Chapter 2
1-Dimensional Motion

Quantities for motion

Scalars	Vectors
Distance	displacement
Speed	velocity
	acceleration

Remember that scalar quantities only have magnitude.

Definitions

- Distance = How far apart two points in space are from each other.

  (only has magnitude)
- Displacement = The relative location of one point in space to another point. (involves direction)

- Speed = How fast an object is traveling.
- Velocity = How fast **and in what direction** an object is traveling.

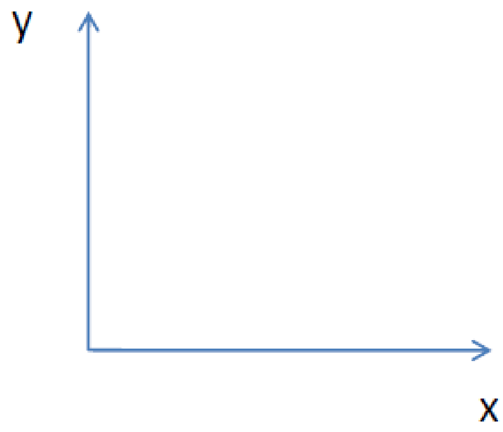
- Acceleration = The changing of an object's velocity with time.

  (This can include the velocity's direction.)
- Examples: Speeding your car up.

  Keeping your speed constant but driving around a curve.

Coordinate systems
When talking about directions, we need to have a reference frame. (coordinate system)

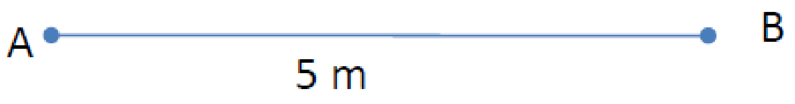
When setting up a coordinate system you can pick any direction to be positive.

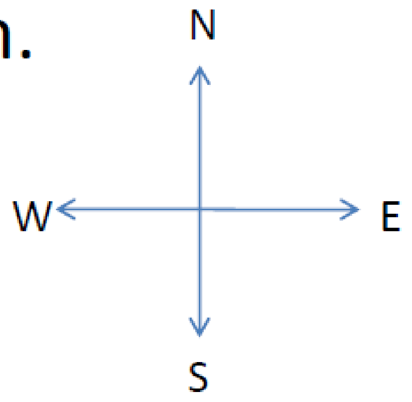


For this coordinate system, x and y are positive in the right and up directions respectively.

Example of distance and displacement

The distance between A and B is 5 m.





The displacement of B with respect to A is 5 m to the right of A.

You could say that point B is 5 m east of point A.

Speed and velocity

- Speed

  S How fast an object is moving at that instant.

  S_{ave} = total distance traveled/total time

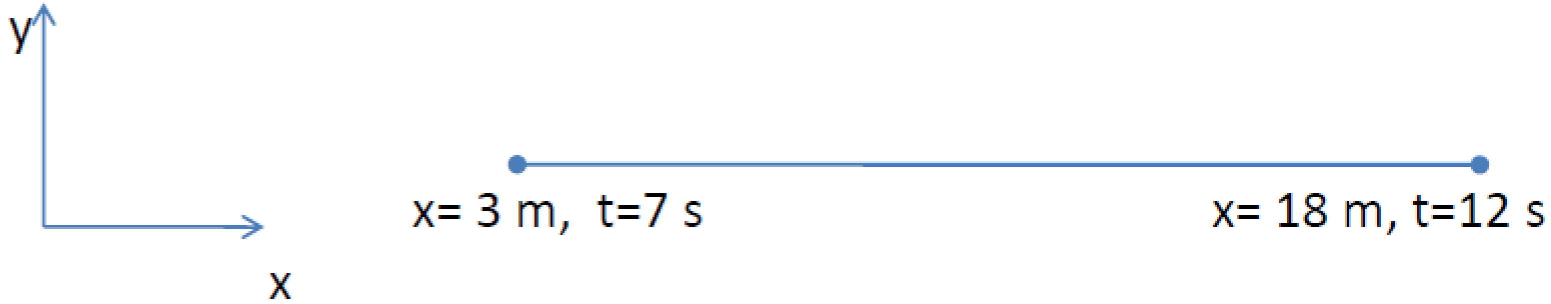
- Velocity

  V How fast and in what direction an object is moving.

  $\mathbf{V_{ave}}$= total displacement/total time

$$v_{ave} = \frac{\Delta \boldsymbol{x}}{\Delta t} = \frac{\boldsymbol{x_f} - \boldsymbol{x_i}}{t_f - t_i}$$

Velocity



$$v_{ave} = \frac{\Delta \boldsymbol{x}}{\Delta t} = \frac{18m - 3m}{12s - 7s} = 3\frac{m}{s}$$

Direction is to the right.

For constant velocity: V_{ave} is just V

$$\boldsymbol{v_{ave}} = \frac{\Delta \boldsymbol{x}}{\Delta \boldsymbol{t}} = \frac{\boldsymbol{x_f - x_i}}{\boldsymbol{t_f - t_i}} \quad \text{becomes} \quad \boldsymbol{V} = \frac{\boldsymbol{x_f - x_i}}{\boldsymbol{t}}$$

Rearranging the equation gives: $\mathbf{x = x_0 + vt}$ for constant velocity

(This is just a matter of notation…
most of the time we will rename x_f => x and x_i => x_o)

x is the value of the position at time t.
x (position) as a function of t (time)

Example

Estimate the average speed of an Apollo spacecraft in m/s if it took 5 days to reach the moon. The moon is 3.8×10^8m from the Earth.

Calculate the average speed of an Apollo spacecraft in m/s if it took 5 days to reach the moon. The moon is 3.8×10^8m from the Earth.

