Chapter- 4 b

Engineering Economics

Nominal and Effective Interest Rates

College of Biomedical Engineering and Applied Science

Nominal Interest Rates:

Nominal interest rate (r) is an interest rate that does not include any consideration of compounding, by definition,

$$r = \text{Interest rate per period} \times \text{Number of Periods}$$

It may be stated for any time period such as 1 year, 6 months, quarter Month, week, day etc.

Nominal Interest Rates:

r = 1.5% per month × 24 months

= 36% per two year period

r = 1.5% per month × 12 months

= 18% per year period

r = 1.5% per month × 6 months

= 9% per Semiannual period

r = 1.5% per month × 3 month

= 4.5% per quarter

Effective Interest Rate:

- Effective interest rate is the actual rate that applies for a stated period of time. The compounding of interest during the time period of the corresponding nominal rate is accounted for by the effective interest rate.
- It is commonly expressed on an annual basis as the effective annual rate but any time basis Can be used.
- An effective rate has the compounding frequency attached to the nominal rate Statement.
- If the compounding frequency is not stated, it is assumed to be the same period of r, in which case, the nominal and effective rates have the same value.

Effective Interest Rate:

Examples

12% per year, compounded monthly

12% per year, compounded quarterly

3 % per year, compounded monthly

6 % per 6 months, compounded weekly

Compounding more often than time period

3 % per quarter, compounded quarterly

Compounding same as than time period

Effective Interest Rate:

General Format

r % per time period t, compounded m-ly

All the interest formulas, factors, tabulated values, and spreadsheet relations must have the effective interest rate to properly account for the time value of money.

Effective Interest Rate:

Definition of Terms

Time Period:

The basic time unit of the interest rate.

Compounding Period (CP):

The time unit used to determine the effect of the interest.

Compounding Frequency (m):

The number of times that compounding occurs within the time Period t.

Effective Interest Rate:

$$\text{Effective rate per CP} = \frac{\mathbf{r}\% \text{per time period } \mathbf{t}}{\mathbf{m} \text{ compounding periods per } \mathbf{t}}$$

Example 1:

The different bank loan rates for three separate electric generation equipment projects are listed below. Determine the effective rate on the basis of the compounding period for each quote.

- **9% per year, compounded quarterly.**
- **9% per year, compounded monthly.**
- **4.5% per 6-months, compounded weekly.**

Solution: Example

- **9% per year, compounded quarterly**

2.25%	**2.25%**	**2.25%**	**2.25%**

$$\mathbf{i} = \frac{\mathbf{r}}{\mathbf{m}} = \frac{9}{4} = 2.25\%$$

Solution: Example

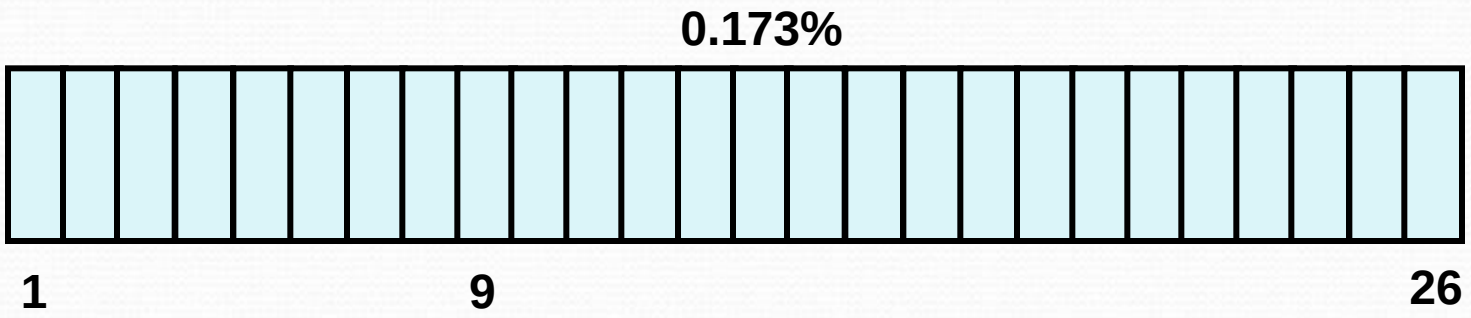
- **9% per year, compounded monthly**

0.75%

$$\mathbf{i} = \frac{\mathbf{r}}{\mathbf{m}} = \frac{9}{12} = 0.75\%$$

Solution: Example

- **4.5% per 6-months, compounded weekly**



$$\mathbf{i} = \frac{\mathbf{r}}{\mathbf{m}} = \frac{9}{26} = 0.173\%$$

Ways to express Nominal and Effective Interest Rates:

Format of Rate Statement:

Nominal Rate Stated, Compounding period Stated

Examples of Statement:

8% per year, compounded quarterly

What about the Effective Rate?

