - R_{f\tau}\right) + s_{pt}SMB_{\tau} + h_{pt}HML_{\tau} + m_{pt}UMD_{\tau} + e_{p\tau}, \qquad (1)$$

where $R_{p\tau}$ is the fund return in month τ, $R_{f\tau}$ is the return on the risk-free rate, $R_{m\tau}$ is the return on a value-weighted market index, SMB_{τ} is the return on a size factor (small minus big stocks), HML_{τ} is the return on a value factor (high minus low book-to-market stocks), and UMD_{τ} is the return on a momentum factor (up minus down stocks). The parameters β_{pt}, s_{pt}, h_{pt}, and m_{pt} represent the market, size, value, and momentum tilts (respectively) of fund p; α_{pt} is the mean return unrelated to the factor tilts; and $e_{p\tau}$ is a mean zero error term. We then calculate the alpha for the fund in month t as its realized return less returns related to the fund's market, size, value, and momentum exposures in month t:

$$\widehat{\alpha}_{pt} = (R_{pt} - R_{ft}) - \left[\widehat{\beta}_{pt}\left(R_{mt} - R_{ft}\right) + \widehat{s}_{pt}SMB_{t} + \widehat{h}_{pt}HML_{t} + \widehat{m}_{pt}UMD_{t}\right]. \qquad (2)$$

We repeat this procedure for all months (t) and all funds (p) to obtain a time series of monthly alphas and factor-related returns for each fund in our sample.

There is an analogous calculation of alphas for other factor models that we evaluate. For example, we estimate a fund's FP6 alpha using the regression of Equation (1), but add the Pastor and Stambaugh (2003) liquidity factor and Frazzini and Pedersen (2014) betting against beta factor as independent variables. To estimate the CAPM alpha, we retain only the market excess return as an independent variable.

2.1.2 Estimating stock alphas

We build the beta-matched passive portfolios from the return characteristics of individual stocks. We estimate abnormal performance for individual stocks in an analogous manner to that of mutual fund alphas described above. First, we estimate the abnormal return (alpha) for each stock using each of the five performance evaluation models. Alpha estimates are updated monthly based on a rolling estimation window. For example using the Carhart4 model, for each stock in month t, we estimate the following time-series regression using thirty-six months of returns data from months $\tau = t-1, \ldots t-36$ where $R_{q\tau}$ is the stock return in month τ, $R_{f\tau}$ is the return on the risk-free rate, $R_{m\tau}$ is the return on a value-weighted market index, SMB_τ is the return on a size factor (small minus big stocks), HML_τ is the return on a value factor (high minus low book-to-market stocks), and UMD_τ is the return on a momentum factor (up minus down stocks). The parameters β_{qt}, s_{qt}, h_{qt}, and m_{qt} represent the respective market, size, value, and momentum tilts of stock q and $e_{q\tau}$ is a mean zero error term.[5] We then calculate the alpha for the stock in month t as its realized return less

[5] The subscript t denotes the parameter estimates used in month t, which are estimated over the thirty-six months prior to month t.

returns related to the stock's market, size, value, and momentum exposures in month t:

$$\widehat{\alpha}_{qt} = (R_{qt} - R_{ft}) - \left[\widehat{\beta}_{qt}\left(R_{mt} - R_{ft}\right) + \widehat{s}_{qt}SMB_t + \widehat{h}_{qt}HML_t + \widehat{m}_{qt}UMD_t\right]. \tag{3}$$

We repeat this procedure for all months (t) and all stocks (q) to obtain a time series of monthly alphas and factor-related returns for each stock in our sample.

2.1.3 Estimating mutual fund passive alphas

We calculate the passive alpha for each fund in month t using the alphas and market betas from individual stocks as in Equation (3). The passive alpha for each fund is the value-weighted alpha of those individual stocks whose beta are in a 10 percent range around estimated fund beta, such that:

$$\widehat{\beta}_{qt} > 95\% \times \widehat{\beta}_{pt}, \widehat{\beta}_{qt} < 105\% \times \widehat{\beta}_{pt}. \tag{4}$$

Let $\widehat{\theta}_{pt}$ denote the estimate of passive alpha for the fund in month t.[6]

The fund's passive alpha allows us to calculate the active alpha for the fund in month t as the standard alpha for the fund in month t less the passive alpha in month t :

$$\widehat{\delta}_{pt} = \widehat{\alpha}_{pt} - \widehat{\theta}_{pt}, \tag{5}$$

where $\widehat{\delta}_{pt}$ is our active alpha estimate for fund p in month t.

[6] We estimate the passive alpha separately for each asset pricing model.

2.2 Horizon for performance evaluation

To estimate longer horizon alphas, we cumulate monthly alphas by fund-month. For example, to estimate annual standard alpha:

$$A_{pt} = \prod_{s=0}^{11} (1 + \widehat{\alpha}_{p,t-s}) - 1, \tag{6}$$

where the monthly alpha estimates are calculated from a particular asset pricing model.

Analogously, we calculate the fund's annual active alpha as follows:

$$\Delta_{pt} = \prod_{s=0}^{11} \left(1 + \widehat{\delta}_{p,t-s}\right) - 1, \tag{7}$$

where monthly active alpha estimates can also vary depending on the asset pricing model used to generate expected returns.

3 Results

3.1 Mutual fund alphas

In Table 1, Panel B presents summary information on fund standard alphas, estimated as usual, without controlling for any heterogeneity across funds in their betas. Here, standard alphas are measured against four different asset pricing models that researchers have used to estimate fund performance: the CAPM, the Fama-French 3-factor model (FF3), the Carhart4 model, and the PS5 model. Average fund standard alphas based on these models are generally less than 1 basis point per month, with the exception of the CAPM, which produces a slightly more positive average outperformance of 6 basis points per month. These alphas all represent risk-adjusted returns before fees, so that if we subtract the average monthly expense ratio of 9.8 basis points, we would see that the average fund underperforms across all the benchmark models.

Panel C of Table 1 presents the same statistics for active alpha. On average, active alphas are lower than standard alphas for each asset pricing model. After removing the passive alpha component of fund performance, on average mutual fund managers do not show any degree of stock-picking ability. Average active alpha ranges from 1 basis point for the CAPM to -5 basis points for the PS5 benchmark model. Active alphas also have larger heterogeneity across funds than standard alphas do.

3.2 Mutual fund beta anomaly

Table 2 examines the degree to which fund alphas are exposed to the beta anomaly. Again, alphas are measured against the four asset pricing models that we have used in section 3.1. In each month, we sort funds into 10 portfolios by their betas and compute the time-series average alphas for each beta-sorted portfolio. We note that mutual fund alphas are all based on gross returns and so do not represent the net alphas earned by investors.

Panel A reports the standard alpha of each beta decile calculated relative to different asset pricing models. The beta anomaly is clearly evident, with the standard alphas sorted by beta showing a consistently declining pattern. Relative to the CAPM, funds in the lowest beta decile have 250 basis points of average outperformance per year, while funds in the highest beta decile underperform by 99 basis points, which implies an economically large performance spread of 348 basis points. The use of alternative asset pricing models does not materially lower the magnitude of this spread. The often-used Carhart (1997) 4-factor model reduces the spread in alphas between beta-sorted portfolios 1 (P1) and 10 (P10) to 298 basis points. The Fama-French (1993) model and the four-factor model augmented with the Pastor-Stambaugh (2003) liquidity factor do marginally better than the Carhart (1997) model, with the P1-P10 alpha spreads of 253 basis points and 269 basis points, respectively. Clearly, if fund abnormal performance is measured by standard alphas, the low-beta mutual funds would exhibit a great degree of skill, as evidenced by their outperformance relative to the benchmark models. On the other hand, the high-beta mutual funds would predictably underperform and represent poor investment opportunities. This pattern suggests that the beta anomaly within assets held by the fund is an important source of standard alpha differences across funds.

Panel B reports the results for active alpha, which controls for the beta anomaly effect by using a passive beta-matched stock portfolio (Equation (5)) to estimate managerial skill. The pattern of active alphas is markedly different than that of standard alphas. Now, skill tends to increase with beta, suggesting that high-beta portfolio managers actually exhibit higher skills on average than low-beta portfolio managers once we control for the beta anomaly. The active alpha spread is quite large based on the CAPM at 331 basis points per year, but the use of multi-factor models do reduce this spread considerably to a minimum of 156 basis points in the case of the PS5 model.

We present the time series of performance spreads in annualized standard alpha and active alpha between the high-beta and low-beta fund portfolios in Figure 1. Each of the three graphs plots the standard alpha and active alpha spreads for the high- vs. low-beta portfolios using three different asset pricing models, respectively, the CAPM, the Carhart4 model, and the FP6 model.[7] Consistent with the results in Table 2, the active alpha spread is generally positive. Moreover, this spread is generally larger than the corresponding spread in standard alpha.

[7] We restrict this analysis for clarity of exposition. As suggested by the Panel A results, the FF3 model and the PS5 model result in similar plots.

3.2.1 Mutual fund beta anomaly and the BAB factor

Frazzini and Pedersen (2014) contend that the beta anomaly is driven by leverage constraints and propose a betting-against-beta factor (BAB) that captures the return affect related to the tightness of this constraint. Since the BAB factor is intended to be useful as a control variable for the low-beta anomaly, it is natural to ask whether an asset pricing model augmented with the BAB factor suffices to remove the performance-beta relation in mutual fund returns. To the extent that the Frazzini-Pedersen (2014) explanation for the low-beta anomaly is correct, the BAB factor should be related to the size of the anomaly. In turn, it should explain at least some of the low-beta premium in funds' standard alphas.

We proceed to analyze more formally the effect of beta on both standard alpha and active alpha using Fama-MacBeth regressions in Table 3. Since Table 2 shows that the relation between alpha and beta is similar across the four standard asset pricing models in that table, Table 3 only reports, for the sake of brevity, the regression results using the CAPM, the Carhart4 model, the PS5 model, and the PS5 model augmented with the BAB factor (FP6) as performance benchmarks.

The first four columns in Panel A of Table 3 regress standard alpha for each asset pricing model on only a constant and beta as a single regressor. The objective of estimating these regressions is to determine the size and the significance of the alpha-beta relation documented in Table 2, and to examine whether the addition of the BAB factor to existing multi-factor models suffices to account for this relation in fund returns. Column (1) reports that the coefficient on beta for the CAPM is -0.05 and is statistically significant. This result indicates that we would expect a fund with a beta of 0.5 to deliver around 5% improvement annually in standard alpha relative to a fund with a beta of 1.5. The results for the Carhart model in column (2) are similar with a slightly larger increase of 6% in annual alpha per unit decrease in market risk.

