GLENCOE MATHEMATICS

Geometry

Chapter 2 Resource Masters

Glencoe

New York, New York
Columbus, Ohio
Chicago, Illinois
Peoria, Illinois
Woodland Hills, California

Consumable Workbooks

Many of the worksheets contained in the Chapter Resource Masters booklets are available as consumable workbooks.

Study Guide and Intervention Workbook	0-07-860191-6
Skills Practice Workbook	0-07-860192-4
Practice Workbook	0-07-860193-2
Reading to Learn Mathematics Workbook	0-07-861061-3

ANSWERS FOR WORKBOOKS The answers for Chapter 2 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

Glencoe

The McGraw-Hill Companies

Copyright © by The McGraw-Hill Companies, Inc. All rights reserved.
Printed in the United States of America. Permission is granted to reproduce the material contained herein on the condition that such material be reproduced only for classroom use; be provided to students, teachers, and families without charge; and be used solely in conjunction with Glencoe's *Geometry.* Any other reproduction, for use or sale, is prohibited without prior written permission of the publisher.

Send all inquiries to:
The McGraw-Hill Companies
8787 Orion Place
Columbus, OH 43240-4027

ISBN: 0-07-860179-7

Geometry
Chapter 2 Resource Masters

1 2 3 4 5 6 7 8 9 10 009 11 10 09 08 07 06 05 04 03

Contents

Teacher's Guide to Using the Chapter 2 Resource Masters

The ***Fast File*** Chapter Resource system allows you to conveniently file the resources you use most often. The *Chapter 2 Resource Masters* includes the core materials needed for Chapter 2. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing in the *Geometry TeacherWorks* CD-ROM.

Vocabulary Builder Pages vii–viii include a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar.

WHEN TO USE Give these pages to students before beginning Lesson 2-1. Encourage them to add these pages to their Geometry Study Notebook. Remind them to add definitions and examples as they complete each lesson.

Vocabulary Builder Pages ix–x include another student study tool that presents up to fourteen of the key theorems and postulates from the chapter. Students are to write each theorem or postulate in their own words, including illustrations if they choose to do so. You may suggest that students highlight or star the theorems or postulates with which they are not familiar.

WHEN TO USE Give these pages to students before beginning Lesson 2-1. Encourage them to add these pages to their Geometry Study Notebook. Remind them to update it as they complete each lesson.

Study Guide and Intervention

Each lesson in *Geometry* addresses two objectives. There is one Study Guide and Intervention master for each objective.

WHEN TO USE Use these masters as reteaching activities for students who need additional reinforcement. These pages can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

Skills Practice There is one master for each lesson. These provide computational practice at a basic level.

WHEN TO USE These masters can be used with students who have weaker mathematics backgrounds or need additional reinforcement.

Practice There is one master for each lesson. These problems more closely follow the structure of the Practice and Apply section of the Student Edition exercises. These exercises are of average difficulty.

WHEN TO USE These provide additional practice options or may be used as homework for second day teaching of the lesson.

Reading to Learn Mathematics

One master is included for each lesson. The first section of each master asks questions about the opening paragraph of the lesson in the Student Edition. Additional questions ask students to interpret the context of and relationships among terms in the lesson. Finally, students are asked to summarize what they have learned using various representation techniques.

WHEN TO USE This master can be used as a study tool when presenting the lesson or as an informal reading assessment after presenting the lesson. It is also a helpful tool for ELL (English Language Learner) students.

Enrichment There is one extension master for each lesson. These activities may extend the concepts in the lesson, offer an historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. These are not written exclusively for honors students, but are accessible for use with all levels of students.

WHEN TO USE These may be used as extra credit, short-term projects, or as activities for days when class periods are shortened.

Assessment Options

The assessment masters in the *Chapter 2 Resources Masters* offer a wide range of assessment tools for intermediate and final assessment. The following lists describe each assessment master and its intended use.

Chapter Assessment

CHAPTER TESTS

- *Form 1* contains multiple-choice questions and is intended for use with basic level students.
- *Forms 2A and 2B* contain multiple-choice questions aimed at the average level student. These tests are similar in format to offer comparable testing situations.
- *Forms 2C and 2D* are composed of free-response questions aimed at the average level student. These tests are similar in format to offer comparable testing situations. Grids with axes are provided for questions assessing graphing skills.
- *Form 3* is an advanced level test with free-response questions. Grids without axes are provided for questions assessing graphing skills.

  All of the above tests include a free-response Bonus question.
- The **Open-Ended Assessment** includes performance assessment tasks that are suitable for all students. A scoring rubric is included for evaluation guidelines. Sample answers are provided for assessment.
- A **Vocabulary Test**, suitable for all students, includes a list of the vocabulary words in the chapter and ten questions assessing students' knowledge of those terms. This can also be used in conjunction with one of the chapter tests or as a review worksheet.

Intermediate Assessment

- Four free-response **quizzes** are included to offer assessment at appropriate intervals in the chapter.
- A **Mid-Chapter Test** provides an option to assess the first half of the chapter. It is composed of both multiple-choice and free-response questions.

Continuing Assessment

- The **Cumulative Review** provides students an opportunity to reinforce and retain skills as they proceed through their study of Geometry. It can also be used as a test. This master includes free-response questions.
- The **Standardized Test Practice** offers continuing review of geometry concepts in various formats, which may appear on the standardized tests that they may encounter. This practice includes multiple-choice, grid-in, and short-response questions. Bubble-in and grid-in answer sections are provided on the master.

Answers

- Page A1 is an answer sheet for the Standardized Test Practice questions that appear in the Student Edition on pages 122–123. This improves students' familiarity with the answer formats they may encounter in test taking.
- The answers for the lesson-by-lesson masters are provided as reduced pages with answers appearing in red.
- Full-size answer keys are provided for the assessment masters in this booklet.

2 Reading to Learn Mathematics

Vocabulary Builder

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 2. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term. Add these pages to your Geometry Study Notebook to review vocabulary at the end of the chapter.

Vocabulary Term	Found on Page	Definition/Description/Example
biconditional		
conjecture kuhn·JEK·chur		
conjunction		
contrapositive		
converse		
counterexample		
deductive reasoning		
disjunction		
if-then statement		

(continued on the next page)

2 Reading to Learn Mathematics

Vocabulary Builder *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
inductive reasoning		
inverse		
negation		
paragraph proof		
postulate		
statement		
theorem		
truth table		
truth value		
two-column proof		

NAME ____ DATE ___________ PERIOD _____

2 Learning to Read Mathematics

Proof Builder

Proof Builder

This is a list of key theorems and postulates you will learn in Chapter 2. As you study the chapter, write each theorem or postulate in your own words. Include illustrations as appropriate. Remember to include the page number where you found the theorem or postulate. Add this page to your Geometry Study Notebook so you can review the theorems and postulates at the end of the chapter.

Theorem or Postulate	Found on Page	Description/Illustration/Abbreviation
Theorem 2.1 *Midpoint Theorem*		
Theorem 2.2		
Theorem 2.3 *Supplement Theorem*		
Theorem 2.4 *Complement Theorem*		
Theorem 2.5		
Theorem 2.6		
Theorem 2.7		

(continued on the next page)

2 Learning to Read Mathematics

Proof Builder *(continued)*

Theorem or Postulate	Found on Page	Description/Illustration/Abbreviation
Theorem 2.8 *Vertical Angle Theorem*		
Theorem 2.9		
Theorem 2.10		
Theorem 2.11		
Theorem 2.12		
Theorem 2.13		
Postulate 2.9 *Segment Addition Postulate*		

NAME ____ DATE ____________ PERIOD _____

2-1 Study Guide and Intervention

Inductive Reasoning and Conjecture

Make Conjectures A **conjecture** is a guess based on analyzing information or observing a pattern. Making a conjecture after looking at several situations is called **inductive reasoning**.

Example 1 **Make a conjecture about the next number in the sequence 1, 3, 9, 27, 81.**

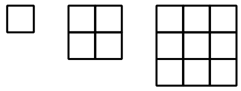
Analyze the numbers:

Notice that each number is a power of 3.

1	3	9	27	81
3^0	3^1	3^2	3^3	3^4

Conjecture: The next number will be 3^5 or 243.

Example 2 **Make a conjecture about the number of small squares in the next figure.**



Observe a pattern: The sides of the squares have measures 1, 2, and 3 units.

Conjecture: For the next figure, the side of the square will be 4 units, so the figure will have 16 small squares.

Exercises

Describe the pattern. Then make a conjecture about the next number in the sequence.

1. −5, 10, −20, 40

2. 1, 10, 100, 1000

3. 1, $\frac{6}{5}$, $\frac{7}{5}$, $\frac{8}{5}$

Make a conjecture based on the given information. Draw a figure to illustrate your conjecture.

4. $A(-1, -1)$, $B(2, 2)$, $C(4, 4)$

5. $\angle 1$ and $\angle 2$ form a right angle.

6. $\angle ABC$ and $\angle DBE$ are vertical angles.

7. $\angle E$ and $\angle F$ are right angles.

© Glencoe/McGraw-Hill

NAME ____ DATE ____________ PERIOD _____

2-1 Study Guide and Intervention *(continued)*

Inductive Reasoning and Conjecture

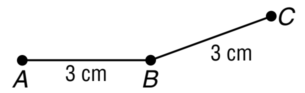
Find Counterexamples A conjecture is false if there is even one situation in which the conjecture is not true. The false example is called a **counterexample**.

Example **Determine whether the conjecture is *true* or *false*. If it is false, give a counterexample.**

Given: $\overline{AB} \cong \overline{BC}$

Conjecture: B is the midpoint of $\overline{AC}$.

Is it possible to draw a diagram with $\overline{AB} \cong \overline{BC}$ such that B is not the midpoint? This diagram is a counterexample because point B is not on $\overline{AC}$. The conjecture is false.



Exercises

Determine whether each conjecture is *true* or *false*. Give a counterexample for any false conjecture.

1. **Given:** Points A, B, and C are collinear.
   Conjecture: $AB + BC = AC$

2. **Given:** $\angle R$ and $\angle S$ are supplementary. $\angle R$ and $\angle T$ are supplementary.
   Conjecture: $\angle T$ and $\angle S$ are congruent.

3. **Given:** $\angle ABC$ and $\angle DEF$ are supplementary.
   Conjecture: $\angle ABC$ and $\angle DEF$ form a linear pair.

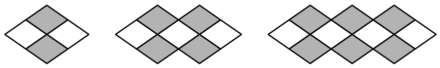
4. **Given:** $\overline{DE} \perp \overline{EF}$
   Conjecture: $\angle DEF$ is a right angle.

© Glencoe/McGraw-Hill

2-1 Skills Practice

Inductive Reasoning and Conjecture

Make a conjecture about the next item in each sequence.

1. 

2. −4, −1, 2, 5, 8

3. 6, $\frac{11}{2}$, 5, $\frac{9}{2}$, 4

4. −2, 4, −8, 16, −32

Make a conjecture based on the given information. Draw a figure to illustrate your conjecture.

5. Points *A*, *B*, and *C* are collinear, and *D* is between *B* and *C*.

6. Point *P* is the midpoint of $\overline{NQ}$.

7. ∠1, ∠2, ∠3, and ∠4 form four linear pairs.

8. ∠3 ≅ ∠4

Determine whether each conjecture is *true* or *false*. Give a counterexample for any false conjecture.

9. Given: ∠*ABC* and ∠*CBD* form a linear pair.
Conjecture: ∠*ABC* ≅ ∠*CBD*

10. Given: $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ are congruent.
Conjecture: *A*, *B*, and *C* are collinear.

11. Given: $AB + BC = AC$
Conjecture: $AB = BC$

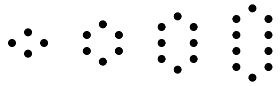
12. Given: ∠1 is complementary to ∠2, and ∠1 is complementary to ∠3.
Conjecture: ∠2 ≅ ∠3

NAME ____ DATE ____________ PERIOD _____

2-1 Practice

Inductive Reasoning and Conjecture

Make a conjecture about the next item in each sequence.

1. 

2. 5, −10, 15, −20

3. $-2, 1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}$

4. 12, 6, 3, 1.5, 0.75

Make a conjecture based on the given information. Draw a figure to illustrate your conjecture.

5. $\angle ABC$ is a right angle.

6. Point S is between R and T.

7. P, Q, R, and S are noncollinear and $\overline{PQ} \cong \overline{QR} \cong \overline{RS} \cong \overline{SP}$.

8. $ABCD$ is a parallelogram.

Determine whether each conjecture is true or false. Give a counterexample for any false conjecture.

9. Given: S, T, and U are collinear and $ST = TU$.
Conjecture: T is the midpoint of $\overline{SU}$.

10. Given: $\angle 1$ and $\angle 2$ are adjacent angles.
Conjecture: $\angle 1$ and $\angle 2$ form a linear pair.

11. Given: $\overline{GH}$ and $\overline{JK}$ form a right angle and intersect at P.
Conjecture: $\overline{GH} \perp \overline{JK}$

12. ALLERGIES Each spring, Rachel starts sneezing when the pear trees on her street blossom. She reasons that she is allergic to pear trees. Find a counterexample to Rachel's conjecture.

NAME ____ DATE ____________ PERIOD _____

2-1 Reading to Learn Mathematics

Inductive Reasoning and Conjecture

Pre-Activity **How can inductive reasoning help predict weather conditions?**

Read the introduction to Lesson 2-1 at the top of page 62 in your textbook.

- What kind of weather patterns do you think meteorologists look at to help predict the weather?
- What is a factor that might contribute to long-term changes in the weather?

Reading the Lesson

1. Explain in your own words the relationship between a conjecture, a counterexample, and inductive reasoning.

2. Make a conjecture about the next item in each sequence.

a. 5, 9, 13, 17

b. $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}$

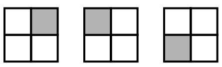
c. 0, 1, 3, 6, 10

d. $8, 3, -2, -7$

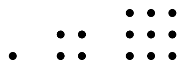
e. 1, 8, 27, 64

f. $1, -2, 4, -8$

g.



h.



3. State whether each conjecture is *true* or *false*. If the conjecture is false, give a counterexample.

a. The sum of two odd integers is even.

b. The product of an odd integer and an even integer is odd.

c. The opposite of an integer is a negative integer.

d. The perfect squares (squares of whole numbers) alternate between odd and even.

Helping You Remember

4. Write a short sentence that can help you remember why it only takes one counterexample to prove that a conjecture is false.

© Glencoe/McGraw-Hill

NAME ____ DATE ____________ PERIOD _____

2-1 Enrichment

Counterexamples

When you make a conclusion after examining several specific cases, you have used **inductive reasoning**. However, you must be cautious when using this form of reasoning. By finding only one **counterexample**, you disprove the conclusion.

Example **Is the statement $\frac{1}{x} \leq 1$ true when you replace x with 1, 2, and 3? Is the statement true for all reals? If possible, find a counterexample.**

$\frac{1}{1} = 1$, $\frac{1}{2} < 1$, and $\frac{1}{3} < 1$. But when $x = \frac{1}{2}$, then $\frac{1}{x} = 2$. This counterexample shows that the statement is not always true.

Answer each question.

1. The coldest day of the year in Chicago occurred in January for five straight years. Is it safe to conclude that the coldest day in Chicago is always in January?

2. Suppose John misses the school bus four Tuesdays in a row. Can you safely conclude that John misses the school bus every Tuesday?

3. Is the equation $\sqrt{k^2} = k$ true when you replace k with 1, 2, and 3? Is the equation true for all integers? If possible, find a counterexample.

4. Is the statement $2x = x + x$ true when you replace x with $\frac{1}{2}$, 4, and 0.7? Is the statement true for all real numbers? If possible, find a counterexample.

5. Suppose you draw four points A, B, C, and D and then draw $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$. Does this procedure give a quadrilateral always or only sometimes? Explain your answers with figures.

6. Suppose you draw a circle, mark three points on it, and connect them. Will the angles of the triangle be acute? Explain your answers with figures.

© Glencoe/McGraw-Hill

2-2 Study Guide and Intervention

Logic

Determine Truth Values A **statement** is any sentence that is either true or false. The truth or falsity of a statement is its **truth value**. A statement can be represented by using a letter. For example,

Statement p: Chicago is a city in Illinois. The truth value of statement *p* is true.

Several statements can be joined in a **compound statement**.

Statement *p* and statement *q* joined by the word *and* is a **conjunction**.	Statement *p* and statement *q* joined by the word *or* is a **disjunction**.	**Negation:** *not p* is the negation of the statement *p*.
Symbols: $p \wedge q$ (Read: *p and q*)	Symbols: $p \vee q$ (Read: *p or q*)	Symbols: $\sim p$ (Read: *not p*)
The conjunction $p \wedge q$ is true only when both *p* and *q* are true.	The disjunction $p \vee q$ is true if *p* is true, if *q* is true, or if both are true.	The statements *p* and $\sim p$ have opposite truth values.

Example 1 **Write a compound statement for each conjunction. Then find its truth value.**

p: An elephant is a mammal.

q: A square has four right angles.

a. $p \wedge q$

Join the statements with *and*: An elephant is a mammal and a square has four right angles. Both parts of the statement are true so the compound statement is true.

b. $\sim p \wedge q$

$\sim p$ is the statement "An elephant is not a mammal." Join $\sim p$ and *q* with the word *and*: An elephant is not a mammal and a square has four right angles. The first part of the compound statement, $\sim p$, is false. Therefore the compound statement is false.

Example 2 **Write a compound statement for each disjunction. Then find its truth value.**

p: A diameter of a circle is twice the radius.

q: A rectangle has four equal sides.

a. $p \vee q$

Join the statements *p* and *q* with the word *or*: A diameter of a circle is twice the radius or a rectangle has four equal sides. The first part of the compound statement, *p*, is true, so the compound statement is true.

b. $\sim p \vee q$

Join $\sim p$ and *q* with the word *or*: A diameter of a circle is not twice the radius or a rectangle has four equal sides. Neither part of the disjunction is true, so the compound statement is false.

Exercises

Write a compound statement for each conjunction and disjunction. Then find its truth value.

p: $10 + 8 = 18$ *q*: September has 30 days. *r*: A rectangle has four sides.

1. *p* and *q*

2. *p* or *r*

3. *q* or *r*

4. *q* and $\sim r$

Lesson 2-2

2-2 Study Guide and Intervention *(continued)*

Logic

Truth Tables One way to organize the truth values of statements is in a **truth table**. The truth tables for negation, conjunction, and disjunction are shown at the right.

Negation	
p	$\sim p$
T	F
F	T

Conjunction		
p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction		
p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Example 1 **Construct a truth table for the compound statement *q* or *r*. Use the disjunction table.**

q	r	q or r
T	T	T
T	F	T
F	T	T
F	F	F

Example 2 **Construct a truth table for the compound statement *p* and (*q* or *r*).**

Use the disjunction table for (q or r). Then use the conjunction table for p and (q or r).

p	q	r	q or r	p and (q or r)
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	F
F	T	F	T	F
F	F	T	T	F
F	F	F	F	F

Exercises

Contruct a truth table for each compound statement.

1. p or r

2. $\sim p \vee q$

3. $q \wedge \sim r$

4. $\sim p \wedge \sim r$

5. (p and r) or q

2-2 Skills Practice

Logic

Use the following statements to write a compound statement for each conjunction and disjunction. Then find its truth value.

p: **$-3 - 2 = -5$**
q: **Vertical angles are congruent.**
r: **$2 + 8 > 10$**
s: **The sum of the measures of complementary angles is 90°.**

1. p and q

2. $p \wedge r$

3. p or s

4. $r \vee s$

5. $p \wedge \sim q$

6. $q \vee \sim r$

Copy and complete each truth table.

7.

p	q	$\sim p$	$\sim p \wedge q$	$\sim(\sim p \wedge q)$
T	T			
T	F			
F	T			
F	F			

8.

p	q	$\sim q$	$p \vee \sim q$
T	T	F	
T	F	T	
F	T	F	
F	F	T	

Construct a truth table for each compound statement.

9. $\sim q \wedge r$

10. $\sim p \vee \sim r$

NAME ______________________________ DATE ____________ PERIOD _____

2-2 Practice

Logic

Use the following statements to write a compound statement for each conjunction and disjunction. Then find its truth value.

p: 60 seconds = 1 minute

q: Congruent supplementary angles each have a measure of 90.

r: $-12 + 11 < -1$

1. $p \wedge q$

2. $q \vee r$

3. $\sim p \vee q$

4. $\sim p \wedge \sim r$

Copy and complete each truth table.

5.

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T			
T	F			
F	T			
F	F			

6.

p	q	$\sim p$	$\sim p \vee q$	$p \wedge (\sim p \vee q)$
T	T			
T	F			
F	T			
F	F			

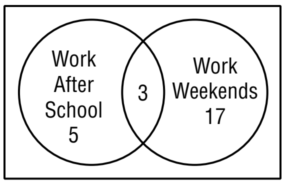
Construct a truth table for each compound statement.

7. $q \vee (p \wedge \sim q)$

8. $\sim q \wedge (\sim p \vee q)$

SCHOOL For Exercises 9 and 10, use the following information.

The Venn diagram shows the number of students in the band who work after school or on the weekends.



9. How many students work after school and on weekends?

10. How many students work after school or on weekends?

© Glencoe/McGraw-Hill

NAME ____ DATE ____________ PERIOD _____

2-2 Reading to Learn Mathematics

Logic

Pre-Activity **How does logic apply to school?**

Read the introduction to Lesson 2-2 at the top of page 67 in your textbook.

How can you use logic to help you answer a multiple-choice question on a standardized test if you are not sure of the correct answer?

Reading the Lesson

1. Supply one or two words to complete each sentence.

a. Two or more statements can be joined to form a _________________ statement.

b. A statement that is formed by joining two statements with the word *or* is called a _________________.

c. The truth or falsity of a statement is called its _________________.

d. A statement that is formed by joining two statements with the word *and* is called a _________________.

e. A statement that has the opposite truth value and the opposite meaning from a given statement is called the _________________ of the statement.

2. Use *true* or *false* to complete each sentence.

a. If a statement is true, then its negation is _________________.

b. If a statement is false, then its negation is _________________.

c. If two statements are both true, then their conjunction is _________________ and their disjunction is _________________.

d. If two statements are both false, then their conjunction is _________________ and their disjunction is _________________.

e. If one statement is true and another is false, then their conjunction is _________________ and their disjunction is _________________.

3. Consider the following statements:

p: Chicago is the capital of Illinois. q: Sacramento is the capital of California.

Write each statement symbolically and then find its truth value.

a. Sacramento is not the capital of California.

b. Sacramento is the capital of California and Chicago is not the capital of Illinois.

Helping You Remember

4. Prefixes can often help you to remember the meaning of words or to distinguish between similar words. Use your dictionary to find the meanings of the prefixes *con* and *dis* and explain how these meanings can help you remember the difference between a *conjunction* and a *disjunction*.

Lesson 2-2

© Glencoe/McGraw-Hill

NAME ______________________________ DATE ____________ PERIOD _____

2-2 Enrichment

Letter Puzzles

An **alphametic** is a computation puzzle using letters instead of digits. Each letter represents one of the digits 0–9, and two different letters cannot represent the same digit. Some alphametic puzzles have more than one answer.

Example **Solve the alphametic puzzle at the right.**

```
  FOUR
+  ONE
  FIVE
```

Since R + E = E, the value of R must be 0. Notice that the thousands digit must be the same in the first addend and the sum. Since the value of I is 9 or less, O must be 4 or less. Use trial and error to find values that work.

F = 8, O = 3, U = 1, R = 0

N = 4, E = 7, I = 6, and V = 5.

```
  8310
+  347
  8657
```

Can you find other solutions to this puzzle?

Find a value for each letter in each alphametic.

1.

```
   HALF
+  HALF
  WHOLE
```

H = ___ A = ___ L = ___

F = ___ W = ___ O = ___

E = ___

2.

```
   TWO
+  TWO
  FOUR
```

T = ___ W = ___ O = ___

F = ___ U = ___ R = ___

3.

```
  THREE
  THREE
+   ONE
  SEVEN
```

T = ___ H = ___ R = ___

E = ___ O = ___ N = ___

S = ___ V = ___

4.

```
   SEND
+  MORE
  MONEY
```

S = ___ E = ___ N = ___

D = ___ M = ___ O = ___

R = ___ Y = ___

5. Do research to find more alphametic puzzles, or create your own puzzles. Challenge another student to solve them.

© Glencoe/McGraw-Hill

NAME ____ DATE ____________ PERIOD _____

2-3 Study Guide and Intervention

Conditional Statements

If-then Statements An if-then statement is a statement such as "If you are reading this page, then you are studying math." A statement that can be written in if-then form is called a **conditional statement**. The phrase immediately following the word *if* is the **hypothesis**. The phrase immediately following the word *then* is the **conclusion**.

A conditional statement can be represented in symbols as $p \rightarrow q$, which is read "p implies q" or "if p, then q."

Example 1 **Identify the hypothesis and conclusion of the statement.**

If $\angle X \cong \angle R$ and $\angle R \cong \angle S$, then $\angle X \cong \angle S$.

hypothesis: $\angle X \cong \angle R$ and $\angle R \cong \angle S$ — conclusion: $\angle X \cong \angle S$

Example 2 **Identify the hypothesis and conclusion. Write the statement in if-then form.**

You receive a free pizza with 12 coupons.

If you have 12 coupons, then you receive a free pizza.

hypothesis: you have 12 coupons — conclusion: you receive a free pizza

Exercises

Identify the hypothesis and conclusion of each statement.

1. If it is Saturday, then there is no school.

2. If $x - 8 = 32$, then $x = 40$.

3. If a polygon has four right angles, then the polygon is a rectangle.

Write each statement in if-then form.

4. All apes love bananas.

5. The sum of the measures of complementary angles is 90.

6. Collinear points lie on the same line.

Determine the truth value of the following statement for each set of conditions.

If it does not rain this Saturday, we will have a picnic.

7. It rains this Saturday, and we have a picnic.

8. It rains this Saturday, and we don't have a picnic.

9. It doesn't rain this Saturday, and we have a picnic.

10. It doesn't rain this Saturday, and we don't have a picnic.

Lesson 2-3

2-3 Study Guide and Intervention *(continued)*

Conditional Statements

Converse, Inverse, and Contrapositive If you change the hypothesis or conclusion of a conditional statement, you form a **related conditional**. This chart shows the three related conditionals, *converse*, *inverse*, and *contrapositive*, and how they are related to a conditional statement.

	Symbols	Formed by	Example
Conditional	$p \to q$	using the given hypothesis and conclusion	If two angles are vertical angles, then they are congruent.
Converse	$q \to p$	exchanging the hypothesis and conclusion	If two angles are congruent, then they are vertical angles.
Inverse	$\sim p \to \sim q$	replacing the hypothesis with its negation and replacing the conclusion with its negation	If two angles are not vertical angles, then they are not congruent.
Contrapositive	$\sim q \to \sim p$	negating the hypothesis, negating the conclusion, and switching them	If two angles are not congruent, then they are not vertical angles.

Just as a conditional statement can be true or false, the related conditionals also can be true or false. A conditional statement always has the same truth value as its contrapositive, and the converse and inverse always have the same truth value.

Exercises

Write the converse, inverse, and contrapositive of each conditional statement. Tell which statements are *true* and which statements are *false*.

1. If you live in San Diego, then you live in California.

2. If a polygon is a rectangle, then it is a square.

3. If two angles are complementary, then the sum of their measures is 90.

NAME _______________________________ DATE ____________ PERIOD _____

2-3 Skills Practice

Conditional Statements

Identify the hypothesis and conclusion of each statement.

1. If you purchase a computer and do not like it, then you can return it within 30 days.

2. If $x + 8 = 4$, then $x = -4$.

3. If the drama class raises $2000, then they will go on tour.

Write each statement in if-then form.

4. A polygon with four sides is a quadrilateral.

5. "Those who stand for nothing fall for anything." (*Alexander Hamilton*)

6. An acute angle has a measure less than 90.

Lesson 2-3

Determine the truth value of the following statement for each set of conditions.
If you finish your homework by 5 P.M., then you go out to dinner.

7. You finish your homework by 5 P.M. and you go out to dinner.

8. You finish your homework by 4 P.M. and you go out to dinner.

9. You finish your homework by 5 P.M. and you do not go out to dinner.

10. Write the converse, inverse, and contrapositive of the conditional statement. Determine whether each statement is true or false. If a statement is false, find a counterexample.
If 89 is divisible by 2, then 89 is an even number.

NAME ____ DATE ____________ PERIOD _____

2-3 Practice

Conditional Statements

Identify the hypothesis and conclusion of each statement.

1. If $3x + 4 = -5$, then $x = -3$.

2. If you take a class in television broadcasting, then you will film a sporting event.

Write each statement in if-then form.

3. "Those who do not remember the past are condemned to repeat it." (*George Santayana*)

4. Adjacent angles share a common vertex and a common side.

Determine the truth value of the following statement for each set of conditions.
If DVD players are on sale for less than $100, then you buy one.

5. DVD players are on sale for $95 and you buy one.

6. DVD players are on sale for $100 and you do not buy one.

7. DVD players are not on sale for under $100 and you do not buy one.

8. Write the converse, inverse, and contrapositive of the conditional statement. Determine whether each statement is true or false. If a statement is false, find a counterexample.
If $(-8)^2 > 0$, *then* $-8 > 0$.

SUMMER CAMP For Exercises 9 and 10, use the following information.
Older campers who attend Woodland Falls Camp are expected to work. Campers who are juniors wait on tables.

9. Write a conditional statement in if-then form.

10. Write the converse of your conditional statement.

© Glencoe/McGraw-Hill

NAME ____ DATE ____________ PERIOD _____

2-3 Reading to Learn Mathematics

Conditional Statements

Pre-Activity **How are conditional statements used in advertisements?**

Read the introduction to Lesson 2-3 at the top of page 75 in your textbook.

Does the second advertising statement in the introduction mean that you will not get a free phone if you sign a contract for only six months of service? Explain your answer.

Reading the Lesson

1. Identify the hypothesis and conclusion of each statement.

a. If you are a registered voter, then you are at least 18 years old.

b. If two integers are even, their product is even.

2. Complete each sentence.

a. The statement that is formed by replacing both the hypothesis and the conclusion of a conditional with their negations is the _________________.

b. The statement that is formed by exchanging the hypothesis and conclusion of a conditional is the _________________.