Find Effective Rate

Ways to express Nominal and Effective Interest Rates:

Format of Rate Statement:

Effective rate stated.

Examples of Statement:

8.243% per year, compounded quarterly.

What about the Effective Rate?

Use effective rate directly.

Ways to express Nominal and Effective Interest Rates:

Format of Rate Statement:

Interest rate stated, no compounding period stated

Examples of Statement:

8% per year or 2% per quarter

What about the Effective Rate?

Rate is effective only for time period stated: find effective rate for all other time periods.

Effective Annual Interest Rates:

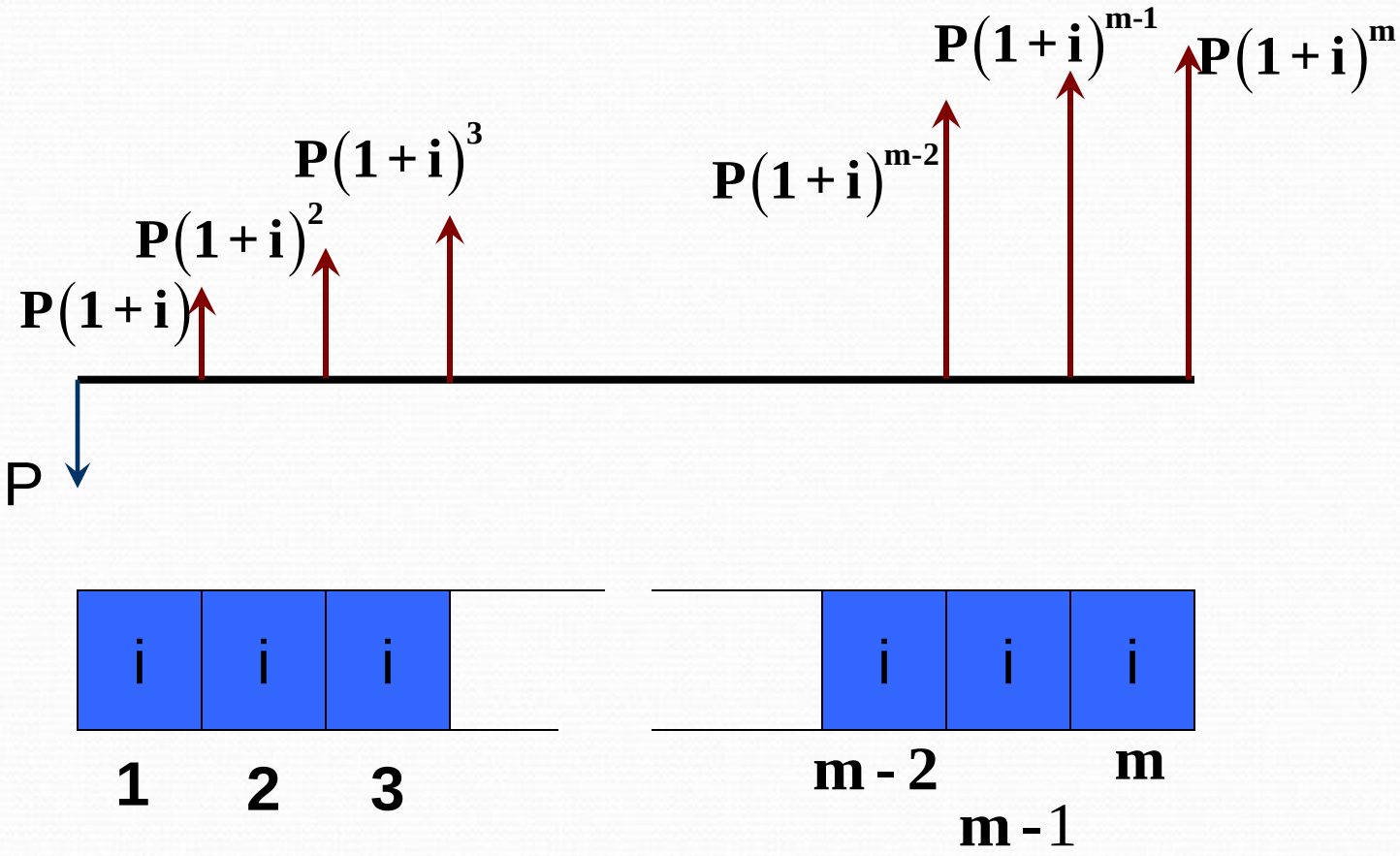
r = nominal interest rate per year.