In column (4), we report the alpha-beta relation using a six-factor model that includes the BAB factor (FP6). As we would expect from Frazzini and Pedersen (2014), the addition of the BAB factor to the benchmark portfolios does reduce the magnitude of this relation between fund alpha and beta. However, the coefficient on beta is 3.1% per year per unit of beta, which continues to be statistically significant. Despite the use of the FP6 model, there still is a significant alpha premium to low-beta mutual funds. This suggests that including the BAB factor to the usual portfolios for fund performance benchmarks does not completely remove the low-beta anomaly in fund alpha. Columns (5)-(8) present multivariate regressions of the same alpha-beta relation, where we include fund size and fund age as controls. These variables proxy for the effects of scale, which Chen et al., (2004), Pastor et al., (2015) and Zhu (2018) discuss as a prominent factor in fund performance. We observe that the coefficients on beta are not significantly affected by the inclusion of these statistically significant controls.

Panel B of Table 3 reports the results of running identical regressions as in Panel A, with active alpha as the dependent variable. The univariate regression results in Columns (1)-(4) indicate that there is a small, positive premium for per unit of beta risk. While this could suggest managers for high-beta funds being more skilled, this relation is statistically significant only using the CAPM. Using the Carhart4, PS5, or FP6 model, it is not significant at the 5% level. The multivariate regressions [Columns (5)-(8)] reveal similar results. Overall, these results indicate that our active alpha successfully removes the beta anomaly in measuring mutual fund performance.

3.3 Persistence of active alpha

We have empirically shown that active alpha is a component of the standard alpha unaffected by the beta anomaly. In turn, it should be a measure of fund skill distinct from any passive persistence due to the beta anomaly. If active alpha really is a measure of managerial skill, it should be repeatable and, thus, persistent. We test this contention in Table 4. Each month

t, we compute the percentile rank based on active alpha, $\widehat{\Delta}_{p,t}$, of each mutual fund p. We then regress the active alpha ranks in the following month, $\widehat{\Delta}_{p,t+1}$, as well as in the next two years, $\widehat{\Delta}_{p,t+12}$ and $\widehat{\Delta}_{p,t+24}$, on $\widehat{\Delta}_{p,t}$. These regressions include controls for fund size, expense ratio, fund age, return volatility and fund flows to control for fund characteristics that could predict active alphas out into the future.

Panel A of Table 4 presents the regression results using the CAPM as the base model for calculating active alpha. We find that active alpha is highly persistent month-to-month. Specifically, the (rank) regression coefficient on $\widehat{\Delta}_{p,t}$ predicting $\widehat{\Delta}_{p,t+1}$ is 0.897 and is statistically significant. In other words, a fund earning a high active alpha in month t is highly likely to continue earning a high active alpha in month $t+1$. This persistence declines with time as the predictability of $\widehat{\Delta}_{p,t+12}$ over one-year horizon is 0.103, which is still statistically significant. Two years out, the coefficient on $\widehat{\Delta}_{p,t}$ falls to only 0.006, and is not statistically significant. The control variables in these regressions are generally insignificant with the exception of return volatility and fund flow, which show some predictive ability at longer horizons, but none of the controls are significant at the one-month horizon.

In Panel B, we see that using the Carhart4 model as the asset pricing model produces similar results. Using this model, the coefficient of $\widehat{\Delta}_{p,t+1}$ on $\widehat{\Delta}_{p,t}$ is 0.898, a number that again indicates significant predictability of active alpha at the one-month horizon. The persistence level again declines with time to 0.15 at the one-year horizon and to 0.035 at the two-year horizon. These coefficients are both statistically significant but economically small.

We obtain similar results in Panel C using the FP6 model as the base model for calculating active alpha. The coefficient of $\widehat{\Delta}_{p,t+1}$ on $\widehat{\Delta}_{p,t}$ is significant 0.900. As in Panel B, statistically significant predictability of active alpha is also evident at the one- and two-year horizons, though the persistence coefficients drop markedly as the prediction horizon increases. Using the FP6 model, none of the control variables significantly predict active alpha at any horizon.

Generally, we observe that the persistence results grow consistently stronger as we account for more factor-related returns. This may be due to the fact that controlling for risk via factor models and controlling for returns related to the beta anomaly via our active alpha are jointly important for producing a clean estimate of true fund manager skill.

The persistence results are illustrated graphically in Figure 2. Panel A shows the persistence of active alpha when calculated using the CAPM. Differences in active alpha persist for about 8 months, though a small amount of outperformance continue to hold until about 14 months out. Consistent with the results in Table 4, the active alpha spread between the highest decile (10) and the lowest decile (1) portfolios (sorted by active alpha today) in the case of Carhart4 or FP6 model is generally larger, and is more persistent. In particular, the active alphas in decile 1 portfolio do not match those in decile 10 portfolio until about 20 months out in the Carhart4 model. When the FP6 model is used, the outperformance of the top active alpha portfolio persists for about 24 months. In summary, regardless of the asset pricing model used to calculate active alpha, the measure exhibits significant persistence, particularly at shorter horizons.

3.4 Fund performance and active alpha

3.4.1 Conventional Performance

Table 5 examines the characteristics of active alpha, derived from each of the CAPM, the Carhart4, and the FP6 asset pricing models. We do this to better understand how active alpha relates to conventional measures of fund performance. Panel A of Table 5 presents 10 portfolios sorted by active alpha constructed from the CAPM. Panel B presents the same 10 sorted portfolios where active alpha is constructed from the Carhart4 model, and Panel C shows the performance of fund portfolios sorted on the active alpha derived from the FP6 model.

For each model we sort the all funds into 10 portfolios formed on the basis of active alpha calculated from that particular model and examine conventional measures of performance for the funds. When we look at gross returns or market-adjusted returns we see similar patterns across all three panels. Lower active alphas correlate with lower return funds. The 10-1 difference in fund returns averages about 30 basis points in Panel A and Panel B and 20 basis points in Panel C. The results in Panel C confirm again that a component of the performance associated with active alpha is associated with the BAB factor, a result demonstrated in Table 3, but a significant component of returns to active alpha is not correlated with BAB factor risk.

When we look at Sharpe ratios and information ratios we find a significant performance difference between high and low active alpha portfolios across all three asset pricing measures. The difference in monthly Sharpe ratios ranges from 0.042 for the FP6 model to 0.063 basis points using the Carhart4 model. This difference is not only statistically significant, but represents a considerable fraction of the monthly average Sharpe ratios which range between 0.148 and 0.211. Examining the information ratio, we find that high active alpha portfolios outperform low active alpha by an amount comparable to the Sharpe ratio outperformance. In the last column we examine the performance of active alpha in obtaining standard alpha outperformance using the standard Carhart4 factor model. Active alpha is associated with a spread of 16 basis points per month in standard alpha using the CAPM, 22 basis points per month using the Carhart4 model and 13 basis points per month using the FP6 model. These differences are statistically significant for both the Carhart four factor model and the FP6 model, but not for the CAPM. This spread in four factor alphas is not that surprising given we benchmark active alpha against standard alpha (Equation (5)). Thus, active alpha and standard alpha tend to be positively correlated.[8]

[8]The correlation between active alpha and standard alpha is between 0.60 and 0.70 across the five asset pricing models we use to calculate the alpha measures, This result holds for both the raw alphas and the percentile ranks.

Overall, these results indicate that mutual fund portfolios with high active alpha are desirable on a range of performance measures desired by investors. Thus, high active alpha portfolios should attract investor capital, provided the investors are sophisticated enough to consider active alpha when allocating cash to a mutual fund. This result we explore below after first examining whether active alpha is associated with other recently developed measures of fund performance.

3.4.2 Sources of outperformance

We find that active alpha is associated with superior performance against a number of asset pricing models and produces superior returns according measured by several conventional performance measures. We next test how active alpha is related to three recent measures found to be associated with fund performance. We examine whether the outperformance in active alpha could come from the return gap (Kacperczyk, Sialm and Zheng, 2008), the active share (Cremers and Petijisto, 2009) and, R^2 (Amihud and Goyenko, 2013). We do this by regressing the percentile rank of a funds active alpha on the return gap, the active share, and the fund's R^2, for active alpha calculated from the CAPM, the Carhart4 model and the FP6 model.

Table 6 presents the results, first by regressing active alpha on each performance measure alone, and then with controls for *fund size*, the *expense ratio*, *fund age*, *return volatility*, *fund flow* and the past *12 month return*. We find that active alpha is significantly positively related to the return gap, the degree of fund manager activity between reporting quarters. Active alpha is also positively related to active share, the degree of deviation from a fund's benchmark. Both of these results hold with and without the control variables. But active alpha is not significantly related to the R^2 measure of fund performance, perhaps because Amihud and Goyenko (2013) found stronger results from double-sorting on R^2 and standard alpha than by sorting on R^2 alone. Overall, we can conclude from Table 6 that high active alpha managers tend to be those with larger return gap and active share, indicating that they are both active and selective managers.

3.5 Fund flows and active alpha

As discussed in the previous section, active alpha predicts superior portfolio performance (Table 5), and we have shown in section 3.3 that active alpha is persistent (Table 4). Therefore, we would expect that active alpha is a desirable fund characteristic. Therefore, we would expect that at least some sophisticated investors would allocate their cash towards funds that exhibit high active alpha performance. On the other hand, the fund literature finds that investors allocate their funds based on standard alpha measures (Barber et al., 2016; Berk and van Binsbergen, 2016). A natural question to ask then is whether there are any investors that allocate funds based on active alpha.

To investigate this question, we run panel regressions of fund flows on the lagged ranks of annualized active alpha and standard alpha. We report the results in Table 7. Following the prior literature on fund flows, we calculate flows for fund p in month t as:

$$Flow_{p,t} = \frac{TNA_{p,t} - TNA_{p,t-1}\left(1 + R_{p,t}\right)}{TNA_{p,t-1}}, \tag{8}$$

so that flows represent the percentage change in the fund's net assets not attributable to its return gains or losses. Specifically, the regression specification that we utilize in Table 7 is

$$Flow_{p,t} = a + bPerformance_{p,t-1} + \mathbf{c}'\mathbf{X}_{p,t-1} + \varepsilon_{p,t}, \tag{9}$$

where $Performance_{p,t-1}$ is measured using the lagged percentile rank for the fund based on either its annualized active alpha ($\widehat{\delta}_{p,t-1}$) or its annualized standard alpha ($\alpha_{p,t-1}$). We include a vector of control variables ($\mathbf{X}_{p,t-1}$), which yields a vector of coefficient estimates ($\mathbf{c}$). As controls, we include lagged fund flows from month $t-13$, a lag of a fund's expense ratio, a fund's return standard deviation estimated over the prior twelve months, lagged fund size, and the log of fund age in month $t-1$. We also include fixed effects for Morningstar *Category* $\times$ *Month*.