3. Consider the following statement:
You live in North America if you live in the United States.

a. Write this conditional statement in if-then form and give its truth value. If the statement is false, give a counterexample.

b. Write the inverse of the given conditional statement in if-then form and give its truth value. If the statement is false, give a counterexample.

c. Write the contrapositive of the given conditional statement in if-then form and give its truth value. If the statement is false, give a counterexample.

d. Write the converse of the given conditional statement in if-then form and give its truth value. If the statement is false, give a counterexample.

Helping You Remember

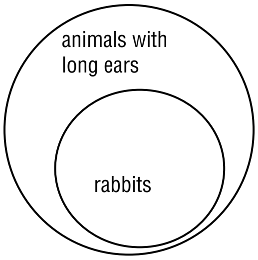
4. When working with a conditional statement and its three related conditionals, what is an easy way to remember which statements are logically equivalent to each other?

2-3 Enrichment

Venn Diagrams

A type of drawing called a **Venn diagram** can be useful in explaining conditional statements. A Venn diagram uses circles to represent sets of objects.

Consider the statement "All rabbits have long ears." To make a Venn diagram for this statement, a large circle is drawn to represent all animals with long ears. Then a smaller circle is drawn inside the first to represent all rabbits. The Venn diagram shows that every rabbit is included in the group of long-eared animals.



The set of rabbits is called a **subset** of the set of long-eared animals.

The Venn diagram can also explain how to write the statement, "All rabbits have long ears," in if-then form. Every rabbit is in the group of long-eared animals, so if an animal is a rabbit, then it has long ears.

For each statement, draw a Venn diagram. Then write the sentence in if-then form.

1. Every dog has long hair.

2. All rational numbers are real.

3. People who live in Iowa like corn.

4. Staff members are allowed in the faculty lounge.

© Glencoe/McGraw-Hill

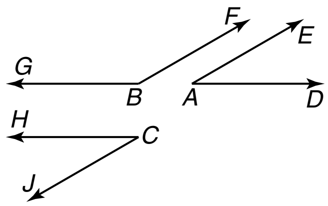
2-4 Study Guide and Intervention

Deductive Reasoning

Law of Detachment **Deductive reasoning** is the process of using facts, rules, definitions, or properties to reach conclusions. One form of deductive reasoning that draws conclusions from a true conditional $p \rightarrow q$ and a true statement p is called the **Law of Detachment**.

Law of Detachment	If $p \rightarrow q$ is true and p is true, then q is true.
Symbols	$[(p \rightarrow q)] \wedge p] \rightarrow q$

Example **The statement *If two angles are supplementary to the same angle, then they are congruent* is a true conditional. Determine whether each conclusion is valid based on the given information. Explain your reasoning.**



a. Given: ∠*A* and ∠*C* are supplementary to ∠*B*.
Conclusion: ∠*A* is congruent to ∠*C*.

The statement ∠*A and* ∠*C are supplementary to* ∠*B* is the hypothesis of the conditional. Therefore, by the Law of Detachment, the conclusion is true.

b. Given: ∠*A* is congruent to ∠*C*.
Conclusion: ∠*A* and ∠*C* are supplementary to ∠*B*.

The statement ∠*A is congruent to* ∠*C* is not the hypothesis of the conditional, so the Law of Detachment cannot be used. The conclusion is not valid.

Exercises

Determine whether each conclusion is valid based on the true conditional given. If not, write *invalid*. Explain your reasoning.

If two angles are complementary to the same angle, then the angles are congruent.

1. **Given:** ∠*A* and ∠*C* are complementary to ∠*B*.
Conclusion: ∠*A* is congruent to ∠*C*.

2. **Given:** ∠*A* ≅ ∠*C*
Conclusion: ∠*A* and ∠*C* are complements of ∠*B*.

3. **Given:** ∠*E* and ∠*F* are complementary to ∠*G*.
Conclusion: ∠*E* and ∠*F* are vertical angles.

Lesson 2-4

NAME ____ DATE ____________ PERIOD _____

2-4 Study Guide and Intervention *(continued)*

Deductive Reasoning

Law of Syllogism Another way to make a valid conclusion is to use the **Law of Syllogism**. It is similar to the Transitive Property.

Law of Syllogism	If $p \rightarrow q$ is true and $q \rightarrow r$ is true, then $p \rightarrow r$ is also true.
Symbols	$[(p \rightarrow q)] \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

Example **The two conditional statements below are true. Use the Law of Syllogism to find a valid conclusion. State the conclusion.**

(1) If a number is a whole number, then the number is an integer.
(2) If a number is an integer, then it is a rational number.

p: A number is a whole number.

q: A number is an integer.

r: A number is a rational number.

The two conditional statements are $p \rightarrow q$ and $q \rightarrow r$. Using the Law of Syllogism, a valid conclusion is $p \rightarrow r$. A statement of $p \rightarrow r$ is "if a number is a whole number, then it is a rational number."

Exercises

Determine whether you can use the Law of Syllogism to reach a valid conclusion from each set of statements.

1. If a dog eats Superdog Dog Food, he will be happy.
Rover is happy.

2. If an angle is supplementary to an obtuse angle, then it is acute.
If an angle is acute, then its measure is less than 90.

3. If the measure of $\angle A$ is less than 90, then $\angle A$ is acute.
If $\angle A$ is acute, then $\angle A \cong \angle B$.

4. If an angle is a right angle, then the measure of the angle is 90.
If two lines are perpendicular, then they form a right angle.

5. If you study for the test, then you will receive a high grade.
Your grade on the test is high.

© Glencoe/McGraw-Hill

NAME ____ DATE ____________ PERIOD _____

2-4 Skills Practice

Deductive Reasoning

Determine whether the stated conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning.

If the sum of the measures of two angles is 180, then the angles are supplementary.

1. **Given:** $m\angle A + m\angle B$ is 180.
   Conclusion: $\angle A$ and $\angle B$ are supplementary.

2. **Given:** $m\angle ABC$ is 95 and $m\angle DEF$ is 90.
   Conclusion: $\angle ABC$ and $\angle DEF$ are supplementary.

3. **Given:** $\angle 1$ and $\angle 2$ are a linear pair.
   Conclusion: $\angle 1$ and $\angle 2$ are supplementary.

Use the Law of Syllogism to determine whether a valid conclusion can be reached from each set of statements. If a valid conclusion is possible, write it.

4. If two angles are complementary, then the sum of their measures is 90.
   If the sum of the measures of two angles is 90, then both of the angles are acute.

5. If the heat wave continues, then air conditioning will be used more frequently.
   If air conditioning is used more frequently, then energy costs will be higher.

Determine whether statement (3) follows from statements (1) and (2) by the Law of Detachment or the Law of Syllogism. If it does, state which law was used. If it does not, write *invalid*.

6. (1) If it is Tuesday, then Marla tutors chemistry.
   (2) If Marla tutors chemistry, then she arrives home at 4 P.M.
   (3) If Marla arrives at home at 4 P.M., then it is Tuesday.

7. (1) If a marine animal is a starfish, then it lives in the intertidal zone of the ocean.
   (2) The intertidal zone is the least stable of the ocean zones.
   (3) If a marine animal is a starfish, then it lives in the least stable of the ocean zones.

Lesson 2-4

NAME ____ DATE ____________ PERIOD _____

2-4 Practice

Deductive Reasoning

Determine whether the stated conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning.

If a point is the midpoint of a segment, then it divides the segment into two congruent segments.

1. **Given:** R is the midpoint of $\overline{QS}$.
   Conclusion: $\overline{QR} \cong \overline{RS}$

2. **Given:** $\overline{AB} \cong \overline{BC}$
   Conclusion: B divides $\overline{AC}$ into two congruent segments.

Use the Law of Syllogism to determine whether a valid conclusion can be reached from each set of statements. If a valid conclusion is possible, write it.

3. If two angles form a linear pair, then the two angles are supplementary.
   If two angles are supplementary, then the sum of their measures is 180.

4. If a hurricane is Category 5, then winds are greater than 155 miles per hour.
   If winds are greater than 155 miles per hour, then trees, shrubs, and signs are blown down.

Determine whether statement (3) follows from statements (1) and (2) by the Law of Detachment or the Law of Syllogism. If it does, state which law was used. If it does not, write *invalid*.

5. (1) If a whole number is even, then its square is divisible by 4.
   (2) The number I am thinking of is an even whole number.
   (3) The square of the number I am thinking of is divisible by 4.

6. (1) If the football team wins its homecoming game, then Conrad will attend the school dance the following Friday.
   (2) Conrad attends the school dance on Friday.
   (3) The football team won the homecoming game.

7. **BIOLOGY** If an organism is a parasite, then it survives by living on or in a host organism. If a parasite lives in or on a host organism, then it harms its host. What conclusion can you draw if a virus is a parasite?

© Glencoe/McGraw-Hill

NAME ______________________________ DATE ____________ PERIOD _____

2-4 Reading to Learn Mathematics

Deductive Reasoning

Pre-Activity **How does deductive reasoning apply to health?**

Read the introduction to Lesson 2-4 at the top of page 82 in your textbook.

Suppose a doctor wants to use the dose chart in your textbook to prescribe an antibiotic, but the only scale in her office gives weights in pounds. How can she use the fact that 1 kilogram is about 2.2 pounds to determine the correct dose for a patient?

Reading the Lesson

If *s*, *t*, and *u* are three statements, match each description from the list on the left with a symbolic statement from the list on the right.

1. negation of t	**a.** $s \vee u$
2. conjunction of s and u	**b.** $[(s \to t) \wedge s] \to t$
3. converse of $s \to t$	**c.** $\sim s \to \sim u$
4. disjunction of s and u	**d.** $\sim u \to \sim s$
5. Law of Detachment	**e.** $\sim t$
6. contrapositive of $s \to t$	**f.** $[(u \to t) \wedge (t \to s)] \to (u \to s)$
7. inverse of $s \to u$	**g.** $s \wedge u$
8. contrapositive of $s \to u$	**h.** $t \to s$
9. Law of Syllogism	**i.** t
10. negation of $\sim t$	**j.** $\sim t \to \sim s$

11. Determine whether statement (3) follows from statements (1) and (2) by the Law of Detachment or the Law of Syllogism. If it does, state which law was used. If it does not, write *invalid*.

a. (1) Every square is a parallelogram.
(2) Every parallelogram is a polygon.
(3) Every square is a polygon.

b. (1)If two lines that lie in the same plane do not intersect, they are parallel.
(2) Lines ℓ and m lie in plane $\mathcal{U}$ and do not intersect.
(3) Lines ℓ and m are parallel.

c. (1) Perpendicular lines intersect to form four right angles.
(2) $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are four right angles.
(3) $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are formed by intersecting perpendicular lines.

Helping You Remember

12. A good way to remember something is to explain it to someone else. Suppose that a classmate is having trouble remembering what the Law of Detachment means?

Lesson 2-4

NAME ____ DATE ____________ PERIOD _____

2-4 Enrichment

Valid and Faulty Arguments

Consider the statements at the right. What conclusions can you make?

(1) Boots is a cat.
(2) Boots is purring.
(3) A cat purrs if it is happy.

From statements 1 and 3, it is correct to conclude that Boots purrs if it is happy. However, it is faulty to conclude from only statements 2 and 3 that Boots is happy. The if-then form of statement 3 is *If a cat is happy, then it purrs*.

Advertisers often use faulty logic in subtle ways to help sell their products. By studying the arguments, you can decide whether the argument is valid or faulty.

Decide if each argument is valid or faulty.

1. (1) If you buy Tuff Cote luggage, it will survive airline travel.
(2) Justin buys Tuff Cote luggage.
Conclusion: Justin's luggage will survive airline travel.

2. (1) If you buy Tuff Cote luggage, it will survive airline travel.
(2) Justin's luggage survived airline travel.
Conclusion: Justin has Tuff Cote luggage.

3. (1) If you use Clear Line long distance service, you will have clear reception.
(2) Anna has clear long distance reception.
Conclusion: Anna uses Clear Line long distance service.

4. (1) If you read the book *Beautiful Braids*, you will be able to make beautiful braids easily.
(2) Nancy read the book *Beautiful Braids*.
Conclusion: Nancy can make beautiful braids easily.

5. (1) If you buy a word processor, you will be able to write letters faster.
(2) Tania bought a word processor.
Conclusion: Tania will be able to write letters faster.

6. (1) Great swimmers wear AquaLine swimwear.
(2) Gina wears AquaLine swimwear.
Conclusion: Gina is a great swimmer.

7. Write an example of faulty logic that you have seen in an advertisement.

NAME ____ DATE ____________ PERIOD _____

2-5 Study Guide and Intervention

Postulates and Paragraph Proofs

Points, Lines, and Planes In geometry, a **postulate** is a statement that is accepted as true. Postulates describe fundamental relationships in geometry.

Postulate: Through any two points, there is exactly one line.
Postulate: Through any three points not on the same line, there is exactly one plane.
Postulate: A line contains at least two points.
Postulate: A plane contains at least three points not on the same line.
Postulate: If two points lie in a plane, then the line containing those points lies in the plane.
Postulate: If two lines intersect, then their intersection is exactly one point.
Postulate: If two planes intersect, then their intersection is a line.

Example **Determine whether each statement is *always*, *sometimes*, or *never* true.**

a. There is exactly one plane that contains points *A*, *B*, and *C*.
Sometimes; if *A*, *B*, and *C* are collinear, they are contained in many planes. If they are noncollinear, then they are contained in exactly one plane.

b. Points *E* and *F* are contained in exactly one line.
Always; the first postulate states that there is exactly one line through any two points.

c. Two lines intersect in two distinct points *M* and *N*.
Never; the intersection of two lines is one point.

Exercises

Use postulates to determine whether each statement is *always*, *sometimes*, or *never* true.

1. A line contains exactly one point.

2. Noncollinear points *R*, *S*, and *T* are contained in exactly one plane.

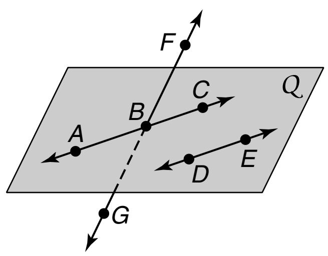
3. Any two lines ℓ and m intersect.

4. If points *G* and *H* are contained in plane $\mathcal{M}$, then $\overline{GH}$ is perpendicular to plane $\mathcal{M}$.

5. Planes $\mathcal{R}$ and $\mathcal{S}$ intersect in point *T*.

6. If points *A*, *B*, and *C* are noncollinear, then segments $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$ are contained in exactly one plane.

In the figure, $\overline{AC}$ and $\overline{DE}$ are in plane *Q* and $\overline{AC} \parallel \overline{DE}$. State the postulate that can be used to show each statement is true.



7. Exactly one plane contains points *F*, *B*, and *E*.

8. $\overleftrightarrow{BE}$ lies in plane *Q*.

Lesson 2-5

© Glencoe/McGraw-Hill

2-5 Study Guide and Intervention *(continued)*

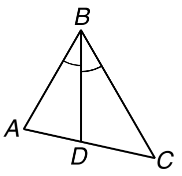
Postulates and Paragraph Proofs

Paragraph Proofs A statement that can be proved true is called a **theorem**. You can use undefined terms, definitions, postulates, and already-proved theorems to prove other statements true.

A logical argument that uses deductive reasoning to reach a valid conclusion is called a **proof**. In one type of proof, a **paragraph proof**, you write a paragraph to explain why a statement is true.

Example **In $\triangle ABC$, $\overline{BD}$ is an angle bisector. Write a paragraph proof to show that $\angle ABD \cong \angle CBD$.**

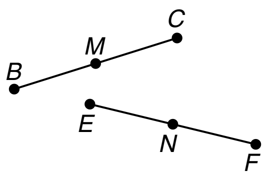
By definition, an angle bisector divides an angle into two congruent angles. Since $\overline{BD}$ is an angle bisector, $\angle ABC$ is divided into two congruent angles. Thus, $\angle ABD \cong \angle CBD$.



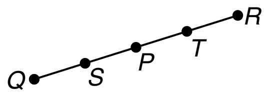
Exercises

1. Given that $\angle A \cong \angle D$ and $\angle D \cong \angle E$, write a paragraph proof to show that $\angle A \cong \angle E$.

2. It is given that $\overline{BC} \cong \overline{EF}$, M is the midpoint of $\overline{BC}$, and N is the midpoint of $\overline{EF}$. Write a paragraph proof to show that $BM = EN$.



3. Given that S is the midpoint of $\overline{QP}$, T is the midpoint of $\overline{PR}$, and P is the midpoint of $\overline{ST}$, write a paragraph proof to show that $QS = TR$.

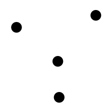


2-5 Skills Practice

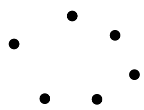
Postulates and Paragraph Proofs

Determine the number of line segments that can be drawn connecting each pair of points.

1.



2.



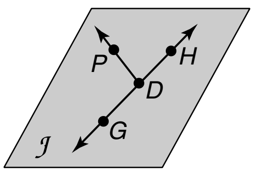
Determine whether the following statements are *always*, *sometimes*, or *never* true. Explain.

3. Three collinear points determine a plane.

4. Two points A and B determine a line.

5. A plane contains at least three lines.

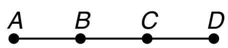
In the figure, $\overleftrightarrow{DG}$ and $\overrightarrow{DP}$ lie in plane $\mathcal{J}$ and H lies on $\overleftrightarrow{DG}$. State the postulate that can be used to show each statement is true.



6. G and P are collinear.

7. Points D, H, and P are coplanar.

8. PROOF In the figure at the right, point B is the midpoint of $\overline{AC}$ and point C is the midpoint of $\overline{BD}$. Write a paragraph proof to prove that $AB = CD$.



Lesson 2-5

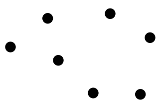
NAME ____ DATE ____________ PERIOD _____

2-5 Practice

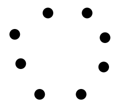
Postulates and Paragraph Proofs

Determine the number of line segments that can be drawn connecting each pair of points.

1.



2.

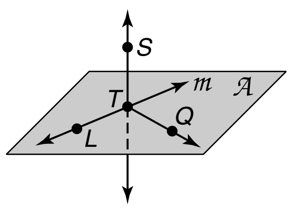


Determine whether the following statements are *always*, *sometimes*, or *never* true. Explain.

3. The intersection of two planes contains at least two points.

4. If three planes have a point in common, then they have a whole line in common.

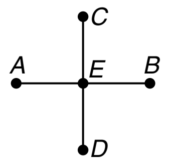
In the figure, line m and $\overleftrightarrow{TQ}$ lie in plane $\mathcal{A}$. State the postulate that can be used to show that each statement is true.



5. L, T, and line m lie in the same plane.

6. Line m and $\overline{ST}$ intersect at T.

7. In the figure, E is the midpoint of $\overline{AB}$ and $\overline{CD}$, and $AB = CD$. Write a paragraph proof to prove that $\overline{AE} \cong \overline{ED}$.



8. LOGIC Points A, B, and C are not collinear. Points B, C, and D are not collinear. Points A, B, C, and D are not coplanar. Describe two planes that intersect in line BC.

© Glencoe/McGraw-Hill

NAME ____ DATE ____________ PERIOD _____

2-5 Reading to Learn Mathematics

Postulates and Paragraph Proofs

Pre-Activity **How are postulates used by the founding fathers of the United States?**

Read the introduction to Lesson 2-5 at the top of page 89 in your textbook.

Postulates are often described as statements that are so basic and so clearly correct that people will be willing to accept them as true without asking for evidence or proof. Give a statement about numbers that you think most people would accept as true without evidence.

Reading the Lesson

1. Determine whether each of the following is a *correct* or *incorrect* statement of a geometric postulate. If the statement is incorrect, replace the underlined words to make the statement correct.

a. A plane contains at least <u>two points</u> that do not lie on the same line.

b. If <u>two planes</u> intersect, then the intersection is a line.

c. Through any <u>four points</u> not on the same line, there is exactly one plane.

d. A line contains at least <u>one point</u>.

e. If two lines <u>are parallel</u>, then their intersection is exactly one point.

f. Through any two points, there is <u>at most</u> one line.

2. Determine whether each statement is *always*, *sometimes*, or *never* true. If the statement is not always true, explain why.

a. If two planes intersect, their intersection is a line.

b. The midpoint of a segment divides the segment into two congruent segments.

c. There is exactly one plane that contains three collinear points.

d. If two lines intersect, their intersection is one point.

3. Use the walls, floor, and ceiling of your classroom to describe a model for each of the following geometric situations.

a. two planes that intersect in a line

b. two planes that do not intersect

c. three planes that intersect in a point

Helping You Remember

4. A good way to remember a new mathematical term is to relate it to a word you already know. Explain how the idea of a mathematical *theorem* is related to the idea of a scientific *theory*.

Lesson 2-5

NAME ____ DATE ____________ PERIOD _____

2-5 Enrichment

Logic Problems

The following problems can be solved by eliminating possibilities. It may be helpful to use charts such as the one shown in the first problem. Mark an X in the chart to eliminate a possible answer.

Solve each problem.

1. Nancy, Olivia, Mario, and Kenji each have one piece of fruit in their school lunch. They have a peach, an orange, a banana, and an apple. Mario does not have a peach or a banana. Olivia and Mario just came from class with the student who has an apple. Kenji and Nancy are sitting next to the student who has a banana. Nancy does not have a peach. Which student has each piece of fruit?

	Nancy	Olivia	Mario	Kenji
Peach				
Orange				
Banana				
Apple				

2. Victor, Leon, Kasha, and Sheri each play one instrument. They play the viola, clarinet, trumpet, and flute. Sheri does not play the flute. Kasha lives near the student who plays flute and the one who plays trumpet. Leon does not play a brass or wind instrument. Which student plays each instrument?

3. Mr. Guthrie, Mrs. Hakoi, Mr. Mirza, and Mrs. Riva have jobs of doctor, accountant, teacher, and office manager. Mr. Mirza lives near the doctor and the teacher. Mrs. Riva is not the doctor or the office manager. Mrs. Hakoi is not the accountant or the office manager. Mr. Guthrie went to lunch with the doctor. Mrs. Riva's son is a high school student and is only seven years younger than his algebra teacher. Which person has each occupation?

4. Yvette, Lana, Boris, and Scott each have a dog. The breeds are collie, beagle, poodle, and terrier. Yvette and Boris walked to the library with the student who has a collie. Boris does not have a poodle or terrier. Scott does not have a collie. Yvette is in math class with the student who has a terrier. Which student has each breed of dog?

2-6 Study Guide and Intervention

Algebraic Proof

Algebraic Proof The following properties of algebra can be used to justify the steps when solving an algebraic equation.

Property	Statement
Reflexive	For every number a, $a = a$.
Symmetric	For all numbers a and b, if $a = b$ then $b = a$.
Transitive	For all numbers a, b, and c, if $a = b$ and $b = c$ then $a = c$.
Addition and Subtraction	For all numbers a, b, and c, if $a = b$ then $a + c = b + c$ and $a - c = b - c$.
Multiplication and Division	For all numbers a, b, and c, if $a = b$ then $a \cdot c = b \cdot c$, and if $c \neq 0$ then $\frac{a}{c} = \frac{b}{c}$.
Substitution	For all numbers a and b, if $a = b$ then a may be replaced by b in any equation or expression.
Distributive	For all numbers a, b, and c, $a(b + c) = ab + ac$.

Example **Solve $6x + 2(x - 1) = 30$.**

Algebraic Steps	Properties
$6x + 2(x - 1) = 30$	Given
$6x + 2x - 2 = 30$	Distributive Property
$8x - 2 = 30$	Substitution
$8x - 2 + 2 = 30 + 2$	Addition Property
$8x = 32$	Substitution
$\frac{8x}{8} = \frac{32}{8}$	Division Property
$x = 4$	Substitution

Exercises

Complete each proof.

1. Given: $\frac{4x + 6}{2} = 9$
Prove: $x = 3$

Statements	Reasons
a. $\frac{4x + 6}{2} = 9$	**a.** ____________
b. $\_\left(\frac{4x + 6}{2}\right) = 2(9)$	**b.** Mult. Prop.
c. $4x + 6 = 18$	**c.** ____________
d. $4x + 6 - 6 = 18 - 6$	**d.** ____________
e. $4x =$ ____________	**e.** Substitution
f. $\frac{4x}{4} =$ ____________	**f.** Div. Prop.
g. ____________	**g.** Substitution

2. Given: $4x + 8 = x + 2$
Prove: $x = -2$

Statements	Reasons
a. $4x + 8 = x + 2$	**a.** ____________
b. $4x + 8 - x = x + 2 - x$	**b.** ____________
c. $3x + 8 = 2$	**c.** Substitution
d. ____________	**d.** Subtr. Prop.
e. ____________	**e.** Substitution
f. $\frac{3x}{3} = \frac{-6}{3}$	**f.** ____________
g. ____________	**g.** Substitution

NAME ______________________ DATE __________ PERIOD _____

2-6 Study Guide and Intervention *(continued)*

Algebraic Proof

Geometric Proof Geometry deals with numbers as measures, so geometric proofs use properties of numbers. Here are some of the algebraic properties used in proofs.

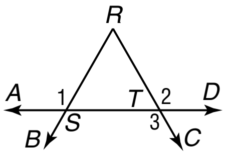
Property	Segments	Angles
Reflexive	$AB = AB$	$m\angle A = m\angle A$
Symmetric	If $AB = CD$, then $CD = AB$.	If $m\angle A = m\angle B$, then $m\angle B = m\angle A$.
Transitive	If $AB = CD$ and $CD = EF$, then $AB = EF$.	If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $m\angle 1 = m\angle 3$.

Example **Write a two-column proof.**

Given: $m\angle 1 = m\angle 2$, $m\angle 2 = m\angle 3$

Prove: $m\angle 1 = m\angle 3$

Proof:



Statements	Reasons
1. $m\angle 1 = m\angle 2$	**1.** Given
2. $m\angle 2 = m\angle 3$	**2.** Given
3. $m\angle 1 = m\angle 3$	**3.** Transitive Property

Exercises

State the property that justifies each statement.

1. If $m\angle 1 = m\angle 2$, then $m\angle 2 = m\angle 1$.

2. If $m\angle 1 = 90$ and $m\angle 2 = m\angle 1$, then $m\angle 2 = 90$.

3. If $AB = RS$ and $RS = WY$, then $AB = WY$.

4. If $AB = CD$, then $\frac{1}{2}AB = \frac{1}{2}CD$.

5. If $m\angle 1 + m\angle 2 = 110$ and $m\angle 2 = m\angle 3$, then $m\angle 1 + m\angle 3 = 110$.

6. $RS = RS$

7. If $AB = RS$ and $TU = WY$, then $AB + TU = RS + WY$.

8. If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $m\angle 1 = m\angle 3$.

9. A formula for the area of a triangle is $A = \frac{1}{2}bh$. Prove that bh is equal to 2 times the area of the triangle.

© Glencoe/McGraw-Hill

2-6 Skills Practice

Algebraic Proof

State the property that justifies each statement.

1. If $80 = m\angle A$, then $m\angle A = 80$.

2. If $RS = TU$ and $TU = YP$, then $RS = YP$.

3. If $7x = 28$, then $x = 4$.

4. If $VR + TY = EN + TY$, then $VR = EN$.

5. If $m\angle 1 = 30$ and $m\angle 1 = m\angle 2$, then $m\angle 2 = 30$.

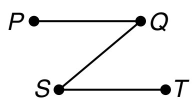
Complete the following proof.

6. Given: $8x - 5 = 2x + 1$
Prove: $x = 1$
Proof:

Statements	Reasons
a. $8x - 5 = 2x + 1$	**a.** ____________
b. $8x - 5 - 2x = 2x + 1 - 2x$	**b.** ____________
c. ____________	**c.** Substitution Property
d. ____________	**d.** Addition Property
e. $6x = 6$	**e.** ____________
f. $\frac{6x}{6} = \frac{6}{6}$	**f.** ____________
g. ____________	**g.** ____________

Write a two-column proof for the following.

7. If $\overline{PQ} \cong \overline{QS}$ and $\overline{QS} \cong \overline{ST}$, then $PQ = ST$.

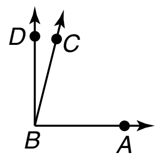


2-6 Practice

Algebraic Proof

PROOF Write a two-column proof.

1. If $m\angle ABC + m\angle CBD = 90$, $m\angle ABC = 3x - 5$, and $m\angle CBD = \frac{x + 1}{2}$, then $x = 27$.



2. FINANCE The formula for simple interest is $I = prt$, where I is interest, p is principal, r is rate, and t is time. Solve the formula for r and justify each step.

NAME ______________________________ DATE ____________ PERIOD _____

2-6 Reading to Learn Mathematics

Algebraic Proof

Pre-Activity **How is mathematical evidence similar to evidence in law?**

Read the introduction to Lesson 2-6 at the top of page 94 in your textbook.

What are some of the things that lawyers might use in presenting their closing arguments to a trial jury in addition to evidence gathered prior to the trial and testimony heard during the trial?

Reading the Lesson

1. Name the property illustrated by each statement.

a. If $a = 4.75$ and $4.75 = b$, then $a = b$.

b. If $x = y$, then $x + 8 = y + 8$.

c. $5(12 + 19) = 5 \cdot 12 + 5 \cdot 19$

d. If $x = 5$, then x may be replaced with 5 in any equation or expression.

e. If $x = y$, then $8x = 8y$.

f. If $x = 23.45$, then $23.45 = x$.

g. If $5x = 7$, then $x = \frac{7}{5}$.

h. If $x = 12$, then $x - 3 = 9$.

2. Give the reason for each statement in the following two-column proof.

Given: $5(n - 3) = 4(2n - 7) - 14$

Prove: $n = 9$

Statements	Reasons
1. $5(n - 3) = 4(2n - 7) - 14$	**1.** ____________________
2. $5n - 15 = 8n - 28 - 14$	**2.** ____________________
3. $5n - 15 = 8n - 42$	**3.** ____________________
4. $5n - 15 + 15 = 8n - 42 + 15$	**4.** ____________________
5. $5n = 8n - 27$	**5.** ____________________
6. $5n - 8n = 8n - 27 - 8n$	**6.** ____________________
7. $-3n = -27$	**7.** ____________________
8. $\frac{-3n}{-3} = \frac{-27}{-3}$	**8.** ____________________
9. $n = 9$	**9.** ____________________

Helping You Remember

3. A good way to remember mathematical terms is to relate them to words you already know. Give an everyday word that is related in meaning to the mathematical term *reflexive* and explain how this word can help you to remember the Reflexive Property and to distinguish it from the Symmetric and Transitive Properties.