m = number of compounding periods per year.

i = effective interest rate per CP.

i_a = effective interest rate per year.

Effective Annual Interest Rates:



Effective Annual Interest Rates:

If I_a be the annual effective interest rates then

$$F = P\left(1 + i_a\right)$$

The rate i per CP must be compounded through all m Periods to obtain the total effect of compounding by the end of the year.

$$F = P\left(1 + i\right)^m$$

Effective Annual Interest Rates:

Comparing the two, we get

$$i_a = (1 + i)^m - 1$$

Above equation calculates the effective annual interest rate for any number of compounding periods when I is the rate for one compounding period.

!!! It is also possible to determine the nominal annual interest rate

Example 2:

Jacki obtained a new credit card from a national bank, MBNA with a stated rate of 18% per year, compounded monthly. For a $1000 balance at the beginning of the year, find the effective annual rate and the total amount owed to MBNA after 1 year, provided no payments are made during the year.

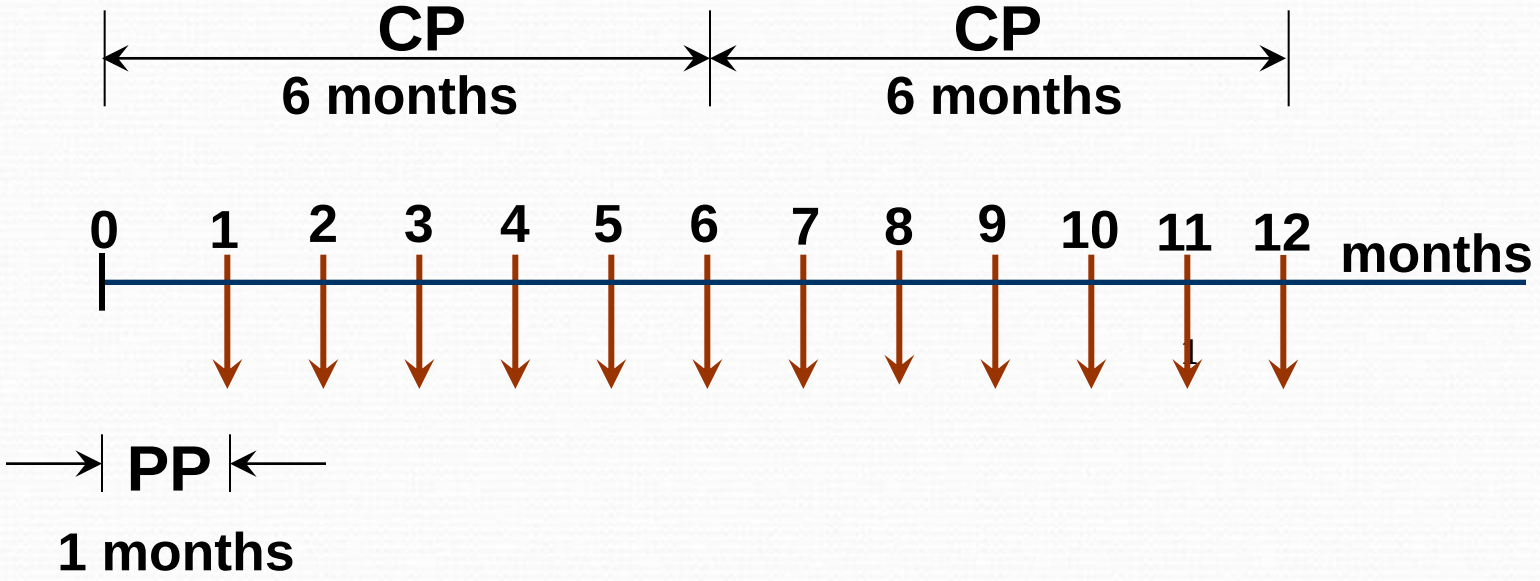
Solution: Example

$$i_a = (1 + i)^m - 1$$

$$= (1 + 0.015)^{12} - 1 = 0.19562$$

$$\mathbf{F = \$1000(1.19562) = \$1195.62}$$

Effective Interest Rates for Any Time Period:



It is important to distinguish between payment period and compounding period because in many circumstances they do not coincide.

To evaluate the cash flows that occur more frequently than Annually, that is, PP < 1 year, the effective interest rate over the PP must be used in the engineering economy analysis.

Effective Interest Rate Can be Generalized:

$$\text{Effective } i = \left(1 + r/m\right)^{m} - 1$$

Where

r = Nominal Interest Rate Per Payment Period (PP)
m = number of compounding periods per payment period

Example 3:

Visteon, a spin-off company of Ford Motor Company, supplies major automobile components to auto manufactures worldwide and is Ford's largest supplier. An engineer is on a Visteon committee to evaluate bids for new generation coordinate measuring machinery to be directly linked to the automated manufacturing of high-precision components. Three vendors bids include the interest rates. Visteon will make payments on semiannual basis only. The engineer is confused about the effective interest rates- what they are annually and over the payment period of 6 months.

Bid # 1: 9% per year, compounded quarterly
Bid # 2: 3% per quarter, compounded quarterly
Bid # 3: 8.8 % per year, compounded monthly

a) **Determine the effective interest rate for each bid on the basis of semiannual payments, and construct cash flow diagrams for each bid rates**
b) **What are the effective annual rates? These are to be a part of the final bid selection.**
c) **Which bid has the lowest effective annual rate?**

Solutions

(a) **Payment Period PP = 6 month**

Nominal Interest Rates r = 9% per year
= 4.5% per 6 months

CP, m = 2 quarters per 6-months

Effective i% per 6-months =

$$\left(1+\frac{0.045}{2}\right)^{2}-1=1.0455-1=4.55\%$$

Semiannual Rates

Bid	Nominal per 6 Months, r	CP per PP m	Effective i
1	4.5%	2	4.55%
2	6.0%	2	6.09%
3	4.4%	6	4.48%

b) For the effective annual rate

Payment Period PP = 1 year

Nominal Interest Rates r = 9% per year

CP, m = 4 quarters per

Effective i% per year =

$$\left(1+\frac{0.09}{4}\right)^{4}-1=1.0931-1=9.31\%$$

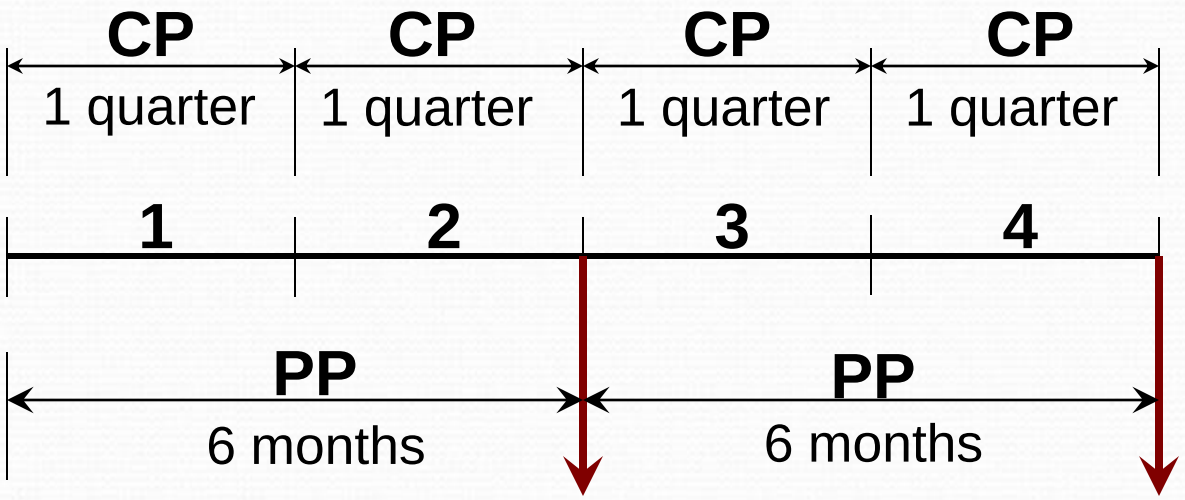
Annual Rates

Bid	Nominal per year, r	CP per year m	Effective i
1	9.0%	4	9.31%
2	12.0%	4	12.55%
3	8.8%	12	9.16%

c)