1 day is 60 x 60 x 24 seconds or 86400 s

So 5 days is 432000 s

Average Speed = distance/time

$= (3.8 \times 10^8 m)/(4.32 \times 10^5 s)$

$= 880$ m/s

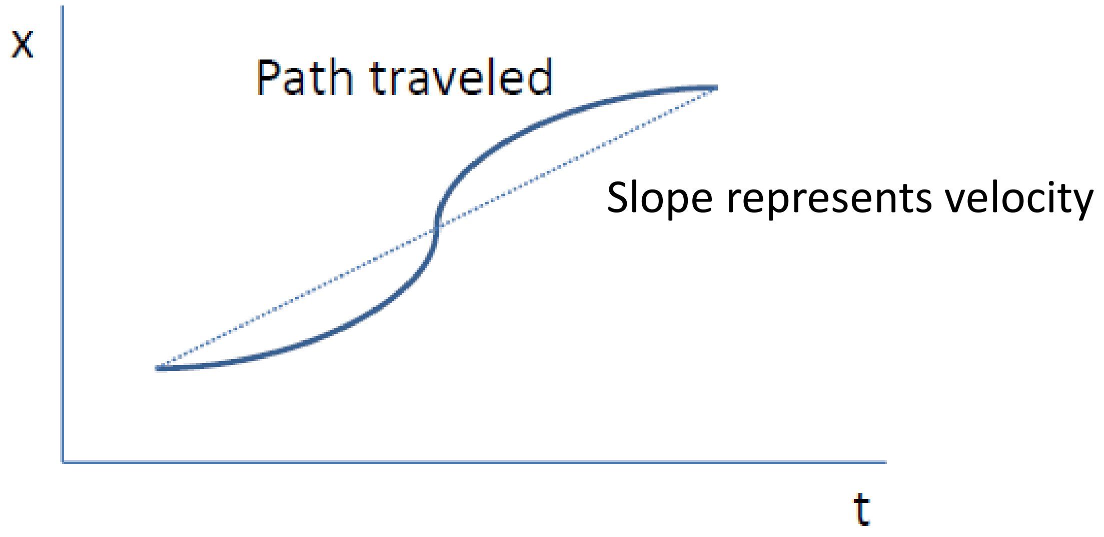
Graphical Interpretation of velocity

If you have a graph of **position versus time**, the **slope** of the graph between any two points is the **average velocity** in that time interval.

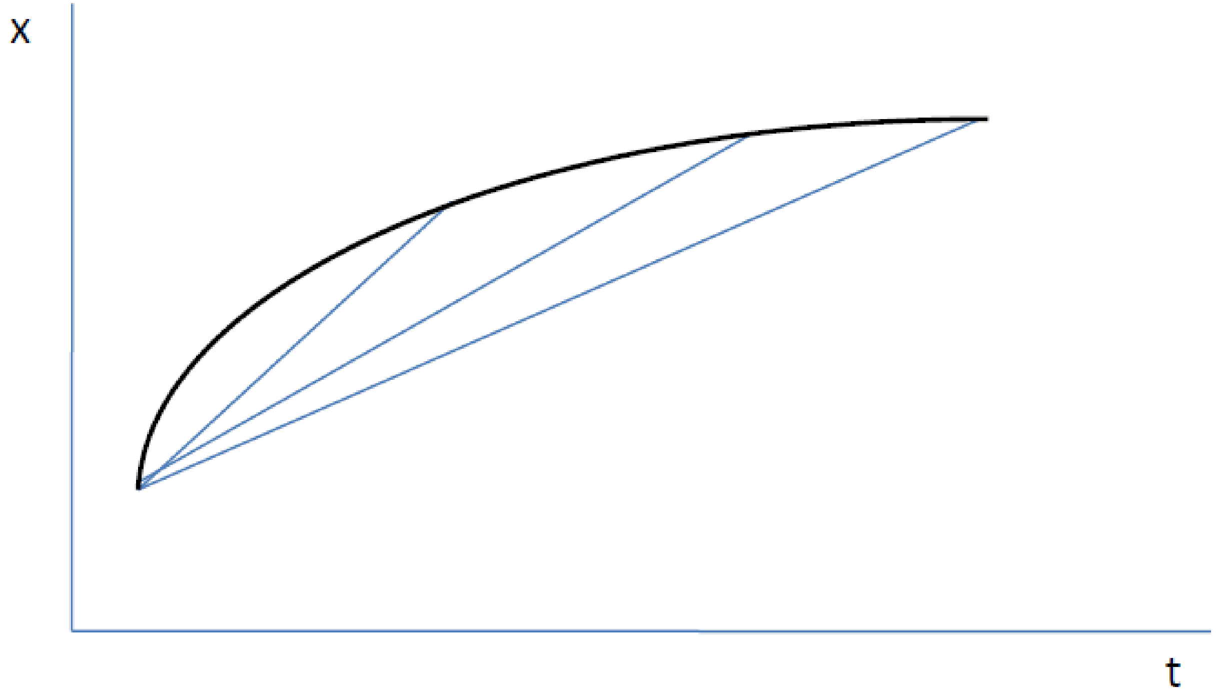
As the two points on the graph approach each other, the slope between them becomes the instantaneous velocity at that time.

Graphical Representation of velocity

If you have a graph of position versus time, the slope of the line connecting two endpoints represents velocity.