The results of estimating equation (9) using the overall performance rank as the regressor are presented in Panels A and B of Table 7. We present the regression coefficients of fund flow on performance rank, where the standard alpha and active alpha are estimated using three different asset pricing models: the CAPM, the Carhart4 model, and the FP6 model. Since our active alpha is, by construction, a component of standard alpha, we begin in Panel A by estimating the effects of standard alpha and active alpha independently to better understand the strength of each measure in attracting fund flows. We see that fund flows are significantly positively related to past performance as measured by either standard alpha or active alpha. For both performance measures, the flow-performance relation weakens slightly as we add more factors to our performance benchmarks, but in all six regressions, standard alpha and active alpha significantly attract fund flows.[9] In particular, the coefficients on active alpha are approximately two-thirds of the magnitude of those on standard alpha. This result implies that the skill component of standard alpha generate significant flows, but also that there is a significant fraction of investors who allocate flows to the passive component of standard alpha. The larger coefficients on standard alpha and in turn, its relative strength in predicting fund flows is not surprising, as standard alpha is the more familiar performance measure.

Of course, one alternative hypothesis is to argue that the results in Panel A are consistent with investors chasing only standard alpha, but the significant coefficients on active alpha driven by the correlation between active alpha and standard alpha. We further investigate whether there are any investors attending to active alpha by jointly estimate the effects of

[9] This is consistent with the recent literature (Berk and van Binsbergen, 2016; Barber et al., 2016) that finds alphas from more sophisticated models do not explain fund flows as well as the CAPM alpha.

standard alpha and active alpha in attracting fund flows. To address potential concerns about multicollinearity in this specification, we compute the variance inflation factor (VIF) for the active alpha percentile rank. Across alternative asset-pricing models, the VIF turns out to be no larger than 2, which suggests that multicollinearity is not an issue.[10] Moreover, in the appendix, we present a simple model that formally identifies conditions for the existence of investors chasing active alpha by testing the significance of the *partial* coefficient associated with active alpha. Across alternative asset-pricing models, we find that the partial coefficient on active alpha is appreciably smaller than two-thirds of the magnitude of that on standard alpha, which indicates that indeed the size of the coefficient on active alpha in Panel A is partly due to its high correlation with active alpha. On the other hand, fund flows are jointly significantly positively related to both standard alpha and active alpha. The results indicate that there exists a subset of investors, who are apparently aware that passive alpha should not necessarily be rewarded and allocate capital based on active alpha.

Panels C and D are identical to Panels A and B except that, following the practice of Sirri and Tufano (1998), they replace the overall performance rank with the within-category performance rank as the regressor. Specifically, funds are ordered within the nine categories corresponding to Morningstar's 3×3 stylebox based on their active alphas or standard alphas. This allows us to test whether our results in Panels A and B are simply driven by mutual fund investors chasing styles, rather than them chasing fund performance per se. However, we continue to find that both standard alpha and active alpha continue to generate similar flow responses, and the coefficients on active alpha is smaller in magnitude than those on standard alpha. This alternative ranking decreases the gap between active alpha and standard alpha in attracting fund flows.

The results in Table 7 confirm that large flows chase standard alpha. It is apparent that

[10] As a rule of thumb, a regression model may be subject to multicollinearity worries if a variable has VIF values greater than 10 (or 5 to be conservative).

the bulk of these flows do not discriminate between the passive alpha component (obtained from the beta anomaly) and the active alpha component. While we maintain that any flows allocated to passive alpha are not rewarding managerial outperformance, it can be consistent with rational choices by some constrained investors.[11]

3.5.1 Investor sophistication and active alpha

Thus far, we have found that flows respond to both the standard alpha and the skill component of it, active alpha. We view these results as suggestive that mutual fund investors are heterogeneous in their ability to adjust for returns related to the beta anomaly. We argue that most fund investors allocate their capital based on the standard alpha, but that some fraction of investors are sophisticated and will seek out active alphas. In this section, we test and find strong support for this conjecture. We do so by comparing the flow-performance relations for fund flows from retail share classes, where less sophisticated investors arguably represent a higher proportion of fund investors, to fund flows from institutional share classes.

Evans and Fahlenbrach (2012) report that institutional investors are more sensitive to high fees and poor-risk adjusted performance, which they argue is consistent with greater ability to select and monitor managers by institutional investors. If institutional investors are more knowledgeable than retail investors, they likely would use more sophisticated benchmarks when evaluating fund performance and, thus, we anticipate that they will respond more strongly to active alpha than will retail investors.

To test this conjecture, we repeat the analysis of the flow-performance relations in Panel B of Table 7, separating out flows to retail share classes and institutional share classes. To do so, we first classify a share class as institutional if it is identified by CRSP or Morningstar as

[11]Specifically, in our model, investors who cannot short the passive benchmark do not discriminate between the two components of standard alpha, active and passive, and, in turn, they produce flows chasing standard alpha.

an institutional share class. Otherwise, a share class is classified as retail. We then calculate flows at the share class level, aggregating the flows for the retail share classes for that fund each month into a single retail fund-month observation and do a similar aggregation for the institutional share classes. All other variables continue to be computed by aggregating all share classes of the same fund each month.

To test for the heterogeneity in flow-performance relations across share types, we modify the regression specification of Table 7, Panel B, by interacting each of the performance ranks of a fund (one for annualized active alpha, the other for annualized standard alpha) with dummies for the share types (one for institutional share type, the other for retail share type). We also include fund fixed effects, which absorb variations in average fund flows across different share types within a fund.

The results of this flow analysis are presented in Table 8. Comparing the institutional share class coefficient estimates to the retail coefficient estimates, we find that institutional share class flows are more sensitive to fund performance than retail flows. The difference between the retail and institutional response to performance is significantly negative for both the standard alpha and active alpha, regardless of the asset pricing model.[12] This is consistent with greater monitoring by institutional investors as in Evans and Fahlenbrach (2012). Additionally, flows are significantly more sensitive to standard alphas than to the active component of alpha for both the retail and institutional share types, showing that our findings in Table 7 are robust: most fund investors, retail and institutional, are not sophisticated enough to discriminate between the active versus passive components of standard alpha.

We find that flows from institutional share classes respond strongly to active alpha. This

[12] Specifically, the difference is statistically significant at the 1% level in all cases, except for the differential response to active alpha using the Carhart4 model, which is significant at the 5% level. Otherwise, the t-statistics on the difference tests would range from -3.5 (in the case of active alpha using the FP6 model) to -9.3 (in the case of standard alpha using the Carhart4 model).

response is statistically highly significant at the 1% level, regardless of the asset pricing model. On the contrary, the flow response from retail share classes to active alpha is comparatively less or is statistically insignificant. In particular, when the CAPM is used, the institutional share class coefficient estimate on active alpha represents over 40% of the magnitude of that on standard alpha, whereas the corresponding ratio for the retail shares is less than 10%; the difference is statistically and economically significant. In the case of Carhart4 or FP6 model, the retail share class coefficient estimates on active alpha produce t-statistics that are smaller or insignificant.

Taken together, these results provide strong support for our hypothesis that more sophisticated investors, who use more sophisticated models to assess fund manager skill, will also account for returns related to the beta anomaly and, thus, will seek out active alpha when allocating capital to mutual funds.

3.6 Explaining the role of active alpha in generating fund flows

We began our empirical investigation of active alpha by establishing that it is a persistent fund characteristic that can be used to pick funds with higher risk-adjusted performance, arguing that it is a measure of fund manager skill. Consistent with this argument, we show that active alpha attracts fund flows, and fund flows respond to active alpha in and beyond standard alpha. We also show that the strength of active alpha (relative to that of standard alpha) in attracting fund flows is significantly higher in the case of the, presumably more sophisticated, institutional share classes.

We contend that this empirical evidence is consistent with an active management industry, which serves investors of varying sophistication. While investor sophistication has many dimensions, our empirical results point to heterogeneous ability on the part of investors to account for the beta anomaly in evaluating mutual fund performance.

In the appendix, we formalize this argument by describing a simple model with the presence of both sophisticated and naive investors. In addition to the actively managed fund, sophisticated investors have available an alternative investment opportunity in the passive benchmark (with the same beta risk). Naive investors make risky investments only with the mutual fund. In this model, sophisticated investors care about difference in alphas between fund alpha (standard alpha) and passive benchmark (passive alpha), which corresponds to active alpha. Because they can short the passive benchmark portfolio, they only care if the manager can provide risk-adjusted return in and beyond that offered by passive benchmark. On the other hand, naive investors do not care about the source of the alpha outperformance.

3.6.1 Calibration

Our empirical evidence is consistent with a simple model of heterogeneous investor sophistication, in which two types of investors coexist. In this section, we use the model to obtain guidance as to how large active alpha chasers as a group might be by calibrating the model to the data. We find that 24% of investors are sophisticated, which is quite large considering that active alpha is a novel measure and suggests reasonably high sophistication on the part of fund investors.

In the model, both components of the fund's alpha are assumed to evolve as AR(1):

$$\delta_t = \left(1 - \phi^A\right) \delta^* + \phi^A \delta_{t-1} + \tau_t, \tag{10}$$

$$\theta_t = \left(1 - \phi^P\right) \theta^* + \phi^P \theta_{t-1} + \upsilon_t, \tag{11}$$

where τ_t and υ_t are *i.i.d.*, respectively, $N\left(0, \sigma_\tau^2\right)$ and $N\left(0, \sigma_\upsilon^2\right)$. δ^* and θ^* represent, respectively, the unconditional expectations of active alpha and passive alpha.[13]

[13] Examination of the partial correlation plots of our active and passive alpha estimates show that AR(1) is a valid description of the persistence of these measures. No month other than the most recent month has a partial correlation coefficient that makes a significant contribution to the time series processes of these measures.