Bid #3 includes the lowest effective annual rate of 9.16%, which is equivalent to an effective semiannual rate of 4.48%

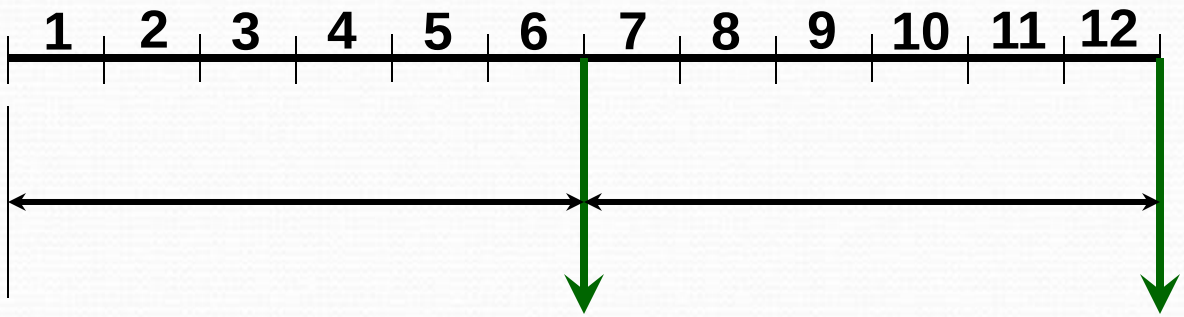
Cash Flow Diagram



Quarterly Compounding

Cash Flow Diagram

CP = 1 months



Monthly compounding

Example 4:

A dot com company plans to place money in a new venture capital fund that currently returns 18% year, compounded daily. What effective interest rate is this (a) yearly and (b) semiannually?

Solutions

a) r = 18% m = 365 days

$$\text{Effective } i\% \text{ per year} = \left(1+\frac{0.18}{365}\right)^{365} - 1 = 19.716\%$$

b) r = 9% m = 182 days

$$\text{Effective } i\% \text{ per year} = \left(1+\frac{0.09}{182}\right)^{182} - 1 = 9.415\%$$

Equivalence Relations (Single Payment)

$$PP \geq CP$$

When only single-amount cash flows are involved, there are two equally correct ways to determine *I* and *n* for P/F and F/P factors.

Method 1:

- **Determine the effective interest over the compounding period CP.**
- **Set *n* equal to the number of compounding periods between P and F.**

The relations to calculate P and F are:

P = F(P/F, effective i% per CP, total number of periods n)

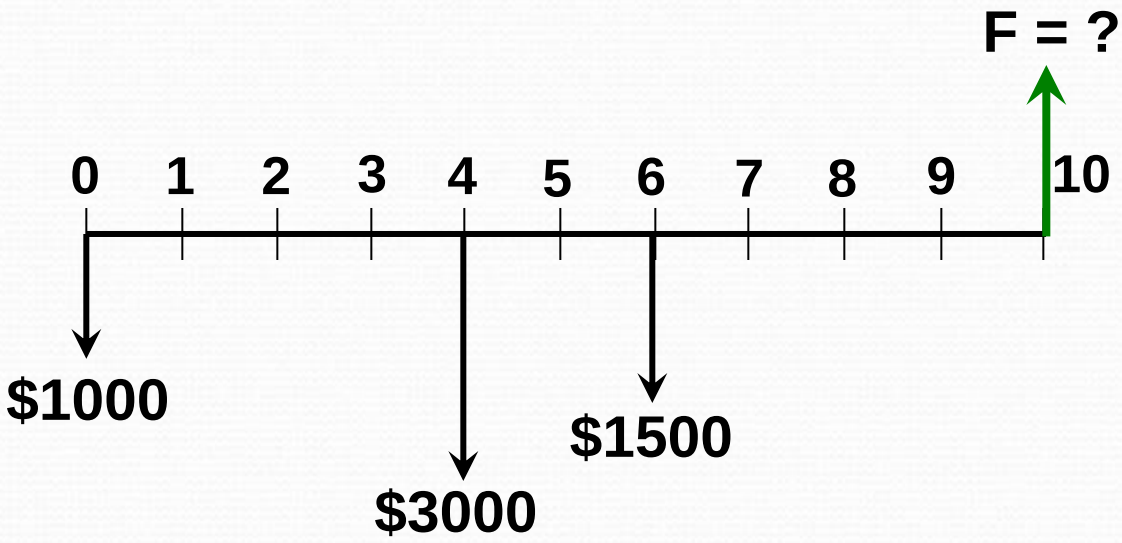
F = P(P/F, effective i% per CP, total number of periods n)