© Glencoe/McGraw-Hill

NAME ____ DATE _____________ PERIOD _____

2-6 Enrichment

Symmetric, Reflexive, and Transitive Properties

Equality has three important properties.

Reflexive	$a = a$
Symmetric	If $a = b$, then $b = a$.
Transitive	If $a = b$ and $b = c$, then $a = c$.

Other relations have some of the same properties. Consider the relation "is next to" for objects labeled X, Y, and Z. Which of the properties listed above are true for this relation?

X is next to X. *False*
If X is next to Y, then Y is next to X. *True*
If X is next to Y and Y is next to Z, then X is next to Z. *False*

Only the symmetric property is true for the relation "is next to."

For each relation, state which properties (*symmetric*, *reflexive*, *transitive*) are true.

1. is the same size as

2. is a family descendant of

3. is in the same room as

4. is the identical twin of

5. is warmer than

6. is on the same line as

7. is a sister of

8. is the same weight as

9. Find two other examples of relations, and tell which properties are true for each relation.

© Glencoe/McGraw-Hill

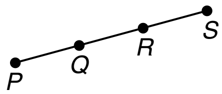
2-7 Study Guide and Intervention

Proving Segment Relationships

Segment Addition Two basic postulates for working with segments and lengths are the Ruler Postulate, which establishes number lines, and the Segment Addition Postulate, which describes what it means for one point to be between two other points.

Ruler Postulate	The points on any line or line segment can be paired with real numbers so that, given any two points *A* and *B* on a line, *A* corresponds to zero and *B* corresponds to a positive real number.
Segment Addition Postulate	*B* is between *A* and *C* if and only if $AB + BC = AC$.

Example **Write a two-column proof.**



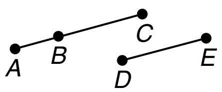
Given: Q is the midpoint of $\overline{PR}$.
R is the midpoint of $\overline{QS}$.
Prove: $PR = QS$

Statements	Reasons
1. Q is the midpoint of $\overline{PR}$.	**1.** Given
2. $PQ = QR$	**2.** Definition of midpoint
3. R is the midpoint of $\overline{QS}$.	**3.** Given
4. $QR = RS$	**4.** Definition of midpoint
5. $PQ + QR = QR + RS$	**5.** Addition Property
6. $PQ + QR = PR$, $QR + RS = QS$	**6.** Segment Addition Postulate
7. $PR = QS$	**7.** Substitution

Exercises

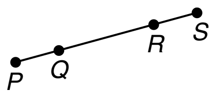
Complete each proof.

1. Given: $BC = DE$
Prove: $AB + DE = AC$



Statements	Reasons
a. $BC = DE$	**a.** ____________
b. ____________	**b.** Seg. Add. Post.
c. $AB + DE = AC$	**c.** ____________

2. Given: Q is between P and R, R is between Q and S, $PR = QS$.
Prove: $PQ = RS$



Statements	Reasons
a. Q is between P and R.	**a.** Given
b. $PQ + QR = PR$	**b.** ____________
c. R is between Q and S.	**c.** ____________
d. ____________	**d.** Seg. Add. Post.
e. $PR = QS$	**e.** ____________
f. $PQ + QR = QR + RS$	**f.** ____________
g. $PQ + QR - QR = QR + RS - QR$	**g.** ____________
h. ____________	**h.** Substitution

NAME ____ DATE ____________ PERIOD _____

2-7 Study Guide and Intervention *(continued)*

Proving Segment Relationships

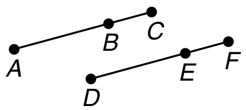
Segment Congruence Three properties of algebra—the Reflexive, Symmetric, and Transitive Properties of Equality—have counterparts as properties of geometry. These properties can be proved as a theorem. As with other theorems, the properties can then be used to prove relationships among segments.

Segment Congruence Theorem	Congruence of segments is reflexive, symmetric, and transitive.
Reflexive Property	$\overline{AB} \cong \overline{AB}$
Symmetric Property	If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.
Transitive Property	If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

Example **Write a two-column proof.**

Given: $\overline{AB} \cong \overline{DE}$; $\overline{BC} \cong \overline{EF}$

Prove: $\overline{AC} \cong \overline{DF}$



Statements	**Reasons**
1. $\overline{AB} \cong \overline{DE}$	**1.** Given
2. $AB = DE$	**2.** Definition of congruence of segments
3. $\overline{BC} \cong \overline{EF}$	**3.** Given
4. $BC = EF$	**4.** Definition of congruence of segments
5. $AB + BC = DE + EF$	**5.** Addition Property
6. $AB + BC = AC$, $DE + EF = DF$	**6.** Segment Addition Postulate
7. $AC = DF$	**7.** Substitution
8. $\overline{AC} \cong \overline{DF}$	**8.** Definition of congruence of segments

Exercises

Justify each statement with a property of congruence.

1. If $\overline{DE} \cong \overline{GH}$, then $\overline{GH} \cong \overline{DE}$.

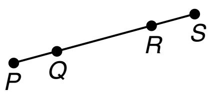
2. If $\overline{AB} \cong \overline{RS}$ and $\overline{RS} \cong \overline{WY}$, then $\overline{AB} \cong \overline{WY}$.

3. $\overline{RS} \cong \overline{RS}$.

4. Complete the proof.

Given: $\overline{PR} \cong \overline{QS}$

Prove: $\overline{PQ} \cong \overline{RS}$



Statements	**Reasons**
a. $\overline{PR} \cong \overline{QS}$	**a.** ____________________
b. $PR = QS$	**b.** ____________________
c. $PQ + QR = PR$	**c.** ____________________
d. ____________________	**d.** Segment Addition Postulate
e. $PQ + QR = QR + RS$	**e.** ____________________
f. ____________________	**f.** Subtraction Property
g. ____________________	**g.** Definition of congruence of segments

© Glencoe/McGraw-Hill

2-7 Skills Practice

Proving Segment Relationships

Justify each statement with a property of equality, a property of congruence, or a postulate.

1. $QA = QA$

2. If $\overline{AB} \cong \overline{BC}$ and $\overline{BC} \cong \overline{CE}$, then $\overline{AB} \cong \overline{CE}$.

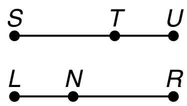
3. If Q is between P and R, then $PR = PQ + QR$.

4. If $AB + BC = EF + FG$ and $AB + BC = AC$, then $EF + FG = AC$.

Complete each proof.

5. Given: $\overline{SU} \cong \overline{LR}$
$\overline{TU} \cong \overline{LN}$

Prove: $\overline{ST} \cong \overline{NR}$



Proof:

Statements	**Reasons**
a. $\overline{SU} \cong \overline{LR}$, $\overline{TU} \cong \overline{LN}$	**a.** ____________
b. ____________	**b.** Definition of $\cong$ segments
c. $SU = ST + TU$ $LR = LN + NR$	**c.** ____________
d. $ST + TU = LN + NR$	**d.** ____________
e. $ST + LN = LN + NR$	**e.** ____________
f. $ST + LN - LN = LN + NR - LN$	**f.** ____________
g. ____________	**g.** Substitution Property
h. $\overline{ST} \cong \overline{NR}$	**h.** ____________

6. Given: $\overline{AB} \cong \overline{CD}$

Prove: $\overline{CD} \cong \overline{AB}$

Proof:

Statements	**Reasons**
a. ____________	**a.** Given
b. $AB = CD$	**b.** ____________
c. $CD = AB$	**c.** ____________
d. ____________	**d.** Definition of $\cong$ segments

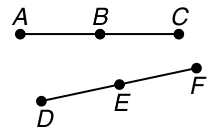
NAME ____ DATE ____________ PERIOD _____

2-7 Practice

Proving Segment Relationships

Complete the following proof.

1. Given: $\overline{AB} \cong \overline{DE}$
B is the midpoint of $\overline{AC}$.
E is the midpoint of $\overline{DF}$.

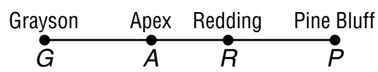


Prove: $\overline{BC} \cong \overline{EF}$

Proof:

Statements	**Reasons**
a. ____________________ ____________________ ____________________	**a.** Given
b. $AB = DE$	**b.** ____________________
c. ____________________ ____________________	**c.** Definition of Midpoint
d. $AC = AB + BC$ $DF = DE + EF$	**d.** ____________________
e. $AB + BC = DE + EF$	**e.** ____________________
f. $AB + BC = AB + EF$	**f.** ____________________
g. $AB + BC - AB = AB + EF - AB$	**g.** Subtraction Property
h. $BC = EF$	**h.** ____________________
i. ____________________	**i.** ____________________

2. TRAVEL Refer to the figure. DeAnne knows that the distance from Grayson to Apex is the same as the distance from Redding to Pine Bluff. Prove that the distance from Grayson to Redding is equal to the distance from Apex to Pine Bluff.



NAME ____ DATE ____________ PERIOD _____

2-7 Reading to Learn Mathematics

Proving Segment Relationships

Pre-Activity **How can segment relationships be used for travel?**

Read the introduction to Lesson 2-7 at the top of page 101 in your textbook.

- What is the total distance that the plane will fly to get from San Diego to Dallas?
- Before leaving home, a passenger used a road atlas to determine that the distance between San Diego and Dallas is about 1350 miles. Why is the flying distance greater than that?

Reading the Lesson

1. If E is between Y and S, which of the following statements are *always* true?

A. $YS + ES = YE$ **B.** $YS - ES = YE$

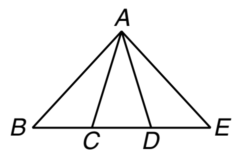
C. $YE > ES$ **D.** $YE \cdot ES = YS$

E. $SE + EY = SY$ **F.** E is the midpoint of $\overline{YS}$.

2. Give the reason for each statement in the following two-column proof.

Given: C is the midpoint of $\overline{BD}$.
D is the midpoint of $\overline{CE}$.

Prove: $\overline{BD} \cong \overline{CE}$



Statements	Reasons
1. C is the midpoint of $\overline{BD}$.	**1.** ______________
2. $BC = CD$	**2.** ______________
3. D is the midpoint of $\overline{CE}$.	**3.** ______________
4. $CD = DE$	**4.** ______________
5. $BC = DE$	**5.** ______________
6. $BC + CD = CD + DE$	**6.** ______________
7. $BC + CD = BD$ $CD + DE = CE$	**7.** ______________
8. $BD = CE$	**8.** ______________
9. $\overline{BD} \cong \overline{CE}$	**9.** ______________

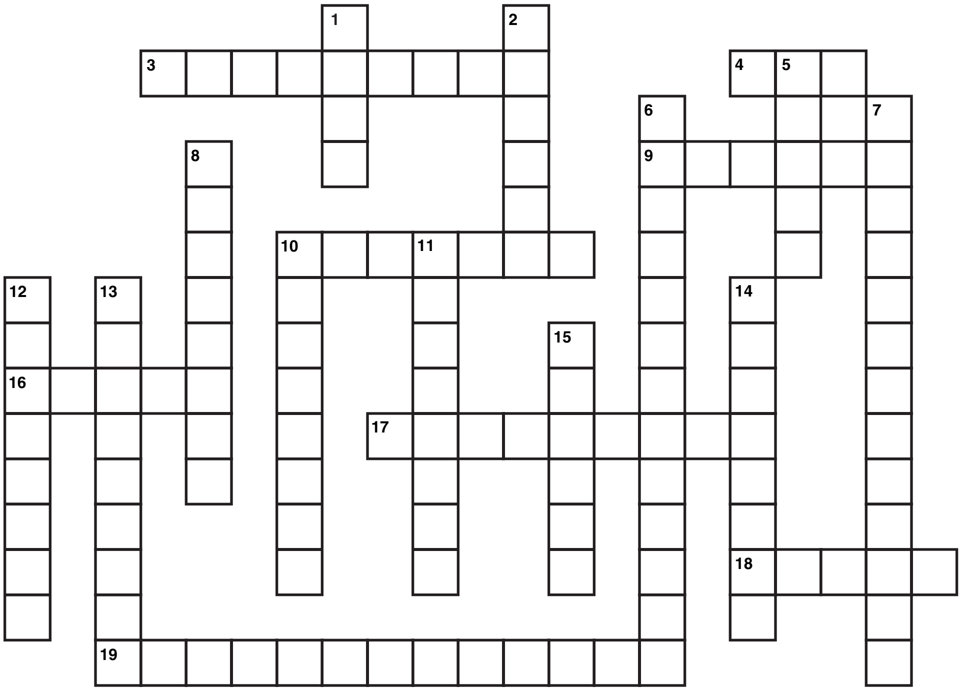
Helping You Remember

3. One way to keep the names of related postulates straight in your mind is to associate something in the name of the postulate with the content of the postulate. How can you use this idea to distinguish between the Ruler Postulate and the Segment Addition Postulate?

NAME ______________________________ DATE ____________ PERIOD _____

2-7 Enrichment

Geometry Crossword Puzzle



ACROSS

3. Points on the same line are ______.

4. A point on a line and all points of the line to one side of it.

9. An angle whose measure is greater than 90.

10. Two endpoints and all points between them.

16. A flat figure with no thickness that extends indefinitely in all directions.

17. Segments of equal length are ______ segments.

18. Two noncollinear rays with a common endpoint.

19. If $m\angle A + m\angle B = 180$, then $\angle A$ and $\angle B$ are ______ angles.

DOWN

1. The set of all points collinear to two points is a ______.

2. The point where the x- and y-axis meet.

5. An angle whose measure is less than 90.

6. If $m\angle A + m\angle D = 90$, then $\angle A$ and $\angle D$ are ______ angles.

7. Lines that meet at a 90° angle are ______.

8. Two angles with a common side but no common interior points are ______.

10. An "angle" formed by opposite rays is a ______ angle.

11. The middle point of a line segment.

12. Points that lie in the same plane are ______.

13. The four parts of a coordinate plane.

14. Two nonadjacent angles formed by two intersecting lines are ______ angles.

15. In angle ABC, point B is the ______.

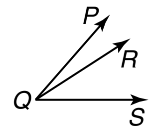
NAME ____ DATE ____________ PERIOD _____

2-8 Study Guide and Intervention

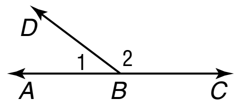
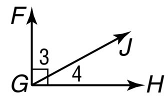
Proving Angle Relationships

Supplementary and Complementary Angles There are two basic postulates for working with angles. The Protractor Postulate assigns numbers to angle measures, and the Angle Addition Postulate relates parts of an angle to the whole angle.

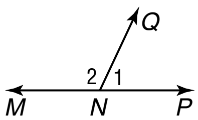
Protractor Postulate	Given $\overrightarrow{AB}$ and a number r between 0 and 180, there is exactly one ray with endpoint A, extending on either side of $\overrightarrow{AB}$, such that the measure of the angle formed is r.
Angle Addition Postulate	R is in the interior of $\angle PQS$ if and only if $m\angle PQR + m\angle RQS = m\angle PQS$.



The two postulates can be used to prove the following two theorems.

Supplement Theorem	If two angles form a linear pair, then they are supplementary angles. If $\angle 1$ and $\angle 2$ form a linear pair, then $m\angle 1 + m\angle 2 = 180$.	
Complement Theorem	If the noncommon sides of two adjacent angles form a right angle, then the angles are complementary angles. If $\overleftrightarrow{GF} \perp \overleftrightarrow{GH}$, then $m\angle 3 + m\angle 4 = 90$.	

Example 1 **If $\angle 1$ and $\angle 2$ form a linear pair and $m\angle 2 = 115$, find $m\angle 1$.**

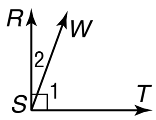


$m\angle 1 + m\angle 2 = 180$ Suppl. Theorem

$m\angle 1 + 115 = 180$ Substitution

$m\angle 1 = 65$ Subtraction Prop.

Example 2 **If $\angle 1$ and $\angle 2$ form a right angle and $m\angle 2 = 20$, find $m\angle 1$.**



$m\angle 1 + m\angle 2 = 90$ Compl. Theorem

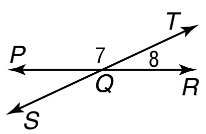
$m\angle 1 + 20 = 90$ Substitution

$m\angle 1 = 70$ Subtraction Prop.

Exercises

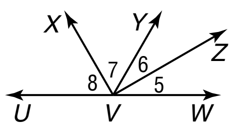
Find the measure of each numbered angle.

1.



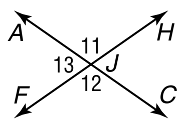
$m\angle 7 = 5x + 5,$
$m\angle 8 = x - 5$

2.



$m\angle 5 = 5x, m\angle 6 = 4x + 6,$
$m\angle 7 = 10x,$
$m\angle 8 = 12x - 12$

3.



$m\angle 11 = 11x,$
$m\angle 12 = 10x + 10$

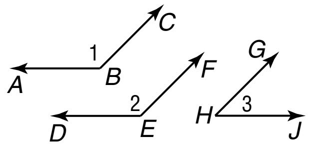
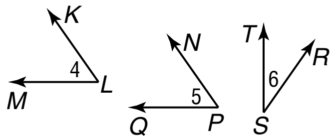
Lesson 2-8

NAME ______________________________ DATE ____________ PERIOD _____

2-8 Study Guide and Intervention *(continued)*

Proving Angle Relationships

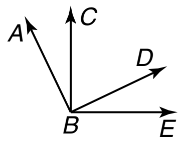
Congruent and Right Angles Three properties of angles can be proved as theorems.

Congruence of angles is reflexive, symmetric, and transitive.	
Angles supplementary to the same angle or to congruent angles are congruent.	Angles complementary to the same angle or to congruent angles are congruent.
	
If ∠1 and ∠2 are supplementary to ∠3, then ∠1 ≅ ∠2.	If ∠4 and ∠5 are complementary to ∠6, then ∠4 ≅ ∠5.

Example **Write a two-column proof.**

Given: ∠*ABC* and ∠*CBD* are complementary.
∠*DBE* and ∠*CBD* form a right angle.

Prove: ∠*ABC* ≅ ∠*DBE*



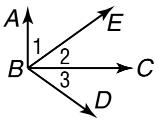
Statements	**Reasons**
1. ∠*ABC* and ∠*CBD* are complementary. ∠*DBE* and ∠*CBD* form a right angle.	**1.** Given
2. ∠*DBE* and ∠*CBD* are complementary.	**2.** Complement Theorem
3. ∠*ABC* ≅ ∠*DBE*	**3.** Angles complementary to the same ∠ are ≅.

Exercises

Complete each proof.

1. Given: $\overline{AB} \perp \overline{BC}$; ∠1 and ∠3 are complementary.

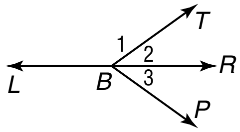
Prove: ∠2 ≅ ∠3



Statements	**Reasons**
a. $\overline{AB} \perp \overline{BC}$	**a.** ______________
b. ______________	**b.** Definition of ⊥
c. $m\angle 1 + m\angle 2 = m\angle ABC$	**c.** ______________
d. ∠1 and ∠2 form a rt ∠.	**d.** ______________
e. ∠1 and ∠2 are compl.	**e.** ______________
f. ______________	**f.** Given
g. ∠2 ≅ ∠3	**g.** ______________

2. Given: ∠1 and ∠2 form a linear pair. $m\angle 1 + m\angle 3 = 180$

Prove: ∠2 ≅ ∠3



Statements	**Reasons**
a. ∠1 and ∠2 form a linear pair. $m\angle 1 + m\angle 3 = 180$	**a.** Given
b. ______________	**b.** Suppl. Theorem
c. ∠1 is suppl. to ∠3.	**c.** ______________
d. ______________	**d.** ∠s suppl. to the same ∠ are ≅.

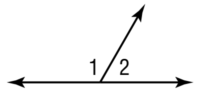
NAME ______________________________ DATE ____________ PERIOD _____

2-8 Skills Practice

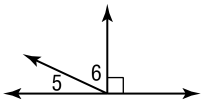
Proving Angle Relationships

Find the measure of each numbered angle.

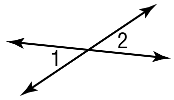
1. $m\angle 2 = 57$



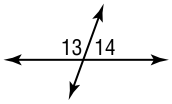
2. $m\angle 5 = 22$



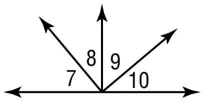
3. $m\angle 1 = 38$



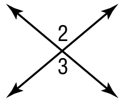
4. $m\angle 13 = 4x + 11$,
$m\angle 14 = 3x + 1$



5. $\angle 9$ and $\angle 10$ are complementary.
$\angle 7 \cong \angle 9$, $m\angle 8 = 41$



6. $m\angle 2 = 4x - 26$,
$m\angle 3 = 3x + 4$

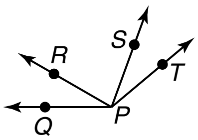


Determine whether the following statements are *always*, *sometimes*, or *never* true.

7. Two angles that are supplementary form a linear pair.

8. Two angles that are vertical are adjacent.

9. Copy and complete the following proof.
Given: $\angle QPS \cong \angle TPR$
Prove: $\angle QPR \cong \angle TPS$
Proof:



Statements	Reasons
a. ____________	**a.** ____________
b. $m\angle QPS = m\angle TPR$	**b.** ____________
c. $m\angle QPS = m\angle QPR + m\angle RPS$ $m\angle TPR = m\angle TPS + m\angle RPS$	**c.** ____________
d. ____________ ____________	**d.** Substitution
e. ____________	**e.** ____________
f. ____________	**f.** ____________

Lesson 2-8

© Glencoe/McGraw-Hill

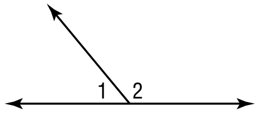
NAME ______________________________ DATE __________ PERIOD _____

2-8 Practice

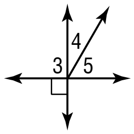
Proving Angle Relationships

Find the measure of each numbered angle.

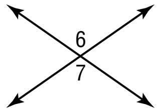
1. $m\angle 1 = x + 10$
$m\angle 2 = 3x + 18$



2. $m\angle 4 = 2x - 5$
$m\angle 5 = 4x - 13$



3. $m\angle 6 = 7x - 24$
$m\angle 7 = 5x + 14$



Determine whether the following statements are *always*, *sometimes*, or *never* true.

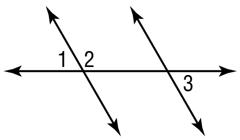
4. Two angles that are supplementary are complementary.

5. Complementary angles are congruent.

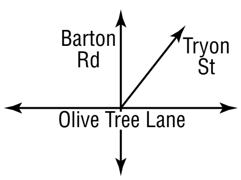
6. Write a two-column proof.

Given: $\angle 1$ and $\angle 2$ form a linear pair.
$\angle 2$ and $\angle 3$ are supplementary.

Prove: $\angle 1 \cong \angle 3$



7. STREETS Refer to the figure. Barton Road and Olive Tree Lane form a right angle at their intersection. Tryon Street forms a 57° angle with Olive Tree Lane. What is the measure of the acute angle Tryon Street forms with Barton Road?



NAME ____ DATE ____________ PERIOD _____

2-8 Reading to Learn Mathematics

Proving Angle Relationships

Pre-Activity **How do scissors illustrate supplementary angles?**

Read the introduction to Lesson 2-8 at the top of page 107 in your textbook.

Is it possible to open a pair of scissors so that the angles formed by the two blades, a blade and a handle, and the two handles, are all congruent? If so, explain how this could happen.

Reading the Lesson

1. Complete each sentence to form a statement that is always true.

a. If two angles form a linear pair, then they are adjacent and __________________.

b. If two angles are complementary to the same angle, then they are __________________.

c. If D is a point in the interior of $\angle ABC$, then $m\angle ABC = m\angle ABD +$ ________________.

d. Given $\overrightarrow{RS}$ and a number x between ________ and ________, there is exactly one ray with endpoint R, extended on either side of RS, such that the measure of the angle formed is x.

e. If two angles are congruent and supplementary, then each angle is a(n) __________________ angle.

f. __________________ lines form congruent adjacent angles.

g. “Every angle is congruent to itself” is a statement of the __________________ Property of angle congruence.

h. If two congruent angles form a linear pair, then the measure of each angle is ________.

i. If the noncommon sides of two adjacent angles form a right angle, then the angles are __________________.

2. Determine whether each statement is *always*, *sometimes*, or *never* true.

a. Supplementary angles are congruent.

b. If two angles form a linear pair, they are complementary.

c. Two vertical angles are supplementary.

d. Two adjacent angles form a linear pair.

e. Two vertical angles form a linear pair.

f. Complementary angles are congruent.

g. Two angles that are congruent to the same angle are congruent to each other.

h. Complementary angles are adjacent angles.

Helping You Remember

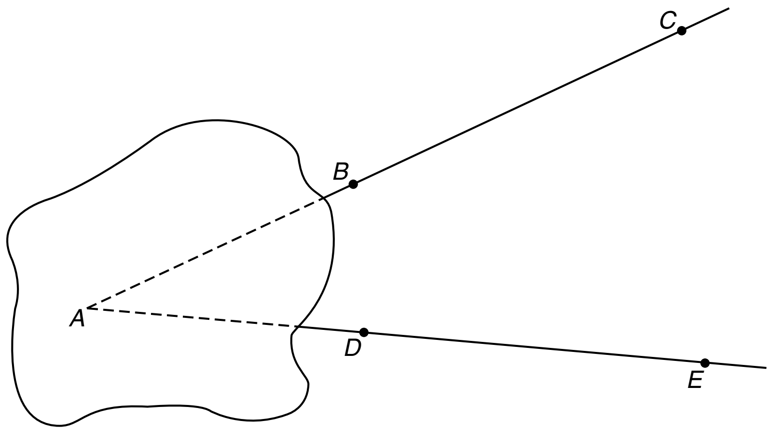
3. A good way to remember something is to explain it to someone else. Suppose that a classmate thinks that two angles can only be *vertical* angles if one angle lies above the other. How can you explain to him the meaning of vertical angles, using the word *vertex* in your explanation?

NAME ____ DATE ____________ PERIOD _____

2-8 Enrichment

Bisecting a Hidden Angle

The vertex of ∠*BAD* at the right is hidden in a region. Within the region, you are not allowed to use a compass. Can you bisect the angle?



Follow these instructions to bisect ∠*BAD*.

1. Use a straightedge to draw lines *CE* and *BD*.

2. Construct the bisectors of ∠*DEC* and ∠*BCE*.

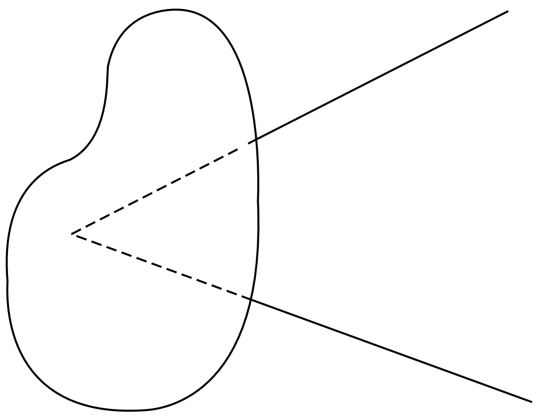
3. Label the intersection of the two bisectors as point *P*.

4. Construct the bisectors of ∠*BDE* and ∠*DBC*.

5. Label the intersection of the two previous bisectors as point *Q*.

6. Use a straightedge to draw line *PQ*, which bisects the hidden angle.

7. Another hidden angle is shown at right. Construct the bisector using the method above, or devise your own method.



© Glencoe/McGraw-Hill

NAME ______________________________ DATE ____________ PERIOD ______

2 Chapter 2 Test, Form 1

SCORE ______

Write the letter for the correct answer in the blank at the right of each question.

1. Make a conjecture about the next term in this sequence: 92, 87, 82, 77, 72. **1.** ______

A. -5 **B.** 62 **C.** 67 **D.** 77

2. Make a conjecture given that M is the midpoint of $\overline{BC}$. **2.** ______

A. $BM = BC$ **B.** $BM = MC$ **C.** $MC = BC$ **D.** M bisects $\angle C$

3. Given: $a + b \leq 8$ and $a = 2$ **3.** ______

Conjecture: $b \leq 5$

Which of the following would be a counterexample?

A. $b = 3$ **B.** $b = 5$ **C.** $b = 6$ **D.** $b = a$

4. If p is true and q is false, what is the truth value of p or q? **4.** ______

A. true **B.** false **C.** 0 **D.** 1

For Questions 5 and 6, use this truth table.

p	q	$\sim p$	$\sim p \vee q$
T	T		
T	F		
F	T		
F	F		

5. Which would be the values in the $\sim p$ column? **5.** ______

A. F T F T **B.** F F T T

C. T F F T **D.** T T F F

6. Which would be the values in the $\sim p \vee q$ column? **6.** ______

A. F F T F **B.** T T T F

C. T T T T **D.** T F T T

7. Identify the hypothesis of the statement *If* $x + 4 = 5$*, then* $x = 1$. **7.** ______

A. If $x = 1$, then $x + 4 = 5$. **B.** If $x + 4 \neq 5$, then $x \neq 1$.

C. $x + 4 = 5$ **D.** $x = 1$

8. Identify the converse of the statement *If cats fly, then ducks bark*. **8.** ______

A. If cats don't fly, then ducks don't bark.

B. If ducks don't bark, then cats don't fly.

C. If cats bark, then ducks fly.

D. If ducks bark, then cats fly.

9. Identify the inverse of the statement *If a triangle has 3 equal sides, then it is equilateral*. **9.** ______

A. If a triangle does not have 3 equal sides, then it is not equilateral.

B. If a triangle is equilateral, then it has 3 equal sides.

C. If a triangle is not equilateral, then it does not have 3 equal sides.

D. If a triangle has 2 equal sides, then it is isosceles.

10. Which of the following illustrates the Law of Detachment? **10.** ______

A. $[(p \rightarrow q) \vee (q \rightarrow r)] \rightarrow (p \rightarrow r)$ **B.** $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

C. $[(p \rightarrow q) \wedge q] \rightarrow p$ **D.** $[(p \rightarrow q) \wedge p] \rightarrow q$

Assessments

NAME ______________________ DATE __________ PERIOD ______

2 Chapter 2 Test, Form 1 *(continued)*

11. Which of the following illustrates the Law of Syllogism? **11.** ______

A. $[(p \rightarrow q) \vee (q\ (\ r)] \rightarrow (p \rightarrow r)$ **B.** $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

C. $[(p \rightarrow q) \wedge q] \rightarrow p$ **D.** $[(p \rightarrow q) \wedge p] \rightarrow q$

12. Which best describes the statement *A plane contains at least 3 points not on the same line*? **12.** ______

A. always true **B.** sometimes true

C. never true **D.** cannot tell

13. Which is a type of proof where you write a paragraph to explain why a conjecture for a given situation is true? **13.** ______

A. argument **B.** explanation

C. paragraph proof **D.** two-column proof

For Questions 14–16, choose the property that justifies the statement.