We begin by estimating the model parameters that can be inferred directly from Equations (10) and (11). Regressing annualized active alpha or annualized passive alpha on its lag with fund fixed effects, the autoregressive coefficients are estimated around 0.91, so we use $\phi^A = 0.91$ and $\phi^P = 0.91$. Berk and Green (2004) infer the parameters that govern the distribution of skill level (mean of the prior, ϕ_0, and prior standard deviation, γ) by calibrating their model. They report that the flow-performance relationship is consistent with high average levels of skills ($\phi_0 = 6.5\%$ per year, or 0.5% per month, and γ is similar in magnitude to ϕ_0). So we will draw both components of the fund's average alpha (δ^* and θ^*) from a lognormal distribution with mean 0.26% per month and standard deviation 0.26% per month, which implies the fund's average alpha is drawn from a distribution with mean 0.52% per month. These numbers are consistent with both components of the fund's average alpha contributing equally and being drawn from diffuse distributions.[14]

We also set $\sigma_\tau = 0.4$ percent per month, which is the estimate of the standard deviation of residuals obtained from estimating (10). We note that we would set $\sigma_\upsilon = 0.3$ percent (per month), if we were to use the same reasoning for the fund's passive alpha, but instead we will use a σ_υ prior that allows us to match the empirical correlation between the sign of the change in expected active alpha and the sign of the change in expected standard alpha (see below).

In the model, the excess return on the actively managed fund is $r_t = \alpha_t + \beta^* r_t^M$, where r_t^M is the excess return on the market portfolio, and β^* is the fund beta. The passive benchmark portfolio's excess return has mean θ_t and the same risk as the fund, so $r_t^P = \theta_t + \beta^* r_t^M$. In other words, this is a CAPM world, and idiosyncratic risk is negligible since we are analyzing diversified U.S. equity mutual funds. To determine the parameter β^*, we

[14]The average value of the fixed effects from estimating (10) is higher than that from estimating (11), so apparently δ^* is drawn from a distribution with lower mean than θ^*. Our use of equal means for δ^* and θ^* aids the efficiency of our simulation exercise. In each sample, we assume that it ends whenever investor expects the fund's alpha or the active alpha to be negative going forward, so a slightly higher average for δ^* allows for a larger effective sample size given the number of simulations.

appeal to Proposition 1 from Frazzini and Pedersen (2014), which shows that a security's alpha with respect to the market is $\alpha_t^s = \psi_t (1 - \beta_t^s)$ (in equilibrium). Running the Fama-MacBeth regression of annualized alpha on one minus beta in the sample of U.S. equities and suppressing the constant term, the estimated coefficient is around 0.006, so we infer the beta of a fund with average passive alpha of θ^* as $\beta^* = 1 - \theta^*/0.006$. Finally, the volatility of monthly market excess returns is higher than 4 percent, so we specify $Std\left(r_t^M\right) = 0.05$, or 5 percent, and in turn, specify $\sigma_\epsilon = \beta^* \times Std\left(r_t^M\right)$ in each sample.

We simulate the model more than 2.5 million times over 500 months. In each sample, we assume that it ends whenever investor expects the fund's alpha or the active alpha negative going forward, at which point, at least one type of investors stop investing in the fund. This allows us to construct roughly 10,000 samples of simulated time-series data, with an average sample length of more than 100 months. In each sample, the model produces the total net flow into the fund as

$$Flow_t = q\frac{\widehat{\delta}_{t+1|t} - \widehat{\delta}_{t|t-1}}{\widehat{\delta}_{t|t-1}} + (1-q)\frac{\widehat{\alpha}_{t+1|t} - \widehat{\alpha}_{t|t-1}}{\widehat{\alpha}_{t|t-1}},$$

where q is the fraction of sophisticated investors. Intuitively, fund flows are fully determined by changes in investors' expectations of the fund's alpha and its active alpha, while the relative importance of the two expectations in attracting fund flows is determined by the fraction of sophisticated investors. We cannot expect to realistically quantify the magnitude of the capital response or the changes in investors' beliefs, all of which are likely to also vary due to factors beyond the scope of our model. Following Berk and van Binsbergen (2016), rather than making further assumptions necessary for quantitatively matching the data, we sidestep this issue by focusing only on the direction of the capital response and those of the changes in investors' beliefs.

Specifically, we first infer the parameter σ_v by matching the correlation between the directions of investors' update on active alpha and standard alpha:

$$corr\left(sign\left(\widehat{\delta}_{t+1|t}-\widehat{\delta}_{t|t-1}\right), sign\left(\widehat{\alpha}_{t+1|t}-\widehat{\alpha}_{t|t-1}\right)\right),$$

which we estimate in our data and in the simulated data.[15] Empirically, this correlation is about 0.43. We set $\sigma_{v} = 0.7$ percent per month, which produces an average simulated correlation of 0.44 across simulated samples. Note that we can match the parameter σ_{v} from this process without worrying about q because both types of investors rationally update their beliefs such that the fraction of sophisticated investors does not change this simulated correlation.

Finally, we match the parameter q governing the fraction of sophisticated investors by matching the ratio of the empirical correlations between the sign of the capital inflows and the sign of the update on $\widehat{\delta}_{t+1|t}$ to the analogous correlation using $\widehat{\alpha}_{t+1|t}$:

$$\frac{corr(sign\left(Flow_{t}\right), sign(\widehat{\delta}_{t+1|t}-\widehat{\delta}_{t|t-1}))}{corr(sign\left(Flow_{t}\right), sign\left(\widehat{\alpha}_{t+1|t}-\widehat{\alpha}_{t|t-1}\right))}. \tag{12}$$

Ratio (12) captures the strength of active alpha in predicting fund flows (relative to that of standard alpha).

Panel A of Figure 3 shows how the numerator (the green lines with diamond markers) versus the denominator (the orange lines with no marker) of the ratio (12) vary with the fraction of sophisticated investors. The figure plots the median (the solid lines) as well as the 25th and 75th percentiles (the dotted lines) of the estimated correlations based on signs

[15] The function $sign\left(x\right)$ returns the sign of a real number, taking values 1 for a positive number, -1 for a negative number and zero for zero:

$$sign\left(x\right) \equiv \begin{cases} \frac{x}{|x|} & x \neq 0 \\ 0 & x = 0 \end{cases}.$$

across simulated samples. As expected, the direction of capital response is primarily driven by changes in investors' expectation about standard alpha for small q (most investors are unsophisticated) and by changes in investors' expectation about active alpha for large q (most investors are sophisticated). Moreover, the correlation between fund flows and active alpha decreases monotonically to a significantly positive 0.44 even as investors chasing active alpha vanish away, while the correlation between fund flows and standard alphas also decreases monotonically to a significantly positive 0.44 even as investors chasing standard alpha vanish away. Thus, even if no investors attend to active alpha, the positive correlation between standard alpha and active alpha induces a positive correlation between fund flows and active alpha.

Our simulated moment of interest (12) is plotted in Panel B of Figure 2 as a function of q (the blue lines) with its empirical estimate (the red line) of 0.63. Again, the figure plots the median (the solid line) as well as the 25th and 75th percentiles (the dotted lines) of the simulated moment. The relative strength of active alpha in predicting fund flows grows monotonically stronger when mutual fund investors as a group becomes more sophisticated, as expected. Comparing the blue and red lines, we see that a fraction of sophisticated investors at 24% is consistent with the empirical estimate of (12), and the 25th and 75th percentiles for q are 15% and 33%.

Considering that active alpha is a new performance measure, we interpret this result as positive evidence for a substantial degree of investor sophistication in evaluating fund performance.

4 Conclusion

Mutual fund managers can earn positive alphas passively by allocating resources to low beta assets to take advantage of the low-beta anomaly. This positive relation between beta and standard alpha is significant over a number of different asset pricing models, including a six-factor model that includes the four factors in the Carhart (1997) model plus a liquidity factor and the betting against beta factor of Frazzini and Pedersen (2014). To correct for the passive alphas that exist regardless of the asset pricing model, we develop a measure of alpha called active alpha that subtracts the outperformance from a beta-matched passive portfolio from the fund's standard alpha. We contend that active alpha is a useful measure of managerial skill since it isolates outperformance that is distinct from the outperformance that can be obtained from the low-beta anomaly.

A high active alpha is associated with positive portfolio properties including overall returns, market-adjusted returns and high Sharpe ratios. Active alpha is also predictable, in that past active alphas are significantly correlated with future active alphas for at least 12 months into the future. Given the positive properties of high active alpha portfolios and the fact that it is to some extent persistent, sophisticated investors should allocate their capital to high active alpha funds. We find evidence that active alpha does attract cash flows, particularly from more sophisticated investors who are presumably aware of the low-beta anomaly.

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Appendix: The model

In this appendix we present a simple model to highlight the impact of the relative composition of sophisticated versus naive investors on the flow-performance relation. We use this model in Section 3.6.1 to infer the fraction of sophisticated investors consistent with allocations to active alpha, rather than just standard alpha, in the data.

There is an actively managed mutual fund, whose manager has the potential ability to generate expected returns in excess of those provided by a passive benchmark — an equally-risky alternative investment opportunity available to some investors. The expected passive alpha on the benchmark portfolio and the manager's ability to outperform it are unknown to investors, who learn about this ability and the actual passive alpha by observing the return history of the two portfolios. Let $r_t = \alpha_t + \beta^* r_t^M$ denote the return, in excess of the risk-free rate, on the actively managed fund, where β^* is the fund beta. The parameter α_t is the fund's expected alpha, a noisy signal of managerial ability since the passive alpha component of this return is also unknown. The market portfolio's excess return, r_t^M, is normally distributed with mean zero and variance σ_M^2 and is independently distributed through time. The passive benchmark portfolio's excess return has mean θ_t and the same risk as the fund, i.e., $r_t^P = \theta_t + \beta^* r_t^M$. Note that the model is partial equilibrium.[16]

In the model,

$$\delta_t = \alpha_t - \theta_t = r_t - r_t^P \tag{A1}$$

where δ_t is the risk-adjusted return to investors over what would be earned on the passive benchmark, and corresponds to active alpha. Of course, active alpha is the same as the standard alpha measure if the passive benchmark has zero alpha, but when the passive

[16] The benchmark portfolio's returns are assumed to be exogenously given, and we do not model the source of successful managers' abilities. In that sense, our approach is similar to that in Berk and Green (2004) and Huang et al. (2012). We are describing the simplest model, which produces the sensitivity of mutual fund flows not only to the standard alpha measure, but also to active alpha that is an alternative measure of fund manager skill controlling for passive alpha.

alpha is non-zero the standard alpha and active alpha will diverge. Note that α_t, θ_t (and in turn δ_t) vary over time. Specifically, both components of the alpha are assumed to evolve as AR(1):

$$\delta_t = \left(1 - \phi^A\right)\delta^* + \phi^A \delta_{t-1} + \tau_t, \tag{A2a}$$

$$\theta_t = \left(1 - \phi^P\right)\theta^* + \phi^P \theta_{t-1} + \upsilon_t, \tag{A2b}$$

where τ_t and υ_t are *i.i.d.*, respectively, $N\left(0, \sigma_\tau^2\right)$ and $N\left(0, \sigma_\upsilon^2\right)$. δ^* and θ^* represent, respectively, the unconditional expectations of the active alpha and the passive alpha.