Method 2

- Determine the effective interest rate for time period t of the nominal rate,
- set n equal to the total number of periods using this same time period.
- The P and F relations are same with the term effective *i*% per t substituted for the interest rates.

Example 5:

An engineer working as a private consultant made deposits into a special account to cover unreimbursed travel expenses. Cash flow diagram is shown. Find the amount in the account after 10 years at an interest rate of 12% per year, compounded semiannually.



Method I

Effective rate i = 6% per 6 month period

$$F = 1000(F/P,6\%,20) + 3000(F/P,6\%,12) + 1500(F/P,6\%,8)$$
$$= 1000(3.2071) + 3000(2.0122) + 1500(1.5938)$$
$$= \$11,634$$

Method 2

Express the effective annual rate, based on semiannual compounding.

$$\textbf{Effective } i\% \textbf{ per year} = \left(1 + \frac{0.12}{2}\right)^2 - 1 = 12.36\%$$

$$\begin{aligned} F &= 1000(F/P,12.36\%,10) + 3000(F/P,12.36\%,6) + 1500(F/P,12.36\%,4) \\ &= 1000(3.2071) + 3000(2.0122) + 1500(1.5938) \\ &= \$11,634 \end{aligned}$$

Equivalence Relations (Series)

PP ≥ CP

When cash flows involve a series (i.e. A, G, g) and the payment period equals or exceeds the compounding period In length,

- **Find the effective i per payment period**
- **Determine n as the total number of payment periods**

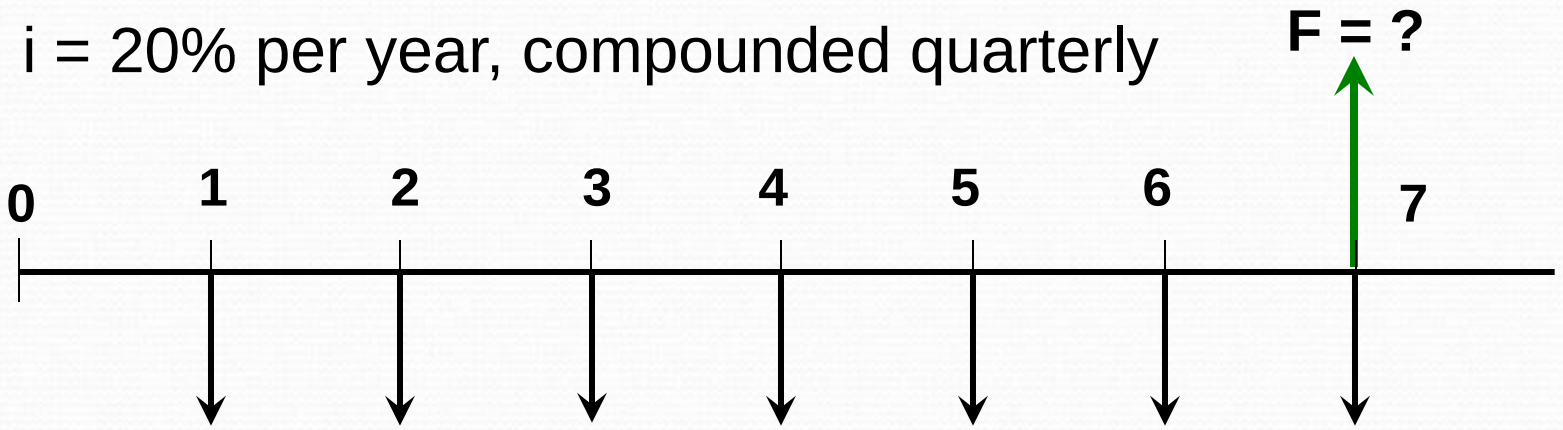
Example 6:

For the past seven years, a quality manager has paid $500 every 6 months for the software maintenance contract of a LAN. What is the equivalent amount after the last payment, if these funds are taken from a pool that has been returning 20% per year, compounded quarterly?