14. If $3x = 6$, then $x = 2$. **14.** ______

A. Addition **B.** Subtraction **C.** Multiplication **D.** Division

15. If $m\angle A = 10$ and $m\angle B = 10$, then $m\angle A = m\angle B$. **15.** ______

A. Reflexive **B.** Symmetric **C.** Substitution **D.** Equality

16. If $\overline{PS} \cong \overline{WX}$, then $PS = WX$. **16.** ______

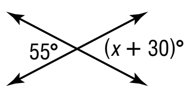
A. Reflexive **B.** Symmetric

C. Definition of congruent segments **D.** Transitive

17. If A, B and N are collinear and $AB + BN = AN$, which point is between the other two? **17.** ______

A. A **B.** B **C.** N **D.** cannot tell

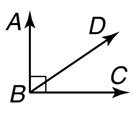
18. Find x. **18.** ______



A. 25 **B.** 35

C. 55 **D.** 125

19. If $m\angle ABD = 56$, find $m\angle DBC$. **19.** ______



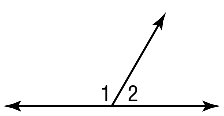
A. 124 **B.** 56

C. 44 **D.** 34

20. If a right angle is trisected, what is the measure of each of the smaller angles? **20.** ______

A. 30 **B.** 45 **C.** 60 **D.** 90

Bonus If $m\angle 1$ is twice $m\angle 2$, find $m\angle 1$. **B:** ______________



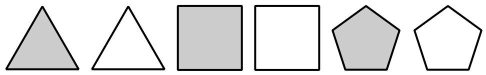
NAME ________________ DATE ________ PERIOD ______

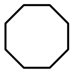
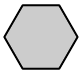
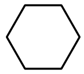
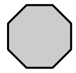
2 Chapter 2 Test, Form 2A

SCORE ______

Write the letter for the correct answer in the blank at the right of each question.

1. Make a conjecture about the next object in this sequence. **1.** ______



A.  **B.**  **C.**  **D.** 

2. **Given:** $|n|$ is a positive number. **Conjecture:** n is a negative number. Which of the following would be a counterexample? **2.** ______

A. -10 **B.** 0 **C.** -1 **D.** 10

3. If p is true and q is false, what is the truth value of p and q? **3.** ______

A. true **B.** false **C.** 0 **D.** 1

For Questions 4 and 5, use this truth table.

p	q	$\sim q$	$p \vee \sim q$
T	T		
T	F		
F	T		
F	F		

4. Which would be the values in the $\sim q$ column? **4.** ______

A. F F T T **B.** T T F F
C. F T F T **D.** T F T F

5. Which would be the values in the $p \wedge \sim q$ column? **5.** ______

A. F T F F **B.** F T T F
C. T T F T **D.** T F T T

6. Identify the conclusion of the statement *Jack will go to school if today is Monday*. **6.** ______

A. Jack will go to school **B.** Jack will not go to school
C. today is Monday **D.** today is not Monday

7. Identify the inverse of the following statement. **7.** ______
If $x = 2$, then $x + 3 = 5$.

A. If $x + 3 = 5$, then $x = 2$. **B.** If $x + 3 \neq 5$, then $x \neq 2$.
C. If $x \neq 2$, then $x + 3 \neq 5$. **D.** $x = 2$ and $x + 3 = 5$.

8. Identify the contrapositive of the following statement. **8.** ______
If $x = 2$, then $x + 3 = 5$.

A. If $x + 3 = 5$, then $x = 2$. **B.** If $x + 3 \neq 5$, then $x \neq 2$.
C. If $x \neq 2$, then $x + 3 \neq 5$. **D.** $x = 2$ and $x + 3 = 5$.

9. Which law can be used to determine that statement (3) is a valid conclusion to statements (1) and (2)? **9.** ______

(1) If an angle is acute, then it cannot be obtuse.
(2) $\angle A$ is acute.
(3) $\angle A$ cannot be obtuse.

A. Law of Detachment **B.** Law of Syllogism
C. Law of Converse **D.** Statement (3) does not follow.

Assessments

2 Chapter 2 Test, Form 2A *(continued)*

10. Which law can be used to determine that statement (3) is a valid conclusion of statements (1) and (2)? **10.** ______
(1) If a figure has 4 right angles, then the figure is a rectangle.
(2) A rectangle has 2 pairs of parallel sides.
(3) If a figure has 4 right angles, then the figure has 2 pair of parallel sides.
A. Law of Detachment **B.** Law of Syllogism
C. Law of Converse **D.** Statement (3) does not follow.

11. Which best describes the statement *If two planes intersect, then their intersection is a point*? **11.** ______
A. always true **B.** sometimes true **C.** never true D. cannot tell

12. Which of the following is an essential part of a good proof? **12.** ______
A. an if-then statement **B.** a postulate
C. using the contrapositive **D.** a system of deductive reasoning

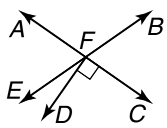
13. Choose the property that justifies the following statement. **13.** ______
If $x = 2$ and $x + y = 3$, then $2 + y = 3$.
A. Reflexive **B.** Symmetric **C.** Transitive **D.** Substitution

14. Choose the property that justifies the statement $m\angle A = m\angle A$. **14.** ______
A. Reflexive **B.** Symmetric **C.** Transitive **D.** Substitution

15. Choose the property that justifies the statement *If* $\overline{GH} \cong \overline{FD}$, *then* $\overline{FD} \cong \overline{GH}$. **15.** ______
A. Reflexive **B.** Symmetric
C. Transitive **D.** Definition of congruent segments

16. If $XY = 6$, $YZ = 4$, and $XZ = 2$, which point is between the other two? **16.** ______
A. X **B.** Y **C.** Z **D.** cannot tell

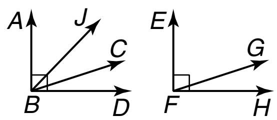
For Questions 17 and 18, use the figure at the right.



17. If $m\angle BFC = 70$, find $m\angle EFD$. **17.** ______
A. 10 **B.** 20
C. 35 **D.** 70

18. If $m\angle AFB = 5x - 10$ and $m\angle BFC = 3x + 20$, find x. **18.** ______
A. 10 **B.** 15 **C.** 21.25 **D.** $23.\overline{3}$

For Questions 19 and 20, use the figure at the right.



19. If $\angle ABC \cong \angle EFG$, and $m\angle ABC = 72$, find $m\angle GFH$. **19.** ______
A. 18 **B.** 72
C. 90 **D.** 108

20. If $m\angle ABJ = 28$, $\angle ABC \cong \angle DBJ$, find $m\angle JBC$. **20.** ______
A. 90 **B.** 56 **C.** 45 **D.** 34

Bonus Angles A and B are vertical angles, $m\angle A = 9x + 10$, and $m\angle B = x^2 + 6x$. Find x. **B:** ______________

NAME ______________________________ DATE ____________ PERIOD ______

2 Chapter 2 Test, Form 2B

SCORE ______

Write the letter for the correct answer in the blank at the right of each question.

1. Make a conjecture based on the information $\overline{AB}$ bisects $\overline{XY}$ at Z. **1.** ______

A. $AZ = ZB$ **B.** $\angle AZX \cong \angle AZY$
C. $AB = XY$ **D.** $XZ = YZ$

2. Given: n is a real number. **2.** ______
Conjecture: $n \geq |n|$
Which of the following would be a counterexample?

A. -2 **B.** 0 **C.** 5 **D.** 10

3. If p is false and q is true, what is the truth value of p or q? **3.** ______

A. true **B.** false **C.** 0 **D.** 1

For Questions 4 and 5, use this truth table.

p	q	$p \wedge q$	$p \vee q$
T	T		
T	F		
F	T		
F	F		

4. What would be the values in the $p \wedge q$ column? **4.** ______

A. T F F T **B.** T F T F
C. T F F F **D.** T T T F

5. What would be the values in the $p \vee q$ column? **5.** ______

A. T F F T **B.** T F T F
C. T F F F **D.** T T T F

6. Identify the conclusion of the statement *Sue will watch the Rose Bowl if today is January 1st*. **6.** ______

A. today is January 1st **B.** today is not January 1st
C. Sue will watch the Rose Bowl **D.** Sue will not watch the Rose Bowl

7. Identify the inverse of the statement. **7.** ______
If $x = 5$, then $x + 8 = 13$.

A. If $x + 8 = 13$, then $x = 5$. **B.** $x = 5$ and $x + 8 = 13$.
C. If $x + 8 \neq 13$, then $x5$. **D.** If $x \neq 5$, then $x + 8 \neq 13$.

8. Identify the contrapositive of the following statement. **8.** ______
If $x = 5$, then $x + 8 = 13$.

A. If $x + 8 = 13$, then $x = 5$. **B.** $x = 5$ and $x + 8 = 13$.
C. If $x + 8 \neq 13$, then $x \neq 5$. **D.** If $x \neq 5$, then $x + 8 \neq 13$.

9. Which law can be used to determine that statement (3) is a valid conclusion to statements (1) and (2)? **9.** ______

(1) All dogs like biscuits.
(2) Sammy is a dog.
(3) Sammy likes biscuits.

A. Law of Detachment **B.** Law of Syllogism
C. Law of Converse **D.** Statement (3) does not follow.

NAME ______________________ DATE ____________ PERIOD ______

2 Chapter 2 Test, Form 2B *(continued)*

10. Which law can be used to determine that statement (3) is a valid conclusion of statements (1) and (2)? **10.** ______
(1) All sparrows fly. (2) All robins fly. (3) All sparrows are robins.
A. Law of Detachment **B.** Law of Syllogism
C. Law of Converse **D.** Statement (3) does not follow.

11. Which best describes the statement *If two points lie in a plane, then the entire line containing those points lies in that plane*? **11.** ______
A. always true **B.** sometimes true **C.** never true **D.** cannot tell

12. Which of the following is an essential part of a good proof? **12.** ______
A. an if-then statement **B.** using the contrapositive
C. listing the given information **D.** defining all the terms

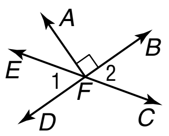
13. Choose the property that justifies the statement. **13.** ______
If $x = y$ and $y = z$, then $x = z$.
A. Conditional **B.** Transitive **C.** Symmetric **D.** Reflexive

14. Choose the property that justifies the following statement. **14.** ______
If $3AB = CD$*, then* $AB = \frac{1}{3}CD$.
A. Addition **B.** Subtraction **C.** Division **D.** Substitution

15. Choose the property that justifies the statement. **15.** ______
If $\overline{GH} \cong \overline{FD}$ *and* $\overline{FD} \cong \overline{CB}$*, then* $\overline{GH} \cong \overline{CB}$.
A. Reflexive **B.** Symmetric
C. Transitive **D.** Def. of ≅ segments

16. If $XY = 6$, $YZ = 8$, and $XZ = 2$, which point is between the other two? **16.** ______
A. X **B.** Y **C.** Z **D.** cannot tell

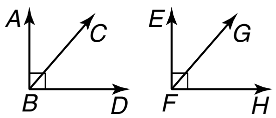
For Questions 17 and 18, use the figure at the right.



17. Find $m\angle AFE$ if $m\angle BFC = 55$. **17.** ______
A. 20 **B.** 35
C. 45 **D.** 55

18. If $m\angle 1 = 5x - 10$ and $m\angle 2 = 3x + 20$, find x. **18.** ______
A. 10 **B.** 15 **C.** 21.25 **D.** $23.\overline{3}$

For Questions 19 and 20, use the figure at the right.



19. If $m\angle ABC = 34$, find $m\angle CBD$. **19.** ______
A. 90 **B.** 56
C. 45 **D.** 34

20. For $\angle ABC \cong \angle EFG$, and $m\angle ABC = 41$, find $m\angle GFH$. **20.** ______
A. 59 **B.** 49 **C.** 41 **D.** 39

Bonus Twenty-two students were asked about drink preferences. Twelve liked grape juice, 15 liked orange juice, and 10 liked both. How many did not like either? **B:** ______________

NAME ________________________ DATE ____________ PERIOD ______

2 Chapter 2 Test, Form 2C

SCORE ______

1. Make a conjecture about the next term in this sequence: $-11, -7, -3, 1, 5$. **1.** ________________

2. Given: $XY = YZ$
Conjecture: Y is a midpoint of $\overline{XZ}$.
Find a counterexample. **2.** ________________

3. What is the truth value of the following statement?
$\sqrt{16} = -4$ and $2 < 2$. **3.** ________________

4. Suppose p is true and q is false. What is the truth value of the disjunction $\sim p \vee \sim q$? **4.** ________________

5. Write the statement *All dogs have four feet* in if-then form. **5.** ________________

6. Identify the hypothesis of the statement *If you live in Chicago, then you live in Illinois*. **6.** ________________

7. Write the converse of the statement *If two lines are perpendicular to the same line, then they are parallel*. **7.** ________________

8. Use the Law of Detachment to write a valid conclusion for the given information. **8.** ________________
(1) If two angles are supplementary, then their measures have a sum of 180.
(2) $\angle X$ and $\angle Y$ are supplementary.

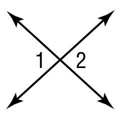
9. Use the Law of Syllogism to write a valid conclusion for the given information. **9.** ________________
(1) If today is January 1, then it is New Year's Day.
(2) If today is New Year's Day, then school is closed.

10. Name the operation that transforms $3x + 6 = 5x - 8$ to $3x = 5x - 14$, then find x. **10.** ________________

11. Complete the proof by supplying the missing information. **11.** ________________
If $2x - 7 = 4$, then $x = \frac{11}{2}$.

Statements	Reasons
1. $2x - 7 = 4$	**1.** Given
2. $2x - 7 + 7 = 4 + 7$	**2.** Addition Property
3. $2x = 11$	**3.** Substitution
4. $\frac{2x}{2} = \frac{11}{2}$	**4.** ________________
5. $x = \frac{11}{2}$	**5.** Substitution

12. If $m\angle 1 = x + 50$ and $m\angle 2 = 3x - 20$, find $m\angle 1$. **12.** ________________



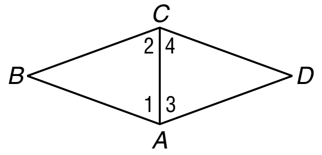
Assessments

Chapter 2 Test, Form 2C *(continued)*

For Questions 13 and 14, complete the proof below by supplying the reasons for each location.

Given: $\overline{AC}$ bisects $\angle BAD$.
$\overline{AC}$ bisects $\angle BCD$.
$\angle 1 \cong \angle 2$

Prove: $\angle 3 \cong \angle 4$



Statements	Reasons	
1. $\overline{AC}$ bisects $\angle BAD$.	**1.** Given	
2. $\overline{AC}$ bisects $\angle BCD$.	**2.** Given	
3. $\angle 1 \cong \angle 2$	**3.** Given	
4. $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$	**4.** (Question 13)	**13.** ______________
5. $\angle 3 \cong \angle 4$	**5.** (Question 14)	**14.** ______________

For Questions 15–20, state the definition, property, postulate, or theorem that justifies each statement.

15. If M is the midpoint of $\overline{AB}$, then $\overline{AM} \cong \overline{MB}$. **15.** ______________

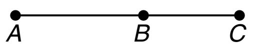
16. If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$. **16.** ______________

17. If $m\angle A + m\angle B = 90$ and $m\angle B = 20$, then $m\angle A + 20 = 90$. **17.** ______________

18. If $\angle X$ and $\angle Y$ are complementary, $\angle Z$ and $\angle Q$ are complementary, and $\angle X \cong \angle Z$, then $\angle Y \cong \angle Q$. **18.** ______________

19. If $\overline{PR} \cong \overline{QT}$, then $PR = QT$. **19.** ______________

20. $AB + BC = AC$



20. ______________

Bonus Write the contrapositive of the statement *A rhombus is a parallelogram.* **B:** ______________

NAME ______________________ DATE __________ PERIOD _____

2 Chapter 2 Test, Form 2D

SCORE _____

1. Make a conjecture about the next term in this sequence: $5, -10, 20, -40$. **1.** ________________

2. Given: $MN = NP$
Conjecture: N is a midpoint of $\overline{MP}$.
Find a counterexample. **2.** ________________

3. What is the truth value of the following statement? **3.** ________________
$\sqrt{25} < -5$ or $3 \leq 3$.

4. Suppose p is true and q is false. What is the truth value of the conjunction $\sim p \wedge \sim q$? **4.** ________________

5. Write the statement *All chickens have two wings* in if-then form. **5.** ________________

6. Identify the conclusion of the statement *If you live in Chicago, then you live in Illinois*. **6.** ________________

7. Write the inverse of the statement *If two lines are parallel to a third line, then they are parallel to each other*. **7.** ________________

8. Use the Law of Detachment to write a valid conclusion for the given information. **8.** ________________
(1) If a football team wins the Big 10 championship, then they will play in the Rose Bowl.
(2) Ohio State wins the Big 10 championship.

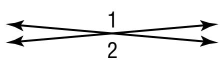
9 Use the Law of Syllogism to write a valid conclusion for the given information. **9.** ________________
(1) If today is Thursday, then *ER* is on television.
(2) If *ER* is on television, then Amy will stay home.

10. Name the operation that transforms $4x - 2 = 7x + 7$ to $4x = 7x + 9$, then find x. **10.** ________________

11. Complete the proof by supplying the missing information. **11.** ________________
If $\frac{x}{3} + 1 = -4$, then $x = -15$.

Statements	**Reasons**
1. $\frac{x}{3} + 1 = -4$	**1.** Given
2. $\frac{x}{3} + 1 - 1 = -4 - 1$	**2.** Subtraction Property
3. $\frac{x}{3} = -5$	**3.** Substitution
4. $\left(\frac{x}{3}\right)3 = (-5)(3)$	**4.** ________________
5. $x = -15$	**5.** Substitution

12. If $m\angle 1 = 5x + 20$ and $m\angle 2 = 3x + 80$, find $m\angle 1$. **12.** ________________

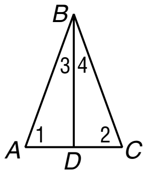


Assessments

NAME ______________________________ DATE ____________ PERIOD ______

2 Chapter 2 Test, Form 2D *(continued)*

For Questions 13 and 14, complete the proof below by supplying the reasons for each location.



Given: $\overline{BD}$ bisects $\angle ABC$.
$\angle 1$ and $\angle 3$ are complementary.
$\angle 2$ and $\angle 4$ are complementary.

Prove: $\angle 1 \cong \angle 2$

Statements	Reasons
1. $\overline{BD}$ bisects $\angle ABC$.	**1.** Given
2. $\angle 1$ and $\angle 3$ are complementary.	**2.** Given
3. $\angle 2$ and $\angle 4$ are complementary.	**3.** Given
4. $\angle 3 \cong \angle 4$	**4.** (Question 13)
5. $\angle 1 \cong \angle 2$	**5.** (Question 14)

13. ____________________

14. ____________________

For Questions 15–20, name the definition, property, postulate, or theorem that justifies each statement.

15. If X is the midpoint of $\overline{ZW}$, then $\overline{ZX} \cong \overline{XW}$. **15.** ____________________

16. If $AB = CD$, then $CD = AB$. **16.** ____________________

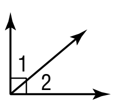
17. If $PQ = RS$, then $PQ + AB = RS + AB$. **17.** ____________________

18. If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$. **18.** ____________________

19. If two angles form a linear pair, then they are supplementary angles. **19.** ____________________

20. If $m\angle A = m\angle B$, then $\angle A \cong \angle B$. **20.** ____________________

Bonus If the ratio of $m\angle 1$ to $m\angle 2$ is 5 to 4, find $m\angle 2$. **B:** ____________________



NAME ______________________ DATE ____________ PERIOD ______

2 Chapter 2 Test, Form 3

SCORE ______

1. Make a conjecture given that $m\angle A = m\angle B$ and $m\angle B = m\angle C$. **1.** ____________

2. Given: $\overline{AB} \parallel \overline{CD}$ and $\overline{BD} \parallel \overline{AC}$ **2.** ____________
Conjecture: *ABDC* is a rectangle.
Find a counterexample.

3. What is the truth value of the statement *A square has 4 congruent sides and a rectangle has 4 parallel sides*? **3.** ____________

4. Suppose *p, q* and *r* are all false. What is the truth value of $(p \vee \sim q) \wedge \sim r$? **4.** ____________

5. In a class of 30 students, 22 like watching football, 17 like watching basketball, and 12 like watching both. Make a Venn diagram of these data. How many of these students do *not* like to watch either football or basketball? **5.** ____________

6. Write the statement *An elephant is a mammal* in if-then form. **6.** ____________

7. Write the contrapositive of the statement *If two angles are supplements of the same angle, then they are congruent*. **7.** ____________

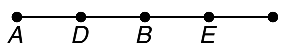
8. Use the Law of Detachment to write a valid conclusion that follows from statements (1) and (2). **8.** ____________

(1) The swim team has practice on Saturday.
(2) Matt is on the swim team.

9. Use the Law of Syllogism to write a valid conclusion that follows from statements (1) and (2). **9.** ____________

(1) If $x + 6 = 10$, then $x = 4$.
(2) If $x = 4$, then $x^2 = 16$.

10. Name the theorem that can be used to state that $\overline{AB} \cong \overline{BC}$, $\overline{AD} \cong \overline{DB}$, and $\overline{BE} \cong \overline{EC}$, given *B* is the midpoint of $\overline{AC}$, *D* the midpoint of $\overline{AB}$, and *E* the midpoint of $\overline{BC}$. **10.** ____________



2 Chapter 2 Test, Form 3 *(continued)*

For Questions 11–14, complete the proofs below by supplying the reasons for each location.

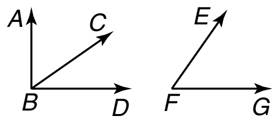
Given: $3 - 2(4 - x) = 11 + 6x$
Prove: $x = -4$

Statements	Reasons
1. $3 - 2(4 - x) = 11 + 6x$	**1.** Given
2. $3 - 8 + 2x = 11 + 6x$	**2.** (Question 11)
3. $-5 + 2x = 11 + 6x$	**3.** Addition of like terms
4. $2x = 16 + 6x$	**4.** (Question 12)
5. $-4x = 16$	**5.** Subtraction Property
6. $x = -4$	**6.** Division Property

11. ____________________

12. ____________________

Given: $\overline{AB} \perp \overline{BD}$
$\angle EFG$ and $\angle CBD$ are complementary.
Prove: $\angle EFG \cong \angle ABC$



Statements	Reasons
1. $\overline{AB} \perp \overline{BD}$	**1.** Given
2. $\angle EFG$ and $\angle CBD$ are complementary.	**2.** Given
3. $\angle ABD$ is a right angle.	**3.** $\perp$ lines form right angles.
4. $m\angle ABD = 90$	**4.** Def. of right angle
5. $\angle ABC + \angle CBD = \angle ABD$	**5.** (Question 13)
6. $m\angle ABC + m\angle CBD = 90$	**6.** Substitution
7. $\angle ABC$ and $\angle CBD$ are complementary.	**7.** Def. of complementary $\angle$s
8. $\angle EFG \cong \angle ABC$	**8.** (Question 14)

13. ____________________

14. ____________________

For Questions 15–20, name the definition, property, postulate, or theorem that justifies each statement.

15. Points A, C, and E are coplanar. **15.** ____________________

16. If $\overline{AB} \cong \overline{CD}$, then $AB + EF = CD + EF$. **16.** ____________________

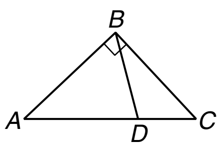
17. If $\overline{AB} \cong \overline{XY}$, then $\overline{XY} \cong \overline{AB}$. **17.** ____________________

18. If $x(y + z) = a$, then $xy + xz = a$. **18.** ____________________

19. If two angles are vertical, then they are congruent. **19.** ____________________

20. If $\angle 1$ and $\angle 2$ are right angles, then $\angle 1 \cong \angle 2$. **20.** ____________________

Bonus If $m\angle ABD = 6y + 2x$, $m\angle DBC = 5x + 8$, $m\angle ADB = 9x + 4y + 3$, and $m\angle CDB = 10y + 4x - 1$, find x and y.



B: ____________________

2 Chapter 2 Open-Ended Assessment

SCORE _____

Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.

1. Make a truth table to prove that an if-then statement is equivalent to its contrapositive and its inverse is equivalent to its converse.

2. Write three statements that illustrate the Law of Syllogism.

3. Write three statements that illustrate the Law of Detachment.

4. Write an example of the Transitive Property and the Substitution Property that illustrates the difference between them.

5. Draw a pair of vertical angles with measures of 40. Label the vertical angles with algebraic expressions involving x. Solve an equation using the expressions and show that each angle measures 40.

6. If two lines intersect, their intersection is a point. Must two lines intersect? Draw a diagram to illustrate what the lines would look like if they do not intersect.

7. Your woodworking teacher suggests you build a stool with only 3 legs because he says it will not rock if it has only 3 legs. Explain why this is true. Use a postulate from this chapter to support your answer.

8. a. Write a true if-then statement for which the converse is false.

b. Write the converse, inverse, and contrapositive of your sentence.

c. Give the truth value of each statement you wrote for part **b**.

NAME ______________________________ DATE ____________ PERIOD ______

2 Chapter 2 Vocabulary Test/Review

SCORE ______

compound statement	deductive argument	inverse	proof
conclusion	deductive reasoning	Law of Detachment	related conditionals
conditional statement	disjunction	Law of Syllogism	statement
conjecture	formal proof	logically equivalent	theorem
conjunction	hypothesis	negation	truth table
contrapositive	if-then statement	paragraph proof	truth value
converse	inductive reasoning	postulate	two-column proof
counterexample	informal proof		

Write whether each sentence is *true* or *false*. If false, replace the underlined word or number to make a true sentence.

1. A postulate is a statement that has been proved. **1.** ______________

2. A theorem is a statement that describes a fundamental relationship between the basic terms of geometry. **2.** ______________

3. If p implies q is true and p is true, then q is also true by the Law of Detachment. **3.** ______________

4. If p implies q and q implies r are true, then p implies r is also true by the Law of Syllogism. **4.** ______________

5. The phrase immediately following the word *then* is called the hypothesis of an if-then statement. **5.** ______________

6. The phrase immediately following the word *if* is called the conclusion of an if-then statement. **6.** ______________

7. A false example is called a counterexample. **7.** ______________

8. A conjecture is an educated guess based on known information. **8.** ______________

9. Statements with the same truth values are said to be logically equivalent. **9.** ______________

10. Inductive reasoning uses facts, rules, definitions, or properties to reach logical conclusions. **10.** ______________

Define each term.

11. conditional statement **11.** ______________

12. truth table **12.** ______________

13. conjunction **13.** ______________

NAME ______________________ DATE ____________ PERIOD ______

2 Chapter 2 Quiz

SCORE ______

(Lessons 2–1 and 2–3)

1. Make a conjecture given that $\triangle ABC$ is an equilateral triangle. **1.** ____________

2. Given: $\angle A$ and $\angle B$ are complementary. **2.** ____________
Conjecture: $m\angle A = 45$
Find a counterexample.

3. What is the truth value of $(p \wedge q) \vee r$? **3.** ____________
p: $(-4)^2 > 0$
q: An isosceles triangle has two congruent sides.
r: Two angles, whose measure have a sum of 90, are supplements.

4. Suppose p and q are both false. What is the truth value of $(p \wedge \sim q) \vee \sim p$? **4.** ____________

5. STANDARDIZED TEST PRACTICE What is the next term in the sequence 1, 4, 9, 16, 25? **5.** ____________

A. 6 **B.** 36
C. 40 **D.** 50

NAME ______________________ DATE ____________ PERIOD ______

2 Chapter 2 Quiz

SCORE ______

(Lessons 2–3 and 2–4)

Assessments

1. Identify the conclusion of the following statement. **1.** ____________
Either $x = 2$ or $x = -2$ if $x^2 = 4$.

For Questions 2 and 3, complete the converse of the statement *If two angles are supplementary and congruent, then they are right angles.* **2.** ____________

3. ____________

If (Question 2), then they are (Question 3).

4. Use the Law of Detachment to tell whether the following reasoning is valid. Write *valid* or *not valid*. **4.** ____________
If you drive at more than 65 miles per hour you will always get a ticket.
Joe got a ticket.
Therefore, Joe drove over 65 miles per hour.

5. Use the Law of Syllogism to write a valid conclusion from this set of statements. **5.** ____________
(1) All isosceles trapezoids are trapezoids.
(2) All trapezoids are quadrilaterals.

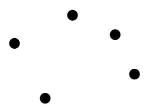
NAME ______________________ DATE ____________ PERIOD ______

2 Chapter 2 Quiz

SCORE ______

(Lessons 2–5 and 2–6)

1. Determine the number of line segments that can be drawn connecting each pair of points.



1. ____________________

2. Complete the following statement.
If AB = BC and A, B, and C are collinear, then B is the ____________________ of $\overline{AC}$.

2. ____________________

For Questions 3–5, name the definition, property, postulate, or theorem that justifies each statement.

3. If $x = 2$, then $2 = x$. **3.** ____________________

4. If $x + 3 = y$, then $x = y - 3$. **4.** ____________________

5. If $m\angle A = m\angle B$ and $m\angle B = m\angle C$, then $m\angle A = m\angle C$. **5.** ____________________

NAME ______________________ DATE ____________ PERIOD ______

2 Chapter 2 Quiz

SCORE ______

(Lessons 2–7 and 2–8)

For Questions 1–4, name the definition, property, postulate, or theorem that justifies each statement.

1. If $\overline{DE} \cong \overline{FG}$, then $\overline{FG} \cong \overline{DE}$. **1.** ____________________

2. If $XY = WZ$, then $XY + TU = WZ + TU$. **2.** ____________________

3. If $m\angle 1 + m\angle 2 = 180$ and $m\angle 2 + m\angle 3 = 180$, then $\angle 1 \cong \angle 3$. **3.** ____________________

4. If $\angle 1$ and $\angle 2$ are vertical angles, then $\angle 1 \cong \angle 2$. **4.** ____________________

5. STANDARDIZED TEST PRACTICE If $m\angle A = 5x - 12$, $m\angle B = 2x + 18$, $\angle A$ and $\angle C$ are supplementary, $\angle B$ and $\angle C$ are supplementary, find x.

A. 5 **B.** 6
C. 10 **D.** 24

5. ____________________

NAME ______________________ DATE ____________ PERIOD ______

2 Chapter 2 Mid-Chapter Test

SCORE ______

(Lessons 2–1 through 2–4)

Part I ***Write the letter for the correct answer in the blank at the right of each question.***

1. Make a conjecture given that points A, B, and C are collinear and $AC + CB = AB$. **1.** ______

A. C is between A and B. **B.** A is between B and C.
C. B is between A and C. **D.** $\triangle ABC$ is equilateral.

2. What is the truth value of $(\sim p \vee q) \wedge r$ if p is true, q is false, and r is true? **2.** ______

A. true **B.** false **C.** 0 **D.** cannot tell

3. What is the truth value of $(\sim p \wedge q) \vee r$ if p is true, q is false, and r is true? **3.** ______

A. true **B.** false **C.** 0 **D.** cannot tell

For Questions 4 and 5, use the statement *If a ray bisects an angle then it divides the angle into two congruent angles* and the given choices.

A. If a ray divides an angle into two congruent angles, then it bisects the angle.
B. A ray bisects an angle if and only if it divides it into two congruent angles.
C. If a ray does not bisect an angle, then it does not divide the angle into two congruent angles.
D. If a ray does not divide an angle into two congruent angles, then it does not bisect the angle.

4. Which choice is the inverse of the given statement? **4.** ______

5. Which choice is the contrapositive of the given statement? **5.** ______

Part II

6. Given: $2a^2 = 72$. **Conjecture:** $a = 6$
Write a counterexample. **6.** ______________

7. Write the statement *All right angles are congruent* in if-then form. **7.** ______________

8. Use the Law of Detachment to write a valid conclusion for statements (1) and (2). **8.** ______________

(1) All fish can swim.
(2) Charlie is a fish.

9. Use the Law of Syllogism to write a valid conclusion for statements (1) and (2). **9.** ______________

(1) If the Giants score a touchdown they will win.
(2) If the Giants win they will play in the Super Bowl.

10. Suppose p is false and q is true. What is the truth value of $(\sim p \vee \sim q) \wedge q$? **10.** ______________

Assessments

NAME ______________________ DATE ____________ PERIOD ______

SCORE ______

2 Chapter 2 Cumulative Review

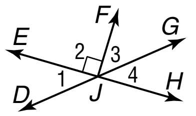
(Chapters 1–2)

1. Draw and label a figure so that $\overleftrightarrow{AB}$ is the intersection of plane R and plane T. (Lesson 1-1)

For Questions 2 and 3, use the Distance Formula to find the distance between each pair of points to the nearest hundredth. (Lesson 1-3)

2. $A(1, -4)$, $B(5, 3)$ **3.** $C(-6, 8)$, $D(4, -1)$

For Questions 4–7, use the figure.



4. Name the vertex of $\angle 3$. (Lesson 1-4)

5. Name the sides of $\angle 2$. (Lesson 1-4)

6. Name the angle that forms a linear pair with $\angle 4$. (Lesson 1-5)

7. Name two acute adjacent angles. (Lesson 1-5)

8. If the perimeter of an n-gon is 4.25 inches, find the perimeter when the length of each side is multiplied by 6. (Lesson 1-6)

9. Make a conjecture based on the statement *$\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel*. Draw a figure to illustrate your conjecture. (Lesson 2-1)

10. Suppose p and r are true and q is false. What is the truth value of the conjunction $(\sim p \vee \sim q) \wedge r$? (Lesson 2-2)

11. Write the *converse* of the statement *If a ray bisects an angle, it extends from the vertex of the angle*. (Lesson 2-3)

12. Use the Law of Syllogism to determine whether a valid conclusion can be reached from the following set of statements.
(1) *If two angles are congruent, they have the same measure.*
(2) $\angle A$ and $\angle B$ are congruent. (Lesson 2-4)

For Questions 13–15, complete the proof of the statement *If $x + 3 = 15x - 53$, then $x = 4$.* (Lesson 2-6)

Statements	Reasons
1. $x + 3 = 15x - 53$	**1.** Given
2. $x - x + 3 = 15x - x - 53$	**2.** Subtraction Property
3. (Question 13)	**3.** Substitution Property
4. $3 + 53 = 14x - 53 + 53$	**4.** (Question 14)
5. $56 = 14x$	**5.** Substitution Property
6. (Question 15)	**6.** Division Property
7. $4 = x$	**7.** Substitution Property
8. $x = 4$	**8.** Symmetric Property

1.
2. ______________
3. ______________
4. ______________
5. ______________
6. ______________
7. ______________
8. ______________
9. ______________
10. ______________
11. ______________
12. ______________
13. ______________
14. ______________
15. ______________

NAME ______________________ DATE ____________ PERIOD ________

2 Standardized Test Practice

(Chapters 1–2)

SCORE ________

Part 1: Multiple Choice

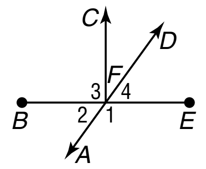
Instructions: Fill in the appropriate oval for the best answer.

1. Find JL if $JK = 17 - x$, $KL = 2x - 7$, and K is the midpoint of $\overline{JL}$. (Lesson 1-3)

A. 8 **B.** 9 **C.** 16 **D.** 18

1. (A) (B) (C) (D)

For Questions 2–4, use the figure at the right.



2. What is another name for $\angle DFE$? (Lesson 1-4)

E. $\angle 1$ **F.** $\angle 2$
G. $\angle 3$ **H.** $\angle 4$

2. (E) (F) (G) (H)

3. Classify $\angle 1$ if $m\angle 1 = 115$. (Lesson 1-4)

A. right **B.** acute **C.** obtuse **D.** scalene

3. (A) (B) (C) (D)

4. What can be assumed from the figure? (Lesson 1-5)

E. $\angle 1 \cong \angle 3$ **F.** $\angle 2 \cong \angle 4$ **G.** $\overline{BF} \cong \overline{FE}$ **H.** $\overline{CF} \perp \overline{BE}$

4. (E) (F) (G) (H)

5. Find the perimeter of a regular octagon if one of its sides is $x + 6$ and another side is $14 - x$. (Lesson 1-6)

A. 4 **B.** 40 **C.** 8 **D.** 80

5. (A) (B) (C) (D)

6. If $p \rightarrow q$ is the conditional, then its converse is ___?___. (Lesson 2-3)

E. $q \rightarrow p$ **F.** $\sim q \rightarrow p$ **G.** $\sim q \rightarrow \sim p$ **H.** $q \rightarrow \sim p$

6. (E) (F) (G) (H)

7. Which statement is always true? (Lesson 2-5)

A. $x = 2$ **B.** $x = x$ **C.** $x = y$ **D.** $x \neq 0$

7. (A) (B) (C) (D)

8. If $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$, then which is a valid conclusion? (Lesson 2-6)

I $m\angle 1 = m\angle 2$
II $\angle 1 \cong \angle 3$
III $m\angle 1 + m\angle 2 = m\angle 3$

E. I, II, and III **F.** II only **G.** I and II **H.** I and III

8. (E) (F) (G) (H)

For Questions 9 and 10, name the property that justifies the given statement.

9. If $AB = CD$ and $CD = 11$, then $AB = 11$. (Lesson 2-7)

A. Transitive **B.** Symmetric **C.** Congruence **D.** Reflexive

9. (A) (B) (C) (D)

10. If $\angle XYZ \cong \angle PQR$, then $\angle PQR \cong \angle XYZ$. (Lesson 2-8)

E. Transitive **F.** Symmetric **G.** Congruence **H.** Reflexive

10. (E) (F) (G) (H)

Assessments

2 Standardized Test Practice *(continued)*

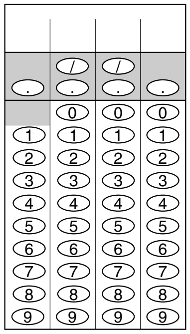
Part 2: Grid In

Instructions: Enter your answer by writing each digit of the answer in a column box and then shading in the appropriate oval that corresponds to that entry.

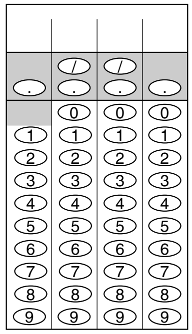
11. Find the measure of $\overline{QR}$ if Q is between points P and R, $PR = 42$, $PQ = 8x$, and $QR = 4x$.
(Lesson 1-2)

12. Use the Distance Formula to find the distance in units between $H(4, -1)$ and $K(-8, 4)$.
(Lesson 1-3)

11.



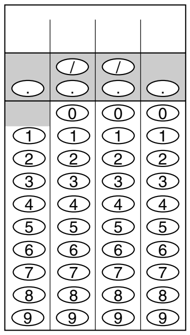
12.



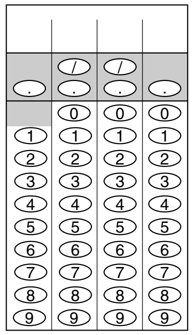
13. If $\angle 1$ and $\angle 2$ are complementary, $m\angle 1 = 6x + 15$ and $m\angle 2 = 2x - 21$, find $m\angle 2$. (Lesson 1-4)

14. Make a conjecture about the next number in the sequence $-2, 4, -8, 16, -32$. (Lesson 2-1)

13.



14.

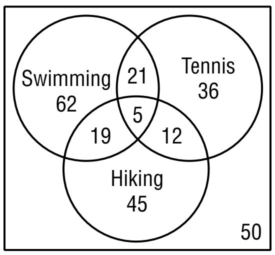


Part 3: Short Response

Instructions: Show your work or explain in words how you found your answer.

For Questions 15 and 16, use the results of a survey of 250 members of a local health club, shown in the Venn diagram.

Exercise Preference



15. How many members enjoy swimming or tennis? (Lesson 2-2)

15. ____________

16. How many members do not enjoy any of the activities? (Lesson 2-2)

16. ____________

17. The owner of Pop's Pizza and Games says that to win the radio/CD player, you must first win 4500 credits. Each time you play the Racetrack game, you win 30 credits. How many times must you play the Racetrack game to win enough credits for the radio/CD player? (Lesson 2-4)

17. ____________

18. If B is in the interior of $\angle DEF$, $m\angle DEB = 27.2$, and $m\angle DEF = 92.5$, find $m\angle BEF$. (Lesson 2-7)

18. ____________

NAME ______________________ DATE ____________ PERIOD ______

2 Standardized Test Practice

Student Record Sheet *(Use with pages 122–123 of the Student Edition.)*

Part 1 *Multiple Choice*

Select the best answer from the choices given and fill in the corresponding oval.

1 (A) (B) (C) (D)	**4** (A) (B) (C) (D)	**7** (A) (B) (C) (D)
2 (A) (B) (C) (D)	**5** (A) (B) (C) (D)	**8** (A) (B) (C) (D)
3 (A) (B) (C) (D)	**6** (A) (B) (C) (D)	

Part 2 *Short Response/Grid In*

Solve the problem and write your answer in the blank.

For Questions 9 and 11, also enter your answer by writing each number or symbol in a box. Then fill in the corresponding oval for that number or symbol.

9 ______________ (grid in)

10 ______________

11 ______________ (grid in)

12 ______________

9

	(/)	(/)	
(.)	(.)	(.)	(.)
	(0)	(0)	(0)
(1)	(1)	(1)	(1)
(2)	(2)	(2)	(2)
(3)	(3)	(3)	(3)
(4)	(4)	(4)	(4)
(5)	(5)	(5)	(5)
(6)	(6)	(6)	(6)
(7)	(7)	(7)	(7)
(8)	(8)	(8)	(8)
(9)	(9)	(9)	(9)

11

	(/)	(/)	
(.)	(.)	(.)	(.)
	(0)	(0)	(0)
(1)	(1)	(1)	(1)
(2)	(2)	(2)	(2)
(3)	(3)	(3)	(3)
(4)	(4)	(4)	(4)
(5)	(5)	(5)	(5)
(6)	(6)	(6)	(6)
(7)	(7)	(7)	(7)
(8)	(8)	(8)	(8)
(9)	(9)	(9)	(9)

Part 3 *Open-Ended*

Record your answers for Questions 13–15 on the back of this paper.

NAME ______________________ DATE __________ PERIOD _____

2-1 Study Guide and Intervention

Inductive Reasoning and Conjecture

Make Conjectures A **conjecture** is a guess based on analyzing information or observing a pattern. Making a conjecture after looking at several situations is called **inductive reasoning**.

Example 1 **Make a conjecture about the next number in the sequence 1, 3, 9, 27, 81.**

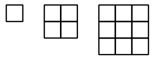
Analyze the numbers:

Notice that each number is a power of 3.

1	3	9	27	81
3^0	3^1	3^2	3^3	3^4

Conjecture: The next number will be 3^5 or 243.

Example 2 **Make a conjecture about the number of small squares in the next figure.**



Observe a pattern: The sides of the squares have measures 1, 2, and 3 units.
Conjecture: For the next figure, the side of the square will be 4 units, so the figure will have 16 small squares.

Exercises

Describe the pattern. Then make a conjecture about the next number in the sequence.

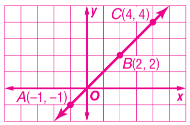
1. −5, 10, −20, 40 **Pattern: Each number is −2 times the previous number. Conjecture: The next number is −80.**

2. 1, 10, 100, 1000 **Pattern: Each number is 10 times the previous number. Conjecture: The next number is 10,000.**

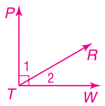
3. 1, $\frac{6}{5}$, $\frac{7}{5}$, $\frac{8}{5}$ **Pattern: Each number is $\frac{1}{5}$ more than the previous number. Conjecture: The next number is $\frac{9}{5}$.**

Make a conjecture based on the given information. Draw a figure to illustrate your conjecture. **4–7. Sample answers are given.**

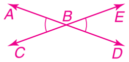
4. $A(-1, -1)$, $B(2, 2)$, $C(4, 4)$
Points *A*, *B*, and *C* are collinear.



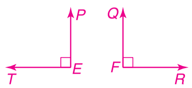
5. ∠1 and ∠2 form a right angle.
∠1 and ∠2 are complementary.



6. ∠*ABC* and ∠*DBE* are vertical angles.
∠*ABC* and ∠*DBE* are congruent.



7. ∠*E* and ∠*F* are right angles.
∠*E* and ∠*F* are congruent.



Lesson 2-1

NAME ______________________ DATE __________ PERIOD _____

2-1 Study Guide and Intervention *(continued)*

Inductive Reasoning and Conjecture

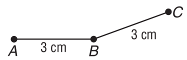
Find Counterexamples A conjecture is false if there is even one situation in which the conjecture is not true. The false example is called a **counterexample**.

Example **Determine whether the conjecture is *true* or *false*. If it is false, give a counterexample.**

Given: $\overline{AB} \cong \overline{BC}$

Conjecture: *B* is the midpoint of $\overline{AC}$.

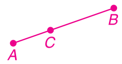
Is it possible to draw a diagram with $\overline{AB} \cong \overline{BC}$ such that *B* is not the midpoint? This diagram is a counterexample because point *B* is not on $\overline{AC}$. The conjecture is false.



Exercises

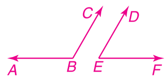
Determine whether each conjecture is *true* or *false*. Give a counterexample for any false conjecture.

1. **Given:** Points *A*, *B*, and *C* are collinear.
Conjecture: $AB + BC = AC$
False; *C* could be between *A* and *B*.



2. **Given:** ∠*R* and ∠*S* are supplementary. ∠*R* and ∠*T* are supplementary.
Conjecture: ∠*T* and ∠*S* are congruent.
true

3. **Given:** ∠*ABC* and ∠*DEF* are supplementary.
Conjecture: ∠*ABC* and ∠*DEF* form a linear pair. **False; the angles could be nonadjacent.**



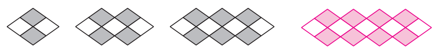
4. **Given:** $\overline{DE} \perp \overline{EF}$
Conjecture: ∠*DEF* is a right angle.
true

NAME ______________________ DATE ________ PERIOD ____

2-1 Skills Practice

Inductive Reasoning and Conjecture

Make a conjecture about the next item in each sequence.

1. 

2. −4, −1, 2, 5, 8 **11**

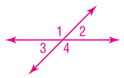
3. 6, $\frac{11}{2}$, 5, $\frac{9}{2}$, 4 **$\frac{7}{2}$**

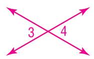
4. −2, 4, −8, 16, −32 **64**

Make a conjecture based on the given information. Draw a figure to illustrate your conjecture. **5–8. Sample answers are given.**

5. Points A, B, and C are collinear, and D is between B and C.
A, B, C, and D are collinear.


6. Point P is the midpoint of $\overline{NQ}$.
$NP = PQ$


7. ∠1, ∠2, ∠3, and ∠4 form four linear pairs.
∠1, ∠2, ∠3, and ∠4 are formed by two intersecting lines.


8. ∠3 ≅ ∠4
∠3 and ∠4 are vertical angles.


Determine whether each conjecture is *true* or *false*. Give a counterexample for any false conjecture.

9. Given: ∠ABC and ∠CBD form a linear pair.
Conjecture: ∠ABC ≅ ∠CBD
False; one of the angles could be acute and the other obtuse.

10. Given: $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ are congruent.
Conjecture: A, B, and C are collinear.
False; $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ could form a triangle.

11. Given: $AB + BC = AC$
Conjecture: $AB = BC$
false; counterexample: 

12. Given: ∠1 is complementary to ∠2, and ∠1 is complementary to ∠3.
Conjecture: ∠2 ≅ ∠3
true

© Glencoe/McGraw-Hill 59 *Glencoe Geometry*

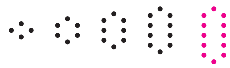
Lesson 2-1

NAME ______________________ DATE ________ PERIOD ____

2-1 Practice (Average)

Inductive Reasoning and Conjecture

Make a conjecture about the next item in each sequence.

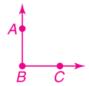
1. 

2. 5, −10, 15, −20 **25**

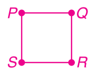
3. −2, 1, $-\frac{1}{2}$, $\frac{1}{4}$, $-\frac{1}{8}$ **$\frac{1}{16}$**

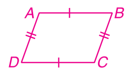
4. 12, 6, 3, 1.5, 0.75 **0.375**

Make a conjecture based on the given information. Draw a figure to illustrate your conjecture. **5–8. Sample answers are given.**

5. ∠ABC is a right angle.
$\overrightarrow{BA} \perp \overrightarrow{BC}$


6. Point S is between R and T.
$RS + ST = RT$


7. P, Q, R, and S are noncollinear and $\overline{PQ} \cong \overline{QR} \cong \overline{RS} \cong \overline{SP}$.
The segments form a square.


8. $ABCD$ is a parallelogram.
$AB = CD$ and $BC = AD$.


Determine whether each conjecture is true or false. Give a counterexample for any false conjecture.

9. Given: S, T, and U are collinear and $ST = TU$.
Conjecture: T is the midpoint of $\overline{SU}$.
true

10. Given: ∠1 and ∠2 are adjacent angles.
Conjecture: ∠1 and ∠2 form a linear pair.
False; ∠1 and ∠2 could each measure 60°.

11. Given: $\overline{GH}$ and $\overline{JK}$ form a right angle and intersect at P.
Conjecture: $\overline{GH} \perp \overline{JK}$
true

12. **ALLERGIES** Each spring, Rachel starts sneezing when the pear trees on her street blossom. She reasons that she is allergic to pear trees. Find a counterexample to Rachel's conjecture.
Sample answer: Rachel could be allergic to other types of plants that blossom when the pear trees blossom.

© Glencoe/McGraw-Hill 60 *Glencoe Geometry*

Answers

NAME ______________________ DATE ________ PERIOD _____

2-1 Reading to Learn Mathematics

Inductive Reasoning and Conjecture

Pre-Activity **How can inductive reasoning help predict weather conditions?**

Read the introduction to Lesson 2-1 at the top of page 62 in your textbook.

- What kind of weather patterns do you think meteorologists look at to help predict the weather? **Sample answer: patterns of high and low temperatures, including heat spells and cold spells; patterns of precipitation, including wet spells and dry spells**
- What is a factor that might contribute to long-term changes in the weather? **Sample answer: global warming due to high usage of fossil fuels**

Reading the Lesson

1. Explain in your own words the relationship between a conjecture, a counterexample, and inductive reasoning.
Sample answer: A conjecture is an educated guess based on specific examples or information. A counterexample is an example that shows that a conjecture is false. Inductive reasoning is the process of making a conjecture based on specific examples or information.

2. Make a conjecture about the next item in each sequence.

a. 5, 9, 13, 17 **21**

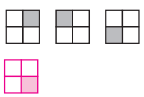
b. $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}$ $\mathbf{\frac{1}{81}}$

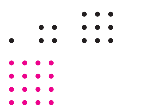
c. 0, 1, 3, 6, 10 **15**

d. 8, 3, −2, −7 **−12**

e. 1, 8, 27, 64 **125**

f. 1, −2, 4, −8 **16**

g. 

h. 

3. State whether each conjecture is *true* or *false*. If the conjecture is false, give a counterexample.

a. The sum of two odd integers is even.
true

b. The product of an odd integer and an even integer is odd.
False; sample answer: 5 · 8 = 40, which is even.

c. The opposite of an integer is a negative integer. **False; sample answer: The opposite of the integer −5 is 5, which is a positive integer.**

d. The perfect squares (squares of whole numbers) alternate between odd and even.
true

Helping You Remember

4. Write a short sentence that can help you remember why it only takes one counterexample to prove that a conjecture is false.
Sample answer: True means *always* true.

Lesson 2-1

NAME ______________________ DATE ________ PERIOD _____

2-1 Enrichment

Counterexamples

When you make a conclusion after examining several specific cases, you have used **inductive reasoning**. However, you must be cautious when using this form of reasoning. By finding only one **counterexample**, you disprove the conclusion.

Example **Is the statement $\frac{1}{x} \leq 1$ true when you replace x with 1, 2, and 3? Is the statement true for all reals? If possible, find a counterexample.**

$\frac{1}{1} = 1$, $\frac{1}{2} < 1$, and $\frac{1}{3} < 1$. But when $x = \frac{1}{2}$, then $\frac{1}{x} = 2$. This counterexample shows that the statement is not always true.

Answer each question.

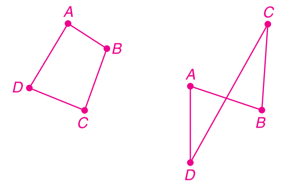
1. The coldest day of the year in Chicago occurred in January for five straight years. Is it safe to conclude that the coldest day in Chicago is always in January? **no**

2. Suppose John misses the school bus four Tuesdays in a row. Can you safely conclude that John misses the school bus every Tuesday? **no**

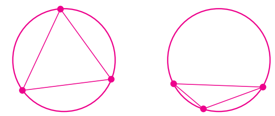
3. Is the equation $\sqrt{k^2} = k$ true when you replace k with 1, 2, and 3? Is the equation true for all integers? If possible, find a counterexample.
It is true for 1, 2, and 3. It is not true for negative integers. Sample answer: −2

4. Is the statement $2x = x + x$ true when you replace x with $\frac{1}{2}$, 4, and 0.7? Is the statement true for all real numbers? If possible, find a counterexample.
It is true for all real numbers.

5. Suppose you draw four points A, B, C, and D and then draw $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$. Does this procedure give a quadrilateral always or only sometimes? Explain your answers with figures.
only sometimes
Example: Counterexample:



6. Suppose you draw a circle, mark three points on it, and connect them. Will the angles of the triangle be acute? Explain your answers with figures.
no, only sometimes
Example: Counterexample:



NAME ______________________ DATE __________ PERIOD _____

2-2 Study Guide and Intervention

Logic

Determine Truth Values A **statement** is any sentence that is either true or false. The truth or falsity of a statement is its **truth value**. A statement can be represented by using a letter. For example,

Statement p: Chicago is a city in Illinois. The truth value of statement p is true.

Several statements can be joined in a **compound statement**.

Statement p and statement q joined by the word *and* is a **conjunction**.	Statement p and statement q joined by the word *or* is a **disjunction**.	**Negation:** *not p* is the negation of the statement p.
Symbols: $p \wedge q$ (Read: *p and q*)	Symbols: $p \vee q$ (Read: *p or q*)	Symbols: $\sim p$ (Read: *not p*)
The conjunction $p \wedge q$ is true only when both p and q are true.	The disjunction $p \vee q$ is true if p is true, if q is true, or if both are true.	The statements p and $\sim p$ have opposite truth values.

Example 1 **Write a compound statement for each conjunction. Then find its truth value.**

p: An elephant is a mammal.

q: A square has four right angles.

a. $p \wedge q$

Join the statements with *and*: An elephant is a mammal and a square has four right angles. Both parts of the statement are true so the compound statement is true.

b. $\sim p \wedge q$

$\sim p$ is the statement "An elephant is not a mammal." Join $\sim p$ and q with the word *and*: An elephant is not a mammal and a square has four right angles. The first part of the compound statement, $\sim p$, is false. Therefore the compound statement is false.

Example 2 **Write a compound statement for each disjunction. Then find its truth value.**

p: A diameter of a circle is twice the radius.

q: A rectangle has four equal sides.

a. $p \vee q$

Join the statements p and q with the word *or*: A diameter of a circle is twice the radius or a rectangle has four equal sides. The first part of the compound statement, p, is true, so the compound statement is true.

b. $\sim p \vee q$

Join $\sim p$ and q with the word *or*: A diameter of a circle is not twice the radius or a rectangle has four equal sides. Neither part of the disjunction is true, so the compound statement is false.

Exercises

Write a compound statement for each conjunction and disjunction. Then find its truth value.

p: 10 + 8 = 18 q: September has 30 days. r: A rectangle has four sides.

1. p and q **10 + 8 = 18 and September has 30 days; true.**

2. p or r **10 + 8 = 18 or a rectangle has four sides; true.**

3. q or r **September has 30 days or a rectangle has four sides; true.**

4. q and $\sim r$ **September has 30 days and a rectangle does not have four sides; false.**

Lesson 2-2

NAME ______________________ DATE __________ PERIOD _____

2-2 Study Guide and Intervention *(continued)*

Logic

Truth Tables One way to organize the truth values of statements is in a **truth table**. The truth tables for negation, conjunction, and disjunction are shown at the right.

Negation

p	$\sim p$
T	F
F	T

Conjunction

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Example 1 **Construct a truth table for the compound statement q or r. Use the disjunction table.**

q	r	q or r
T	T	T
T	F	T
F	T	T
F	F	F

Example 2 **Construct a truth table for the compound statement p and (q or r).**

Use the disjunction table for (q or r). Then use the conjunction table for p and (q or r).

p	q	r	q or r	p and (q or r)
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	F
F	T	F	T	F
F	F	T	T	F
F	F	F	F	F

Exercises

Contruct a truth table for each compound statement.

1. p or r

p	r	p or r
T	T	T
T	F	T
F	T	T
F	F	F

2. $\sim p \vee q$

p	$\sim p$	q	$\sim p \vee q$
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	T

3. $q \wedge \sim r$

q	r	$\sim r$	$q \wedge \sim r$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

4. $\sim p \wedge \sim r$

p	r	$\sim p$	$\sim r$	$\sim p \wedge \sim r$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

5. (p and r) or q

p	q	r	p and r	(p and r) or q
T	T	T	T	T
T	T	F	F	T
T	F	T	T	T
T	F	F	F	F
F	T	T	F	T
F	T	F	F	T
F	F	T	F	F
F	F	F	F	F

NAME ______________________ DATE __________ PERIOD _____

2-2 Skills Practice

Logic

Use the following statements to write a compound statement for each conjunction and disjunction. Then find its truth value.

p: $-3 - 2 = -5$
q: Vertical angles are congruent.
r: $2 + 8 > 10$
s: The sum of the measures of complementary angles is 90°.

1. p and q **$-3 - 2 = -5$ and vertical angles are congruent; true.**

2. $p \wedge r$ **$-3 - 2 = -5$ and $2 + 8 > 10$; false.**

3. p or s **$-3 - 2 = -5$ or the sum of the measures of complementary angles is 90°; true.**

4. $r \vee s$ **$2 + 8 > 10$ or the sum of the measures of complementary angles is 90°; true.**

5. $p \wedge \sim q$ **$-3 - 2 = -5$ and vertical angles are not congruent; false.**

6. $q \vee \sim r$ **Vertical angles are congruent or $2 + 8 \leq 10$; true.**

Copy and complete each truth table.

7.

p	q	$\sim p$	$\sim p \wedge q$	$\sim(\sim p \wedge q)$
T	T	**F**	**F**	**T**
T	F	**F**	**F**	**T**
F	T	**T**	**T**	**F**
F	F	**T**	**F**	**T**

8.

p	q	$\sim q$	$p \vee \sim q$
T	T	F	**T**
T	F	T	**T**
F	T	F	**F**
F	F	T	**T**

Construct a truth table for each compound statement.

9. $\sim q \wedge r$

q	r	$\sim q$	$\sim q \wedge r$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

10. $\sim p \vee \sim r$

p	r	$\sim p$	$\sim r$	$\sim p \vee \sim r$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

Lesson 2-2

NAME ______________________ DATE __________ PERIOD _____

2-2 Practice (Average)

Logic

Use the following statements to write a compound statement for each conjunction and disjunction. Then find its truth value.

p: 60 seconds = 1 minute
q: Congruent supplementary angles each have a measure of 90.
r: $-12 + 11 < -1$

1. $p \wedge q$ **60 seconds = 1 minute and congruent supplementary angles each have a measure of 90; true.**

2. $q \vee r$ **Congruent supplementary angles each have a measure of 90 or $-12 + 11 < -1$; true.**

3. $\sim p \vee q$ **60 seconds ≠ 1 minute or congruent supplementary angles each have a measure of 90; true.**

4. $\sim p \wedge \sim r$ **60 seconds ≠ 1 minute and $-12 + 11 \geq -1$; false.**

Copy and complete each truth table.

5.

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T	**F**	**F**	**F**
T	F	**F**	**T**	**T**
F	T	**T**	**F**	**T**
F	F	**T**	**T**	**T**

6.

p	q	$\sim p$	$\sim p \vee q$	$p \wedge (\sim p \vee q)$
T	T	**F**	**T**	**T**
T	F	**F**	**F**	**F**
F	T	**T**	**T**	**F**
F	F	**T**	**T**	**F**

Construct a truth table for each compound statement.

7. $q \vee (p \wedge \sim q)$

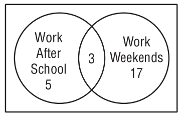
p	q	$\sim q$	$p \wedge \sim q$	$q \vee (p \wedge \sim q)$
T	T	F	F	T
T	F	T	T	T
F	T	F	F	T
F	F	T	F	F

8. $\sim q \wedge (\sim p \vee q)$

p	q	$\sim p$	$\sim q$	$\sim p \vee q$	$\sim q \wedge (\sim p \vee q)$
T	T	F	F	T	F
T	F	F	T	F	F
F	T	T	F	T	F
F	F	T	T	T	T

SCHOOL For Exercises 9 and 10, use the following information.

The Venn diagram shows the number of students in the band who work after school or on the weekends.



9. How many students work after school and on weekends? **3**

10. How many students work after school or on weekends? **25**

NAME ______________________ DATE __________ PERIOD _____

2-2 Reading to Learn Mathematics

Logic

Pre-Activity How does logic apply to school?

Read the introduction to Lesson 2-2 at the top of page 67 in your textbook.

How can you use logic to help you answer a multiple-choice question on a standardized test if you are not sure of the correct answer? **Sample answer: Eliminate the choices that you know are wrong. Then choose the one you think is most likely correct from the ones that are left.**

Reading the Lesson

1. Supply one or two words to complete each sentence.

a. Two or more statements can be joined to form a **compound** statement.

b. A statement that is formed by joining two statements with the word *or* is called a **disjunction**.

c. The truth or falsity of a statement is called its **truth value**.

d. A statement that is formed by joining two statements with the word *and* is called a **conjunction**.

e. A statement that has the opposite truth value and the opposite meaning from a given statement is called the **negation** of the statement.

2. Use *true* or *false* to complete each sentence.

a. If a statement is true, then its negation is **false**.

b. If a statement is false, then its negation is **true**.

c. If two statements are both true, then their conjunction is **true** and their disjunction is **true**.

d. If two statements are both false, then their conjunction is **false** and their disjunction is **false**.

e. If one statement is true and another is false, then their conjunction is **false** and their disjunction is **true**.

3. Consider the following statements:
p: Chicago is the capital of Illinois. *q*: Sacramento is the capital of California.
Write each statement symbolically and then find its truth value.

a. Sacramento is not the capital of California. **$\sim q$; false**

b. Sacramento is the capital of California and Chicago is not the capital of Illinois.
$q \wedge \sim p$; true

Helping You Remember

4. Prefixes can often help you to remember the meaning of words or to distinguish between similar words. Use your dictionary to find the meanings of the prefixes *con* and *dis* and explain how these meanings can help you remember the difference between a *conjunction* and a *disjunction*. **Sample answer: *Con* means *together* and *dis* means *apart*, so a *conjunction* is an *and* (or *both together*) statement and a *disjunction* is an *or* statement.**

NAME ______________________ DATE __________ PERIOD _____

2-2 Enrichment

Letter Puzzles

An **alphametic** is a computation puzzle using letters instead of digits. Each letter represents one of the digits 0–9, and two different letters cannot represent the same digit. Some alphametic puzzles have more than one answer.

Example **Solve the alphametic puzzle at the right.**

```
  FOUR
+  ONE
  FIVE
```

Since R + E = E, the value of R must be 0. Notice that the thousands digit must be the same in the first addend and the sum. Since the value of I is 9 or less, O must be 4 or less. Use trial and error to find values that work.

F = 8, O = 3, U = 1, R = 0

N = 4, E = 7, I = 6, and V = 5.

```
  8310
+  347
  8657
```

Can you find other solutions to this puzzle?

Find a value for each letter in each alphametic. Sample answers are shown

1.

```
   HALF      9703
+  HALF   +  9703
  WHOLE     19406
```

H = **9** A = **7** L = **0**
F = **3** W = **1** O = **4**
E = **6**

2.

```
   TWO      734
+  TWO   +  734
  FOUR     1468
```

T = **7** W = **3** O = **4**
F = **1** U = **6** R = **8**

3.

```
  THREE     43277
  THREE     43277
+   ONE   +   517
  SEVEN     87071
```

T = **4** H = **3** R = **2**
E = **7** O = **5** N = **1**
S = **8** V = **0**

4.

```
   SEND      9567
+  MORE   +  1085
  MONEY     10652
```

S = **9** E = **5** N = **6**
D = **7** M = **1** O = **0**
R = **8** Y = **2**

5. Do research to find more alphametic puzzles, or create your own puzzles. Challenge another student to solve them. **See students' work.**

NAME ______________________ DATE __________ PERIOD _____

2-3 Study Guide and Intervention

Conditional Statements

If-then Statements An if-then statement is a statement such as "If you are reading this page, then you are studying math." A statement that can be written in if-then form is called a **conditional statement**. The phrase immediately following the word *if* is the **hypothesis**. The phrase immediately following the word *then* is the **conclusion**.

A conditional statement can be represented in symbols as $p \rightarrow q$, which is read "p implies q" or "if p, then q."

Example 1 **Identify the hypothesis and conclusion of the statement.**

If $\angle X \cong \angle R$ and $\angle R \cong \angle S$, then $\angle X \cong \angle S$.
(hypothesis: $\angle X \cong \angle R$ and $\angle R \cong \angle S$; conclusion: $\angle X \cong \angle S$)

Example 2 **Identify the hypothesis and conclusion. Write the statement in if-then form.**

You receive a free pizza with 12 coupons.

If you have 12 coupons, then you receive a free pizza.
(hypothesis: you have 12 coupons; conclusion: you receive a free pizza)

Exercises

Identify the hypothesis and conclusion of each statement.

1. If it is Saturday, then there is no school. **H: it is Saturday; C: there is no school**
2. If $x - 8 = 32$, then $x = 40$. **H: $x - 8 = 32$; C: $x = 40$**
3. If a polygon has four right angles, then the polygon is a rectangle.
   H: a polygon has four right angles; C: the polygon is a rectangle

Write each statement in if-then form.

4. All apes love bananas.
   If an animal is an ape, then it loves bananas.
5. The sum of the measures of complementary angles is 90. **If two angles are complementary, then the sum of their measures is 90.**
6. Collinear points lie on the same line.
   If points are collinear, then they lie on the same line.

Determine the truth value of the following statement for each set of conditions.
If it does not rain this Saturday, we will have a picnic.

7. It rains this Saturday, and we have a picnic. **true**
8. It rains this Saturday, and we don't have a picnic. **true**
9. It doesn't rain this Saturday, and we have a picnic. **true**
10. It doesn't rain this Saturday, and we don't have a picnic. **false**

Lesson 2-3

NAME ______________________ DATE __________ PERIOD _____

2-3 Study Guide and Intervention *(continued)*

Conditional Statements

Converse, Inverse, and Contrapositive If you change the hypothesis or conclusion of a conditional statement, you form a **related conditional**. This chart shows the three related conditionals, *converse*, *inverse*, and *contrapositive*, and how they are related to a conditional statement.

	Symbols	Formed by	Example
Conditional	$p \rightarrow q$	using the given hypothesis and conclusion	If two angles are vertical angles, then they are congruent.
Converse	$q \rightarrow p$	exchanging the hypothesis and conclusion	If two angles are congruent, then they are vertical angles.
Inverse	$\sim p \rightarrow \sim q$	replacing the hypothesis with its negation and replacing the conclusion with its negation	If two angles are not vertical angles, then they are not congruent.
Contrapositive	$\sim q \rightarrow \sim p$	negating the hypothesis, negating the conclusion, and switching them	If two angles are not congruent, then they are not vertical angles.

Just as a conditional statement can be true or false, the related conditionals also can be true or false. A conditional statement always has the same truth value as its contrapositive, and the converse and inverse always have the same truth value.

Exercises

Write the converse, inverse, and contrapositive of each conditional statement. Tell which statements are *true* and which statements are *false*.

1. If you live in San Diego, then you live in California.
   Converse: If you live in California, then you live in San Diego. Inverse: If you do not live in San Diego, then you do not live in California. Contrapositive: If you do not live in California, then you do not live in San Diego. True: statement and contrapositive; false: converse and inverse
2. If a polygon is a rectangle, then it is a square.
   Converse: If a polygon is a square, then it is a rectangle. Inverse: If a polygon is not a rectangle, then it is not a square. Contrapositive: If a polygon is not a square, then it is not a rectangle. True: converse and inverse; false: statement and contrapositive
3. If two angles are complementary, then the sum of their measures is 90.
   Converse: If the sum of the measures of two angles is 90, then the angles are complementary. Inverse: If two angles are not complementary, then the sum of their measures is not 90. Contrapositive: if the sum of the measures of two angle is not 90, then the angles are not complementary. True: all; false: none

NAME ______________________ DATE ________ PERIOD _____

2-3 Skills Practice

Conditional Statements

Identify the hypothesis and conclusion of each statement.

1. If you purchase a computer and do not like it, then you can return it within 30 days.
H: you purchase a computer and do not like it;
C: you can return it within 30 days

2. If $x + 8 = 4$, then $x = -4$.
H: $x + 8 = 4$; C: $x = -4$

3. If the drama class raises $2000, then they will go on tour.
H: the drama class raises $2000; C: they will go on tour

Write each statement in if-then form.

4. A polygon with four sides is a quadrilateral.
If a polygon has four sides, then it is a quadrilateral.

5. "Those who stand for nothing fall for anything." (*Alexander Hamilton*)
If you stand for nothing, then you will fall for anything.

6. An acute angle has a measure less than 90.
If an angle is acute, then its measure is less than 90.

Determine the truth value of the following statement for each set of conditions.
If you finish your homework by 5 P.M., then you go out to dinner.

7. You finish your homework by 5 P.M. and you go out to dinner. **true**

8. You finish your homework by 4 P.M. and you go out to dinner. **true**

9. You finish your homework by 5 P.M. and you do not go out to dinner. **false**

10. Write the converse, inverse, and contrapositive of the conditional statement. Determine whether each statement is true or false. If a statement is false, find a counterexample.
If 89 is divisible by 2, then 89 is an even number.
Converse: If 89 is an even number, then 89 is divisible by 2; true.
Inverse: If 89 is not divisible by 2, then 89 is not an even number; true.
Contrapositive: If 89 is not an even number, then 89 is not divisible by 2; true.

NAME ______________________ DATE ________ PERIOD _____

2-3 Practice (Average)

Conditional Statements

Identify the hypothesis and conclusion of each statement.

1. If $3x + 4 = -5$, then $x = -3$.
H: $3x + 4 = -5$; C: $x = -3$

2. If you take a class in television broadcasting, then you will film a sporting event.
H: you take a class in television broadcasting;
C: you will film a sporting event

Write each statement in if-then form.

3. "Those who do not remember the past are condemned to repeat it." (*George Santayana*)
If you do not remember the past, then you are condemned to repeat it.

4. Adjacent angles share a common vertex and a common side.
If two angles are adjacent, then they share a common vertex and a common side.

Determine the truth value of the following statement for each set of conditions.
If DVD players are on sale for less than $100, then you buy one.

5. DVD players are on sale for $95 and you buy one. **true**

6. DVD players are on sale for $100 and you do not buy one. **true**

7. DVD players are not on sale for under $100 and you do not buy one. **true**

8. Write the converse, inverse, and contrapositive of the conditional statement. Determine whether each statement is true or false. If a statement is false, find a counterexample.
If $(-8)^2 > 0$, then $-8 > 0$.
Converse: If $-8 > 0$, then $(-8)^2 > 0$; true.
Inverse: If $(-8)^2 \leq 0$, then $-8 \leq 0$; true.
Contrapositive: If $-8 \leq 0$, then $(-8)^2 \leq 0$; false.

SUMMER CAMP For Exercises 9 and 10, use the following information.
Older campers who attend Woodland Falls Camp are expected to work. Campers who are juniors wait on tables.

9. Write a conditional statement in if-then form.
Sample answer: If you are a junior, then you wait on tables.

10. Write the converse of your conditional statement.
If you wait on tables, then you are a junior.

NAME ______________ DATE ________ PERIOD ____

2-3 Reading to Learn Mathematics

Conditional Statements

Pre-Activity **How are conditional statements used in advertisements?**

Read the introduction to Lesson 2-3 at the top of page 75 in your textbook.

Does the second advertising statement in the introduction mean that you will not get a free phone if you sign a contract for only six months of service? Explain your answer. **No; it only tells you what happens if you sign up for one year.**

Reading the Lesson

1. Identify the hypothesis and conclusion of each statement.

a. If you are a registered voter, then you are at least 18 years old. **Hypothesis: you are a registered voter; Conclusion: you are at least 18 years old**

b. If two integers are even, their product is even. **Hypothesis: two integers are even; Conclusion: their product is even**

2. Complete each sentence.

a. The statement that is formed by replacing both the hypothesis and the conclusion of a conditional with their negations is the **inverse**.

b. The statement that is formed by exchanging the hypothesis and conclusion of a conditional is the **converse**.

3. Consider the following statement:
You live in North America if you live in the United States.

a. Write this conditional statement in if-then form and give its truth value. If the statement is false, give a counterexample. **If you live in the United States, then you live in North America; false: You live in Hawaii.**

b. Write the inverse of the given conditional statement in if-then form and give its truth value. If the statement is false, give a counterexample. **If you do not live in the United States, then you do not live in North America; false; sample answer: You live in Mexico.**

c. Write the contrapositive of the given conditional statement in if-then form and give its truth value. If the statement is false, give a counterexample. **If you do not live in North America, then you do not live in the United States; false: You live in Hawaii.**

d. Write the converse of the given conditional statement in if-then form and give its truth value. If the statement is false, give a counterexample. **If you live in North America, then you live in the United States; false; sample answer: You live in Canada.**

Helping You Remember

4. When working with a conditional statement and its three related conditionals, what is an easy way to remember which statements are logically equivalent to each other? **Sample answer: The two statements whose names contain *verse* (the converse and the inverse) are a logically equivalent pair. The other two (the original conditional and the contrapositive) are the other logically equivalent pair.**

Lesson 2-3

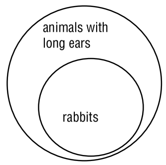
NAME ______________ DATE ________ PERIOD ____

2-3 Enrichment

Venn Diagrams

A type of drawing called a **Venn diagram** can be useful in explaining conditional statements. A Venn diagram uses circles to represent sets of objects.

Consider the statement "All rabbits have long ears." To make a Venn diagram for this statement, a large circle is drawn to represent all animals with long ears. Then a smaller circle is drawn inside the first to represent all rabbits. The Venn diagram shows that every rabbit is included in the group of long-eared animals.



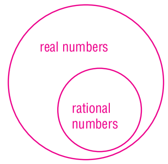
The set of rabbits is called a **subset** of the set of long-eared animals.

The Venn diagram can also explain how to write the statement, "All rabbits have long ears," in if-then form. Every rabbit is in the group of long-eared animals, so if an animal is a rabbit, then it has long ears.

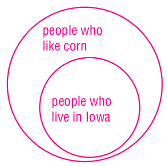
For each statement, draw a Venn diagram. Then write the sentence in if-then form.

1. Every dog has long hair.
If an animal is a dog, then it has long hair.

2. All rational numbers are real.
If a number is rational, then it is real.



3. People who live in Iowa like corn.



If a person lives in Iowa, then the person likes corn.

4. Staff members are allowed in the faculty lounge.

If a person is a staff member, then the person is allowed in the faculty cafeteria.

NAME ______________________ DATE __________ PERIOD _____

2-4 Study Guide and Intervention

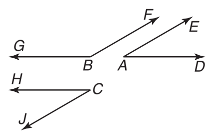
Deductive Reasoning

Law of Detachment **Deductive reasoning** is the process of using facts, rules, definitions, or properties to reach conclusions. One form of deductive reasoning that draws conclusions from a true conditional $p \rightarrow q$ and a true statement p is called the **Law of Detachment**.

Law of Detachment	If $p \rightarrow q$ is true and p is true, then q is true.
Symbols	$[(p \rightarrow q)] \wedge p] \rightarrow q$

Example **The statement *If two angles are supplementary to the same angle, then they are congruent* is a true conditional. Determine whether each conclusion is valid based on the given information. Explain your reasoning.**

a. Given: $\angle A$ and $\angle C$ are supplementary to $\angle B$.
Conclusion: $\angle A$ is congruent to $\angle C$.



The statement *$\angle A$ and $\angle C$ are supplementary to $\angle B$* is the hypothesis of the conditional. Therefore, by the Law of Detachment, the conclusion is true.

b. Given: $\angle A$ is congruent to $\angle C$.
Conclusion: $\angle A$ and $\angle C$ are supplementary to $\angle B$.

The statement *$\angle A$ is congruent to $\angle C$* is not the hypothesis of the conditional, so the Law of Detachment cannot be used. The conclusion is not valid.

Exercises

Determine whether each conclusion is valid based on the true conditional given. If not, write *invalid*. Explain your reasoning.

If two angles are complementary to the same angle, then the angles are congruent.

1. Given: $\angle A$ and $\angle C$ are complementary to $\angle B$.
Conclusion: $\angle A$ is congruent to $\angle C$.
The given statement is the hypothesis of the conditional statement. Since the conditional is true, the conclusion $\angle A \cong \angle C$ is true.

2. Given: $\angle A \cong \angle C$
Conclusion: $\angle A$ and $\angle C$ are complements of $\angle B$.
The given statement is not the hypothesis of the conditional. Therefore, the conclusion is invalid.

3. Given: $\angle E$ and $\angle F$ are complementary to $\angle G$.
Conclusion: $\angle E$ and $\angle F$ are vertical angles.
While the given statement is the hypothesis of the conditional statement, the statement that $\angle E$ and $\angle F$ are vertical angles is not the conclusion of the conditional. The conclusion is invalid.

Lesson 2-4

NAME ______________________ DATE __________ PERIOD _____

2-4 Study Guide and Intervention *(continued)*

Deductive Reasoning

Law of Syllogism Another way to make a valid conclusion is to use the **Law of Syllogism**. It is similar to the Transitive Property.

Law of Syllogism	If $p \rightarrow q$ is true and $q \rightarrow r$ is true, then $p \rightarrow r$ is also true.
Symbols	$[(p \rightarrow q)] \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

Example **The two conditional statements below are true. Use the Law of Syllogism to find a valid conclusion. State the conclusion.**

(1) If a number is a whole number, then the number is an integer.
(2) If a number is an integer, then it is a rational number.

p: A number is a whole number.
q: A number is an integer.
r: A number is a rational number.

The two conditional statements are $p \rightarrow q$ and $q \rightarrow r$. Using the Law of Syllogism, a valid conclusion is $p \rightarrow r$. A statement of $p \rightarrow r$ is "if a number is a whole number, then it is a rational number."

Exercises

Determine whether you can use the Law of Syllogism to reach a valid conclusion from each set of statements.

1. If a dog eats Superdog Dog Food, he will be happy.
Rover is happy.
No conclusion is possible.

2. If an angle is supplementary to an obtuse angle, then it is acute.
If an angle is acute, then its measure is less than 90.
A conclusion is possible: If an angle is supplementary to an obtuse angle, then its measure is less than 90.

3. If the measure of $\angle A$ is less than 90, then $\angle A$ is acute.
If $\angle A$ is acute, then $\angle A \cong \angle B$.
A conclusion is possible: If the measure of $\angle A$ is less 90, then $\angle A \cong \angle B$.

4. If an angle is a right angle, then the measure of the angle is 90.
If two lines are perpendicular, then they form a right angle.
A conclusion is possible: If two lines are perpendicular, then the angle they form has measure 90.

5. If you study for the test, then you will receive a high grade.
Your grade on the test is high.
No conclusion is possible.

Answers

NAME ______________________ DATE ________ PERIOD _____

2-4 Skills Practice

Deductive Reasoning

Determine whether the stated conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning.

If the sum of the measures of two angles is 180, then the angles are supplementary.

1. Given: $m\angle A + m\angle B$ is 180.
Conclusion: $\angle A$ and $\angle B$ are supplementary.
Valid; the conclusion follows by the Law of Detachment.

2. Given: $m\angle ABC$ is 95 and $m\angle DEF$ is 90.
Conclusion: $\angle ABC$ and $\angle DEF$ are supplementary.
Invalid; the given information shows that the sum of the angle measures is 185, not 180. You cannot use $\sim p$ and $p \to q$ to conclude q.

3. Given: $\angle 1$ and $\angle 2$ are a linear pair.
Conclusion: $\angle 1$ and $\angle 2$ are supplementary.
Valid; since the sum of the measures of the angles of a linear pair is 180, the angles are supplementary.

Use the Law of Syllogism to determine whether a valid conclusion can be reached from each set of statements. If a valid conclusion is possible, write it.

4. If two angles are complementary, then the sum of their measures is 90.
If the sum of the measures of two angles is 90, then both of the angles are acute.
If two angles are complementary, then both of the angles are acute.

5. If the heat wave continues, then air conditioning will be used more frequently.
If air conditioning is used more frequently, then energy costs will be higher.
If the heat wave continues, then energy costs will be higher.

Determine whether statement (3) follows from statements (1) and (2) by the Law of Detachment or the Law of Syllogism. If it does, state which law was used. If it does not, write *invalid*.

6. (1) If it is Tuesday, then Marla tutors chemistry.
(2) If Marla tutors chemistry, then she arrives home at 4 P.M.
(3) If Marla arrives at home at 4 P.M., then it is Tuesday.
invalid

7. (1) If a marine animal is a starfish, then it lives in the intertidal zone of the ocean.
(2) The intertidal zone is the least stable of the ocean zones.
(3) If a marine animal is a starfish, then it lives in the least stable of the ocean zones.
yes; Law of Syllogism

NAME ______________________ DATE ________ PERIOD _____

2-4 Practice (Average)

Deductive Reasoning

Determine whether the stated conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning.

If a point is the midpoint of a segment, then it divides the segment into two congruent segments.

1. Given: R is the midpoint of $\overline{QS}$.
Conclusion: $\overline{QR} \cong \overline{RS}$
Valid; since R is the midpoint of $\overline{QS}$, the Law of Detachment indicates that it divides $\overline{QS}$ into two congruent segments.

2. Given: $\overline{AB} \cong \overline{BC}$
Conclusion: B divides $\overline{AC}$ into two congruent segments.
Invalid; the points A, B, and C may not be collinear, and if they are not, then B will not be the midpoint of $\overline{AC}$.

Use the Law of Syllogism to determine whether a valid conclusion can be reached from each set of statements. If a valid conclusion is possible, write it.

3. If two angles form a linear pair, then the two angles are supplementary.
If two angles are supplementary, then the sum of their measures is 180.
If two angles form a linear pair, then the sum of their measures is 180.

4. If a hurricane is Category 5, then winds are greater than 155 miles per hour.
If winds are greater than 155 miles per hour, then trees, shrubs, and signs are blown down.
If a hurricane is Category 5, then trees, shrubs, and signs are blown down.

Determine whether statement (3) follows from statements (1) and (2) by the Law of Detachment or the Law of Syllogism. If it does, state which law was used. If it does not, write *invalid*.

5. (1) If a whole number is even, then its square is divisible by 4.
(2) The number I am thinking of is an even whole number.
(3) The square of the number I am thinking of is divisible by 4.
yes; Law of Detachment

6. (1) If the football team wins its homecoming game, then Conrad will attend the school dance the following Friday.
(2) Conrad attends the school dance on Friday.
(3) The football team won the homecoming game.
invalid

7. BIOLOGY If an organism is a parasite, then it survives by living on or in a host organism. If a parasite lives in or on a host organism, then it harms its host. What conclusion can you draw if a virus is a parasite?
If a virus is a parasite, then it harms its host.

NAME ______________________ DATE __________ PERIOD ____

2-4 Reading to Learn Mathematics

Deductive Reasoning

Pre-Activity How does deductive reasoning apply to health?

Read the introduction to Lesson 2-4 at the top of page 82 in your textbook.

Suppose a doctor wants to use the dose chart in your textbook to prescribe an antibiotic, but the only scale in her office gives weights in pounds. How can she use the fact that 1 kilogram is about 2.2 pounds to determine the correct dose for a patient? **Sample answer: The doctor can divide the patient's weight in pounds by 2.2 to find the equivalent mass in kilograms. She can then use the dose chart.**

Reading the Lesson

If *s*, *t*, and *u* are three statements, match each description from the list on the left with a symbolic statement from the list on the right.

1. negation of t **e**	**a.** $s \vee u$
2. conjunction of s and u **g**	**b.** $[(s \to t) \wedge s] \to t$
3. converse of $s \to t$ **h**	**c.** $\sim s \to \sim u$
4. disjunction of s and u **a**	**d.** $\sim u \to \sim s$
5. Law of Detachment **b**	**e.** $\sim t$
6. contrapositive of $s \to t$ **j**	**f.** $[(u \to t) \wedge (t \to s)] \to (u \to s)$
7. inverse of $s \to u$ **c**	**g.** $s \wedge u$
8. contrapositive of $s \to u$ **d**	**h.** $t \to s$
9. Law of Syllogism **f**	**i.** t
10. negation of $\sim t$ **i**	**j.** $\sim t \to \sim s$

11. Determine whether statement (3) follows from statements (1) and (2) by the Law of Detachment or the Law of Syllogism. If it does, state which law was used. If it does not, write *invalid*.

a. (1) Every square is a parallelogram.
(2) Every parallelogram is a polygon.
(3) Every square is a polygon. **yes; Law of Syllogism**

b. (1)If two lines that lie in the same plane do not intersect, they are parallel.
(2) Lines ℓ and m lie in plane $\mathcal{U}$ and do not intersect.
(3) Lines ℓ and m are parallel. **yes; Law of Detachment**

c. (1) Perpendicular lines intersect to form four right angles.
(2) $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are four right angles.
(3) $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are formed by intersecting perpendicular lines. **invalid**

Helping You Remember

12. A good way to remember something is to explain it to someone else. Suppose that a classmate is having trouble remembering what the Law of Detachment means? **Sample answer: The word *detach* means to take something off of another thing. The Law of Detachment says that when a conditional and its hypothesis are both true, you can detach the conclusion and feel confident that it too is a true statement.**

Lesson 2-4

NAME ______________________ DATE __________ PERIOD ____

2-4 Enrichment

Valid and Faulty Arguments

Consider the statements at the right. What conclusions can you make?

(1) Boots is a cat.
(2) Boots is purring.
(3) A cat purrs if it is happy.

From statements 1 and 3, it is correct to conclude that Boots purrs if it is happy. However, it is faulty to conclude from only statements 2 and 3 that Boots is happy. The if-then form of statement 3 is *If a cat is happy, then it purrs.*

Advertisers often use faulty logic in subtle ways to help sell their products. By studying the arguments, you can decide whether the argument is valid or faulty.

Decide if each argument is valid or faulty.

1. (1) If you buy Tuff Cote luggage, it will survive airline travel.
(2) Justin buys Tuff Cote luggage.
Conclusion: Justin's luggage will survive airline travel. **valid**

2. (1) If you buy Tuff Cote luggage, it will survive airline travel.
(2) Justin's luggage survived airline travel.
Conclusion: Justin has Tuff Cote luggage. **faulty**

3. (1) If you use Clear Line long distance service, you will have clear reception.
(2) Anna has clear long distance reception.
Conclusion: Anna uses Clear Line long distance service. **faulty**

4. (1) If you read the book *Beautiful Braids*, you will be able to make beautiful braids easily.
(2) Nancy read the book *Beautiful Braids*.
Conclusion: Nancy can make beautiful braids easily. **valid**

5. (1) If you buy a word processor, you will be able to write letters faster.
(2) Tania bought a word processor.
Conclusion: Tania will be able to write letters faster. **valid**

6. (1) Great swimmers wear AquaLine swimwear.
(2) Gina wears AquaLine swimwear.
Conclusion: Gina is a great swimmer. **faulty**

7. Write an example of faulty logic that you have seen in an advertisement. **See students' work.**

Answers

NAME ______________________ DATE __________ PERIOD _____

2-5 Study Guide and Intervention

Postulates and Paragraph Proofs

Points, Lines, and Planes In geometry, a **postulate** is a statement that is accepted as true. Postulates describe fundamental relationships in geometry.

Postulate: Through any two points, there is exactly one line.
Postulate: Through any three points not on the same line, there is exactly one plane.
Postulate: A line contains at least two points.
Postulate: A plane contains at least three points not on the same line.
Postulate: If two points lie in a plane, then the line containing those points lies in the plane.
Postulate: If two lines intersect, then their intersection is exactly one point.
Postulate: If two planes intersect, then their intersection is a line.

Example **Determine whether each statement is *always*, *sometimes*, or *never* true.**

a. There is exactly one plane that contains points *A*, *B*, and *C*.
Sometimes; if *A*, *B*, and *C* are collinear, they are contained in many planes. If they are noncollinear, then they are contained in exactly one plane.

b. Points *E* and *F* are contained in exactly one line.
Always; the first postulate states that there is exactly one line through any two points.

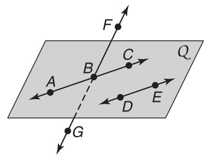
c. Two lines intersect in two distinct points *M* and *N*.
Never; the intersection of two lines is one point.

Exercises

Use postulates to determine whether each statement is *always*, *sometimes*, or *never* true.

1. A line contains exactly one point. **never**
2. Noncollinear points *R*, *S*, and *T* are contained in exactly one plane. **always**
3. Any two lines ℓ and m intersect. **sometimes**
4. If points *G* and *H* are contained in plane $\mathcal{M}$, then $\overline{GH}$ is perpendicular to plane $\mathcal{M}$. **never**
5. Planes $\mathcal{R}$ and $\mathcal{S}$ intersect in point *T*. **never**
6. If points *A*, *B*, and *C* are noncollinear, then segments $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$ are contained in exactly one plane. **always**

In the figure, $\overline{AC}$ and $\overline{DE}$ are in plane *Q* and $\overline{AC} \parallel \overline{DE}$. State the postulate that can be used to show each statement is true.



7. Exactly one plane contains points *F*, *B*, and *E*. **Through any three points not on the same line, there is exactly one plane.**
8. $\overleftrightarrow{BE}$ lies in plane *Q*. **If two points lie in a plane, then the line containing those points lies in the plane.**

Lesson 2-5

NAME ______________________ DATE __________ PERIOD _____

2-5 Study Guide and Intervention *(continued)*

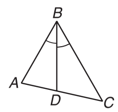
Postulates and Paragraph Proofs

Paragraph Proofs A statement that can be proved true is called a **theorem**. You can use undefined terms, definitions, postulates, and already-proved theorems to prove other statements true.

A logical argument that uses deductive reasoning to reach a valid conclusion is called a **proof**. In one type of proof, a **paragraph proof**, you write a paragraph to explain why a statement is true.

Example **In $\triangle ABC$, $\overline{BD}$ is an angle bisector. Write a paragraph proof to show that $\angle ABD \cong \angle CBD$.**

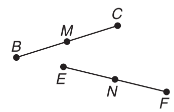
By definition, an angle bisector divides an angle into two congruent angles. Since $\overline{BD}$ is an angle bisector, $\angle ABC$ is divided into two congruent angles. Thus, $\angle ABD \cong \angle CBD$.



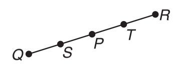
Exercises

1. Given that $\angle A \cong \angle D$ and $\angle D \cong \angle E$, write a paragraph proof to show that $\angle A \cong \angle E$.
Since $\angle A \cong \angle D$ and $\angle D \cong \angle E$, then $m\angle A = m\angle D$ and $m\angle D = m\angle E$ by the def. of congruence. So $m\angle A = m\angle E$ by the Transitive Property, and $\angle A \cong \angle E$ by the def. of congruence.

2. It is given that $\overline{BC} \cong \overline{EF}$, *M* is the midpoint of $\overline{BC}$, and *N* is the midpoint of $\overline{EF}$. Write a paragraph proof to show that $BM = EN$.
M is the midpoint of $\overline{BC}$ and N is the midpoint of $\overline{EF}$, so $BM = \frac{1}{2}BC$ and $EN = \frac{1}{2}EF$. $\overline{BC} \cong \overline{EF}$ so $BC = EF$ by the def. of congruence, and $\frac{1}{2}BC = \frac{1}{2}EF$ by the mult. prop. Thus $BM = EN$ by the Transitive Property.



3. Given that *S* is the midpoint of $\overline{QP}$, *T* is the midpoint of $\overline{PR}$, and *P* is the midpoint of $\overline{ST}$, write a paragraph proof to show that $QS = TR$.
P is the midpoint of $\overline{QR}$ so $QP = PR$. $\frac{1}{2}QP = \frac{1}{2}PR$ by the Multiplication Property. S and T are the midpoints of $\overline{QP}$ and $\overline{PR}$, respectively, so $QS = \frac{1}{2}QP$ and $TR = \frac{1}{2}PR$. Then $TR = \frac{1}{2}QP$ by substitution, and $QS = TR$ by the Transitive Property.



NAME ______________________ DATE __________ PERIOD _____

2-5 Skills Practice

Postulates and Paragraph Proofs

Determine the number of line segments that can be drawn connecting each pair of points.

1.  **6**

2.  **15**

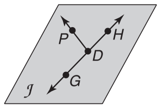
Determine whether the following statements are *always*, *sometimes*, or *never* true. Explain.

3. Three collinear points determine a plane.
Never; 3 noncollinear points determine a plane.

4. Two points A and B determine a line.
Always; through any two points there is exactly one line.

5. A plane contains at least three lines.
Always; a plane contains at least three points not on the same line, and each pair of these determines a line.

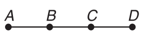
In the figure, $\overleftrightarrow{DG}$ and $\overleftrightarrow{DP}$ lie in plane $\mathcal{J}$ and H lies on $\overleftrightarrow{DG}$. State the postulate that can be used to show each statement is true.



6. G and P are collinear.
Postulate 2.1: through any two points, there is exactly one line.

7. Points D, H, and P are coplanar.
Postulate 2.2; Through any three points not on the same line, there is exactly one plane.

8. PROOF In the figure at the right, point B is the midpoint of $\overline{AC}$ and point C is the midpoint of $\overline{BD}$. Write a paragraph proof to prove that $AB = CD$.



Given: B is the midpoint of $\overline{AC}$.
C is the midpoint of $\overline{BD}$.

Prove: $AB = CD$

Proof: Since B is the midpoint of $\overline{AC}$ and C is the midpoint of $\overline{BD}$, we know by the Midpoint Theorem that $\overline{AB} \cong \overline{BC}$ and $\overline{BC} \cong \overline{CD}$. Since congruent segments have equal measures, $AB = BC$ and $BC = CD$. Thus, by the Transitive Property of Equality, $AB = CD$.

NAME ______________________ DATE __________ PERIOD _____

2-5 Practice (Average)

Postulates and Paragraph Proofs

Determine the number of line segments that can be drawn connecting each pair of points.

1.  **21**

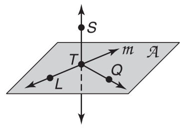
2.  **28**

Determine whether the following statements are *always*, *sometimes*, or *never* true. Explain.

3. The intersection of two planes contains at least two points.
Always; the intersection of two planes is a line, and a line contains at least two points.

4. If three planes have a point in common, then they have a whole line in common.
Sometimes; they might have only that single point in common.

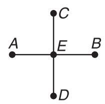
In the figure, line m and $\overleftrightarrow{TQ}$ lie in plane $\mathcal{A}$. State the postulate that can be used to show that each statement is true.



5. L, T, and line m lie in the same plane.
Postulate 2.5: If two points lie in a plane, then the entire line containing those points lies in that plane.

6. Line m and $\overline{ST}$ intersect at T.
Postulate 2.6: If two lines intersect, then their intersection is exactly one point.

7. In the figure, E is the midpoint of $\overline{AB}$ and $\overline{CD}$, and $AB = CD$. Write a paragraph proof to prove that $\overline{AE} \cong \overline{ED}$.



Given: E is the midpoint of $\overline{AB}$ and $\overline{CD}$
$AB = CD$

Prove: $\overline{AE} \cong \overline{ED}$

Proof: Since E is the midpoint of $\overline{AB}$ and $\overline{CD}$, we know by the Midpoint Theorem, that $\overline{AE} \cong \overline{EB}$ and $\overline{CE} \cong \overline{ED}$. By the definition of congruent segments, $AE = EB = \frac{1}{2}AB$ and $CE = ED = \frac{1}{2}CD$. Since $AB = CD$, $\frac{1}{2}AB = \frac{1}{2}CD$ by the Multiplication Property. So $AE = ED$, and by the definition of congruent segments, $\overline{AE} \cong \overline{ED}$.

8. LOGIC Points A, B, and C are not collinear. Points B, C, and D are not collinear. Points A, B, C, and D are not coplanar. Describe two planes that intersect in line BC.
the plane that contains A, B, and C and the plane that contains B, C, and D

NAME ______________________ DATE ____________ PERIOD _____

2-5 Reading to Learn Mathematics

Postulates and Paragraph Proofs

Pre-Activity **How are postulates used by the founding fathers of the United States?**

Read the introduction to Lesson 2-5 at the top of page 89 in your textbook.

Postulates are often described as statements that are so basic and so clearly correct that people will be willing to accept them as true without asking for evidence or proof. Give a statement about numbers that you think most people would accept as true without evidence. **Sample answer: Every number is equal to itself.**

Reading the Lesson

1. Determine whether each of the following is a *correct* or *incorrect* statement of a geometric postulate. If the statement is incorrect, replace the underlined words to make the statement correct.

a. A plane contains at least two points that do not lie on the same line. **incorrect; three points**

b. If two planes intersect, then the intersection is a line. **correct**

c. Through any four points not on the same line, there is exactly one plane. **incorrect; three points**

d. A line contains at least one point. **incorrect; two points**

e. If two lines are parallel, then their intersection is exactly one point. **incorrect; intersect**

f. Through any two points, there is at most one line. **incorrect; exactly**

2. Determine whether each statement is *always*, *sometimes*, or *never* true. If the statement is not always true, explain why.

a. If two planes intersect, their intersection is a line. **always**

b. The midpoint of a segment divides the segment into two congruent segments. **always**

c. There is exactly one plane that contains three collinear points. **never; Sample answer: There are infinitely many planes if the three points are collinear, but only one plane if the points are noncollinear.**

d. If two lines intersect, their intersection is one point. **always**

3. Use the walls, floor, and ceiling of your classroom to describe a model for each of the following geometric situations.

a. two planes that intersect in a line **Sample answer: two adjacent walls that intersect at an edge of both walls in the corner of the room**

b. two planes that do not intersect **Sample answer: the ceiling and the floor (or two opposite walls)**

c. three planes that intersect in a point **Sample answer: the floor (or ceiling) and two adjacent walls that intersect at a corner of the floor (or ceiling)**

Helping You Remember

4. A good way to remember a new mathematical term is to relate it to a word you already know. Explain how the idea of a mathematical *theorem* is related to the idea of a scientific *theory*. **Sample answer: Scientists do experiments to prove theories; mathematicians use deductive reasoning to prove theorems. Both processes involve using evidence to show that certain statements are true.**

Lesson 2-5

NAME ______________________ DATE ____________ PERIOD _____

2-5 Enrichment

Logic Problems

The following problems can be solved by eliminating possibilities. It may be helpful to use charts such as the one shown in the first problem. Mark an X in the chart to eliminate a possible answer.

Solve each problem.

1. Nancy, Olivia, Mario, and Kenji each have one piece of fruit in their school lunch. They have a peach, an orange, a banana, and an apple. Mario does not have a peach or a banana. Olivia and Mario just came from class with the student who has an apple. Kenji and Nancy are sitting next to the student who has a banana. Nancy does not have a peach. Which student has each piece of fruit?

	Nancy	Olivia	Mario	Kenji
Peach	X	X	X	
Orange	X	X		X
Banana	X		X	X
Apple		X	X	X

Nancy—apple, Olivia—banana, Mario—orange, Kenji—peach

2. Victor, Leon, Kasha, and Sheri each play one instrument. They play the viola, clarinet, trumpet, and flute. Sheri does not play the flute. Kasha lives near the student who plays flute and the one who plays trumpet. Leon does not play a brass or wind instrument. Which student plays each instrument?

Victor—flute, Leon—viola, Kasha—clarinet, Sheri—trumpet

3. Mr. Guthrie, Mrs. Hakoi, Mr. Mirza, and Mrs. Riva have jobs of doctor, accountant, teacher, and office manager. Mr. Mirza lives near the doctor and the teacher. Mrs. Riva is not the doctor or the office manager. Mrs. Hakoi is not the accountant or the office manager. Mr. Guthrie went to lunch with the doctor. Mrs. Riva's son is a high school student and is only seven years younger than his algebra teacher. Which person has each occupation?

Mr. Guthrie—teacher, Mrs. Hakoi—doctor, Mr. Mirza—office manager, Mrs. Riva—accountant

4. Yvette, Lana, Boris, and Scott each have a dog. The breeds are collie, beagle, poodle, and terrier. Yvette and Boris walked to the library with the student who has a collie. Boris does not have a poodle or terrier. Scott does not have a collie. Yvette is in math class with the student who has a terrier. Which student has each breed of dog?

Yvette—poodle, Lana—collie, Boris—beagle, Scott—terrier

NAME ________________ DATE ________ PERIOD ____

2-6 Study Guide and Intervention

Algebraic Proof

Algebraic Proof The following properties of algebra can be used to justify the steps when solving an algebraic equation.

Property	Statement
Reflexive	For every number a, $a = a$.
Symmetric	For all numbers a and b, if $a = b$ then $b = a$.
Transitive	For all numbers a, b, and c, if $a = b$ and $b = c$ then $a = c$.
Addition and Subtraction	For all numbers a, b, and c, if $a = b$ then $a + c = b + c$ and $a - c = b - c$.
Multiplication and Division	For all numbers a, b, and c, if $a = b$ then $a \cdot c = b \cdot c$, and if $c \neq 0$ then $\frac{a}{c} = \frac{b}{c}$.
Substitution	For all numbers a and b, if $a = b$ then a may be replaced by b in any equation or expression.
Distributive	For all numbers a, b, and c, $a(b + c) = ab + ac$.

Example Solve $6x + 2(x - 1) = 30$.

Algebraic Steps	Properties
$6x + 2(x - 1) = 30$	Given
$6x + 2x - 2 = 30$	Distributive Property
$8x - 2 = 30$	Substitution
$8x - 2 + 2 = 30 + 2$	Addition Property
$8x = 32$	Substitution
$\frac{8x}{8} = \frac{32}{8}$	Division Property
$x = 4$	Substitution

Exercises

Complete each proof.

1. Given: $\frac{4x + 6}{2} = 9$
Prove: $x = 3$

Statements	Reasons
a. $\frac{4x + 6}{2} = 9$	a. **Given**
b. $2\left(\frac{4x + 6}{2}\right) = 2(9)$	b. Mult. Prop.
c. $4x + 6 = 18$	c. **Subst.**
d. $4x + 6 - 6 = 18 - 6$	d. **Subtr. Prop.**
e. $4x = 12$	e. Substitution
f. $\frac{4x}{4} = \frac{12}{4}$	f. Div. Prop.
g. $x = 3$	g. Substitution

2. Given: $4x + 8 = x + 2$
Prove: $x = -2$

Statements	Reasons
a. $4x + 8 = x + 2$	a. **Given**
b. $4x + 8 - x = x + 2 - x$	b. **Subtr. Prop.**
c. $3x + 8 = 2$	c. Substitution
d. $3x + 8 - 8 = 2 - 8$	d. Subtr. Prop.
e. $3x = -6$	e. Substitution
f. $\frac{3x}{3} = \frac{-6}{3}$	f. **Div. Prop.**
g. $x = -2$	g. Substitution

Lesson 2-6

NAME ________________ DATE ________ PERIOD ____

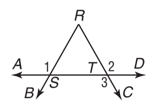
2-6 Study Guide and Intervention *(continued)*

Algebraic Proof

Geometric Proof Geometry deals with numbers as measures, so geometric proofs use properties of numbers. Here are some of the algebraic properties used in proofs.

Property	Segments	Angles
Reflexive	$AB = AB$	$m\angle A = m\angle A$
Symmetric	If $AB = CD$, then $CD = AB$.	If $m\angle A = m\angle B$, then $m\angle B = m\angle A$.
Transitive	If $AB = CD$ and $CD = EF$, then $AB = EF$.	If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $m\angle 1 = m\angle 3$.

Example **Write a two-column proof.**



Given: $m\angle 1 = m\angle 2$, $m\angle 2 = m\angle 3$
Prove: $m\angle 1 = m\angle 3$
Proof:

Statements	Reasons
1. $m\angle 1 = m\angle 2$	1. Given
2. $m\angle 2 = m\angle 3$	2. Given
3. $m\angle 1 = m\angle 3$	3. Transitive Property

Exercises

State the property that justifies each statement.

1. If $m\angle 1 = m\angle 2$, then $m\angle 2 = m\angle 1$. **Symmetric Property**
2. If $m\angle 1 = 90$ and $m\angle 2 = m\angle 1$, then $m\angle 2 = 90$. **Substitution**
3. If $AB = RS$ and $RS = WY$, then $AB = WY$. **Transitive Property**
4. If $AB = CD$, then $\frac{1}{2}AB = \frac{1}{2}CD$. **Multiplication Property**
5. If $m\angle 1 + m\angle 2 = 110$ and $m\angle 2 = m\angle 3$, then $m\angle 1 + m\angle 3 = 110$. **Substitution**
6. $RS = RS$ **Reflexive Property**
7. If $AB = RS$ and $TU = WY$, then $AB + TU = RS + WY$. **Addition Property**
8. If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $m\angle 1 = m\angle 3$. **Transitive Property**
9. A formula for the area of a triangle is $A = \frac{1}{2}bh$. Prove that bh is equal to 2 times the area of the triangle.

Given: $A = \frac{1}{2}bh$
Prove: $2A = bh$

Statements	Reasons
1. $A = \frac{1}{2}bh$	1. Given
2. $2A = 2\left(\frac{1}{2}bh\right)$	2. Mult. Property
3. $2A = bh$	3. Substitution

NAME ______________________ DATE ________ PERIOD ____

2-6 Skills Practice

Algebraic Proof

State the property that justifies each statement.

1. If $80 = m\angle A$, then $m\angle A = 80$. **Symmetric Property**

2. If $RS = TU$ and $TU = YP$, then $RS = YP$. **Transitive Property**

3. If $7x = 28$, then $x = 4$. **Division Property**

4. If $VR + TY = EN + TY$, then $VR = EN$. **Subtraction Property**

5. If $m\angle 1 = 30$ and $m\angle 1 = m\angle 2$, then $m\angle 2 = 30$. **Substitution Property**

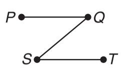
Complete the following proof.

6. Given: $8x - 5 = 2x + 1$
Prove: $x = 1$
Proof:

Statements	Reasons
a. $8x - 5 = 2x + 1$	a. **Given**
b. $8x - 5 - 2x = 2x + 1 - 2x$	b. **Subtraction Property**
c. **$6x - 5 = 1$**	c. Substitution Property
d. **$6x - 5 + 5 = 1 + 5$**	d. Addition Property
e. $6x = 6$	e. **Substitution Property**
f. $\frac{6x}{6} = \frac{6}{6}$	f. **Division Property**
g. **$x = 1$**	g. **Substitution Property**

Write a two-column proof for the following.

7. If $\overline{PQ} \cong \overline{QS}$ and $\overline{QS} \cong \overline{ST}$, then $PQ = ST$.



Given: $\overline{PQ} \cong \overline{QS}$ and $\overline{QS} \cong \overline{ST}$
Prove: $PQ = ST$
Proof:

Statements	Reasons
1. $\overline{PQ} \cong \overline{QS}$, $\overline{QS} \cong \overline{ST}$	1. Given
2. $PQ = QS$, $QS = ST$	2. Definition of congruent segments
3. $PQ = ST$	3. Transitive Property

Lesson 2-6

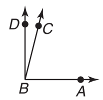
NAME ______________________ DATE ________ PERIOD ____

2-6 Practice (Average)

Algebraic Proof

PROOF Write a two-column proof.

1. If $m\angle ABC + m\angle CBD = 90$, $m\angle ABC = 3x - 5$, and $m\angle CBD = \frac{x+1}{2}$, then $x = 27$.



Given: $m\angle ABC + m\angle CBD = 90$
$m\angle ABC = 3x - 5$
$m\angle CBD = \frac{x+1}{2}$
Prove: $x = 27$
Proof:

Statements	Reasons
1. $m\angle ABC + m\angle CBD = 90$, $m\angle ABC = 3x - 5$, $m\angle CBD = \frac{x+1}{2}$	1. Given
2. $3x - 5 + \frac{x+1}{2} = 90$	2. Substitution Property
3. $(2)(3x - 5) + (2)\left(\frac{x+1}{2}\right) = (2)90$	3. Multiplication Property
4. $6x - 10 + x + 1 = 180$	4. Substitution Property
5. $7x - 9 = 180$	5. Substitution Property
6. $7x - 9 + 9 = 180 + 9$	6. Addition Property
7. $7x = 189$	7. Substitution Property
8. $\frac{7x}{7} = \frac{189}{7}$	8. Division Property
9. $x = 27$	9. Substitution Property

2. FINANCE The formula for simple interest is $I = prt$, where I is interest, p is principal, r is rate, and t is time. Solve the formula for r and justify each step.

Given: $I = prt$
Prove: $r = \frac{I}{pt}$
Proof:

Statements	Reasons
1. $I = prt$	1. Given
2. $\frac{I}{pt} = \frac{prt}{pt}$	2. Division Property
3. $\frac{I}{pt} = r$	3. Substitution Property
4. $r = \frac{I}{pt}$	4. Symmetric Property

NAME ______________________ DATE __________ PERIOD _____

2-6 Reading to Learn Mathematics

Algebraic Proof

Pre-Activity **How is mathematical evidence similar to evidence in law?**

Read the introduction to Lesson 2-6 at the top of page 94 in your textbook.

What are some of the things that lawyers might use in presenting their closing arguments to a trial jury in addition to evidence gathered prior to the trial and testimony heard during the trial? **Sample answer: They might tell the jury about laws related to the case, court rulings, and precedents set by earlier trials.**

Reading the Lesson

1. Name the property illustrated by each statement.

a. If $a = 4.75$ and $4.75 = b$, then $a = b$. **Transitive Property of Equality**

b. If $x = y$, then $x + 8 = y + 8$. **Addition Property of Equality**

c. $5(12 + 19) = 5 \cdot 12 + 5 \cdot 19$ **Distributive Property**

d. If $x = 5$, then x may be replaced with 5 in any equation or expression. **Substitution Property of Equality**

e. If $x = y$, then $8x = 8y$. **Multiplication Property of Equality**

f. If $x = 23.45$, then $23.45 = x$. **Symmetric Property of Equality**

g. If $5x = 7$, then $x = \frac{7}{5}$. **Division Property of Equality**

h. If $x = 12$, then $x - 3 = 9$. **Subtraction Property of Equality**

2. Give the reason for each statement in the following two-column proof.

Given: $5(n - 3) = 4(2n - 7) - 14$

Prove: $n = 9$

Statements	Reasons
1. $5(n - 3) = 4(2n - 7) - 14$	**1.** **Given**
2. $5n - 15 = 8n - 28 - 14$	**2.** **Distributive Property**
3. $5n - 15 = 8n - 42$	**3.** **Substitution Property**
4. $5n - 15 + 15 = 8n - 42 + 15$	**4.** **Addition Property**
5. $5n = 8n - 27$	**5.** **Substitution Property**
6. $5n - 8n = 8n - 27 - 8n$	**6.** **Subtraction Property**
7. $-3n = -27$	**7.** **Substitution Property**
8. $\frac{-3n}{-3} = \frac{-27}{-3}$	**8.** **Division Property**
9. $n = 9$	**9.** **Substitution Property**

Helping You Remember

3. A good way to remember mathematical terms is to relate them to words you already know. Give an everyday word that is related in meaning to the mathematical term *reflexive* and explain how this word can help you to remember the Reflexive Property and to distinguish it from the Symmetric and Transitive Properties. **Sample answer: Reflection: If you look at your reflection, you see *yourself*. The Reflexive Property says that every number is equal to *itself*. The Reflexive Property involves only one number, while the Symmetric and Transitive Properties each involve two or three numbers.**

Lesson 2-6

NAME ______________________ DATE __________ PERIOD _____

2-6 Enrichment

Symmetric, Reflexive, and Transitive Properties

Equality has three important properties.

Reflexive $a = a$
Symmetric If $a = b$, then $b = a$.
Transitive If $a = b$ and $b = c$, then $a = c$.

Other relations have some of the same properties. Consider the relation "is next to" for objects labeled X, Y, and Z. Which of the properties listed above are true for this relation?

X is next to X. *False*
If X is next to Y, then Y is next to X. *True*
If X is next to Y and Y is next to Z, then X is next to Z. *False*

Only the symmetric property is true for the relation "is next to."

For each relation, state which properties (*symmetric*, *reflexive*, *transitive*) are true.

1. is the same size as
symmetric, reflexive, transitive

2. is a family descendant of
transitive

3. is in the same room as
symmetric, reflexive, transitive

4. is the identical twin of
symmetric, transitive

5. is warmer than
transitive

6. is on the same line as
symmetric, reflexive

7. is a sister of
transitive

8. is the same weight as
symmetric, reflexive transitive

9. Find two other examples of relations, and tell which properties are true for each relation.
See students' work.

Answers

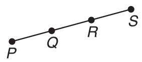
2-7 Study Guide and Intervention

Proving Segment Relationships

Segment Addition Two basic postulates for working with segments and lengths are the Ruler Postulate, which establishes number lines, and the Segment Addition Postulate, which describes what it means for one point to be between two other points.

Ruler Postulate	The points on any line or line segment can be paired with real numbers so that, given any two points *A* and *B* on a line, *A* corresponds to zero and *B* corresponds to a positive real number.
Segment Addition Postulate	*B* is between *A* and *C* if and only if $AB + BC = AC$.

Example **Write a two-column proof.**



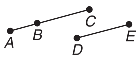
Given: Q is the midpoint of $\overline{PR}$.
R is the midpoint of $\overline{QS}$.
Prove: $PR = QS$

Statements	Reasons
1. Q is the midpoint of $\overline{PR}$.	**1.** Given
2. $PQ = QR$	**2.** Definition of midpoint
3. R is the midpoint of $\overline{QS}$.	**3.** Given
4. $QR = RS$	**4.** Definition of midpoint
5. $PQ + QR = QR + RS$	**5.** Addition Property
6. $PQ + QR = PR$, $QR + RS = QS$	**6.** Segment Addition Postulate
7. $PR = QS$	**7.** Substitution

Exercises

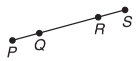
Complete each proof.

1. Given: $BC = DE$
Prove: $AB + DE = AC$



Statements	Reasons
a. $BC = DE$	**a.** Given
b. $AB + BC = AC$	**b.** Seg. Add. Post.
c. $AB + DE = AC$	**c.** Substitution

2. Given: Q is between P and R, R is between Q and S, $PR = QS$.
Prove: $PQ = RS$



Statements	Reasons
a. Q is between P and R.	**a.** Given
b. $PQ + QR = PR$	**b.** Seg. Add. Post.
c. R is between Q and S.	**c.** Given
d. $QR + RS = QS$	**d.** Seg. Add. Post.
e. $PR = QS$	**e.** Given
f. $PQ + QR = QR + RS$	**f.** Substitution
g. $PQ + QR - QR = QR + RS - QR$	**g.** Subtraction Prop.
h. $PQ = RS$	**h.** Substitution

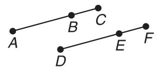
2-7 Study Guide and Intervention *(continued)*

Proving Segment Relationships

Segment Congruence Three properties of algebra—the Reflexive, Symmetric, and Transitive Properties of Equality—have counterparts as properties of geometry. These properties can be proved as a theorem. As with other theorems, the properties can then be used to prove relationships among segments.

Segment Congruence Theorem	Congruence of segments is reflexive, symmetric, and transitive.
Reflexive Property	$\overline{AB} \cong \overline{AB}$
Symmetric Property	If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.
Transitive Property	If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

Example **Write a two-column proof.**



Given: $\overline{AB} \cong \overline{DE}$; $\overline{BC} \cong \overline{EF}$
Prove: $\overline{AC} \cong \overline{DF}$

Statements	Reasons
1. $\overline{AB} \cong \overline{DE}$	**1.** Given
2. $AB = DE$	**2.** Definition of congruence of segments
3. $\overline{BC} \cong \overline{EF}$	**3.** Given
4. $BC = EF$	**4.** Definition of congruence of segments
5. $AB + BC = DE + EF$	**5.** Addition Property
6. $AB + BC = AC$, $DE + EF = DF$	**6.** Segment Addition Postulate
7. $AC = DF$	**7.** Substitution
8. $\overline{AC} \cong \overline{DF}$	**8.** Definition of congruence of segments

Exercises

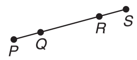
Justify each statement with a property of congruence.

1. If $\overline{DE} \cong \overline{GH}$, then $\overline{GH} \cong \overline{DE}$. **Symmetric Property**

2. If $\overline{AB} \cong \overline{RS}$ and $\overline{RS} \cong \overline{WY}$, then $\overline{AB} \cong \overline{WY}$. **Transitive Prop.**

3. $\overline{RS} \cong \overline{RS}$ **Reflexive Prop.**

4. Complete the proof.
Given: $\overline{PR} \cong \overline{QS}$
Prove: $\overline{PQ} \cong \overline{RS}$



Statements	Reasons
a. $\overline{PR} \cong \overline{QS}$	**a.** Given
b. $PR = QS$	**b.** Definition of congruence of segments
c. $PQ + QR = PR$	**c.** Segment Addition Postulate
d. $QR + RS = QS$	**d.** Segment Addition Postulate
e. $PQ + QR = QR + RS$	**e.** Substitution
f. $PQ = RS$	**f.** Subtraction Property
g. $\overline{PQ} \cong \overline{RS}$	**g.** Definition of congruence of segments

NAME ______________________ DATE __________ PERIOD _____

2-7 Skills Practice

Proving Segment Relationships

Justify each statement with a property of equality, a property of congruence, or a postulate.

1. $QA = QA$
Reflexive Property of Equality

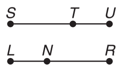
2. If $\overline{AB} \cong \overline{BC}$ and $\overline{BC} \cong \overline{CE}$, then $\overline{AB} \cong \overline{CE}$.
Transitive Property of Congruence

3. If Q is between P and R, then $PR = PQ + QR$.
Segment Addition Postulate

4. If $AB + BC = EF + FG$ and $AB + BC = AC$, then $EF + FG = AC$.
Substitution Property

Complete each proof.

5. Given: $\overline{SU} \cong \overline{LR}$
$\overline{TU} \cong \overline{LN}$
Prove: $\overline{ST} \cong \overline{NR}$
Proof:



Statements	Reasons
a. $\overline{SU} \cong \overline{LR}$, $\overline{TU} \cong \overline{LN}$	a. **Given**
b. **$SU = LR$, $TU = LN$**	b. Definition of ≅ segments
c. $SU = ST + TU$ $LR = LN + NR$	c. **Segment Addition Postulate**
d. $ST + TU = LN + NR$	d. **Substitution Property**
e. $ST + LN = LN + NR$	e. **Substitution Property**
f. $ST + LN - LN = LN + NR - LN$	f. **Subtraction Property**
g. **$ST = NR$**	g. Substitution Property
h. $\overline{ST} \cong \overline{NR}$	h. **Definition of ≅ segments**

6. Given: $\overline{AB} \cong \overline{CD}$
Prove: $\overline{CD} \cong \overline{AB}$
Proof:

Statements	Reasons
a. **$\overline{AB} \cong \overline{CD}$**	a. Given
b. $AB = CD$	b. **Definition of ≅ segments**
c. $CD = AB$	c. **Symmetric Property of =**
d. **$\overline{CD} \cong \overline{AB}$**	d. Definition of ≅ segments

Lesson 2-7

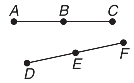
NAME ______________________ DATE __________ PERIOD _____

2-7 Practice (Average)

Proving Segment Relationships

Complete the following proof.

1. Given: $\overline{AB} \cong \overline{DE}$
B is the midpoint of $\overline{AC}$.
E is the midpoint of $\overline{DF}$.
Prove: $\overline{BC} \cong \overline{EF}$
Proof:



Statements	Reasons
a. **$\overline{AB} \cong \overline{DE}$** **$B$ is the midpoint of $\overline{AC}$.** **E is the midpoint of $\overline{DF}$.**	a. Given
b. $AB = DE$	b. **Definition of ≅ segments**
c. **$AB = BC$** **$DE = EF$**	c. Definition of Midpoint
d. $AC = AB + BC$ $DF = DE + EF$	d. **Segment Addition Postulate**
e. $AB + BC = DE + EF$	e. **Substitution Property**
f. $AB + BC = AB + EF$	f. **Substitution Property**
g. $AB + BC - AB = AB + EF - AB$	g. Subtraction Property
h. $BC = EF$	h. **Substitution Property**
i. **$\overline{BC} \cong \overline{EF}$**	i. **Definition of ≅ segments**

2. TRAVEL Refer to the figure. DeAnne knows that the distance from Grayson to Apex is the same as the distance from Redding to Pine Bluff. Prove that the distance from Grayson to Redding is equal to the distance from Apex to Pine Bluff.

Given: $\overline{GA} \cong \overline{RP}$
Prove: $\overline{GR} \cong \overline{AP}$
Proof:

Statements	Reasons
1. $\overline{GA} \cong \overline{RP}$	1. Given
2. $GA = RP$	2. Definition of ≅ segments
3. $GA + AR = AR + RP$	3. Addition Property
4. $GR = GA + AR$, $AP = AR + RP$	4. Segment Addition Postulate
5. $GR = AP$	5. Substitution Property
6. $\overline{GR} \cong \overline{AP}$	6. Definition of ≅ segments

Answers

NAME ______________________ DATE ________ PERIOD _____

2-7 Reading to Learn Mathematics

Proving Segment Relationships

Pre-Activity **How can segment relationships be used for travel?**

Read the introduction to Lesson 2-7 at the top of page 101 in your textbook.

- What is the total distance that the plane will fly to get from San Diego to Dallas? **1430 mi**
- Before leaving home, a passenger used a road atlas to determine that the distance between San Diego and Dallas is about 1350 miles. Why is the flying distance greater than that? **Sample answer: Phoenix is not on a straight line between San Diego and Dallas, so the stop added to the distance traveled. A nonstop flight would have been shorter.**

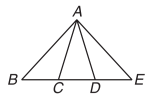
Reading the Lesson

1. If E is between Y and S, which of the following statements are *always* true? **B, E**

A. $YS + ES = YE$ **B.** $YS - ES = YE$
C. $YE > ES$ **D.** $YE \cdot ES = YS$
E. $SE + EY = SY$ **F.** E is the midpoint of $\overline{YS}$.

2. Give the reason for each statement in the following two-column proof.

Given: C is the midpoint of $\overline{BD}$.
D is the midpoint of $\overline{CE}$.
Prove: $\overline{BD} \cong \overline{CE}$



Statements	Reasons
1. C is the midpoint of $\overline{BD}$.	1. **Given**
2. $BC = CD$	2. **Definition of midpoint**
3. D is the midpoint of $\overline{CE}$.	3. **Given**
4. $CD = DE$	4. **Definition of midpoint**
5. $BC = DE$	5. **Transitive Property of Equality**
6. $BC + CD = CD + DE$	6. **Addition Property of Equality**
7. $BC + CD = BD$ $CD + DE = CE$	7. **Segment Addition Postulate**
8. $BD = CE$	8. **Substitution Property**
9. $\overline{BD} \cong \overline{CE}$	9. **Def. of ≅ segments**

Helping You Remember

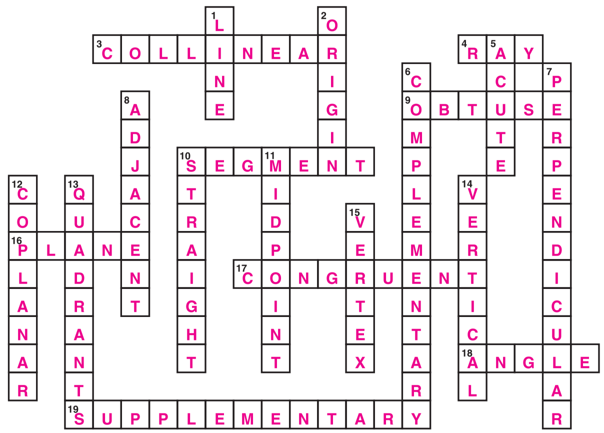
3. One way to keep the names of related postulates straight in your mind is to associate something in the name of the postulate with the content of the postulate. How can you use this idea to distinguish between the Ruler Postulate and the Segment Addition Postulate? **Sample answer: There are two words in "Ruler Postulate" and three words in "Segment Addition Postulate." The statement of the Ruler Postulate mentions two points, and the statement of the Segment Addition Postulate mentions three points.**

Lesson 2-7

NAME ______________________ DATE ________ PERIOD _____

2-7 Enrichment

Geometry Crossword Puzzle



ACROSS

3. Points on the same line are ______.
4. A point on a line and all points of the line to one side of it.
9. An angle whose measure is greater than 90.
10. Two endpoints and all points between them.
16. A flat figure with no thickness that extends indefinitely in all directions.
17. Segments of equal length are ______ segments.
18. Two noncollinear rays with a common endpoint.
19. If $m\angle A + m\angle B = 180$, then $\angle A$ and $\angle B$ are ______ angles.

DOWN

1. The set of all points collinear to two points is a ______.
2. The point where the x- and y-axis meet.
5. An angle whose measure is less than 90.
6. If $m\angle A + m\angle D = 90$, then $\angle A$ and $\angle D$ are ______ angles.
7. Lines that meet at a 90° angle are ______.
8. Two angles with a common side but no common interior points are ______.
10. An "angle" formed by opposite rays is a ______ angle.
11. The middle point of a line segment.
12. Points that lie in the same plane are ______.
13. The four parts of a coordinate plane.
14. Two nonadjacent angles formed by two intersecting lines are ______ angles.
15. In angle ABC, point B is the ______.

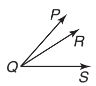
NAME _______________ DATE _______ PERIOD _____

2-8 Study Guide and Intervention

Proving Angle Relationships

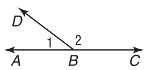
Supplementary and Complementary Angles There are two basic postulates for working with angles. The Protractor Postulate assigns numbers to angle measures, and the Angle Addition Postulate relates parts of an angle to the whole angle.

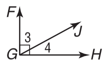
Protractor Postulate	Given $\overrightarrow{AB}$ and a number r between 0 and 180, there is exactly one ray with endpoint A, extending on either side of $\overrightarrow{AB}$, such that the measure of the angle formed is r.
Angle Addition Postulate	R is in the interior of $\angle PQS$ if and only if $m\angle PQR + m\angle RQS = m\angle PQS$.



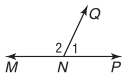
The two postulates can be used to prove the following two theorems.

Supplement Theorem	If two angles form a linear pair, then they are supplementary angles. If $\angle 1$ and $\angle 2$ form a linear pair, then $m\angle 1 + m\angle 2 = 180$.
Complement Theorem	If the noncommon sides of two adjacent angles form a right angle, then the angles are complementary angles. If $\overrightarrow{GF} \perp \overrightarrow{GH}$, then $m\angle 3 + m\angle 4 = 90$.



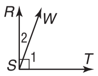


Example 1 **If $\angle 1$ and $\angle 2$ form a linear pair and $m\angle 2 = 115$, find $m\angle 1$.**



$m\angle 1 + m\angle 2 = 180$ Suppl. Theorem
$m\angle 1 + 115 = 180$ Substitution
$m\angle 1 = 65$ Subtraction Prop.

Example 2 **If $\angle 1$ and $\angle 2$ form a right angle and $m\angle 2 = 20$, find $m\angle 1$.**

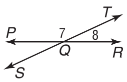


$m\angle 1 + m\angle 2 = 90$ Compl. Theorem
$m\angle 1 + 20 = 90$ Substitution
$m\angle 1 = 70$ Subtraction Prop.

Exercises

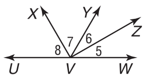
Find the measure of each numbered angle.

1.



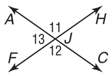
$m\angle 7 = 5x + 5$,
$m\angle 8 = x - 5$
$m\angle 7 = 155$,
$m\angle 8 = 25$

2.



$m\angle 5 = 5x$, $m\angle 6 = 4x + 6$,
$m\angle 7 = 10x$,
$m\angle 8 = 12x - 12$
$m\angle 5 = 30$, $m\angle 6 = 30$,
$m\angle 7 = 60$, $m\angle 8 = 60$

3.



$m\angle 11 = 11x$,
$m\angle 12 = 10x + 10$
$m\angle 11 = 110$,
$m\angle 12 = 110$,
$m\angle 13 = 70$

Lesson 2-8

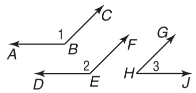
NAME _______________ DATE _______ PERIOD _____

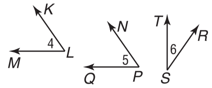
2-8 Study Guide and Intervention *(continued)*

Proving Angle Relationships

Congruent and Right Angles Three properties of angles can be proved as theorems.

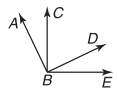
Congruence of angles is reflexive, symmetric, and transitive.	
Angles supplementary to the same angle or to congruent angles are congruent. If $\angle 1$ and $\angle 2$ are supplementary to $\angle 3$, then $\angle 1 \cong \angle 2$.	Angles complementary to the same angle or to congruent angles are congruent. If $\angle 4$ and $\angle 5$ are complementary to $\angle 6$, then $\angle 4 \cong \angle 5$.





Example **Write a two-column proof.**

Given: $\angle ABC$ and $\angle CBD$ are complementary.
$\angle DBE$ and $\angle CBD$ form a right angle.
Prove: $\angle ABC \cong \angle DBE$

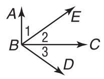


Statements	Reasons
1. $\angle ABC$ and $\angle CBD$ are complementary. $\angle DBE$ and $\angle CBD$ form a right angle.	1. Given
2. $\angle DBE$ and $\angle CBD$ are complementary.	2. Complement Theorem
3. $\angle ABC \cong \angle DBE$	3. Angles complementary to the same $\angle$ are $\cong$.

Exercises

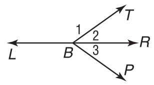
Complete each proof.

1. Given: $\overline{AB} \perp \overline{BC}$;
$\angle 1$ and $\angle 3$ are complementary.
Prove: $\angle 2 \cong \angle 3$



Statements	Reasons
a. $\overline{AB} \perp \overline{BC}$	a. **Given**
b. **$\angle ABC$ is a rt. $\angle$.**	b. Definition of $\perp$
c. $m\angle 1 + m\angle 2 = m\angle ABC$	c. **$\angle$ Add. Post.**
d. $\angle 1$ and $\angle 2$ form a rt $\angle$.	d. **Substitution**
e. $\angle 1$ and $\angle 2$ are compl.	e. **Compl. Theorem**
f. **$\angle 1$ and $\angle 3$ are compl.**	f. Given
g. $\angle 2 \cong \angle 3$	g. **$\angle$s compl. to the same $\angle$ are $\cong$.**

2. Given: $\angle 1$ and $\angle 2$ form a linear pair.
$m\angle 1 + m\angle 3 = 180$
Prove: $\angle 2 \cong \angle 3$



Statements	Reasons
a. $\angle 1$ and $\angle 2$ form a linear pair. $m\angle 1 + m\angle 3 = 180$	a. Given
b. **$\angle 1$ is suppl. to $\angle 2$.**	b. Suppl. Theorem
c. $\angle 1$ is suppl. to $\angle 3$.	c. **Def. of suppl.**
d. **$\angle 2 \cong \angle 3$**	d. $\angle$s suppl. to the same $\angle$ are $\cong$.

Answers

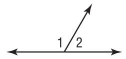
NAME ______________________ DATE __________ PERIOD _____

2-8 Skills Practice

Proving Angle Relationships

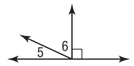
Find the measure of each numbered angle.

1. $m\angle 2 = 57$



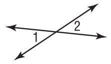
$m\angle 1 = 123$

2. $m\angle 5 = 22$



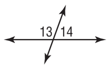
$m\angle 6 = 68$

3. $m\angle 1 = 38$



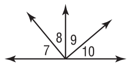
$m\angle 2 = 38$

4. $m\angle 13 = 4x + 11$, $m\angle 14 = 3x + 1$



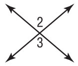
$m\angle 13 = 107$, $m\angle 14 = 73$

5. $\angle 9$ and $\angle 10$ are complementary. $\angle 7 \cong \angle 9$, $m\angle 8 = 41$



$m\angle 7 = 49$, $m\angle 9 = 49$, $m\angle 10 = 41$

6. $m\angle 2 = 4x - 26$, $m\angle 3 = 3x + 4$



$m\angle 2 = 94$, $m\angle 3 = 94$

Determine whether the following statements are *always*, *sometimes*, or *never* true.

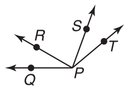
7. Two angles that are supplementary form a linear pair. **sometimes**

8. Two angles that are vertical are adjacent. **never**

9. Copy and complete the following proof.

Given: $\angle QPS \cong \angle TPR$

Prove: $\angle QPR \cong \angle TPS$



Proof:

Statements	Reasons
a. **$\angle QPS \cong \angle TPR$**	**a.** **Given**
b. $m\angle QPS = m\angle TPR$	**b.** **Definition of ≅ angles**
c. $m\angle QPS = m\angle QPR + m\angle RPS$ $m\angle TPR = m\angle TPS + m\angle RPS$	**c.** **Angle Addition Postulate**
d. **$m\angle QPR + m\angle RPS = m\angle TPS + m\angle RPS$**	**d.** Substitution
e. **$m\angle QPR = m\angle TPS$**	**e.** **Subtraction Property**
f. **$\angle QPR \cong \angle TPS$**	**f.** **Definition of ≅ angles**

Lesson 2-8

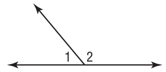
NAME ______________________ DATE __________ PERIOD _____

2-8 Practice (Average)

Proving Angle Relationships

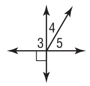
Find the measure of each numbered angle.

1. $m\angle 1 = x + 10$
$m\angle 2 = 3x + 18$



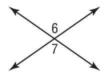
$m\angle 1 = 48$, $m\angle 2 = 132$

2. $m\angle 4 = 2x - 5$
$m\angle 5 = 4x - 13$



$m\angle 3 = 90$, $m\angle 4 = 31$, $m\angle 5 = 59$

3. $m\angle 6 = 7x - 24$
$m\angle 7 = 5x + 14$



$m\angle 6 = 109$, $m\angle 7 = 109$

Determine whether the following statements are *always*, *sometimes*, or *never* true.

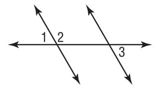
4. Two angles that are supplementary are complementary.
never

5. Complementary angles are congruent.
sometimes

6. Write a two-column proof.

Given: $\angle 1$ and $\angle 2$ form a linear pair.
$\angle 2$ and $\angle 3$ are supplementary.

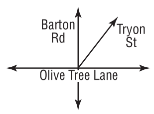
Prove: $\angle 1 \cong \angle 3$



Proof:

Statements	Reasons
1. $\angle 1$ and $\angle 2$ form a linear pair. $\angle 2$ and $\angle 3$ are supplementary.	**1. Given**
2. $\angle 1$ and $\angle 2$ are supplementary.	**2. Supplement Theorem**
3. $\angle 1 \cong \angle 3$	**3. ∠s suppl. to the same ∠ or ≅ ∠s are ≅.**

7. STREETS Refer to the figure. Barton Road and Olive Tree Lane form a right angle at their intersection. Tryon Street forms a 57° angle with Olive Tree Lane. What is the measure of the acute angle Tryon Street forms with Barton Road? **33**



NAME ______________ DATE ________ PERIOD ____

2-8 Reading to Learn Mathematics

Proving Angle Relationships

Pre-Activity **How do scissors illustrate supplementary angles?**

Read the introduction to Lesson 2-8 at the top of page 107 in your textbook.

Is it possible to open a pair of scissors so that the angles formed by the two blades, a blade and a handle, and the two handles, are all congruent? If so, explain how this could happen. **Sample answer: Yes; open the scissors so that the two blades are perpendicular. Then all the angles will be right angles and will be congruent.**

Reading the Lesson

1. Complete each sentence to form a statement that is always true.
   - **a.** If two angles form a linear pair, then they are adjacent and **supplementary**.
   - **b.** If two angles are complementary to the same angle, then they are **congruent**.
   - **c.** If D is a point in the interior of $\angle ABC$, then $m\angle ABC = m\angle ABD +$ **$m\angle DBC$**.
   - **d.** Given $\overrightarrow{RS}$ and a number x between **0** and **180**, there is exactly one ray with endpoint R, extended on either side of RS, such that the measure of the angle formed is x.
   - **e.** If two angles are congruent and supplementary, then each angle is a(n) **right** angle.
   - **f.** **Perpendicular** lines form congruent adjacent angles.
   - **g.** "Every angle is congruent to itself" is a statement of the **Reflexive** Property of angle congruence.
   - **h.** If two congruent angles form a linear pair, then the measure of each angle is **90**.
   - **i.** If the noncommon sides of two adjacent angles form a right angle, then the angles are **complementary**.

2. Determine whether each statement is *always*, *sometimes*, or *never* true.
   - **a.** Supplementary angles are congruent. **sometimes**
   - **b.** If two angles form a linear pair, they are complementary. **never**
   - **c.** Two vertical angles are supplementary. **sometimes**
   - **d.** Two adjacent angles form a linear pair. **sometimes**
   - **e.** Two vertical angles form a linear pair. **never**
   - **f.** Complementary angles are congruent. **sometimes**
   - **g.** Two angles that are congruent to the same angle are congruent to each other. **always**
   - **h.** Complementary angles are adjacent angles. **sometimes**

Helping You Remember

3. A good way to remember something is to explain it to someone else. Suppose that a classmate thinks that two angles can only be *vertical* angles if one angle lies above the other. How can you explain to him the meaning of vertical angles, using the word *vertex* in your explanation? **Sample answer: Two angles are *vertical* angles if they share the same *vertex* and their sides are opposite rays. It doesn't matter how the angles are positioned.**

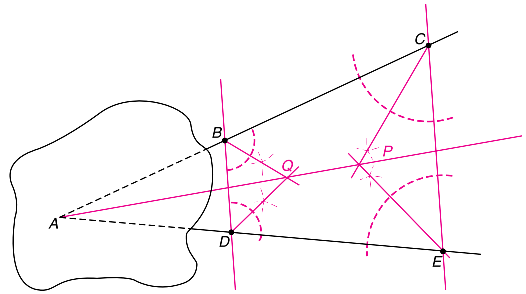
Lesson 2-8

NAME ______________ DATE ________ PERIOD ____

2-8 Enrichment

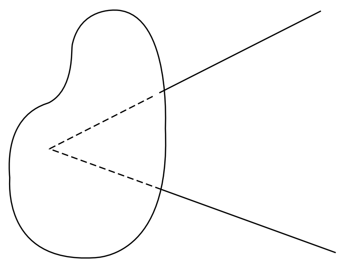
Bisecting a Hidden Angle

The vertex of $\angle BAD$ at the right is hidden in a region. Within the region, you are not allowed to use a compass. Can you bisect the angle?



Follow these instructions to bisect $\angle BAD$.

1. Use a straightedge to draw lines CE and BD.
2. Construct the bisectors of $\angle DEC$ and $\angle BCE$.
3. Label the intersection of the two bisectors as point P.
4. Construct the bisectors of $\angle BDE$ and $\angle DBC$.
5. Label the intersection of the two previous bisectors as point Q.
6. Use a straightedge to draw line PQ, which bisects the hidden angle.
7. Another hidden angle is shown at right. Construct the bisector using the method above, or devise your own method. **See students' work.**



Answers

Chapter 2 Assessment Answer Key

Form 1
Page 105

1. C
2. B
3. C
4. A
5. B
6. D
7. C
8. D
9. A
10. D

Page 106

11. B
12. A
13. C
14. D
15. C
16. C
17. B
18. A
19. D
20. A

B: 120

Form 2A
Page 107

1. B
2. D
3. B
4. C
5. A
6. A
7. C
8. B
9. A

(continued on the next page)

Chapter 2 Assessment Answer Key

Form 2A *(continued)*

Page 108

10. B
11. C
12. D
13. D
14. A
15. B
16. C
17. B
18. C
19. A
20. D

B: 5

Form 2B

Page 109

1. D
2. A
3. A
4. C
5. D
6. C
7. D
8. C
9. A

Page 110

10. D
11. A
12. C
13. B
14. C
15. C
16. A
17. B
18. B
19. B
20. B

B: 5

Answers

Chapter 2 Assessment Answer Key

Form 2C

Page 111

1. 9
2. Sample answer: X, Y, and Z are not collinear.
3. false
4. true
5. If an animal is a dog, then it has four feet.
6. you live in Chicago
7. If two lines are ∥, then they are ⊥ to the same line.
8. $m\angle X + m\angle Y = 180$
9. If today is January 1, then school is closed.
10. subtracting 6 from each side, $x = 7$
11. Division Property
12. 85

Page 112

13. Def. of ∠ bisector
14. Substitution
15. Midpoint Theorem
16. Transitive Property
17. Substitution
18. Angles complementary to same ∠s or ≅ ∠s are ≅.
19. Def. of ≅ segments
20. Segment Addition Postulate

B: If a figure is not a ▱, then it is not a rhombus.

Chapter 2 Assessment Answer Key

Form 2D

Page 113

1. 80
2. Sample answer: M, N, and P are not collinear.
3. true
4. false
5. If a bird is a chicken, then it has two wings.
6. you live in Illinois
7. If two lines are not ∥ to a third line, then they are not ∥ to each other.
8. Ohio State will play in the Rose Bowl.
9. If today is Thursday, then Amy will stay home.
10. adding 2 to each side, $x = -3$
11. Multiplication Property
12. 170

Page 114

13. Def. of ∠ bisector
14. Complements of ≅ ∠s are ≅.
15. Midpoint Theorem
16. Symmetric Property
17. Segment Addition Property
18. Transitive Property
19. Supplementary Angles Theorem
20. Def. of ≅ ∠s

B: 40

Answers

Chapter 2 Assessment Answer Key

Form 3

Page 115

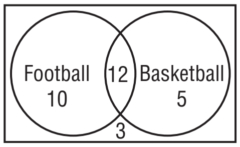
1. $m\angle A = m\angle C$

2. *ABDC* is a parallelogram or a rhombus.

3. false

4. true

5. 3



6. If an animal is an elephant, then it is a mammal.

7. If two ∠s are not ≅, then they are not supplements of the same ∠.

8. Matt has practice on Saturday.

9. If $x + 6 = 10$, then $x^2 = 16$.

10. Midpoint Theorem

Page 116

11. Distributive Property

12. Addition Property

13. ∠ Addition Postulate

14. Complements of the same angle are ≅.

15. Three points lie in the same plane.

16. Seg. Addition Post.

17. Symmetric Prop.

18. Distributive Prop.

19. Vertical ∠s Theorem

20. All right ∠s are ≅.

B: $x = 4$, $y = 9$

Chapter 2 Assessment Answer Key

Page 117, Open-Ended Assessment
Scoring Rubric

Score	General Description	Specific Criteria
4	**Superior** A correct solution that is supported by well-developed, accurate explanations	• Shows thorough understanding of the concepts of *truth tables*, *logic*, *algebraic properties*, *postulates*, *theorems*, *conditional statements*, and *segment and angle relationships*. • Uses appropriate strategies to solve problems. • Computations are correct. • Written explanations are exemplary. • Diagrams and truth tables are accurate and appropriate. • Goes beyond requirements of some or all problems.
3	**Satisfactory** A generally correct solution, but may contain minor flaws in reasoning or computation	• Shows an understanding of the concepts of *truth tables*, *logic*, *algebraic properties*, *postulates*, *theorems*, *conditional statements*, and *segment and angle relationships*. • Uses appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are effective. • Diagrams and truth tables are mostly accurate and appropriate. • Satisfies all requirements of problems.
2	**Nearly Satisfactory** A partially correct interpretation and/or solution to the problem	• Shows an understanding of most of the concepts of *truth tables*, *logic*, *algebraic properties*, *postulates*, *theorems*, *conditional statements*, and *segment and angle relationships*. • May not use appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are satisfactory. • Diagrams and truth tables are mostly accurate. • Satisfies the requirements of most of the problems.
1	**Nearly Unsatisfactory** A correct solution with no supporting evidence or explanation	• Final computation is correct. • No written explanations or work shown to substantiate the final computation. • Diagrams and truth tables may be accurate but lack detail or explanation. • Satisfies minimal requirements of some of the problems.
0	**Unsatisfactory** An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given	• Shows little or no understanding of most of the concepts of *truth tables*, *logic*, *algebraic properties*, *postulates*, *theorems*, *conditional statements*, and *segment and angle relationships*. • Does not use appropriate strategies to solve problems. • Computations are incorrect. • Written explanations are unsatisfactory. • Diagrams and truth tables are inaccurate or inappropriate. • Does not satisfy requirements of problems. • No answer given.

Chapter 2 Assessment Answer Key

Page 117, Open-Ended Assessment
Sample Answers

In addition to the scoring rubric found on page A31, the following sample answers may be used as guidance in evaluating open-ended assessment items.

1. Truth table with the following columns:

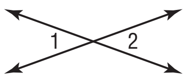
p	q	$\sim p$	$\sim q$	$p \to q$	$q \to p$	$\sim p \to \sim q$	$\sim q \to \sim p$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

Since column 5 is the same as column 8, the conditional is equivalent to its contrapositive and since column 6 is the same as column 7, the converse and the inverse are equivalent.

2. (1) If you are at the Sears Tower, then you are in Chicago.
(2) If you are in Chicago, then you are in Illinois.
(3) Therefore, if you are at the Sears Tower, then you are in Illinois.

3. (1) If two numbers are even, then their sum is even.
(2) The numbers are 4 and 6.
(3) The sum of 4 and 6 is even.

4. An example of the Transitive Property could be *If* $AB = BC$ *and* $BC = EF$, *then* $AB = EF$. A similar example that illustrates the Substitution Property would be *If* $AB = BC$ *and* $AB + EF = GH$, *then* $BC + EF = GH$.

5. 

For $m\angle 1 = 7x - 9$ and $m\angle 2 = 5x + 5$, find x and $m\angle 1$.

$7x - 9 = 5x + 5$
$2x = 14$
$x = 7$

Therefore, $m\angle 1 = 49 - 9 = 40$ and $m\angle 2 = 35 + 5 = 40$.

6. No, two lines do not have to intersect. If they do not intersect they could be parallel or skew. If they do not have to be distinct lines they could also be the same line. Students should draw a diagram showing two parallel lines.

7. Any three distinct points lie in one plane. Therefore, if the stool has 3 legs the bottoms of all 3 legs would always lie in the same plane, i.e. the plane of the floor, so it would not rock. If the stool had 4 legs and they were not all exactly the same length, then only 3 of them could be on the floor at one time and the stool would rock.

8a. Students should write a true statement with a false converse such as *If you are in Houston, then you are in Texas*.

b. Converse: If you are in Texas, then you are in Houston.
Inverse: If you are not in Houston, then you are not in Texas.
Contrapositive: If you are not in Texas, then you are not in Houston.

c. Converse: false
Inverse: false
Contrapositive: true

© Glencoe/McGraw-Hill

Chapter 2 Assessment Answer Key

Vocabulary Test/Review
Page 118

1. false, theorem
2. false, postulate
3. true
4. true
5. false, conclusion
6. false, hypothesis
7. true
8. true
9. true
10. false, deductive reasoning
11. a statement written in if-then form
12. a convenient method for organizing the truth values of statements
13. a compound statement formed by joining two or more statements with the word *and*

Quiz 1
Page 119

1. $AB = BC = AC$
2. Sample answer: $m\angle A = 40$ when $m\angle B = 50$.
3. true
4. true
5. B

Quiz 2
Page 119

1. $x = 2$ or $x = -2$
2. two ∠s are right ∠s
3. supplementary and ≅
4. not valid
5. All isosceles trapezoids are quadrilaterals.

Quiz 3
Page 120

1. 10
2. midpoint
3. Symmetric Property
4. Subtraction Property
5. Transitive Property

Quiz 4
Page 120

1. Symmetric Prop.
2. Segment Addition Prop.
3. ∠s supplementary to the same ∠ are ≅.
4. Vertical ∠s are ≅.
5. C

Answers

Chapter 2 Assessment Answer Key

Mid-Chapter Test
Page 121

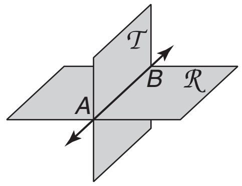
Part I

1. A
2. B
3. A
4. C
5. D

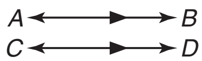
Part II

6. $a = -6$
7. If two $\angle$s are right $\angle$s, then they are $\cong$.
8. Charlie can swim.
9. If the Giants score a touchdown, then they will play in the Super Bowl.
10. true

Cumulative Review
Page 122

1. 

2. 8.06 units
3. 13.45 units
4. point J
5. $\overrightarrow{JE}$ and $\overrightarrow{JF}$
6. $\angle EJG$ or $\angle DJH$
7. $\angle 3$ and $\angle 4$
8. 25.5 in.
9. $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ do not intersect.



10. true
11. If a ray extends from the vertex of an $\angle$, it bisects the $\angle$.
12. $\angle A$ and $\angle B$ have the same measure.
13. $3 = 14x - 53$
14. Addition Property
15. $\frac{56}{14} = \frac{14x}{14}$

© Glencoe/McGraw-Hill

Chapter 2 Assessment Answer Key

Standardized Test Practice

Page 123

1. (A) (B) (C) **(D)**

2. (E) (F) (G) **(H)**

3. (A) (B) **(C)** (D)

4. (E) **(F)** (G) (H)

5. (A) (B) (C) **(D)**

6. **(E)** (F) (G) (H)

7. (A) **(B)** (C) (D)

8. (E) (F) **(G)** (H)

9. **(A)** (B) (C) (D)

10. (E) **(F)** (G) (H)

Page 124

11. 1 4

12. 1 3

13. 3

14. 6 4

15. 155

16. 50

17. 150

18. 65.3

Answers