There are two types of investors: a fraction q of investors are *sophisticated*, indexed by s, who allocate money across all assets (the risk-free asset and the active fund, as well as its passive benchmark). The remaining $1 - q$ fraction of investors are *naive*, indexed by n, who only allocate money between the active fund and the risk-free asset. We note that the behavior of naive investors is consistent with the empirical evidence on limited market participation.

On date $t-1$, investors have priors about δ_t and θ_t. These investors form their posterior expectations of the fund manager's ability as well as of the passive alpha through Bayesian updating. On date t, after observing the period t excess return r_t, they update their priors about δ_t and θ_t, which in turn imply their beliefs about δ_{t+1} and θ_{t+1}. Investors' prior beliefs are assumed to be normally distributed:

$$\begin{bmatrix} \delta_1 \\ \theta_1 \end{bmatrix} \sim N\left(\begin{bmatrix} \widehat{\delta}_{1|0} \\ \widehat{\theta}_{1|0} \end{bmatrix}, \begin{bmatrix} V_{1|0}^{\delta} & 0 \\ 0 & V_{1|0}^{\theta} \end{bmatrix} \right). \tag{A3}$$

Assume that $V_{1|0}^{\theta} = \sigma_\theta^2$, where σ_θ^2 is given by the (unique) real positive solution to the equation

$$\sigma_{\theta}^{2} = \left(\phi^{P}\right)^{2} \frac{\sigma_{\theta}^{2}\beta^{2}\sigma_{M}^{2}}{\sigma_{\theta}^{2} + \beta^{2}\sigma_{M}^{2}} + \sigma_{\upsilon}^{2}. \tag{A4}$$

Then, it is straightforward to show by using standard Bayesian results for updating the moments of a normal distribution that their posterior expectations after observing the history $\left\{r_u, r_u^P\right\}_{u=1}^{t}$ are:

$$\begin{bmatrix} \delta_{t+1} \\ \theta_{t+1} \end{bmatrix} \Big| \left\{r_u, r_u^P\right\}_{u=1}^{t} \sim N\left(\begin{bmatrix} \widehat{\delta}_{t+1|t} \\ \widehat{\theta}_{t+1|t} \end{bmatrix}, \begin{bmatrix} \sigma_{\tau}^{2} & 0 \\ 0 & \sigma_{\theta}^{2} \end{bmatrix}\right) \tag{A5}$$

where

$$\widehat{\delta}_{t+1|t} = \left(1 - \phi^{A}\right)\delta^{*} + \phi^{A}\left(r_t - r_t^{P}\right) \tag{A6a}$$

$$\widehat{\theta}_{t+1|t} = \left(1 - \phi^{P}\right)\theta^{*} + \phi^{P}\left(w\widehat{\theta}_{t|t-1} + (1-w)\, r_t^{P}\right) \tag{A6b}$$

and $w = \beta^{2}\sigma_{M}^{2} / \left(\sigma_{\theta}^{2} + \beta^{2}\sigma_{M}^{2}\right)$. Similarly, this implies the posterior about α_{t+1} is normally distributed with a mean of $\widehat{\alpha}_{t+1|t} = \left(\widehat{\delta}_{t+1|t} + \widehat{\theta}_{t+1|t}\right)$ and a variance of $(\sigma_{\tau}^{2} + \sigma_{\theta}^{2})$.

We consider an overlapping-generations (OLG) economy in which investors of type $i \in \{s, n\}$ are born each time period t with wealth $W_{i,t}$ and live for two periods. Each time period t, young investors have a constant absolute risk aversion (CARA) utility over their period $t+1$ wealth, $e^{-\gamma_i W_{i,t+1}}$, where $W_{i,t+1} = W_{i,t} + X_{i,t}r_{t+1} + X_{i,t}^{P}r_{t+1}^{P}$, $X_{i,t}$ is the dollar allocation to the mutual fund at time t, and $X_{i,t}^{P}$ is the dollar allocation to the passive benchmark. Since naive investors are assumed to make risky investments only with the mutual fund, $X_{n,t}^{P} = 0$.

Given CARA utility, it is easy to show that the optimal mutual fund holdings are

$$X_{s,t} = \frac{\widehat{\delta}_{t+1|t}}{\gamma_s \sigma_\tau^2} \tag{A7a}$$

$$X_{n,t} = \frac{\widehat{\alpha}_{t+1|t}}{\gamma_n \left(\sigma_\tau^2 + \sigma_\theta^2 + \beta^2 \sigma_M^2\right)} \tag{A7b}$$

Imposing the restriction that $X_{s,t}$ and $X_{n,t}$ are nonnegative (no shorting of funds), we have

$$X_{s,t} = \frac{\max\left(\widehat{\delta}_{t+1|t}\,, 0\right)}{\gamma_s \sigma_\tau^2} \tag{A8a}$$

$$X_{n,t} = \frac{\max\left(\widehat{\alpha}_{t+1|t}\,, 0\right)}{\gamma_n \left(\sigma_\tau^2 + \sigma_\theta^2 + \beta^2 \sigma_M^2\right)} \tag{A8b}$$

Intuitively, when choosing their optimal allocation to the fund, sophisticated investors will consider only active alpha, since they have the ability to short the passive benchmark portfolio and in turn extract only the performance truly attributable to managerial ability. On the other hand, naive investors will attend to the standard alpha measure, since they cannot short sell the benchmark asset and in turn care equally about all sources of alpha.

We define the flow into the fund from investors of type i on date t as

$$F_{i,t} = \frac{X_{i,t} - X_{i,t-1}}{X_{i,t-1}}. \tag{A9}$$

The total net flow into the fund is then

$$F_t = qF_{s,t} + (1-q)\,F_{n,t} = q\frac{\max\left(\widehat{\delta}_{t+1|t}\,, 0\right)}{\max\left(\widehat{\delta}_{t|t-1}\,, 0\right)} + (1-q)\,\frac{\max\left(\widehat{\alpha}_{t+1|t}\,, 0\right)}{\max\left(\widehat{\alpha}_{t|t-1}\,, 0\right)} - 1. \tag{A10}$$

For simplicity, we assume the history of observed returns is such that $\widehat{\delta}_{t|t-1}\,, \widehat{\alpha}_{t|t-1} > 0$. Hence, both types of investors started with positive dollar holdings, $X_{s,t-1}, X_{n,t-1} > 0$, in the mutual fund at time $t-1$.

Looking forward, it is useful to note two facts that follows immediately from equation (A10). If all investors are naive, $q=0$, then the partial effect of active alpha on fund flows is null. On the other hand, if all investors are sophisticated, $q=1$, then the partial effect of standard alpha on fund flows is null. Essentially, an empirical observation that flows respond positively to both active alpha and the standard alpha measure would suffice to show that at least some investors are sophisticated and not all investors are sophisticated, i.e., $q \in (0,1)$. Moreover, how strongly flows respond to active alpha vs. the standard alpha measure would be informative of q, the fraction of investors who are sophisticated.

Table 1: Summary Statistics

This table summarizes the statistics across fund-month observations from Jan. 1983 to Dec. 2014. Panel A reports fund characteristics such as net return, flows, fund size, expense ratio, age, and return volatility. Percentage fund flow is percentage change TNA from month t-1 to t adjusted for the fund return in month t. Return volatility is calculated as the standard deviation of prior 12 month fund returns. All variables are winsorized at the 1% and 99% levels. Panel B presents the estimated alphas from 36-month rolling regressions using various factor models. Panel C presents of estimated active alphas using various factor models.

	#obs	Mean	SD	25th perc	Median	75th perc
Panel A: Fund Characteristics						
Monthly net return	298,055	0.779%	5.130%	-1.870%	1.250%	3.830%
Percentage fund flow	298,055	-0.097%	4.160%	-1.490%	-0.409%	0.832%
Fund size ($mil)	298,055	1,277	2,838	105.5	324.6	1,041
Log fund size	298,055	5.847	1.612	4.659	5.783	6.948
Expense ratio (per month)	298,055	0.098%	0.110%	0.078%	0.096%	0.117%
Age (months)	298,055	198.9	161.7	92	147	238
Return volatility (t-12 to t-1)	298,055	4.635	2.093	3.03	4.231	5.769
Fund Beta	298,055	0.998	0.165	0.909	0.998	1.083
Panel B: Fund Performance - Standard Alpha (per month)						
CAPM alpha	298,055	0.059%	2.310%	-0.997%	0.019%	1.050%
FF3 alpha	298,055	0.005%	1.830%	-0.866%	-0.001%	0.860%
Carhart4 alpha	298,055	-0.006%	1.800%	-0.856%	-0.007%	0.840%
PS5 alpha	298,055	0.003%	1.830%	-0.854%	0.004%	0.859%
Panel C: Fund Performance - Active Alpha (per month)						
CAPM active alpha	297,926	0.011%	3.020%	-1.530%	-0.019%	1.500%
FF3 active alpha	297,977	-0.048%	2.620%	-1.450%	-0.046%	1.370%
Carhart4 active alpha	297,981	-0.038%	2.640%	-1.380%	-0.032%	1.350%
PS5 active alpha	297,970	-0.050%	2.620%	-1.410%	-0.026%	1.340%

Table 2: Beta Anomaly in Mutual Fund Returns

Panel A reports average annualized abnormal return (alpha) for deciles of mutual funds sorted according to market beta exposures. We estimate the beta for a mutual fund using each of the four performance evaluation models. Market beta exposures are updated monthly based on a rolling regression using prior thirty-six months of returns data. Panel B reports average annualized active alphas for deciles of mutual funds sorted according to market beta exposures. Each month, alphas and active alphas are estimated according to Section 2 for each of the four performance evaluation models.