Solution

PP > CP

i = 20% per year, compounded quarterly



A = $500

Effective i% per 6-months =

$$\left(1+\frac{0.1}{2}\right)^2-1=10.25\%$$

$$\mathbf{F = A(F/A,10.25\%,14)}$$

$$\mathbf{= 500(28.4895)}$$

$$\mathbf{= \$14,244.50}$$

Example 7:

Suppose you plan to purchase a car and carry a loan of $12,500 at 9% per year, compounded monthly. Payments will made monthly for 4 years. Determine the monthly payment.

Solution

Effective i% per months =

$$\frac{r}{m} = \frac{9\%}{12} = 0.75\%$$

$$\mathbf{A = \$12{,}500(A/P, 0.75\%, 48)}$$

$$\mathbf{= 12{,}500(0.02489) = \$311.13}$$

Equivalence Relations: Single amounts and Series with

PP < CP

How inter period compounding is handled?

There are two policies:

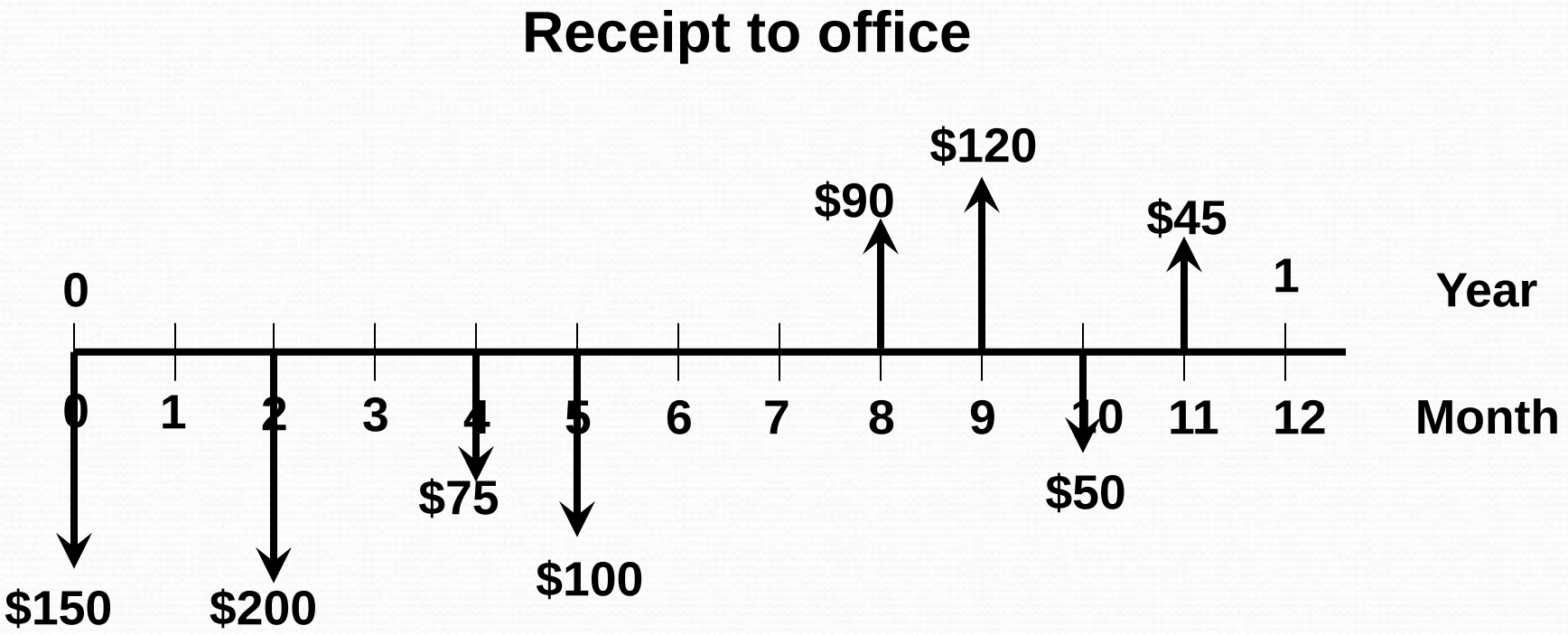
Deposits is regarded as deposited at the end of the compounding period.

Withdrawals are all regarded as with drawn at the beginning.

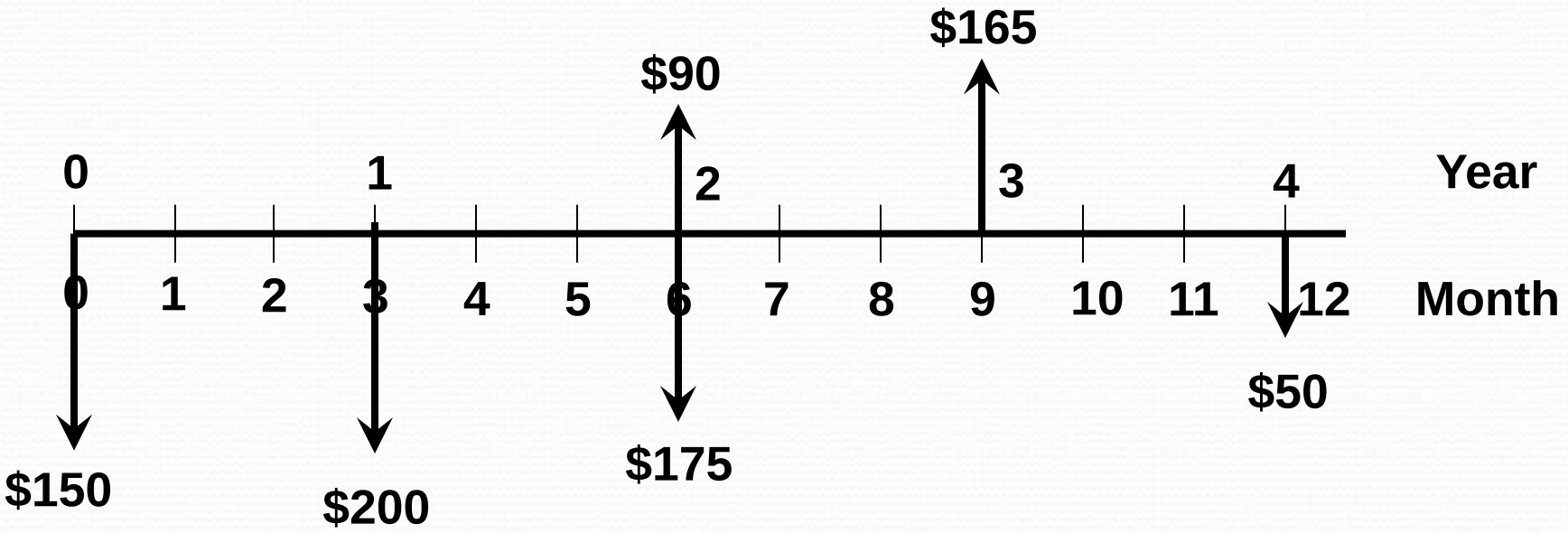
Example 8:

Rob is the on-site coordinating engineer for Alcoa Aluminum, where an under-renovation mine has new ore refining equipment being installed by a local contractor. Rob developed A cash flow diagram in figure in $1000 units from the project perspectives. Included are payments to the contractor he has authorized for the current year and approved advances from Alcoa's home office. He knows that the interest rate on equipment "field projects" such as this is 12% per year, compounded quarterly, and that Alcoa does not bother with Inter period compounding of interest. Will Rob's project finances Be in "red" or the "black" at the end of the year? By How much?

Cash Flow Diagram



Moved Cash Flow Diagram



Effective interest rate = 3%

$$F = 1000[-150(F/P,3\%,4) - 200(F/P,3\%,3)$$
$$+ (-175 + 90)(F/P,3\%,2) + 165(F/P,3\%,1) - 50]$$

$$= \$ - 357,592$$