Graphical Representation of velocity

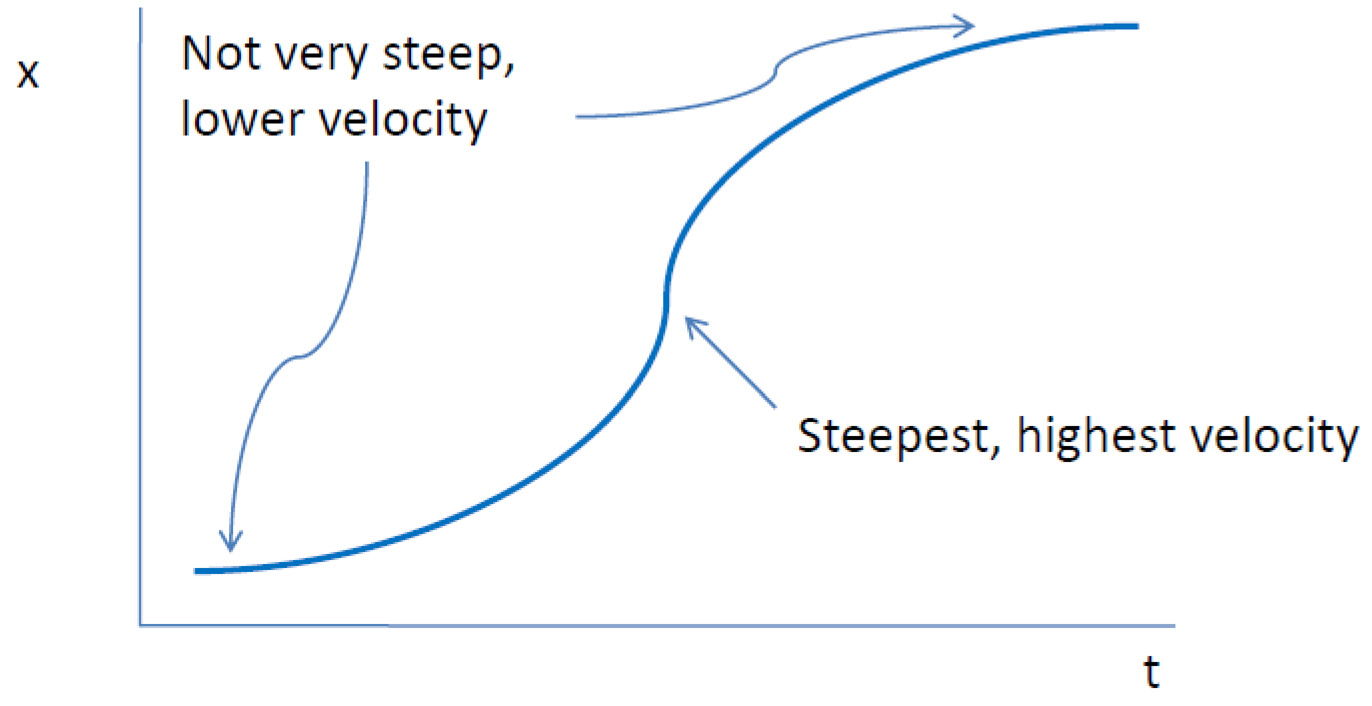


As the time separation is changed, the slope of the blue lines change. As the time interval approaches zero, the slope of the tangent line represents the instantaneous velocity.

Draw this better on the board.

Instantaneous Velocity

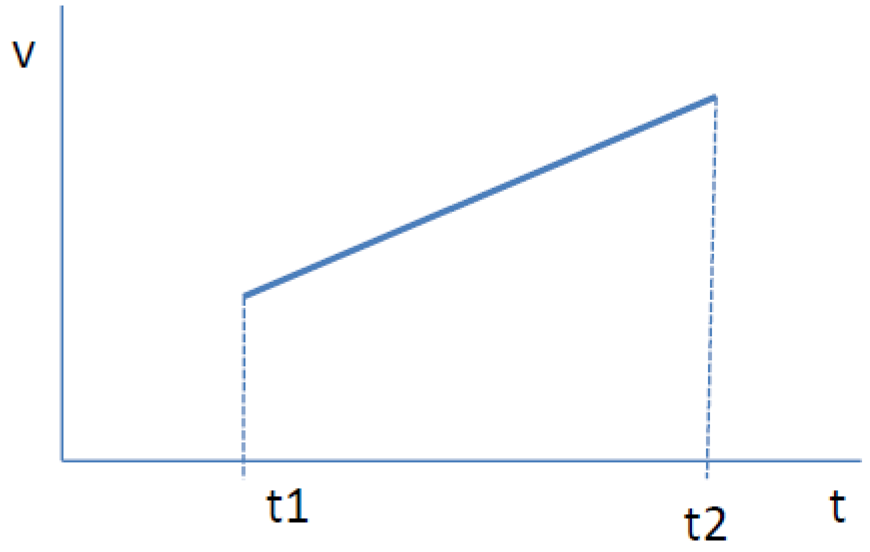
Determined by the slope of a line tangent to the position vs. time curve, at that instant.



Graphical Representation of Velocity

If you have a graph of velocity versus time.

The area under the curve represents the net displacement.



Positive area above the horizontal axis represents positive displacement.

Negative area below the horizontal axis represents negative displacement.

Track example

Usain Bolt owns the world record in the 100 meters with a time of 9.58 seconds.

Quickly estimate his average speed in m/s.

(round the time to 10s)

What was his average speed?

Baseball player hits a home run

The batter stands at home plate, hits a homerun and runs around the bases. The distance between each base is 90 feet.

After returning to home plate…

a)What is the player’s total distance traveled?

b)His displacement?

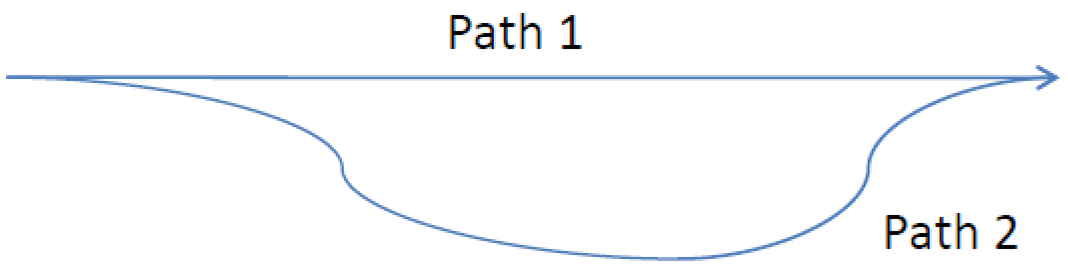
c)Average velocity? (Hint, the time involved doesn’t matter.)

Drag Race

Watching a drag race from overhead you see two cars taking two different paths from the same starting and end points. If they arrive at the end point at the same time:

a)Do they have the same average velocity?

b)Do they have the same average speed?