	Panel A: Alpha				Panel B: Active Alpha			
Beta Group	CAPM	FF3	Carhart4	PS5	CAPM	FF3	Carhart4	PS5
1 (low)	2.50%	1.80%	1.80%	1.81%	-1.02%	-1.29%	-1.42%	-1.27%
2	1.56%	1.10%	0.92%	0.94%	-1.52%	-1.01%	-0.74%	-0.49%
3	1.06%	0.91%	0.55%	0.77%	-1.40%	-1.46%	-0.18%	-1.37%
4	0.72%	0.70%	0.46%	0.64%	-0.34%	-1.04%	0.17%	-1.10%
5	0.52%	0.54%	0.35%	0.48%	-0.69%	-0.50%	-0.26%	-0.43%
6	0.60%	0.19%	0.03%	0.19%	0.35%	-0.13%	-0.83%	-0.45%
7	0.50%	0.08%	-0.16%	0.03%	0.97%	0.30%	-0.82%	-0.12%
8	-0.04%	-0.18%	-0.44%	-0.19%	0.57%	0.07%	-0.74%	0.04%
9	-0.36%	0.19%	-0.19%	-0.17%	1.36%	0.67%	0.06%	0.42%
10 (high)	-0.99%	-0.73%	-1.18%	-0.89%	2.29%	1.19%	0.59%	0.30%
High-Low	-3.48%	-2.53%	-2.98%	-2.69%	3.31%	2.48%	2.01%	1.56%

Table 3: Beta Anomaly in Mutual Fund Returns

Panel A shows the Fama-MacBeth (1973) estimates of annualized alpha regressed on market beta exposure. We estimate the beta for a mutual fund using each of the four performance evaluation models. Panel B shows the Fama-MacBeth (1973) estimates of annualized active alpha regressed on market beta exposure. Market beta exposures are updated monthly based on a rolling regression using prior thirty-six months of returns data. The controls in Column (1) - (4) are nine Morningstar category dummies. The controls in Column (5) - (8) are nine Morningstar category dummies, size, and age. Standard errors are presented in parentheses. *, **, and *** denote 10%, 5%, and 1% significance, respectively.

Panel A: Alpha	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	CAPM	Carhart4	PS5	FP6	CAPM	Carhart4	PS5	FP6
Beta	-0.050**	-0.060***	-0.058***	-0.031**	-0.051**	-0.061***	-0.059***	-0.032***
	(0.020)	(0.012)	(0.012)	(0.013)	(0.020)	(0.012)	(0.012)	(0.012)
Size					0.002***	0.002***	0.002***	0.002***
					(0.000)	(0.001)	(0.001)	(0.000)
Age					-0.003**	-0.004***	-0.004***	-0.004***
					(0.001)	(0.000)	(0.001)	(0.001)
Constant	0.063***	0.067***	0.066***	0.0422***	0.062***	0.072***	0.073***	0.053***
	(0.023)	(0.011)	(0.011)	(0.012)	(0.022)	(0.010)	(0.010)	(0.011)
Style fixed effects	YES	YES	YES	YES	YES	YES	YES	YES
Observations	270,266	270,266	270,266	270,266	269,714	269,714	269,714	269,714
R-squared	0.360	0.148	0.143	0.122	0.371	0.160	0.156	0.134

Panel B: Active Alpha	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	CAPM	Carhart4	PS5	FP6	CAPM	Carhart4	PS5	FP6
Beta	0.036***	0.032*	0.029	0.021	0.035***	0.030	0.027	0.020
	(0.013)	(0.019)	(0.020)	(0.019)	(0.013)	(0.019)	(0.019)	(0.019)
Size					0.002***	0.002***	0.002***	0.002***
					(0.000)	(0.000)	(0.001)	(0.001)
Age					-0.004***	-0.004***	-0.004***	-0.004***
					(0.001)	(0.001)	(0.001)	(0.001)
Constant	-0.030**	-0.029	-0.027	-0.019	-0.025	-0.024	-0.019	-0.007
	(0.014)	(0.019)	(0.021)	(0.020)	(0.016)	(0.018)	(0.019)	(0.020)
Style fixed effects	YES	YES	YES	YES	YES	YES	YES	YES
Observations	270,119	270,179	270,159	270,153	269,567	269,627	269,607	269,601
R-squared	0.222	0.110	0.104	0.099	0.232	0.122	0.114	0.110

Table 4: Active Alpha Persistence

This table reports the results of Fama-MacBeth regressions of the future annualized active alpha rank on the annualized active alpha rank. Each month t, we compute the percentile rank based on the annulized active alpha,$\hat{\Delta}_{p,t}$. A fund's percentile rank represents its percentile performance relative to other funds within the same Morningstar-style box during each month. The rank ranges from 0 to 1. Column (2) reports the monthly cross-sectional regression of annualized active alpha rank on prior month's annualized active alpha rank. We use Newey-West (1987) standard errors with twenty-four lags for column (2). Column (3) reports the monthly cross-sectional regression of annualized active alpha rank on prior year's annualized active alpha rank. We use Newey-West (1987) standard errors with eighteen lags for column (3). Column (4) reports the monthly cross-sectional regression of annualized active alpha on prior two year's annualized active alpha rank. We use Newey-West (1987) standard errors with twelve lags for column (4). In Panel A, active alpha is based on CAPM model. In Panel B, active alpha is based on four-factor model. In Panel C, active alpha is based on six-factor model. Standard errors are presented in parentheses. *, **, and *** denote 10%, 5%, and 1% significance, respectively.

	Panel A: CAPM Active Alpha			Panel B: Carhart4 Active Alpha			Panel C: FP6 Active Alpha		
	$\hat{\Delta}_{p,t+1}$	$\hat{\Delta}_{p,t+12}$	$\hat{\Delta}_{p,t+24}$	$\hat{\Delta}_{p,t+1}$	$\hat{\Delta}_{p,t+12}$	$\hat{\Delta}_{p,t+24}$	$\hat{\Delta}_{p,t+1}$	$\hat{\Delta}_{p,t+12}$	$\hat{\Delta}_{p,t+24}$
$\hat{\Delta}_{p,t}$	0.897***	0.103***	0.006	0.898***	0.150***	0.035***	0.900***	0.152***	0.059***
	(0.004)	(0.013)	(0.014)	(0.003)	(0.014)	(0.010)	(0.004)	(0.014)	(0.011)
Log Fund Size	0.000	0.000	0.002	0.000	0.0013	0.002	0.000	-0.001	-0.001
	(0.000)	(0.003)	(0.003)	(0.000)	(0.003)	(0.003)	(0.000)	(0.002)	(0.003)
Log Exp. Ratio	-0.079	-0.118	0.342	-0.154	-1.154	-1.233	-0.059	-0.359	-0.763
	(0.116)	(0.785)	(0.816)	(0.100)	(0.729)	(0.819)	(0.114)	(0.830)	(0.720)
Log Age	-0.001	-0.004	-0.001	-0.001	-0.009**	-0.011**	-0.001	-0.006	-0.006
	(0.001)	(0.003)	(0.004)	(0.001)	(0.004)	(0.004)	(0.001)	(0.004)	(0.005)
Return Volatility	0.0017	0.012**	0.011**	0.001	0.007	0.004	0.000	0.0054	0.005
	(0.001)	(0.005)	(0.005)	(0.001)	(0.005)	(0.005)	(0.001)	(0.004)	(0.004)
Flow	-0.013	-0.342***	-0.094**	0.005	-0.168**	-0.099	0.017	-0.144*	-0.019
	(0.024)	(0.132)	(0.043)	(0.02)	(0.084)	(0.060)	(0.015)	(0.079)	(0.068)
Constant	0.049***	0.424***	0.443***	0.052***	0.453***	0.540***	0.054***	0.447***	0.495***
	(0.006)	(0.037)	(0.036)	(0.005)	(0.028)	(0.038)	(0.005)	(0.027)	(0.035)
R-squared	0.817	0.056	0.039	0.817	0.068	0.043	0.820	0.064	0.042
Observations	267,316	245,578	221,225	267,374	245,616	221,260	267,349	245,616	221,260
Correlation	0.901	0.103	0.018	0.901	0.146	0.040	0.900	0.130	0.047

Table 5: Active Alpha Sort Portfolio

This table reports performance of active-alpha sorted calendar-time mutual fund portfolios. Each month, mutual funds are assigned to one of ten deciles mutual fund portfolios based on prior month's annualized active alpha. Panel A reports CAPM active alpha sort results. Panel B reports Carhart4 active alpha sort results. Panel C reports FP6 active alpha sort results. All mutual funds are equally weighted within a given portfolio, and the portfolios are rebalanced every month to maintain equal weights. Column (2) - column (5) report mutual fund portfolio's time series average of gross return, market adjusted return, Sharpe ratio, information ratio, and Carhart4 alpha. We use Newey-West (1987) standard errors with eighteen lags; t-statistics are presented in parentheses. *, **, and *** denote 10%, 5%, and 1% significance, respectively.

Panel A: CAPM Active Alpha

Active Alpha	Gross Ret	MAR	Shar. R.	Info. R.	Carhart4 Alpha
1 (low)	0.901%	-0.175%	0.1484	0.1479	-0.045%
2	0.949%	-0.122%	0.1653	0.1649	-0.030%
3	0.958%	-0.111%	0.1683	0.1679	-0.018%
4	0.959%	-0.110%	0.1669	0.1666	-0.025%
5	0.973%	-0.096%	0.1689	0.1685	-0.018%
6	0.988%	-0.081%	0.1740	0.1737	0.010%
7	1.026%	-0.044%	0.1859	0.1857	0.021%
8	1.064%	-0.006%	0.1914	0.1911	0.037%
9	1.110%	0.038%	0.2036	0.2034	0.068%
10 (high)	1.199%	0.120%	0.2042	0.2039	0.116%
High-Low	0.298%***	0.295%***	0.0557**	0.0560**	0.161%
t-stats	(2.636)	(2.602)	(2.057)	(2.064)	(0.949)

Panel B: Carhart4 Active Alpha

Active Alpha	Gross Ret	MAR	Shar. R.	Info. R.	Carhart4 Alpha
1 (low)	0.906%	-0.173%	0.1483	0.1479	-0.058%
2	0.975%	-0.097%	0.1643	0.1638	-0.006%
3	0.991%	-0.079%	0.1722	0.1719	-0.017%
4	0.983%	-0.086%	0.1731	0.1728	-0.015%
5	0.995%	-0.073%	0.1738	0.1735	0.006%
6	0.999%	-0.070%	0.1748	0.1744	0.009%
7	1.002%	-0.066%	0.1820	0.1817	-0.002%
8	1.026%	-0.044%	0.1859	0.1856	0.023%
9	1.044%	-0.027%	0.1889	0.1886	0.012%
10 (high)	1.206%	0.128%	0.2112	0.2110	0.166%
High-Low	0.300%***	0.301%***	0.0629***	0.0631***	0.224%***
t-stats	(3.000)	(3.022)	(2.906)	(2.911)	(2.673)

Table 5 continued

Panel C: FP6 Active Alpha

Active Alpha	Gross Ret	MAR	Sharpe R	Info. R	Carhart4 Alpha
1 (low)	0.976%	-0.101%	0.1639	0.1635	0.001%
2	0.990%	-0.080%	0.1697	0.1693	0.002%
3	0.986%	-0.083%	0.1758	0.1754	0.000%
4	0.984%	-0.085%	0.1685	0.1681	-0.037%
5	0.961%	-0.107%	0.1704	0.1701	-0.015%
6	0.996%	-0.073%	0.1770	0.1767	-0.011%
7	0.983%	-0.086%	0.1731	0.1728	-0.022%
8	1.005%	-0.065%	0.1769	0.1766	-0.001%
9	1.061%	-0.012%	0.1941	0.1939	0.063%
10 (high)	1.182%	0.102%	0.2056	0.2053	0.134%
High-Low	0.205%***	0.203%***	0.0417***	0.0419***	0.133%**
t-stats	(2.658)	(2.639)	(2.461)	(2.459)	(1.967)

Table 6: Analysis of Active Alpha

This table reports the results of panel regressions of active alpha on fund performance measures and characteristics. The dependent variable is the percentaile rank of annualized active alpha. Return gap is calculated following Kacperczyk, Sialm, and Zheng (2008) as the difference between a fund's monthly returns and returns on its most recently reported holdings, Active share is computed as in Cremers and Petajisto (2009). R-squared is calculated following Amihud and Goyenko (2013) as the log transformed R-squared from Carhart (1997) four-factor regressions using monthly returns covering the most recent twenty-four months. Standard errors (double-clustered by fund and month) are presented in parentheses. *, **, and *** denote 10%, 5%, and 1% significance, respectively.

Panel A: Percentile rank of CAPM Active Alpha

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Return Gap	0.0188***			0.0148***			
	(0.00561)			(0.00490)			
Active Share		0.0404***			0.0276**		
		(0.0116)			(0.0108)		
R^2			0.00468			0.00559	
			(0.00886)			(0.00830)	
Log Fund Size				0.00416***	0.00474***	0.00477***	0.00494***
				(0.00149)	(0.00151)	(0.00142)	(0.00141)
Log Exp. Ratio				1.046	0.719	1.124*	1.008*
				(0.648)	(0.657)	(0.626)	(0.598)
Log Age				-0.00320	-0.00403	-0.00481	-0.00498*
				(0.00310)	(0.00310)	(0.00306)	(0.00300)
Return Volatility				0.0218***	0.0215***	0.0219***	0.0216***
				(0.00201)	(0.00200)	(0.00198)	(0.00199)
Flow				0.000203	-0.00448	0.00467	0.00234
				(0.0337)	(0.0336)	(0.0312)	(0.0309)
12 month Return				0.412***	0.409***	0.418***	0.411***
				(0.0319)	(0.0319)	(0.0318)	(0.0320)
Constant	0.561***	0.507***	0.532***	0.388***	0.358***	0.391***	0.398***
	(0.0147)	(0.0128)	(0.0118)	(0.0247)	(0.0233)	(0.0240)	(0.0233)
Style fixed effects	YES	YES	YES	YES	YES	YES	YES
R-squared	0.035	0.035	0.034	0.111	0.111	0.113	0.111
Observations	238,466	236,801	263,348	237,824	236,342	262,811	269,408

Table 6 continued

Panel B: Percentile rank of Carhart4 Active Alpha

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Return Gap	0.0285***			0.0258***			
	(0.00436)			(0.00409)			
Active Share		0.0134			0.00692		
		(0.0119)			(0.0118)		
R^2			0.0113			0.0110	
			(0.00968)			(0.00948)	
Log Fund Size				0.00138	0.00186	0.00245*	0.00239*
				(0.00148)	(0.00152)	(0.00148)	(0.00144)
Log Exp. Ratio				-0.537	-0.580	-0.186	-0.528
				(0.614)	(0.617)	(0.583)	(0.575)
Log Age				-0.00358	-0.00399	-0.00414	-0.00531
				(0.00337)	(0.00339)	(0.00328)	(0.00324)
Return Volatility				0.0130***	0.0130***	0.0128***	0.0133***
				(0.00166)	(0.00167)	(0.00164)	(0.00163)
Flow				0.0899***	0.0818**	0.0905***	0.0812**
				(0.0344)	(0.0343)	(0.0324)	(0.0318)
12 month Return				0.277***	0.278***	0.280***	0.278***
				(0.0249)	(0.0249)	(0.0249)	(0.0247)
Constant	0.525***	0.525***	0.529***	0.446***	0.453***	0.441***	0.455***
	(0.0120)	(0.0134)	(0.0126)	(0.0246)	(0.0262)	(0.0237)	(0.0229)
Style fixed effects	YES	YES	YES	YES	YES	YES	YES
R-squared	0.004	0.004	0.004	0.039	0.038	0.038	0.038
Observations	238,499	236,860	263,408	237,857	236,401	262,871	269,468

Table 6 continued

Panel C: Percentile rank of FP6 Active Alpha

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Return Gap	0.0320***			0.0295***			
	(0.00414)			(0.00391)			
Active Share		0.0594***			0.0501***		
		(0.0117)			(0.0117)		
R^2			-0.0390***			-0.0374***	
			(0.00911)			(0.00904)	
Log Fund size				0.00176	0.00236	0.00279**	0.00189
				(0.00146)	(0.00148)	(0.00141)	(0.00140)
Log Exp. Ratio				1.256*	0.735	0.641	1.137*
				(0.663)	(0.667)	(0.634)	(0.628)
Log Age				-0.00199	-0.00245	-0.00355	-0.00268
				(0.00352)	(0.00346)	(0.00326)	(0.00328)
Return Volatility				0.0115***	0.0110***	0.0119***	0.0117***
				(0.00149)	(0.00148)	(0.00147)	(0.00145)
Flow				0.0853**	0.0734**	0.0725**	0.0817***
				(0.0337)	(0.0337)	(0.0316)	(0.0311)
12 month Return				0.257***	0.253***	0.260***	0.258***
				(0.0223)	(0.0222)	(0.0223)	(0.0222)
Constant	0.518***	0.487***	0.538***	0.416***	0.399***	0.448***	0.432***
	(0.0119)	(0.0146)	(0.0105)	(0.0256)	(0.0247)	(0.0225)	(0.0239)
Style fixed effects	YES	YES	YES	YES	YES	YES	YES
R-squared	0.008	0.009	0.008	0.038	0.038	0.038	0.037
Observations	238,499	236,835	263,382	237,857	236,376	262,845	269,442

Table 7: Fund Flows and Competing Measures

This table presents regression coefficient estimates from panel regressions of monthly fund flow (dependent variable) on the lagged rank of annualized active alpha and the lagged rank of annualized alpha. In Panel A and Panel B, a mutual fund's rank represents its percentile performance relative to all other mutual funds during the same month. In Panel C and Panel D, a fund's rank represents its percentile performance relative to other funds within the same Morningstar-style box during the same month. The rank ranges from 0 to 1. We calculate the rank based on annualized alpha and annualized active alphas. Controls include lagged fund flows from month t-13, lagged values of log of fund size, log of fund age, expense ratio, return volatility, and style-month fixed effects. Standard errors (double-clustered by fund and month) are presented in parentheses. *, **, and *** denote 10%, 5%, and 1% significance, respectively.

Dependent Variable: Flow

Panel A (1): Overall Alpha Percentile Rank				Panel A (2): Overall Active Alpha Percentile Rank			
	CAPM	Carhart4	FP6		CAPM	Carhart4	FP6
Alpha	0.039***	0.029***	0.027***	Active Alpha	0.025***	0.020***	0.019***
	(0.001)	(0.001)	(0.001)		(0.001)	(0.001)	(0.001)
Observations	269,610	269,610	269,610		269,463	269,523	269,497
R-squared	0.243	0.232	0.225		0.219	0.213	0.209
Style-month fixed effects	YES	YES	YES		YES	YES	YES
Controls	YES	YES	YES		YES	YES	YES

Panel B : Overall Alpha Percentile Rank & Overall Active Alpha Percentile Rank			
	CAPM	Carhart4	FP6
Active Alpha Percentile Rank	0.006***	0.003***	0.002***
	(0.001)	(0.001)	(0.001)
Alpha Percentile Rank	0.035***	0.028***	0.025***
	(0.001)	(0.001)	(0.001)
Observations	269,463	269,523	269,497
R-squared	0.244	0.232	0.226
Style-month fixed effects	YES	YES	YES
Controls	YES	YES	YES

Table 7 continued

Panel C (1): Alpha Percentile Rank within Style				Panel C (2): Active Alpha Percentile Rank within Style			
	CAPM	Carhart4	FP6		CAPM	Carhart4	FP6
Alpha	0.031***	0.027***	0.025***	Active Alpha	0.022***	0.019***	0.018***
	(0.001)	(0.001)	(0.001)		(0.001)	(0.001)	(0.001)
Observations	269,610	269,610	269,610		269,463	269,523	269,497
R-squared	0.243	0.231	0.225		0.219	0.212	0.209
Style-month fixed effects	YES	YES	YES		YES	YES	YES
Controls	YES	YES	YES		YES	YES	YES

Panel D : Alpha Percentile Rank within Style & Active Alpha Percentile Rank within Style

	CAPM	Carhart4	FP6
Active Alpha Percentile Rank	0.006***	0.003***	0.003***
	(0.001)	(0.001)	(0.001)
Alpha Percentile Rank	0.028***	0.025***	0.023***
	(0.001)	(0.001)	(0.001)
Observations	269,463	269,523	269,497
R-squared	0.244	0.232	0.225
Style-month fixed effects	YES	YES	YES
Controls	YES	YES	YES

Table 8: Investor Sophistication and Flow-Active Alpha Relationship

This table presents regression coefficient estimates from panel regressions of monthly flow to institution/retail share class (dependent variable). A share class is defined as institutional share class if Morningstar share class is INST or CRSP institution fund dummy is 1. For each mutual fund, the flow to its institutional class is the value-weighted flow across fund's multiple institutional classes. Similar, the flow to fund's retail share class is the value-weighted flow across fund's retail classes. Controls include lagged rank of annualized alpha, lagged fund flows from month t-13, lagged values of log of fund size, log of fund age, expense ratio, return volatility, fund fixed effect and style-month fixed effects. Standard errors (double-clustered by fund and month) are presented in parentheses. *, **, and *** denote 10%, 5%, and 1% significance, respectively.