Acceleration

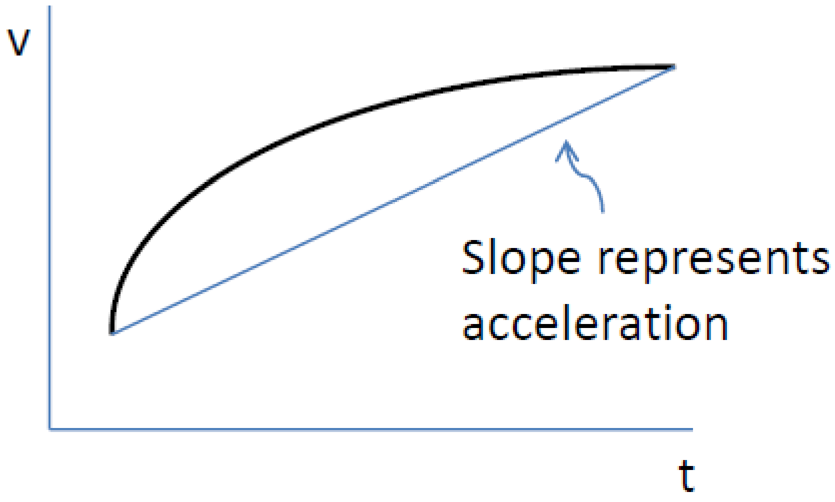
Defined as the time rate of change of the velocity.

Acceleration is a vector quantity.

Acceleration can include the velocity’s magnitude changing, the direction of the velocity changing, or both.

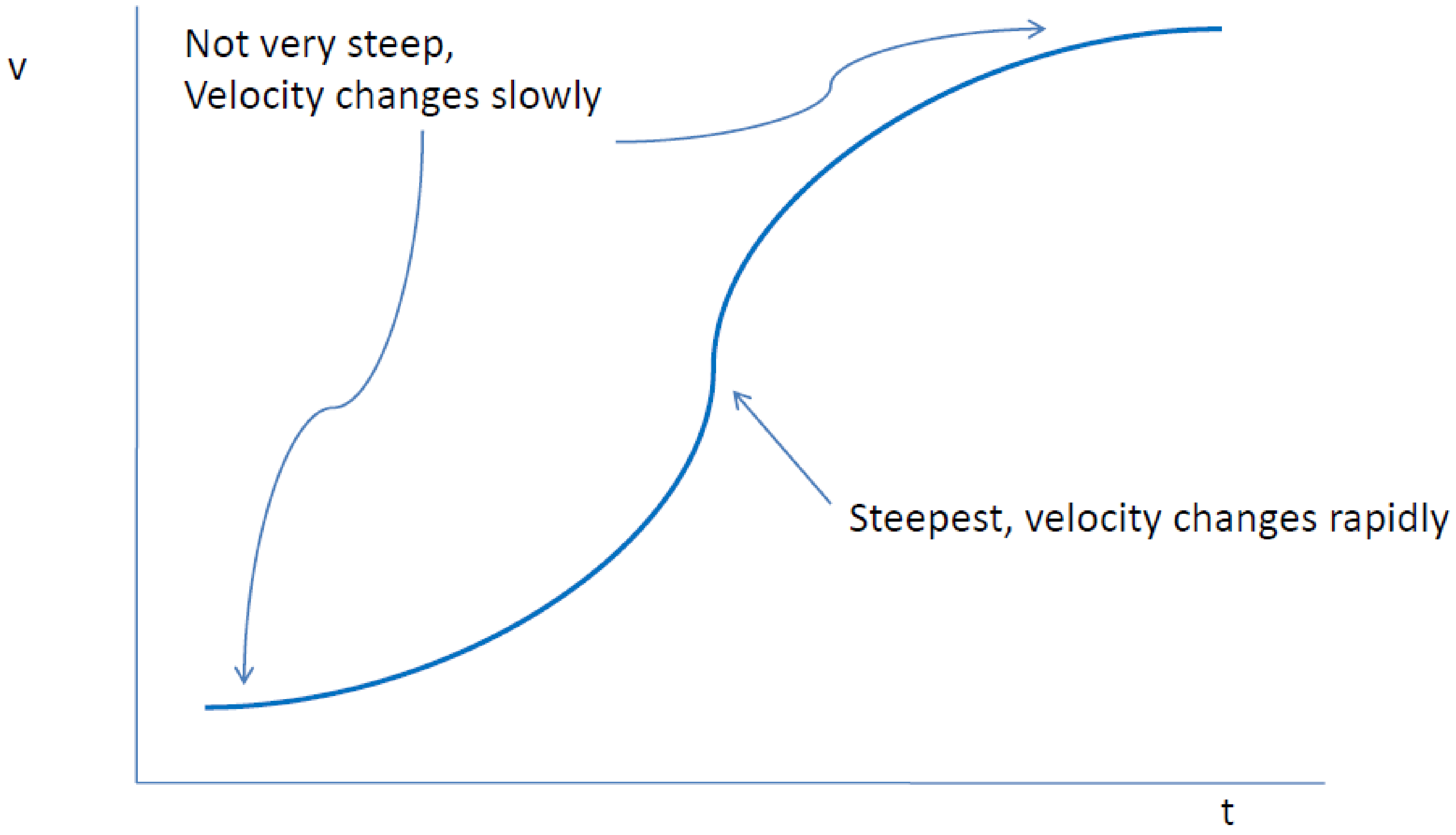
$$a_{ave} = \frac{\Delta \boldsymbol{v}}{\Delta t} = \frac{\boldsymbol{v_f} - \boldsymbol{v_i}}{t_f - t_i}$$

Acceleration

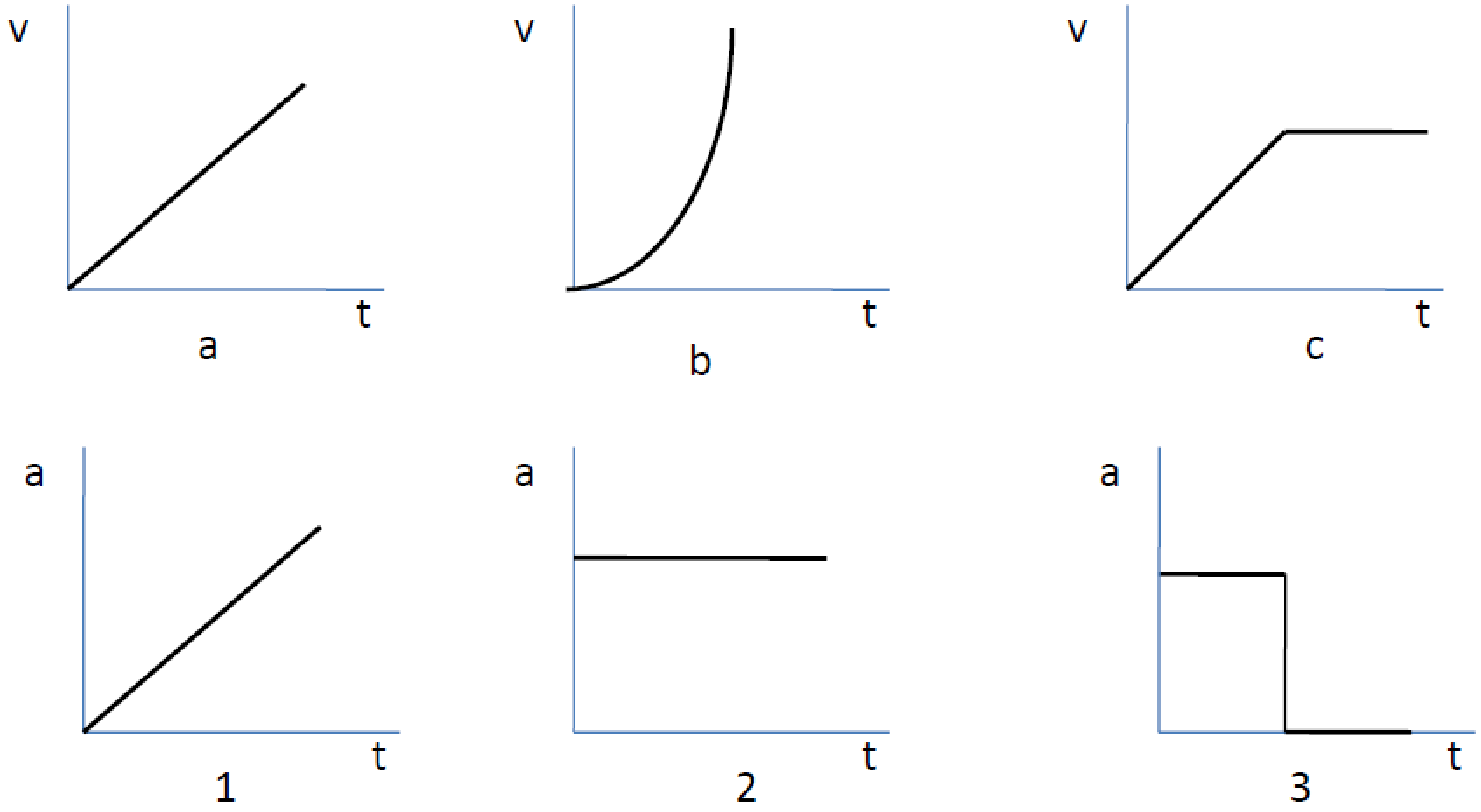


Again, as the time separation is changed, the slope of the blue line changes. As the time interval approaches zero, the slope of the tangent line represents the instantaneous acceleration.

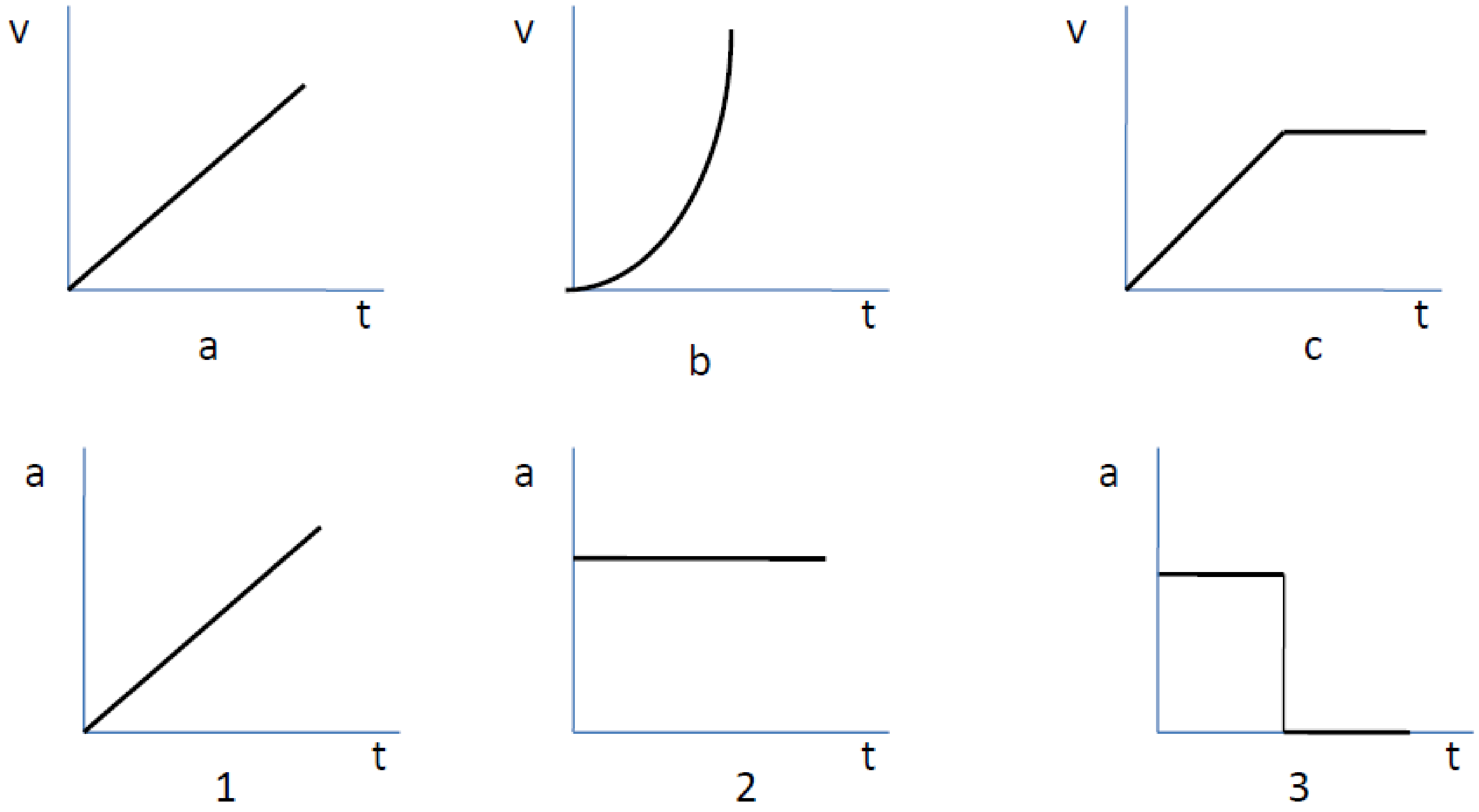
Instantaneous Acceleration



Match the velocity vs. time graph with the corresponding acceleration vs.time graph.



Match the velocity vs. time graph with the corresponding acceleration vs.time graph.



Answers: a-2
b-1
c-3

1-Dimensional motion with <u>Constant Acceleration</u>

$a_{ave} = \frac{\Delta \boldsymbol{v}}{\Delta t} = \frac{\boldsymbol{v}_f - \boldsymbol{v}_i}{t_f - t_i}$ becomes $a = \frac{(v_f - v_i)}{t}$

Rewrite this as $\boxed{v = v_0 + at}$

If **a** is constant, then $v_{ave} = \frac{v_0 + v}{2}$

Since $\Delta x = v_{ave}t$ then $\boxed{\Delta x = \frac{1}{2}(\mathbf{v}_0 + \mathbf{v})t}$

Combining: $\Delta \mathbf{x} = \frac{1}{2}(\mathbf{v}_0 + \mathbf{v})t$ and $\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$

We get: $\Delta \mathbf{x} = \frac{1}{2}(\mathbf{v}_0 + \mathbf{v}_0 + \mathbf{a}t)t$ or $\boxed{\Delta \mathbf{x} = \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2}$

On page 39, by doing some more substituting, we can derive one more equation:

Using: $\Delta \mathbf{x} = \frac{1}{2} (\mathbf{v}_0 + \mathbf{v})t$ and $t = (\mathbf{v} - \mathbf{v}_0)/\mathbf{a}$

We obtain: $\boxed{\mathbf{v}^2 = \mathbf{v}_0^2 + 2\mathbf{a}\Delta \mathbf{x}}$

Three very important equations (found in table 2.4)

For **constant acceleration**

Velocity as function of time.

$$v = v_0 + at$$

Displacement as a function of time.

$$\Delta \mathbf{x} = \mathbf{v}_0 t + \tfrac{1}{2}\,\mathbf{a}t^2$$

Velocity as a function of displacement. Also called timeless equation.

$$\mathbf{v}^2 = \mathbf{v}_0^2 + 2\mathbf{a}\Delta\mathbf{x}$$

Example 2.6 Runway length.

- A plane lands with a speed of 160 mph and can decelerate at a rate of 10 mi/hr/s. If the plane moves with constant speed of 160 mph for 1.0 s after landing before applying the brakes, what is the total runway length needed to come to rest?

Example 2.6 Runway length.

A plane lands with a speed of 160 mph and can decelerate at a rate of 10 mi/hr/s. If the plane moves with constant speed of 160 mph for 1.0 s after landing before applying the brakes, what is the total runway length needed to come to rest?

First convert all quantities to SI units.

v_0= 160 mph (1mph = 0.447 m/s) => 71.5 m/s

a = -10 mi/hr/s = -4.47 m/s^2 (Negative because plane slows down in direction of motion.)

For the 1.0 s the planes travels before applying brakes:

During this time the plane doesn't slow down. $a = 0$

$\Delta x_{coasting} = \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2 = (71.5\ \text{m/s})(1.0\ \text{s}) + \frac{1}{2}(0)(1.0\ \text{s})^2 = 71.5\ \text{m}$

While braking to a stop:

Use $\mathbf{v}^2 = \mathbf{v}_0^2 + 2\mathbf{a}\Delta\mathbf{x}$

$$\Delta \mathbf{x}_{\mathbf{braking}} = (v^2 - v_0^2)/(2a) = \frac{0 - \left(71.5\frac{m}{s}\right)^2}{2\left(-4.47\frac{m}{s^2}\right)} = 572\ \text{m}$$

Total length of runway is 71.5 m + 572 m = 644 m

https://www.youtube.com/watch?v=bVdaFv6kbkw

https://www.youtube.com/watch?v=7EqyVZWrzW4

Free Falling Bodies

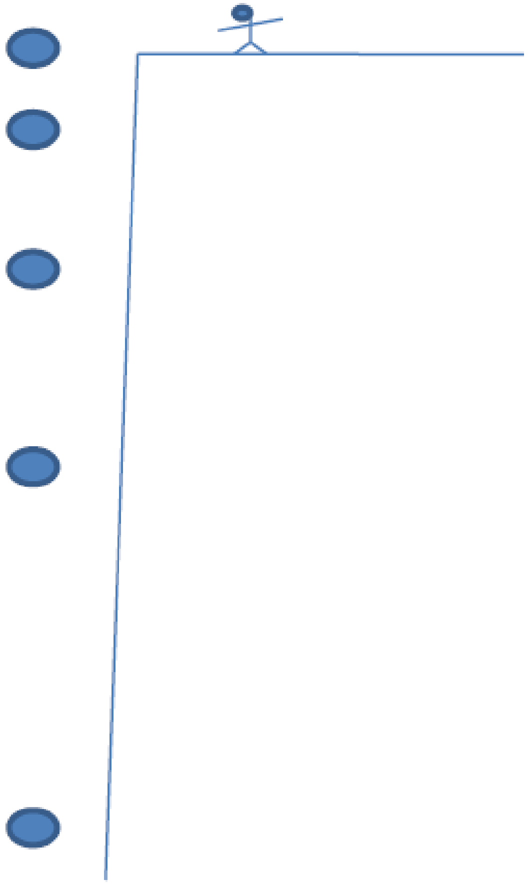
For free falling objects the acceleration is due to gravity; $a = g = 9.8\ m/s^2$ in the downward direction. (If the coordinate system states that up is positive, then gravity is negative.)

For close estimates you can use $g = 10\ m/s^2$

Suppose you dropped a ball from a high cliff. After each second interval, the magnitude of the ball's velocity increases by about 10 m/s.

Notice that in each 1 second time interval, the distance traveled increases.

Time (s)	Velocity (m/s)
0	0
1	10
2	20
3	30
4	40

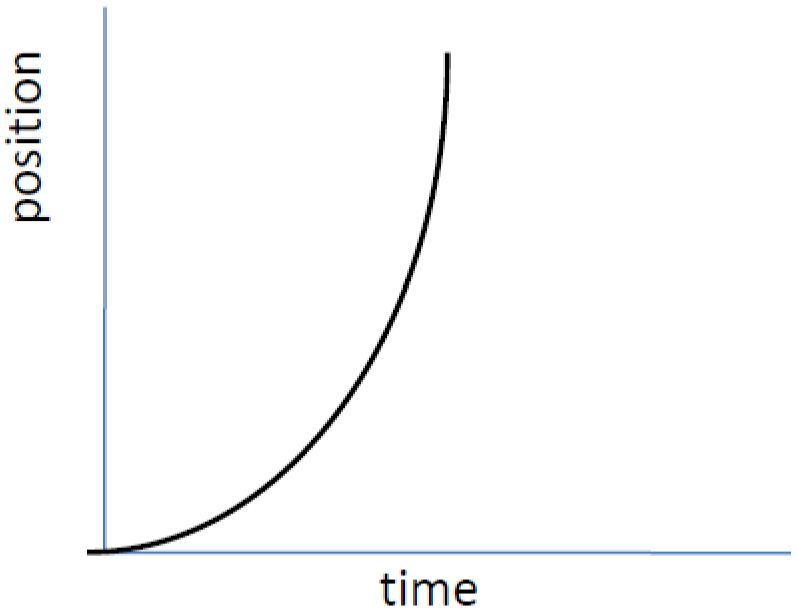


Remember the equation: $\Delta\mathbf{x} = \mathbf{v}_0 t + ½\,\mathbf{a}t^2$

The displacement as a function of time is quadratic.

Free falling object

If acceleration is constant, a graph of position vs. time will look like this:



Ballistic Rocket

(note: I changed the numbers from ex 2.10)

A rocket moves straight upward, starting from rest with an acceleration of +40 m/s^2. It runs out of fuel after 8 seconds and continues upward to a maximum height before falling back to Earth.

a)Find the rocket's velocity and position after 8 s.

b)Find the maximum height the rocket reaches.

c)Find the velocity the instant the rocket hits the ground.

What we know:

Rocket starts from rest. $v_0 = 0$

acceleration = 40 m/s^2 upward

time of acceleration is 8 s.

After 8 seconds, a = g (downward)

A) For the first 8 seconds, the rocket accelerates, use:

$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$ and $\Delta\mathbf{x} = \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$

After 8 seconds:

$v = v_0 + at = 0m/s + (40\ m/s^2)(8\ s) = 320\ m/s$

$\Delta \mathbf{x} = \mathbf{v_0} t + ½\ \mathbf{a} t^2 = (0m/s)(8\ s) + ½\ (40m/s^2)(8\ s)^2$ $\Delta x = 1280\ m$

B) Next the rocket slows down until it reaches the maximum height.

Now: $v_0 = +320\ m/s$, $a = g = -9.8\ m/s^2$

We also know that at the top, the velocity is zero. $v_f = 0m/s$.

Use : $\mathbf{v_f}^2 = \mathbf{v_0}^2 + 2\mathbf{a}\Delta\mathbf{x}$ to find Δx

$$\Delta x = \frac{v_f^2 - v_o^2}{2a} = \frac{0 - \left(320\frac{m}{s}\right)^2}{2(-9.8\frac{m}{s^2})} = 5224m$$

In first 8 seconds the rocket traveled 1280 m.

After running out of fuel the rocket traveled another 5224 m.

Total height is 6504 m.

Notice in example 2.10, the book used a different method.

They both work.

C) Find the rocket's velocity just as it hits the ground.

At peak of trajectory, v = 0

Rocket drops 6504 m (displacement is negative)

a = g (downward)

Use the equation: $\mathbf{v_f}^2 = \mathbf{v}_0{}^2 + 2\mathbf{a}\Delta\mathbf{x}$

$v^2 = (0\ m/s)^2 + 2(-9.8\ m/s^2)(-6504\ m) = 127478\ (m/s)^2$

Taking the square root gives us v = 357 m/s.

What about the sign?? The velocity when it hits the ground is downward. All the other downward quantities in this problem were negative. The final velocity will be negative. (-357 m/s).

Example problem:

Two cars are on a road.

Car A is traveling with a constant velocity of 10 m/s east.

When Car B is passed by Car A, Car B is traveling 5 m/s east and is also accelerating at a rate of 5 m/s^2.

After what time interval will the two cars be side-by-side?

How far will they travel during this time?

- Car A $v = 10$ m/s
- Car B $v_0 = 5$ m/s, $a = 5$ m/s^2

When the cars are side-by-side they will have equal displacements.

Car A $x_a = x_0 + v_a t$

Car B $x_b = x_0 + v_{0b} t + \frac{1}{2} a_b t^2$

Set these two equations equal to each other and solve.

$$(10\ \text{m/s})\ t = (5\ \text{m/s})\ t + \tfrac{1}{2}\ (5\ \text{m/s}^2)\ t^2$$

This is a quadratic equation.

Rewrite as $ax^2 + bx + c = 0$

$\Rightarrow 2.5t^2 - 5t = 0$

Can solve by using $t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Gives two answers. One of them is zero. So the second solution is the answer that we want.

$t = 2$ seconds

How far did the cars travel?

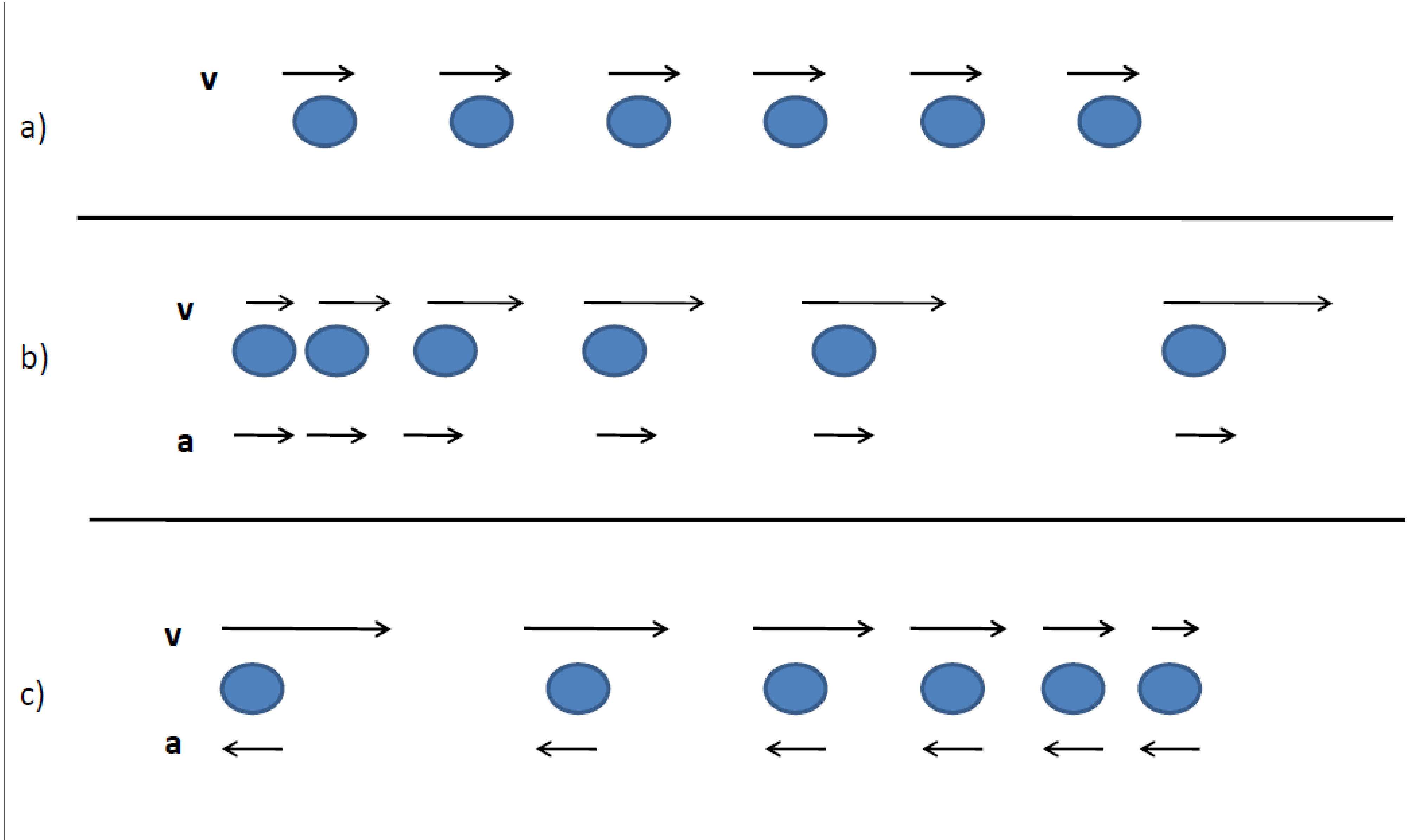
Plug in 2 seconds for the displacement equation for either car.

A: $x_a = v_0 t + \frac{1}{2} a t^2 = (10\ \text{m/s})(2\ \text{s}) = 20\ \text{m}$

B: $x_b = v_0 t + \frac{1}{2} a t^2 = (5\ \text{m/s})(2\ \text{s}) + \frac{1}{2}(5\ \text{m/s}^2)(2\ \text{s})^2 = 20\ \text{m}$

Motion Diagram

similar to page 37



Jet Question

Steam catapult launches a jet at rest, giving it a speed of:

175 mi/hr in 2.5 s.

A)Find the average acceleration of the jet.

B)Assuming that the acceleration is constant, find the distance the plane travels while launched.

a) convert 175 mi/hr to m/s

then use $v = v_0 + a\,t$

to solve for the acceleration

b) use $\Delta x = v_0 t + \frac{1}{2} at^2$

Train Problem

Train traveling down a straight track at 20 m/s when the brakes are hit, resulting in an acceleration of -1.0 m/s^2 as long as the train is moving. How far does the train move during a 40 s time interval starting at the instant the brakes are applied?

Train Problem

Don't use 40 seconds and $\Delta x = v_0 t + ½ at^2$

$\Delta x = (20\ m/s)(40\ s) + ½ (-1\ m/s^2)(40\ s)^2 = 0m$

This would not make sense.

Instead, find out how long the train needs to stop.

$$t = \Delta v/a = \frac{0m/s - 20m/s}{-1.0m/s^2} = 20 \text{ seconds.}$$

Only needs 20 seconds.

Now do $\Delta x = v_0 t + ½ at^2$

$\Delta x = (20\ m/s)(20\ s) + ½\ (-1\ m/s^2)\ (20\ s)^2 = 200m$