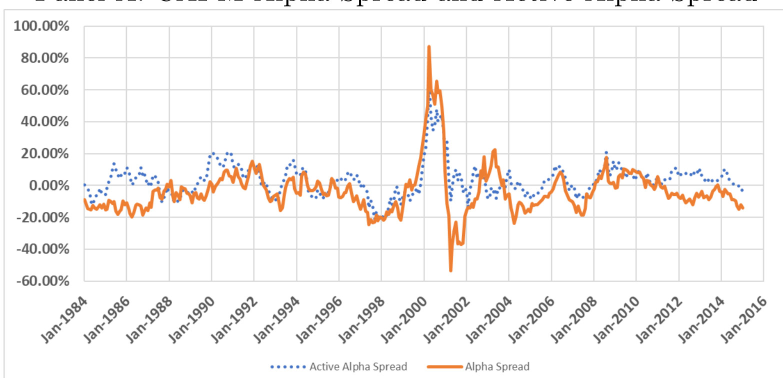
Dependent Variable: Flow

	CAPM	Carhart4	FP6
Ret. Class*Active Alpha	0.00327***	0.00157*	0.000441
	(0.00103)	(0.000905)	(0.000917)
Inst. Class * Active Alpha	0.0176***	0.00590***	0.00845***
	(0.00215)	(0.00214)	(0.00226)
Ret. Class* Alpha	0.0335***	0.0237***	0.0219***
	(0.00174)	(0.00135)	(0.00138)
Inst. Class * Alpha	0.0469***	0.0470***	0.0414***
	(0.090)	(0.091)	(0.090)
R-squared	0.084	0.082	0.080
Observations	382,780	382,867	382,815
Fund fixed effects	YES	YES	YES
Style-month fixed effects	YES	YES	YES

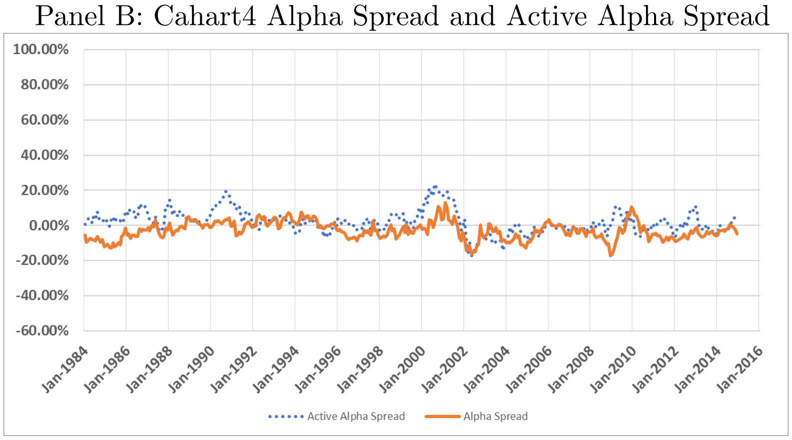
Figure 1: Time series of spreads between high beta and low beta mutual fund portfolios

This figure plots the annualized alpha spreads (solid lines) and annualized active alpha spreads (dotted lines) between highest beta portfolio and lowest beta decile portfolio (decile 10 - decile1). Each month, mutual funds are ranked into equal-weight decile portfolios based on market beta exposures estimate. Mutual fund alphas, active alphas, and market beta exposures are updated monthly based on a rolling regression using prior thirty-six months of returns data. In Panel A, we report annualized alpha and active alpha spreads based on CAPM; in the middle Panel B, we report annualized alpha and active alpha spreads based on Carhart four-factor model; and in Panel C, we report annualized alpha and active alpha spreads based on FP six-factor model.

Panel A: CAPM Alpha Spread and Active Alpha Spread



Panel B: Cahart4 Alpha Spread and Active Alpha Spread



Panel C: FP6 Alpha Spread and Active Alpha Spread

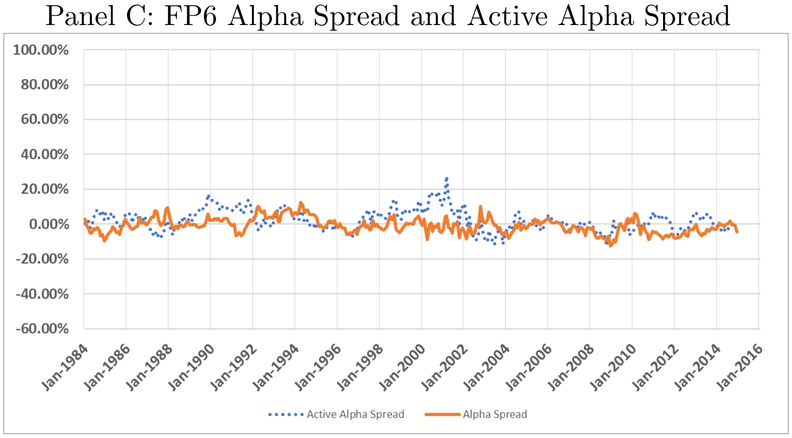
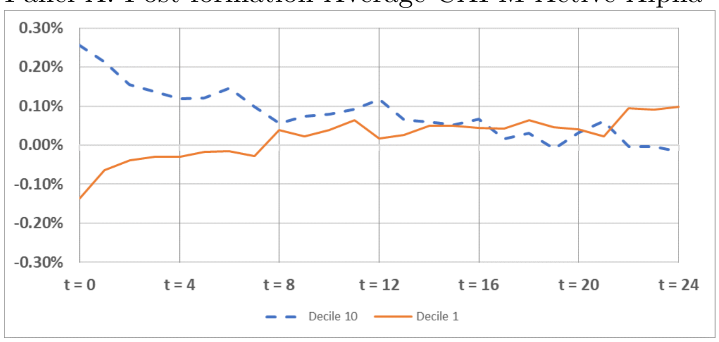


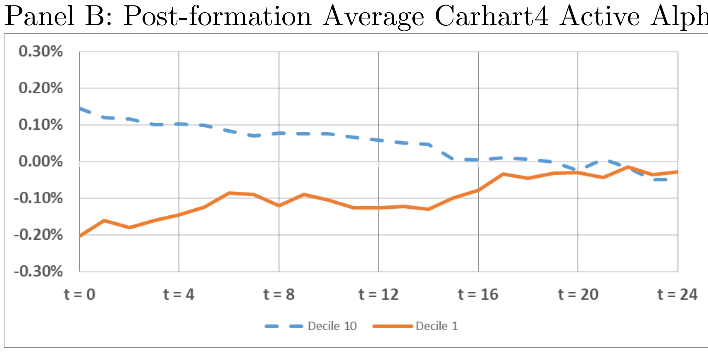
Figure 2: Persistence of Active Alpha

This figure depicts the average monthly active alpha of portfolios tracked over a 2-year period between 1984 and 2014. The portfolios are formed by sorting all the funds into deciles according to lagged annualized active alpha. Subsequently, the top and bottom decile portfolios are tracked over the next 2-year period. The portfolios are equally weighted each month, so the porfolios are readjusted whenever a fund disappears from the sample. In Panel A, we report the CAPM active alphas; in Panel B, we report the Carhart4 active alpha; and in Panel C, we report the FP6 active alpha.

Panel A: Post-formation Average CAPM Active Alpha



Panel B: Post-formation Average Carhart4 Active Alpha



Panel C: Post-formation Average FP6 Active Alpha

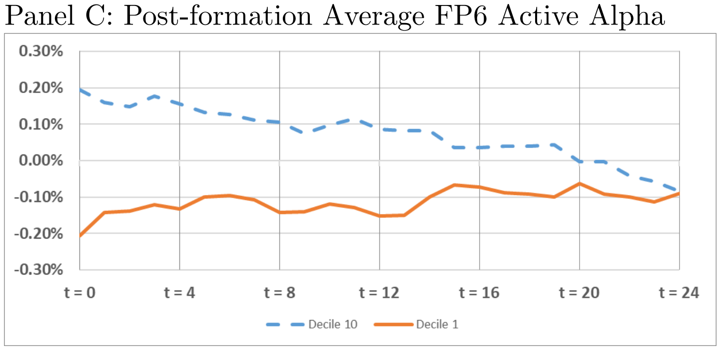
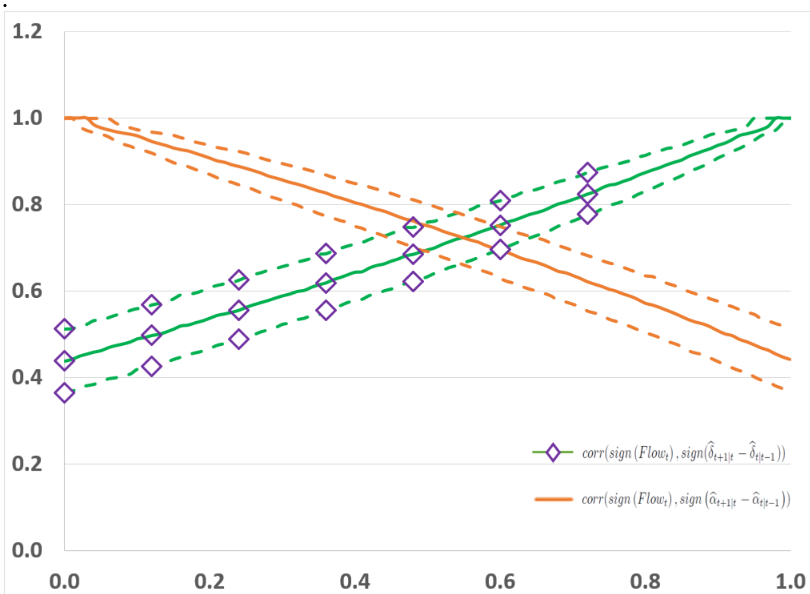


Figure 3: Calibration Results

This figure plots the estimated correlations based on the simulated samples. We plot the median (solid lines) as well as the 25th and 75th percentiles (dotted lines) of the estimated correlations. In Panel A, the green diamond line (orange no-marker line) shows how the correlation between the sign of the capital inflows and the sign of the updates on active alpha (standard alpha) vary with the fraction of sophisticated investors q. In Panel B, the ratio of the correlations between the sign of the capital inflows and the sign of the update on active alpha to the analogous correlation using standard alpha is plotted as a function of the fraction of sophisticated investors q. The red line shows the empirical estimate of this ratio (0.63).

Panel A:



Panel B: