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ABSTRACT

This issue contains abstracts and critical comments for 11 articles. Three articles focus on problem solving; the remainder concern understanding of the equals sign, mathematical structure, mathematical abilities, the role of language in tasks involving sets, sex differences in mathematical errors, subtraction, algorithms as schemes, and mathematical achievement and attitudes in junior high school. An editorial on current educational reform is also included, as well as references to mathematics education research located in RIE and CIJE from April through June 1984. (MNS)

MATHEMATICS EDUCATION INFORMATION REPORT

INVESTIGATIONS IN MATHEMATICS EDUCATION

Volume 17, Number 4 - Fall 1984

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INVESTIGATIONS IN MATHEMATICS EDUCATION

Fall 1984

An editorial comment. . .

The Current Educational Reform

Joe Dan Austin
Rice University

It is an interesting time to be an educator. In the United States there is currently a major effort being made to reform and improve education. It is important that educators, particularly researchers, consider and respond to the changes being made in American education.

Historically, the most recent reform effort in American education will probably be dated from the 1983 publication of "A Nation at Risk." (However, Wise (1979) documents that many of the changes have earlier roots.) This report from a national commission suggests that the opening phases of the current reform effort were made at the national level. However, unlike the previous reform effort following the 1957 Sputnik, the federal government has largely remained on the sidelines. It is, rather, the individual state governments that have undertaken the current educational reforms.

The educational reforms following 1957 included massive federal funding for curriculum writing, teacher retraining, and even educational research. Researchers and educators at all levels were involved from the start of the reform efforts. The educational results of the reforms are perhaps still in some debate. However, it is ironic, in view of the criticism of the "new math," that one current educational concern is the drop in student SAT scores. These scores peaked in 1964 during the height of the 1957 educational reforms.

The current educational reform has generated little involvement of researchers in education and particularly researchers in mathematics education. In fact the current reform effort may have also involved few educators whether they were researchers or not! For example, the most recent education legislation in Texas was opposed by three of the four state teacher unions and has not been endorsed by the Texas Association of School Boards. There were also no research reports generated or cited, to my knowledge, to support the changes legislated. Thus, few educators or researchers seem to have been involved in some of the state efforts to reform education through legislation.

Two major concerns in the current effort to reform and improve education have been low student achievement and teacher competence. Major changes in the states have included some additional funding for education, increased requirements for teacher certification, testing of teachers, legislating what is to be taught, and testing of students for graduation and/or promotions. Some of the changes have been long overdue, as for example the increased expenditures for teacher salaries. However, the cost has been clear in increased paper work for teachers and administrators alike. Some teachers view the previous reform efforts as offering the carrot of additional training (in NSF institutes for example), while the current reform efforts offer the stick of testing to prod improvement in teachers and students. It is an open question as to which produces better results.

Historically American education has had other educational reforms that were largely attempted at the state level. For example, the reform efforts about 1900 were largely made at the state level to reform teacher training and to open American secondary education to students of all ability levels. (See, for example, the 1905 National Society for the Study of Education Yearbook.) The reform efforts around 1900 did produce considerable within-state uniformity in the area of teacher certification. Stiffer certification requirements for teachers, though, did not seem to stimulate larger enrollments in teacher education programs after 1900. It will be interesting to see if the current changes will have this effect as salaries have been improved for teachers. However, the reforms made around 1900 did succeed in opening American secondary schools to a wide spectrum of students. The reform efforts were also considerably aided by major educational research and writing done by writers such as Thorndike and Dewey, among others. Whether the current reform efforts will generate the same level of research interest in education will also be interesting to observe.

Currently it is encouraging to see the additional money and attention given to improving education. However, it is frustrating to see massive changes being made without the input of educational researchers. One specific suggestion that seems critical at this stage of the effort to improve education is the suggestion that we need a major study of educational differences between the states. There are major difficulties in such a large-scale study. The problems, though, do not seem as major as those involved in the major international studies that have already been completed. Also, a major study of between-state differences would likely have a more immediate influence on actual educational practices than have the international studies. A major study of between-state differences would address many important questions affecting each state. Such questions would include but not be limited to the following: Do states that require more years of high school mathematics have better mathematics achievement? How do the type of mathematics courses allowed for graduation affect achievement? How do the mathematics entrance requirements of the major state universities affect the achievement of students in the state? How do the different certification requirements affect student achievement? What is the effect of the varying percent of noncertified teachers on student achievement? There are, of course, many other important questions that a major between-states educational study would and should address. However, without such a study it will be hard for research to appropriately affect state-by-state educational changes.

The educational reforms following 1957 produced lively national debates and national meetings on important educational questions. The reforms and funding produced major research, not all of which supported the reforms being implemented. The current reform efforts have generated as yet few talks at state mathematics meetings other than talks by education agency personnel on how to implement the changes legislated. Researchers need to become involved in assessing the effects of the changes being implemented. Political decisions on issues affecting education can and probably often are made on the results of opinion polls about education. However, substantial improvement in the quality of education that has any lasting value will more likely follow from changes based on evaluation research. Certainly one hopes for the best in the latest educational reforms. However, one would feel better if more research were available and had been used as a basis for the changes being implemented.

It is an interesting time to be an educator.

Reference

Wise, Arthur E. Legislated Learning. Berkeley: University of California Press, 1979.

Af Ekenstam, Adolf and Greger, Karl: SOME ASPECTS OF CHILDREN'S ABILITY TO SOLVE MATHEMATICAL PROBLEMS. Educational Studies in Mathematics 14: 369-384; November 1983.

Abstract and comments prepared for I.M.E. by GERALD KULM, National Institute of Education.

1. Purpose

In elementary school mathematics, problem solving is generally understood to be the solving of routine problems essential to everyday life of the average citizen. The purpose of the study was to describe a method for defining and testing problem-solving ability in children aged 12-13. More specifically, the study proposed to:

- find a working definition of "problem solving ability" for ages 12-13;
- examine childrens' awareness of equivalence of problems with the same text and different numbers;
- explore children's ability to solve two-step problems; and
- study childrens' thinking in handling solved problems, assessing solutions, and constructing problems to fit a given statement.

2. Research Design and Procedures

Both paper and pencil and personal interviews were used to gain insight into childrens' associations and thought paths while solving problems.

A team of teachers and researchers was asked to suggest problems to assess what they considered to be essential to mathematical problem solving for ages 12-13. Analysis and discussion of the problems produced five groups of five problems each. The problem groups reflected the five primary problem-solving abilities adopted as a definition for the study: the ability (1) to choose the correct arithmetic operation in one-step problems, (2) to choose the correct operations in two-step problems, (3) to judge whether an answer is reasonable, (4) to choose relevant information, and (5) to make use of information in problems without a single answer.

The resulting tests was given to about 30 grade 6 classes, chosen at random throughout Sweden. Some children were interviewed by teachers or trainees and tape recorded.

3. Results

The results of the paper-and-pencil tests were not reported. The interviews led to the following conclusions:

1. Most children did not recognize equivalent problems, even when their attention was drawn to them.
2. Although children understood the order of decimals, their performance on problems with decimals was far below that of equivalent whole number problems.
3. In two-step problems, many pupils could carry out the first operation, but several were unable to carry through with the second operation.
4. Nearly all children checked only the computation, but not the reasonableness in relation to the conditions.
5. Hardly anyone drew diagrams.
6. Few children could make up problems for statements involving decimals.
7. Many children could not associate an arithmetic statement with a real-world situation.
8. Many of the problems that children made up involved money. Length and weight were also often used.

Abstractor's Comments

This study attempted to investigate some important ideas in problem solving. Although the focus was on routine word problems, the study of childrens' awareness of equivalent problems and their ability to analyze solutions and construct problems represents work that is very much needed.

Whether or not one agrees with the definition of problem-solving abilities reached by the authors, the approach of constructing carefully designed problem sets to investigate specific processes and abilities is excellent. Equivalent problems with decimals and whole numbers, for example, revealed some critical insights about childrens' misconceptions about the strategy for choosing the correct operation.

The authors' finding that children focus on checking only computations in solutions has important implications for explaining the lack of "Looking Back" processes. The results of childrens' constructing problems underlines the absence of understanding of decimal numbers. It seems impossible to separate a grasp of the meaning of numbers and their operations from the ability to solve problems involving those numbers.

It is unfortunate that at least the means and standard deviations were not reported for the paper-and-pencil tests over the five problem groups. These values, along with the insight gained from the interviews, would be valuable in assessing the relative strengths and relationships between the five abilities proposed by the authors. This type of school-based work, with interviews carried out by teachers, represents a useful way not only to learn about childrens' mathematical processes, but to help teachers begin to find ways of strengthening understanding of mathematical ideas.

Baroody, Arthur J. and Ginsburg, Herbert P. THE EFFECTS OF INSTRUCTION ON CHILDREN'S UNDERSTANDING OF THE "EQUALS" SIGN. Elementary School Journal 84: 199-212; November 1983.

Abstract and comments prepared for I.M.E. by GAIL SPITLER, The University of British Columbia, Vancouver.

1. Purpose

The purpose of this study was to examine the effects of a seven-month, systematic effort to teach a relational meaning of "equals" to children in grades 1, 2, and 3.

2. Rationale

Previous research indicates that children interpret "equals" as an operator and that this limited understanding may continue through high school and college. "One reviewer noted that even the use of hand-held calculators promotes the operator view of 'equals'; the arithmetic problem is punched in first, and then the 'equals' sign key is hit to produce the answer" (p. 209, italics in the original). Two explanations of this phenomena can be found in the literature. The first view is that the operator interpretation is an artifact of early arithmetic instruction and that children may reject equation forms with which they are not familiar. An alternative view holds the interpretation of "equals" is related to stages in cognitive development. For example, a relational interpretation may depend on the consolidation of concrete operational thinking or the advent of formal operational thought.

3. Research Design and Procedures

As part of another study the authors had access to one grade 1, one grade 2, and one grade 3 classroom in a suburban school serving a middle-to-upper-class community in which the mathematics curriculum materials provided thorough and systematic instruction on the relational meaning of the equal sign. The mathematics curriculum under investigation was developed by Wynroth and is an individualized program consisting of a sequence of games. "According to the curriculum guide, 'equals' is defined as 'the same number' in order to avoid the initial learning of 'equals' as 'the answer is' (p. 202). The subjects included 15 children each from a first-, second-, and third-grade class. Students repeating a grade were not included. All of the students had been in the program for seven months; four of the grade 2 and four of grade 3 children had been exposed to the program in the previous year as well.

The authors sought to measure children's conceptions of "equals" along two dimensions: the children's acceptance of the form of an equation and the children's judgment of the validity of an equation. They hypothesized that children "receiving the instruction emphasizing the relational view of 'equals' will accept both the form and the validity of typical equations (e.g., 7 + 6 = 13) and atypical equations (e.g., 7 + 6 = 4 + 9) to which they have been exposed, will accept the validity but perhaps not the form of atypical equations they had not been exposed to and will reject incorrect statements" (p. 201).

After a familiarization session with an interviewer, the children responded to two tasks, administered about a week apart. In each task the child was shown ten separate equations (homework completed by the Cookie Monster) and asked to check them. The equations were presented in random order. In the first task, the child was questioned about correctness of the form of equation, the truth or validity of the equation, and whether the equation should be rated as "right", "wrong" or "in-between". In the second task, the child was asked to equate the truth or validity of the ten separate equations.

Three categories of equations were included in the tasks. Category 1 included equation forms to which a subject was exposed in the course of instruction, including typical forms (e.g., 7 + 6 = 13) and atypical forms (e.g., 13 = 7 + 6). Category 2 included equation forms not seen by the child during the course of instruction including typical forms (e.g., 7 + 6 = 14 - 1) and atypical forms (e.g., 5 + 3 = ~~1111~~ 111). Category 3 included incorrect forms. The equations are listed below by category.

Category 1	Category 1 or 2*	Category 2	Category 3
8 = 8	7 + 6 = 6 + 6 + 1	7 + 6 = 14 - 1	7 + 6 = 6
7 + 6 = 13	6 + 3 = 4 + 4 + 1	5 + 1 = 7 - 1	2 + 2 = 2
13 = 7 + 6	2 + 4 = 3 x 2	7 + 6 = 1111111111111	7 + 6 = 0
7 = 5 + 2		5 + 3 = ~~1111~~ 111	4 + 2 = 42
7 + 6 = 6 + 7		7 + 6 = XIII	
4 + 3 = 3 + 4		3 + 2 = V	
7 + 6 = 4 + 9			
6 + 4 = 5 + 5			

*Some of the grade 1 children had not been exposed to equations of these forms.

4. Findings

Category 1 equation forms were seen by most of the children as being familiar and as being sensible. About half of the students in each grade found the Category 2 equations sensible. All but a very small percentage of the children indicated that the Category 3 equations did not make sense. During the interviews the children repeatedly indicated that they distinguished between the unfamiliarity of a form and whether it made sense. When asked for their definition of the "equals" sign, 20% of the grade 1, 47% of the grade 2, and 33% of the grade 3 children answered that "equals" meant "the same". Similarly, 60% of the grade 1, 27% of the grade 2, and 67% of the grade 3 children used the relational meaning of "equals" in at least one justification.

The authors also examined the response consistency of the children in judging whether atypical Category 1 equation forms made sense. Overall, 44% of the children were classified as inconsistent (meaning that two to half of the trials were judged as making no sense). The first graders appeared to be the most consistent and the second graders to be the least consistent.

Some unique responses made to the equation 8 = 8 were noted.

5. Interpretations

The results of this study support the instruction-related rather than the cognitive development position concerning the basic (is the same as) relational meaning of "equals".

The authors posed that it is neither desirable nor possible to eliminate the operator view of "equals", but rather that these results, along with other training studies, have shown that it is possible to broaden children's view of "equals" to include a relational meaning.

Abstractor's Comments

The authors are to be commended on both the study itself and the written report of the study. The article is well written and relates necessary detail clearly and succinctly. All of my "... but what about ...?" queries were dealt with as I read on. In my opinion the high quality of the article is, in part, a function of the amount of space devoted to various aspects of the report. In particular, the report devotes as many pages to the discussions of the problem and the related literature, the significant and unique features of the instructional program, and the conclusions as are devoted to reporting other specific details of the study. The report of the details of the study includes both the "hard", quantitative data and well-chosen vignettes from the dialogues of actual interviews. However, it is the completeness of the discussion of the nature of the problem (and the related literature) and the authors' interpretations of the findings that commend this article. After finishing the article, I felt that I fully understood the research study and had gained significant insight into the teaching and learning about "equals".

Baroody, Arthur J.; Ginsburg, Herbert P. and Waxman, Barbara. CHILDREN'S USE OF MATHEMATICAL STRUCTURE. Journal for Research in Mathematics Education 14: 156-168; May 1983.

Abstract and comments prepared for I.M.E. by LINDA JENSEN SHEFFIELD, Northern Kentucky University.

1. Purpose

The purpose of the study was to "examine the use of the commutativity, addition-subtraction complement, and N + 1 progression principles in solving number combinations by capable first, second and third graders" (p. 157). The investigators wished to ascertain whether primary school children made use of mathematical structures in computation.

2. Rationale

Even though the use of mathematical structure can aid in computation, literature reviews by Gelman and Starkey (1979) and Suydam and Weaver (1975) show little research done in this area. Some anecdotal evidence indicates that preschoolers can learn to use the commutativity principle (Baroody, 1982), although preschoolers (Ginsburg, 1982) and elementary students (Holt, 1962) may not see opportunities to do so. Other evidence suggests primary students may use the principle of commutativity with small combinations but not with larger ones (Ginsburg, 1982). This study examined the use of addition-subtraction complement and N + 1 progression principles as well as the commutative property of addition by capable primary students.

3. Research Design and Procedures

a) Sample

Eighteen children in each of grades 1, 2, and 3 from two first-second grade combination classes and two second-third grade combination classes in one school from a middle to upper-middle class suburban community were chosen for the study. Children repeating a grade and children with extremely high or low achievement test scores were not included in the study.

b) Procedure

In the preliminary session, children met the experimenters and took a computational test on 18 addition and 4 subtraction combinations. The test data were used to ensure that the items to be used in the experiment were not too easy or too difficult. Items were presented in a game format. Mean accuracy for first graders was 78%; for second graders, 90%, and for third graders, 96%. Subtraction accuracy means were 43%, 61%, and 92% respectively.

In the experimental session, a game format was again used. A Math Baseball game was used which had three innings, one which dealt with the commutativity principle, one with the addition-subtraction principle, and another with progression by 1. The innings were presented with the six different orders (CAP, CPA, ACP, APC, PCA, PAC) balanced between the grades. The target items were presented as every other exercise and were related by the principle of the inning to the items immediately preceding them. For example, 13+6 was followed by 6+13 in the commutativity inning, 9+9 was followed by 18-9 in the addition-subtraction inning, and 6+7 was followed by 6+8 in the N+1 progression principle inning. The last item in each inning was a test item in which the principle would lead to an incorrect response to check that the children were not blindly following the rule. Interviews were tape-recorded and transcribed and responses were scored as to the strategies used.

4. Findings

Most of the children at all three grades used the commutativity principle on at least three of the four target trials. Over 80% of the third graders used the addition-subtraction principle in a majority of the trials, but less than 40% of the first and second graders did so. The N+1 pattern was not frequently used.

5. Interpretations

The fact that commutativity was used by more than 70% of the children at all grade levels suggests that it may be well known to young children even though the first-grade teachers indicated that they had not taught it formally. Advanced strategies such as inventing or memorizing addition algorithms are not necessary for discovering commutativity. It may be particularly suitable to foster by discovery learning in young children.

The fact that the addition-subtraction complement was used less frequently than commutativity may support the hypothesis that the part-whole scheme underlies the complement principle. Resnick (1983) has found that the part-whole scheme is available in at least a primitive form before the child enters school. These data seem to indicate that this scheme is only localized to cases where the addition combinations are well known and is not yet a general principle. Further research is necessary to test the principle's generality.

The low usage of the N+1 principle may have been due to a number of factors. Such a sequential presentation of problems is unlikely to occur except in a school context and students may have been unfamiliar with its use. Third graders often used other more familiar efficient strategies to find the answers. Instructional techniques which encourage looking for creative solutions rather than one single correct method may encourage students to create and use efficient strategies such as this.

Many children felt they were cheating in the game if they used their knowledge of the principles to find a shortcut. The answer to the previous problem was always available on the used pile and some children would look furtively at the pile and then claim that they did not look. Children may have previously been punished for looking for such shortcuts and may need encouragement to look for and use patterns and creative techniques.

Abstractor's Comments

The study of children's use of mathematical structure is certainly important for any elementary teacher or math educator. It would be nearly impossible for a primary student to memorize 100 basic addition facts without the use of patterns and structure. The relationships between addition and subtraction greatly simplify the learning of subtraction facts. Knowledge of which principles children use naturally in a game setting can be useful as a first step in planning instruction.

Follow-up research to answer some of the following questions would be helpful:

1. What are the effects of instruction on the use of structure? Does it help students to be directly instructed in the use of principles such as commutativity, complements, and progression or is it best to encourage children to discover these concepts on their own? Are some concepts better discovered and others better taught?

2. Do children differ in their use of principles depending on whether their instruction has emphasized creativity or one right algorithm? Does instruction which encourages problem solving and pattern-seeking aid children in the use of principles?

3. Would the results of the study have been different if the children had specifically been told to find a pattern or shortcut? As the authors noted, some of the children felt they were cheating if they used the pattern from their previous answers. Other children may have been able to use mathematical structure, but had refrained because they thought they were not supposed to. Encouragement to look for these shortcuts may have found higher percentages of the students able to use the principles. This also would have implications for teaching methods which too often discourage these types of creative problem solving.

4. As the authors noted, the use of problems with different numbers, either larger or smaller, may affect the use of the principles and it would be useful to do further research in this area.

5. Would children of less ability or lower SES perform in the same way? Because these children performed well above the mean on computation achievement at each grade level, it would be useful to know whether this level of use of structure would also be true of children with less ability.

Overall, this study is a good extension of some earlier anecdotal evidence of the use of the commutativity principle. The look at principles could be expanded to include the use of the identity or the associativity principle. It is hoped that these authors and others will continue this line of research and will expand it to the effects of instruction to make it even more useful to classroom teachers.

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Carpenter, T. P.; Hiebert, J; and Mose., J. M. THE EFFECT OF INSTRUCTION ON CHILDREN'S SOLUTIONS OF ADDITION AND SUBTRACTION WORD PROBLEMS. Educational Studies in Mathematics 14: 55-72: 1983.

Abstract and comments prepared for I.M.E. by OTTO BASSLER, Vanderbilt University.

1. Purpose

First-grade children were studied to investigate the effects of instruction in addition and subtraction on the processes used to solve verbal problems involving these operations. Four semantically different classes of problems were investigated. They were:

Change--some direct or implied action causes a change in an initial quantity

Combine--two quantities are considered as parts of a whole

Compare--the relative size of two objects are compared

Equalize--quantities are compared and then one is changed to become equal with the other.

2. Rationale

Prior to formal instruction children exhibit success in solving simple addition and subtraction word problems. These children use strategies that directly model the relationship described in the problem to solve different addition and subtraction problems. This study questions whether this eclectic approach continues as children get older and receive instruct in addition and subtraction.

3. Research Design and Procedures

A sample of 43 children from two first-grade classes of a parochial school were tested in early February and in May. Prior to the February testing the students had no formal instruction in the symbolic representation of addition and subtraction. They had studied numbers, mathematical sentences, and topics from measurement and geometry. Between February and May, two instructional units on addition and subtraction were presented. The Developing Mathematical Processes (DMP) mathematics program was used. DMP emphasizes analysis of verbal problems using a part-whole relationship, modeling numbers with sets of objects, and solving addition or subtraction situations using various forms of counting.

The tests were administered using individual interviews. The February interview consisted of 10 verbal problems in addition and subtraction for students to solve without pencil and paper. In May, six of the 10 problems were readministered in a similar manner. Several days later in May, students were asked to write an arithmetic sentence before they solved each of a set of parallel problems. Problems were read and reread as often as necessary to individual students who were supplied with a set of red and white Unifix cubes. Children were encouraged to solve the problems without the cubes but could use them if necessary. The investigator coded the responses of the subjects, noting the strategy used, whether a correct solution was obtained, and, if incorrect, the type error made. If the strategy used by the student was not apparent to the investigator, the child was asked to describe the method that was used.

Problems represented in both interviews included combine and compare situations for addition, and separate, combine, compare, and equalize situations for subtraction. Number triples for the problems were restricted to six basic facts where each addend was greater than 2 and less than 10; the sum of the addends was greater than 11; and the absolute value of the difference of the addends was greater than 1.

4. Findings

Results were given for the six problems common to the two interviews and the sentence writing problems administered in May. No statistical hypotheses were tested.

The combine-addition problem was solved correctly by 38 of the 43 subjects in February, so there was little room for improvement. All students used a correct strategy on this problem in May. The compare-addition problem was difficult in both testings, but substantial gains in correct strategies and solutions were made following instruction. For both interviews the most common strategy for solving addition problems was "counting all." There was a shift in the error pattern on the compare-addition problem. In the February testing most errors were made when students responded with a given number. In the May testing the percent of students using an incorrect operation increased.

For the four subtraction problems, correct strategies were applied by a large proportion of the subjects in the February testing, so there was little margin for improvement. There was, however a shift in correct strategies selected. In the February interview students used a variety of different strategies to solve different subtraction problems. In the May interview, "separating" was the most frequently used strategy to solve all subtraction problems. The error patterns for both testings were similar.

When students were asked to write number sentences to solve the problem, almost all of them wrote a correct sentence for combine-addition and separate-subtraction problems; about three-fourths responded correctly to compare-addition and combine-subtraction problems; and less than one-half wrote correct sentences for compare-subtraction and equalize-subtraction problems. About one-fourth of the students generally solved the problem before writing the number sentence despite directions to the contrary.

5. Interpretations

Prior to instruction most children modeled directly the actions described in the problem. They do not, however, recognize that different strategies can be used interchangeably. Following instruction most children began to use a single strategy. Most children also learn to write number sentences for verbal problems, but do not realize that these are an aid for solving problems.

Abstractor's Comments

This was an exploratory study which investigated an important problem, that is, strategies that first-grade students use to solve verbal addition and subtraction problems and how these strategies change following instruction. Since no statistical hypotheses were tested, only tentative conclusions can be drawn. These and other speculations provided in the conclusions of the paper offer excellent hypotheses to be tested in future investigations. Other limitations include a non-typical sample of students, small sample size, lack of control on addition and subtraction instruction, and uncontrolled teacher variable. The strategies used by students were carefully described and there appeared to be a difference in the way in which students solved problems before and after instructional units in addition and subtraction were taught. Unfortunately, the authors do not describe the content of the instructional units nor the strategies which the teachers emphasized in instruction in sufficient detail. Hence it is not possible to look at relations between instruction and children's solutions to addition and subtraction word problems.

Fredrick, Dennis; Mishler, Carol; and Hogan, Thomas P. COLLEGE FRESHMEN MATHEMATICS ABILITIES: ADULTS VERSUS YOUNGER STUDENTS. School Science and Mathematics 84: 327-336; April 1984.

Abstract and comments prepared for I.M.E. by THOMAS O'SHEA, Simon Fraser University.

1. Purpose

The study was designed to determine whether adults entering college as freshmen differed from younger freshmen in their mathematical abilities.

2. Rationale

One recent study found adult SAT mathematics scores to be well below those of younger students entering college. Another reported lack of basic mathematical comeptencies in entering adult freshmen. Other studies have indicated that adult freshmen feel anxious about their level of mathematical skills. Specific knowledge of adults' strengths and weaknesses would assist college instructors who face increasing numbers of returning adults.

3. Research Design and Procedures

The study was carried out at the University of Wisconsin-Green Bay in September 1980. An adult freshman was defined as one 25 years of age or more at that time. Almost all freshmen participated in the Freshmen Testing Program for placement purposes, and the tests of interest for the study consisted of the mathematics subtest of the Survey of College Achievement, the mathematics subtest of the Advanced Level 2 form of the Metropolitan Achievement Test (MAT) battery, and an additional 10-item Algebra test.

A total of 73 adult freshmen, two-thirds of whom were female, were identified. The contrast group consisted of 738 younger freshmen, of whom the large majority were 17- or 18-year-olds entering college directly from high school. T-tests were used to assess differences in overall performance between the two groups on the three tests.

For the item analysis portion of the study, a random sample of 100 scores of younger freshmen was drawn (50 men and 50 women). For each item on the MAT mathematics subtest, t-tests were applied to the percentage of adults and younger students responding correctly.

Finally, the MAT items on which the two groups showed similar performance were grouped into three categories of difficulty: those answered correctly by 80 percent or more of each group, those answered correctly by 50 to 79 percent of each group, and those answered correctly by less than 50 percent of each group.

4. Findings

The overall performance of the adults was significantly lower ($p < .05$) than their younger colleagues on all three mathematics tests. It was significantly higher ($p < .05$) on the Humanities subtest of the Survey of College Achievement battery. No significant differences were apparent for any other subtest.

Differences in performance were also found for some of the MAT mathematics items. Adults scored higher ($p < .05$) on one item which involved the use of English measures. On 14 items adult performance was inferior (p-values ranged from .05 to .001). On the remaining 35 items no significant differences occurred. Of the 50 items, 3 were answered correctly by less than half of each group, 12 items were answered correctly by 50 to 79 percent of each group, and 22 items were answered correctly by over 79 percent of each group.

5. Interpretations

Neither group could be termed mathematically illiterate. The two groups showed equal ability in basic whole number operations, in working with fractions, decimals and percents, in solving word problems, and in reading and interpreting graphs.

Younger freshmen were better in using negative numbers and exponential notation, in working with set theory and Venn diagrams, in recalling geometry formulae, and in solving linear and quadratic equations. These skills are often taught in high school geometry, advanced algebra, and trigonometry.

The reason for adults' poorer performance may be lack of practice, or it may be that the adults had less exposure to the material. In either event, the size of the differences in mathematical performace suggests that the gap is not insurmountable. Several review sessions, or remedial coursework, might be used to bring their skills up to those of younger freshmen.

The results cannot be generalized to other adult groups such as graduate students or casual learners not seeking a degree.

Abstractor's Comments

The problem under investigation in this study is one of considerable interest to my own university in which students who enter directly from high school are the exception rather than the norm. Unfortunately, the results published in this study will not be of much practical value to us or to anyone else.

The study suffers from the usual lack of generalizability in reports of this type which simply publish the results of local testing programs. The results may be unique to Wisconsin as a result of its educational structure, university entrance requirements, and mathematics curriculum. The authors take care to point this out, but the problem is not alleviated by such an admission. The issue is further confounded by lack of information on the entrance requirements for adults to the university. For example, at our institution, we have a 'mature student policy' by which adults over the age of 23 may be admitted without having met the academic qualifications required of immediate post-secondary students. It would also have been valuable to know which faculties the adult students were hoping to enter. If older students really do have feelings of mathematics anxiety and inferiority, the sample may have been biased if the adults in general were entering courses of study which did not require mathematical competence.

A second issue of importance is the analysis and reporting of item differences. A sample of 100 younger freshman was selected in order roughly to equate sample sizes for adults and the contrast group. In the contrast group the proportion of females to males was deliberately set at 0.5. If it is true that females tend to do less well in mathematics, one wonders why the sample was not selected on the gender variable to parallel that of the adults, in which two-thirds of the sample were female. Such lack of comparability would tend to produce lower test scores for the adult group.

Why is it that years of critiques in IME and other reviews have not alerted researchers and editors to the unsatisfactory nature of multiple t-tests? Here again, 50 t-tests were carried out and items on which "significant" differences were found were identified. Out of the 50 items, adults performed "significantly" less well on 14. Of the 14, five were at the .05 level. Who knows which performances were truly different and which were spuriously different? At the very least, the p-value for significance on items should be adjusted to yield an acceptable experimentwise error rate for the collection of items (cf. Kirk, 1968, pp. 82-86). In the present study which contained 50 items, a p-value of .001 at the item level would result in an experimentwise error rate of .05. On this basis, performance on 8 items instead of the reported 15 would have been declared different.

There are further problems in the reporting of the results. Table 2 lists items on which the results of the adults and younger students differed. Table 3 purports to group the remaining items, on which similar competency was displayed, into three levels of difficulty. Yet five items in Table 3 were also contained in Table 2, thereby confounding what was meant by "similar" performance.

The final difficulty with this piece of research is that a single item is an inadequate basis on which reliably to assess differences in performance. For example, apparently on the basis of one item, the authors maintain that all new freshmen can solve word problems. Can this skill really "...be taken for granted by college instructors of these freshmen" (p. 332) as the authors suggest? Furthermore, the conclusion that "younger freshmen did much better than did adult freshmen in ... recalling geometry formulae" (p. 334) is downright misleading when, out of five items which might be construed as measuring this objective, the group did not differ on two of them.

A more defensible procedure in determining differences in performance between groups is to group the items into clusters which, on an a priori basis, are thought to measure different components of a basic mathematical concept or skill. For example, in a study carried out several years ago to report achievement and to assess change in mathematics performance over a period of time, we (O'Shea, 1981) required a minimum of six items for each objective on which performance was evaluated, in order to overcome item idiosyncracy and to ensure adequate reliability. For the more sensitive problem of assessing change we used a minimum of 10 items per reporting category.

The authors might have addressed the problem of reliability of conclusions by using the results on the two other mathematics tests to cross-validate their findings on the MAT items. There were 82 mathematics items on the Survey of College Achievement Tests, some of which might have at least confirmed the finding of no difference on basic arithmetic operations. Performance on the items on the Algebra test might also have helped to shed light on the reported difference in ability to solve equations.

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Hudson, Tom. CORRESPONDENCE AND NUMERICAL DIFFERENCES BETWEEN DISJOINT SETS. Child Development 54: 84-90; February 1983.

Abstract and comments prepared for I.M.E. by LIONEL PEREIRA-MENDOZA, Memorial University, St. John's, Newfoundland, Canada.

1. Purpose

The purpose of the study was to investigate the role of language when young children solve problems involving correspondences and numerical differences between disjoint sets.

2. Rationale

The author notes that a number of young children perform poorly when asked questions of the form "How many more ... than ...?". One explanation is consistent with Piaget's statements regarding one-to-one correspondence. An alternative explanation is that the students do not have problems with the correspondence; rather, they misinterpret the "How many more ... than ...?" construction. The existence of this alternative explanation forms the basis for this study.

The article reports on three interrelated experiments.

Experiment 1

3. Research Design and Procedures

The subjects were 28 first-grade children. There were two tasks ('More' and 'Won't Get') involved in this expériment. The materials consisted of two series of eight cards, each containing two sets of objects (e.g., birds and worms, kids and bikes) representing numbers of different size. The items in the two sets were arranged so that, there was no obvious visual pairing for the sets, and the largest number was always on the left. In the 'More' task the question to the subject was phrased in the form "How many more ... than ...?", while in the 'Won't Get' task the question was of the form "How many ... won't get ...?"

Each subject was tested individually on the two tasks. One-half received the 'More' task first, while the other was given the 'Won't Get' task first. Thus, each subject received a total of 16 items.

Each response was scored as either correct, absolute (meaning the subject indicated that the largest number was the correct solution), or as a processing error (any other response). Overall, a subject was said to respond correctly if he or she had at least six correct responses on the task.

4. Findings

On the 'Won't Get' task 100% of the subjects responded correctly, while only 64% responded correctly on the 'More' task (significant at the 0.001 level - sign test). Furthermore, it should be noted that the order of presentation did not affect the performance.

Experiment 2

3. Research Design and Procedures

The subjects were 12 nursery and 24 kindergarten children. There were three tasks involved in the experiment ('More', 'Won't Get', and 'Comparative-Terms'). There were two subtasks in the 'Comparative-Terms' task ('Sets displayed' and 'Sets not displayed'). In the 'Sets displayed' task each drawing consisted of two vertical stacks (four items) or two horizontal rows (four items), arranged in such a way as to highlight visually the appropriate one-to-one correspondence. In the 'Sets not displayed' task each card contained just a pair of numerals, with the larger numeral always in the upper left-hand corner and the smaller in the upper-right hand corner. Below each numeral was a picture appropriate to the question to be asked (e.g., hand-drawn face). In the 'Set displayed' subtask eight questions were asked: four of the form "How many more ... than ...?"; two of the form "How many ... taller than ...?"; and two of the form "How many ... longer than ...?". In the 'Sets not displayed' subtask eight questions were asked: two of each of four forms (e.g., "How many years older is ... than ...?";"How many more is ... than ...?").

Each subject was tested individually on the three tasks with the order of tasks being counterbalanced across subjects. The 'Comparative-Terms' task questions from each of the subtasks were alternated. The scoring and administrative procedures were parallel to those of experiment 1.

4. Findings

Eighty-three percent of the nursery children and 96% of the kindergarten children responded correctly on the 'Won't Get' task, with 17 and 25 percent being the corresponding results on the 'More' task. Every subject who responded correctly on the 'More' task responded correctly on the 'Won't Get' Task. The 'More' task was significantly more difficult than the 'Won't Get' task (significant at the 0.001 level - sign test).

The subjects performed badly on the 'Comparative-Terms' task. The percentages of correct responses for each of the appropriate comparative adjectives were 26% (use of more in 'Sets displayed' subtask); 27% (use of taller/longer in 'Sets displayed' subtask); 28% (use of more in the 'Sets not displayed' subtask); and 29% (use of older in the 'Sets not displayed' subtask).

Experiment 3

3. Research Design and Procedures

The subjects were 30 kindergarten children. There were two tasks involved in this experiment, a 'Conservation' task and a 'Numerical Difference' task. The 'Conservation' task was of the classical form involving two sets of equal numbers. The sets are initially spread out identically in two parallel rows, then one row is spread out so that the visual impression is of a row with one more chip than the other. The 'Numerical Difference' task involved a series of nine drawings of different objects (e.g., birds and worms, dogs and bones). Each drawing contained two sets, with the larger set being on the left and arranged vertically and the smaller set on the right and being arranged horizontally. In order to eliminate length as a clue to the solution the distance between objects was different between items.

Each subject was tested individually on the two tasks. The mode of questioning for the 'Numerical Difference' task was the same as that used in the 'Won't Get' tasks of experiments 1 and 2. However, a correct response on the first one or two items was positively reinforced if the child seemed unsure.

Each response on the 'Numerical difference' task was coded as either correct or incorrect and according to one of three strategies: (i) the pairing strategy involved drawing imaginary lines for one-to-one correspondence; (ii) the covering counting strategy involved a subject either covering a subset of the larger set equivalent to the smaller set or equal in size to the difference between the sets; and (iii) the counting strategy. There are two strategies referred to under the label 'counting strategy': namely, counting whole numbers (subject counts both sets) and counting out an equivalent subset (counts off a subset equivalent to the smaller set from the larger set and states the difference).

4. Findings

Two hundred eighteen (81%) of the numerical difference items were answered correctly, and for these 218 responses a solution strategy was observed in 97 cases. The pairing strategy was used 22 times, while the dominant strategy was counting out an equivalent subset (57 times). Twenty subjects used an observable strategy at least once and 15 used the counting out an equivalent subset. The author notes that posttest questioning indicated that the counting strategy had been used by several of the children for whom no observable strategy had been reported during the experiment.

5. Interpretations

In discussing the various experiments, the author draws the following major conclusions:

a) The evidence of these experiments suggests that difficulties with "How many more ... than ...?" do not arise from lack of appropriate correspondence skills, but involve misinterpretations of comparative structures. The author draws this conclusion from the fact that subjects can answer the 'Won't Get' task and have difficulty with both the 'More' and the 'Comparative-Terms' tasks.

b) "... young children's understanding of correspondences and numerical differences cannot be viewed as consisting merely of perceptually driven role procedures" (p.89). They use sophisticated counting strategies that indicate they comprehend correspondence.

Abstractor's Comments

The question of language and how it related to childrens' answers is an important question for mathematics educators. Whether problems are related to the mathematical concepts involved or the syntatic structure of the questions has major implications for the teaching of mathematics. While an investigation of the "How many more ... than ...?" style question is not new, the author provides strong evidence for the view that incorrect responses to this question cannot automatically be utilized as evidence of a one-to-one correspondence problem. As the author notes, such a result is consistent with other recent research. This finding should lead mathematics educators to undertake further research designed to investigate the relationship between mathematics and language (a direction which is currently attracting attention).

While the study was well planned and reported, there were some points at which additional information would have been useful.

1. In experiment 3 the author indicated that in only 97 of the 218 correct responses was he able to determine the strategy. Why? How were the subjects approaching the problem? Did they just pick the correct answer 'out of the air'? Information on this would have provided more insight into the sophistication of the subjects. It may be that there is a strategy or strategies other than the three identified.

2. The author mentions that a 'Conservation' task was given in experiment 3, but provides no information on the results. Such information would have helped a reader in determining the role of conservation in interpreting the results.

3. The results indicate that subjects often chose the absolute answer as the incorrect response. Why? Does this shed light on how the subject is interpreting the problem? At one point in the paper the author indicated that a posttest interview was carried out. The author might have further developed this area in the report.

Marshall, Sandra P. SEX DIFFERENCES IN MATHEMATICAL ERRORS: AN ANALYSIS OF DISTRACTER CHOICES. Journal for Research in Mathematics Education 14: 325-336; November 1983.

Abstract and comments prepared for I.M.E. by JOANNE ROSSI BECKER, San Jose State University.

1. Purpose

This study investigated the errors made by sixth-grade boys and girls on individual items of a mathematics test to determine how boys' and girls' errors differ.

2. Rationale

Most studies of gender differences in mathematics achievement have focussed on total test scores or on scores of subtests of particular types of questions. Studies of the interaction of gender and item type have rather consistently found girls performing better in computational skill and boys in verbal problem solving. However, little information has been gathered comparing the approaches boys and girls use in solving problems, and the errors they make. Such detailed information would be helpful in designing instruction to help students.

3. Research Design and Procedures

The subjects were all sixth-grade students who took the Survey of Basic Skills through the California Assessment Program in the years 1976-1979, a total of over 1.2 million children, approximately half boys and half girls. The test has 16 forms, each of which includes 10 mathematics items. These 160 items test the strands of measurement and graphing, number concepts, whole number arithmetic, fraction arithmetic, decimal arithmetic, geometry, and probability and statistics.

Data analysis consisted of two parts: the first to check the stability of boys' and girls' errors, the second to identify and classify those errors. To investigate stability, a three-way contingency table was constructed for each item, using sex by distracter by year. Correct responses were excluded. Because items had either four or five alternatives, excluding correct responses but including no responses led to either 2x4x4 or 2x5x4 contingency tables. These were analyzed by multi-way loglinear extensions of a chi X^2 test of association containing both main effects and interaction effects. The three main effects, three two-factor interaction effects, and the one three-factor interaction effect were used to form nine models: one with main effects only; three with the main effects plus one two-way interaction; three with the main effects plus pairs of the two-way interactions; and the full model with the three main effects plus the three two-way interaction effects, plus the three-way interaction. Each test item contingency table was analyzed by a chi X^2 likelihood ratio test to determine which of the nine models fit the data for that item best.

The second part of the analysis classified student errors into five categories proposed by Radatz: semantics, spatial visualization, mastery, association, and use of irrelevant rules. The author developed hypotheses for the reasons for errors in each category, using a set of criteria to specify at least one distracter for each item. The same item was included in several hypotheses if it had distractors that met the criteria for those hypotheses. Using only boys and girls who erred on an item, the items which met each hypothesis then were grouped depending on whether girls or boys had a higher probability of that specific error type. A sign test was used to test the significance of the difference in the number of items in each group for each hypothesis.

4. Findings

In the analysis of the models of responses, it was found that the model with the two-factor interaction effects of distracter by sex and distracter by year fit the data best for 80% of the items. Another 8% of the items had data best fit by one of the three other models including the distracter by sex interaction effect but not the three-way interaction of distracter by sex by year. Thus, there were consistent sex differences in all but 19 of the 160 items.

The error classification analysis found that girls were significantly more likely to make: spatial errors of scale on a graph or figure; errors of mastery involving choice of operation; errors of association involving transfer in which a number pattern is incorrectly applied; errors in key word association; and errors of use of the irrelevant rule of selecting the smallest value. Boys made significantly more errors in only one category, perseverance, or persisting to the end of a computation. The other fourteen hypotheses showed no significant sex differences in selection of distracters.

5. Interpretations

The author concludes that there were consistent sex differences in the children's errors and that these were stable over the years. The interaction between distracter and sex shows that girls and boys selected different answers, and that this pattern applied equally to the different strands of items.

Concerning the type of errors made, the author concludes that girls make more errors due to the misuse of spatial information, use of irrelevant rules, choice of incorrect operation, negative transfer, and key word association; boys are more likely to make errors of perseverance and formula interference. Both sexes made language-related errors but of a different kind.

Abstractor's Comments

This study represents an interesting approach to investigating gender differences in mathematics achievement by examining in detail the errors that children make. Although the author seemed interested in how students attempt to solve problems as well as in the errors they make, I think this study really sheds light only on the latter issue. I think the paper left a few questions unanswered on that aspect as well.

First, some details were left out about the test. Presumably, the same 160 items were used each of the four years, and each test form sampled the seven strands among its 10 items.

In the error classification, it was unclear how the hypotheses to explain errors were developed. Were these hypotheses formed logically from examining the distracters? Or were the test constructors consulted to determine how they chose the distracters in developing the items? Also, how were the criteria used to specify the nature of the distracters developed? To the author's credit, a sample item is included in the article to illustrate each hypothesized error type. The hypotheses seem reasonable but presumably are not exhaustive.

I found myself very curious as to how many items actually showed sex differences. I understand the author's desire to examine errors only, but I assume one reason to look for sex differences in errors is to determine why there are sex differences in overall achievement.

Does the fact that the model that fit most items included both distracter-by-sex and distracter-by-year interactions mean anything about the choice of distracters from one year to another?

I am a little puzzled by the author's conclusions concerning gender differences in errors, which go a bit beyond the significant findings.

However, I would have liked a little more discussion of the significantly different errors. Do these relate at all to other findings of sex differences in test-taking? Is there any evidence in these data to support other findings that girls are less willing to take risks in answering questions? Do they help at all to explain overall achievement differences?

How was overall error rate controlled for the number of tests of significance done?

These questions are relatively minor, however, compared to the interesting questions this study raises. The author's main question, how girls and boys attempt to solve problems, remains unanswered, and would be best answered by interviewing children to determine what processes they use. This study provides some aspects on which to focus, identified by the different errors boys and girls were found to have made.

Matthews, Julia. A SUBTRACTION EXPERIMENT WITH SIX AND SEVEN YEAR OLD CHILDREN. Educational Studies in Mathematics 14: 139-154; May 1983.

Abstract and comments prepared for I.M.E. by JAMES M. MOSER, Wisconsin Department of Public Instruction, Madison.

1. Purpose

To investigate the effects of two short teaching programs on 6- and 7-year old children's ability to solve verbal subtraction problems dealing with "take-away" and comparison situations and to write correct number sentences to represent those problems.

2. Rationale

Earlier research by the author had suggested difficulties in understanding the various interpretations of subtraction in subjects much older than the present group of 6- and 7-year olds. These younger children are at the interface between pre-operational and early concrete operational thought and the author wanted to investigate the possiblity of helping them forward from one stage to the next by means of specific teaching programs.

3. Research Design and Procedures

One hundred seventy-six young children from four schools in the greater London, England area were initially screened by two verbal subtraction problems, one on "take-away" and the other on comparison. Throughout the experiment, number size was restricted to sums less than 10. Those who could not answer both successfully were given teaching program A, which consisted of five individually administered verbal lessons on how to solve the two types of problems. Forty-four children who had originally passed the two screening items and 30 who passed the post test of program A were assigned to program B, half experimental and half control. Program B consisted of five verbal problems dealing with addition and subtraction, each given in two parts. The first part required a verbal solution and the second part required the writing of a number sentence. These five two-part problems were administered as a pre- and posttest. In the interval between pre- and posttests, the experimental group received individual instruction on interpretation and representation of various verbal subtraction situations.

4. Findings

Children assigned to the experimental group in program A performed significantly better on a posttest. In program B, results were mixed. On questions dealing with "take-away" subtraction and simple addition, both groups performed equally well, pre and post, though less successfully on writing than on solving. On the remaining items (two with comparison and one with missing minuend), when performance was better on the posttest, it favored the experimental group. In general, performance for all subjects was better on solving than on symbolic writing.

5. Interpretations

It is possible to assess a level of understanding in subtraction quickly and accurately. Through individualized teaching programs it is possible to teach prerequisite skills needed to establish a sound foundation for subtraction. However, an attempt to symbolize too soon is a danger that should be avoided.

Abstractor's Comments

A person who wishes to read this article in its complete form should be warned that there are errors in several of the tables of data. Aside from that, I would say that the experiment appeared to be well executed. The author is to be commended for her industriousness in that she did all of the individualized teaching herself. However, let me express a few concerns. First, I wish she had reported process as well as product. In other words, it would be interesting to know how the children solved the verbal problems. Second, I feel there is some weakness in the reported results because all administrations of a particular problem type were given with the same number pair assigned to a problem statement. To rule out any possible experimental effect due to a particular number pair, the author should have randomized assignment of number pairs.

Finally, a major conclusion was that not all children of age 6 or 7 are ready for symbolizing. This conclusion was based on her inability to get improved performance on symbolic representation from her experimental group's participation in program B teaching activities. While that may well be a true conclusion, I am not sure it is valid on the basis of the author's experiment. The experiment was too short and the author has not ruled out the possiblity confounding effect of prior instruction in writing number sentences.

Steinberg, Esther R. PROBLEM COMPLEXITY AND THE TRANSFER OF STRATEGIES IN COMPUTER-PRESENTED PROBLEMS. American Educational Research Journal 20: 13-28; Spring 1983.

Abstract and comments prepared for I.M.E. by WILLIAM H. KRAUS, Wittenberg University.

1. Purpose

Two studies are described in the report. The first examined the effects of experience in solving small- or intermediate-size problems on performance in solving large-size problems. The second study examined the effects of experience in solving problems set in a familiar or unfamiliar context on performance in solving problems in another unfamiliar context.

2. Rationale

Problems are easier to solve if they involve a small number of components or if they are set in a familiar context. If solving such problems produced strategies that were transferable to more difficult problems, then these simpler problems could be used to facilitate instruction in solving the more difficult problems. If, on the other hand, solving such problems resulted in only size- or context-specific strategies, then use of the simpler problems in instruction would be less effective.

3. Research Design and Procedures

All problems were presented through the computer-assisted instruction system, PLATO. Three variations of Mastermind-type problems were used.

In the first variation, Pico-fomi, a 2-, 3-, or 4-digit number was randomly chosen by the computer; the digit "9" was not used in the 4-digit problems and the digit "0" was not used in any of the problems. The subject was asked to guess the number, then was told (1) how many digits in the guess were correct digits in the correct place and (2) how many were correct digits but in the wrong place. The subject was then asked to guess again, the process continuing until the number was correctly guessed or eight guesses were used.

In the second variation, Meters, subjects were shown eight (or nine) circular meters with two, three, or four possible settings. In the three-setting version, the subject is to determine, through a guessing system similar to that in Pico-fomi, which meter is set to "a," which is set to "b," and which is set to "c." This is equivalent to Pico-fomi, but gives the appearance of being more complex.

The third variation, Robots, was used only in the second study. It closely resembles Meters, and was presented using a touch-sensitive screen.

<u>Experiment 1</u>. Subjects were 100 college students who responded to an advertisement and who had no previous experience with problems similar to those used in the study. Subjects were paid.

A randomized 2 x 2 factorial design was used. In the resulting four treatments, subjects attempted to solve 15 problems of either 2- or 3-digit size and either Pico-fomi or Meters context.

In a second session, a week later, subjects attempted to solve 10 problems of 4-digit size in the same context they had encountered in the first session. The dependent variables were the number of problems solved and the mean time spent on each problem. A 10-item questionnaire was administered at the end of the second session to gather information about problem-solving strategies used.

Experiment 2. Subjects were 140 college students and military personnel selected in the same way as in Experiment 1.

A randomized 3 x 2 factorial design was used. In the resulting six treatments, subjects attempted to solve 10 problems of 2-, 3-, or 4-digit size and either Pico-fomi or Meters context.

In a second session, subjects attempted to solve 10 problems in the Robots context and of the same size they had encountered in the first session. As in Experiment 1, the dependent variables were the number of problems solved and the mean time per problem, and a 10-item questionnaire was administered.

4. Findings

Analysis of variance was used for most statistical analyses in both studies.

Experiment 1. During the first session (treatment), the number of problems solved was high for all four groups, ranging from 13.7 to 14.5 out of 15 problems. The means in the Meters context were significantly higher ($p < .05$) than in the Pico-fomi context. Mean time per problem in the Pico-fomi groups was 30 and 96 seconds for the 2- and 3-digit groups, respectively, and in the Meters groups was 55 and 134 seconds for the 2- and 3-digit groups, respectively. Both Meters groups spent significantly more time per problem ($p < .01$) than the corresponding Pico-fomi groups.

In the second session (transfer problems), the mean number of 4-digit problems solved by subjects who had worked on 3-digit problems during the first session was significantly greater ($p < .05$) than the mean number solved by subjects who had worked on 2-digit problems. Mean time per problem was not affected by the size of problem in the first session.

Experiment 2. During the first session (treatment), the mean number of problems solved was significantly lower ($p < .01$) for the 4-digit groups than for the 2- or 3-digit groups. Similarly, the mean time per problem was significantly higher ($p < .01$) for the 4-digit groups than for the 2- or 3-digit groups.

During the second session (transfer problems), the mean number of problems solved was significantly lower ($p < .01$) and the mean time per problem was significantly higher ($p < .01$) for the 4-digit groups than for the 2- or 3-digit groups. There were no differences in performance due to differences in context experienced in the first session.

5. Interpretations

The report presents three main conclusions. First, experience solving intermediate-sized problems helped improve performance on large-size problems more than experience solving small-size problems. Second, the context of the problems presented initially (either familiar or unfamiliar) had no effect on performance on the transfer problems. Third, since subjects generally performed better on the problems set in an unfamiliar context (the Meters problems), the greater apparent difficulty of these problems may have forced subjects to spend more time on the problems, thus improving their performance.

The report hypothesizes that when preparing students to solve complex problems set in unfamiliar contexts, instruction should begin with intermediate-size problems in the same context and then proceed to direct instruction in the larger problems.

Abstractor's Comments

The researcher is to be commended for carefully conducted studies and a clearly written report. The rationale for the studies is convincingly presented, the design is sound and is reported in sufficient detail for the reader to judge its merits, illustrations and tables supplement the text well, and the conclusions are generally supported by the data. The report is concise and complete.

The major conclusion of the report, that performance on large-size problems is improved more by experience with intermediate-size problems than by experience with small-size problems, needs to be refined if it is to be generally useful. How is size of a problem to be defined, and how do we determine the point at which increasing the size of a smaller problem produces diminishing returns? Since subjects in both studies were generally successful with both small- and intermediate-size problems, is the most effective size problem the largest possible that can be solved with a level of effort clearly within the abilities of the solver? (The relative difficulty of the 2-, 3-, and 4-digit problems is not addressed in the report. Since the progressive difficulty would appear to be geometrical or exponential, the 4-digit problems may have been too difficult.)

In the second study, there was no effect on the transfer task from the context of the treatment tasks. It is not clear from the research why this is the case. A 3 x 3 factorial design would have allowed the researcher to determine whether both contexts in the treatments worked equally <u>well</u> or equally <u>poorly</u>; either a posttest-only group or, more interestingly, a "Robots context" group would have clarified the matter. The dissimilarity between the Pico-fomi context and the Meters context and the similarity between the Meters context and the Robots context appear to be sufficient to have caused an effect on performance on the transfer task, leaving the reader (and quite possibly the researcher) at a loss to explain the lack of effect.

The finding of both studies that subjects generally performed better on the more complex-appearing Meters problems than on the equivalent, but simpler-appearing, Pico-fomi problems raises many questions and may prove to be a fruitful area for future studies.

Steffe, Leslie P. CHILDREN'S ALGORITHMS AS SCHEMES. Educational Studies in Mathematics 14: 109-125; May 1983.

Abstract and comments prepared for I.M.E. by DOUGLAS H. CLEMENTS, Kent State University, Kent, Ohio.

1. Purpose

The purpose was to present the hypothesis that counting abilities are "the coordinated schemes of actions and operations the child has constructed at a particular point in time" (p. 109), to distinguish between figurative schemes such as intuitive extension and operative schemes such as numerical extension, and to present illustrations of and evidence for these hypotheses in the form of qualitative analyses of the work of two children involved in a teaching experiment.

2. Rationale

Evidence is reviewed that many primary grade students have difficulties in developing arithmetical operations and, more importantly, that many construct their own conceptions of these operations. Unfortunately, these concepts are often unrelated to standard computational algorithms. Steffe argues that this lack of connection is not attributable to the educational methods utilized, but to the preceding developmental levels at which the concepts are formed. From this constructivist perspective, he states that it is critical to understand children's mathematical concepts and methods because they are the foundation of sophisticated methods.

The hypothesis posed in the article is that children's methods should be viewed as schemes. Schemes are defined as organized patterns of behavior consisting of three parts: (a) an initial trigger or occasion; (b) an action or operation, which might be conceptual; and (c) a result of the activity. Schemes can be figural or operative. The latter must involve mental operations, interiorized actions, or actions carried out mentally.

Counting is viewed as the second part of a scheme. In counting, the child coordinates two actions: the production of the number word sequence (e.g., saying or thinking "one, two, three...") and the production of a unit item (for a description of the latter, see Steffe, von Glaserfeld, Richards, & Cobb, 1983). The trigger for counting is an awareness that the numerosity of a group is not known. This "group", or composite unity, is also constructed by the child as separate items are intentionally combined in thought, a process termed integration. The third part of the scheme, the result, is the production of the numerosity of the group.

3. Research Design and Procedures

Six 7-year-old children were interviewed as a part of a two-year teaching experiment. The counting procedures of two of these children were analyzed in this article; one child was hypothesized to possess an operative counting scheme, the other a figurative counting scheme.

4. Findings and Interpretations

Operative counting scheme: Scenetra. Several protocols are offered as evidence that Scenetra displayed the mental operation of integration. For example, presented with 12 tiles, 7 of which were covered, Scenetra was told which tile was the ninth one and was asked how many were covered. She pointed to the ninth tile for five seconds with her hand over her eyes. Then she pointed to the end of the cloth cover and said, "Seven." Asked how she knew, she replied, indicating the eighth tile, "'eight and then seven' (pointing over the cloth again)" (p. 113). That Scenetra counted backward and understood that "seven" referred to all the covered tiles was taken to indicate that each of the number words preceding "nine" implied the number words that preceded it. In other words, Scenetra understood that "seven" implied seven counting acts, and thus the cardinal number of the covered tiles (and therefore performed an integration).

Similarly, in solving a missing addend task, 11 + _ = 19, Scenetra stared at the blank space, then put up eight fingers one at a time while whispering number words. Because she did not start with "one," it was concluded that the "11" implied all the preceding counting acts for Scenetra. Awareness of an unknown numerosity triggered the counting, which was part of a scheme for finding sums. Raising fingers in correspondence with saying "twelve, thirteen,..." is an example of the scheme of numerical extension. Scenetra realized ahead of time that she could use the counting scheme to determine the unknown numerosity, and she purposely planned to record the counting acts. This "knowing in advance" is taken as further evidence that Scenetra's numerical extension was an anticipatory, operative scheme. Because of this, and the improbability that it had been modeled by an adult, it is also an example of a child-generated algorithm. An additional protocol illustrates Scenetra's incorporation of a new unit, ten, into her numerical extension scheme. Asked to add 35 and 26, she partitioned 35 into 30 and 5, and 26 into 20 and 6, and then counted the decades and the digits separately,

Steffe concludes that understanding such operative schemes is critical for understanding children's arithmetical knowledge. Instead of viewing children's procedures merely as a basis for teaching mature--that is, standard--forms of arithmetical algorithms, he suggests that the mature forms themselves should be reorganizations of children's operative schemes. The important point is not that the child-generated algorithms can aid in the construction of standard algorithms, but that they should be allowed to develop into increasingly sophisticated schemes.

Figurative counting scheme: James. The case studies illustrate that children can complete arithmetical tasks without possessing the mental operation of integration. A typical example of this is children's propensity to start counting at "one" even when the first few items of a collection are covered and their numerosity known.

James was given sets of eight and four, revealed and then covered, and asked to tell how many there were altogether. He touched the first cloth eight times counting, "one, two,..., eight," moved to the other cover, and uttered, "nine, ten, eleven, twelve" in synchrony with touching a square pattern associated with "four." He did not perform an integration, but started counting at "one."

That James continued to count past "eight" revealed his ability to perform an intuitive extension. James extended his counting, using a subscheme of pointing in a square pattern to keep track of how many times he counted on past eight. This type of figural pattern was necessary; James invariably lost count whenever he did not use one. But because figural structures (e.g., a triangle for three, domino pattern for five) are not related to each other (i.e., not subpatterns of one another), James could not be aware of the structure of the counting activity in advance. Therefore, unlike Scenetra, he could not anticipate the results of the counting.

In another task, James realized that a multibase "long" represented ten; however, to find out how many blocks were in two multibase longs, he counted from "one." Steffe concludes that this lack of accommodation in his counting scheme implies that the imposition of a standard algorithm for two-digit addition could result only in James constructing a non-arithmetical, "brittle" algorithm. Thus, James' intuitive extension lacked the anticipatory nature and the accommodation characteristic of Scenetra's numerical extension.

Abstractor's Comments

The work of Steffe and his colleagues has made a major contribution to our understanding of young children's counting and related arithmetical knowledge. The present article offers an interesting slice of this work. The nature of the study--limited protocols from only two children--makes writing a critique difficult. However, it can be said that the discussion includes insightful and important observations on children's arithmetical thinking.

The theoretical foundation for the work is the constructivist theory of Piaget. As with Piaget, perhaps the most important insight teachers can gain from exposure to Steffe's observations and perspective is also the most elusive: Children create their own knowledge and understandings. Steffe cites a 1670 passage from Caramuel: "The intellect...does not find numbers but makes them" (p. 111). More poetically, "He who is versed in the science of numbers can tell of the regions of weight and measurement, but he cannot conduct you thither...If he is indeed wise he does not bid you enter the house of his wisdom, but rather leads you to the threshold of your own mind" (Gibran, 1923/1981, pp. 56-57). Fully understood, this idea could radically change a teacher's approach to mathematics.

Similarly, the suggestion that schemes should be viewed as comprising a critical component of children's mathematical knowledge is difficult to deny. In addition, it is important to realize that one cannot assume that children who appear to use similar methods are operating at the same level. However, Steffe also suggests that the construction of mature algorithms should be de-emphasized; instead, children's algorithms "should be nurtured and allowed to grow into increasingly powerful and sophisticated schemes" (p. 119). It has yet to be shown that this pedagogical method will result in efficient mastery of formal mathematics, or even that mathematics instruction should be altered to replicate children's spontaneous development (cf. Carpenter, 1980). Nevertheless, the hypothesis certainly has not been disproven, and there is a dire need for more research in this area. Steffe's research provides a balanced picture of both the untapped potential of children's constructivity and the difficulties in helping them acquire new knowledge.

As mentioned, this work is rooted in the epistemological formulations of Piaget. As with Piaget's writings, it offers rich, intricate, and fascinating notions; however, it is occasionally difficult reading. More important, it is difficult to construct empirical tests which would permit differentiation between alternate theoretical explanations for the children's behaviors. The parallels with Piaget's work also suggest that differences in task variables may greatly affect children's responses and strategies. A related problem is that research has consistently found exceptions to the categorizations and predictions of any single theory of children's development of arithmetical knowledge.

In a similar vein, the basis for the differentiation between constructs is not easily seen. For example, even though James came to curtail his counting of the first collection of objects, "the quality of the curtailment was not numerical. It could easily be explained by curtailment in uttering number word sequences rather than by reflective subtraction" (p. 122). If this is so, it is not absolutely clear how the operation of integration could reliably be attributed to one child and not another.

Because much of the article consists of exposition of a theoretical position, it is impossible to summarize it fully; those interested should read the original article. In addition, readers unfamiliar with this line of work may find it beneficial to consult other writings (e.g., Steffe et al., 1983). For example, the concept of the "production of a unit item" is not elaborated in this article, although it is essential to the main hypotheses.

It is interesting to speculate on possible relationships between the theory presented in this article and other theories, such as information-processing perspectives. The triarchic definition of a scheme corresponds neatly to the information-processing structure of input-processing-output. Observations of children's awareness of the structure of counting activities, along with the constructs of "hindsight" and "anticipation," suggest parallels to the componential theory of Sternberg (1984). That Scenetra knew in advance of her plan to use a counting procedure and of the results of that procedure strongly suggests the presence of functioning metacomponents or executive processes. The observation that Scenetra "could compare the progress she made within a scheme much better than she could compare across schemes" (p. 118) suggests differential strengths of various metacomponents (Sternberg, 1984).

The insights provided stem from a compelling theory. Educators should look forward to additional applications and tests of the theory, especially research which permits discrimination between alternate explanations and generalization to a wider population.

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Abstract and comments prepared for I.M.E. by BARBARA J. PENCE, San Jose University.

1. Purpose

The purpose of this study was to examine the (a) dependence that mathematics achievement and attitude have on each other, and (b) the dependence of mathematics achievement and attitude on experience, sex, race/ethnicity, home environment, father's education, and mother's education.

2. Rationale

The influence of productivity factors (student background and experience) on achievement and attitude is inconsistent, but seems to vary as a function of the size and age of the sample and the measures used. Productivity factors most closely connected with learning include the nine categories of: student age, student ability, student motivation, amount of instruction, quality of instruction, the psychological environments of the home and school, peer-group outside the school, and exposure to mass media, particularly television (Walberg, 1981). Analysis of the National Assessment of Educational Progress (NAEP) survey of high school students yielded significant factors accounting for 59% of the variance in mathematics achievement. Analysis of the NAEP data for 13-year-olds was seen as a necessary link in the attempt to develop a consistent synthesis and generalization across existing information.

3. Research Design and Procedures

Analysis of variance and multiple regression analysis were computed using NAEP data collected in 1977-1978 on a sample of 2,368 13-year-olds. NAEP employed a multistage sampling design with over-sampling of low-income and rural areas. Eight NAEP scales were used to define the variables. A brief summary of each scale including, when appropriate, alpha internal consistency reliabilities is as follows:

Achievement	74 mathematics achievement items; reliability = .92; coded as correct = 1 and incorrect = 0.
Attitude	14 items; reliability = .25; coded according to five choices ranging from strongly disagree to strongly agree.
Experience	8 items surveying student experience with the calculator and the metric system; reliability = .16; coded according to four options.
Home	4 items surveying the reading material available in the home; reliability = .44.
Sex	
Race/Ethnicity	
Father's education	
Mother's education	

4. Findings

Results of the F test for the null hypothesis were (a) achievement was significantly related to all of the factors, and (b) attitude was significantly associated with seven of ten factors (each of the four questions on reading material in the home was entered as a separate scale).

When factors were controlled for one another in the general linear model, the results were as follows:

F Values and R^2 for Achievement and Attitude Regressions

Achievement	F Value	R^2
Sex	5.83**	.323***
Ethnicity	84.68***	
Father's education	11.17***	
Mother's education	6.96***	
Home environment	10.46***	
Experience	1.98*	
Attitude	24.89**	
Attitude		
Sex	3.20*	.082*
Ethnicity	3.36**	
Father's education	.17	
Mother's education	1.53	
Home environment	2.34*	
xperience	2.48**	
Achievement	25.62**	

* = $\alpha < .10$

** = $\alpha < .05$

*** = $\alpha < .01$

The set of first-order interactions was insignificant.

5. Interpretations

These results, even with the reservations concerning the instrumentation, missing factors, and the instability of responses gained from this age group, support the relationship between student mathematics achievement and productivity factors. All factors proved significant when controlled for one another and for socio-economic status and ethnicity.

One implication is that family background is influential in learning. The stimulation afforded by parents with higher education together with verbal materials such as books and magazines support not only reading but mathematics as well.

In addition, the relation between attitude and achievement may be reciprocal in nature: the more one learns, the higher the attitude, and the higher the attitude the greater the amount one learns.

Abstractor's Comments

The major thrust of this investigation was the analysis of a large-scale data set to aid in the synthesis of previous results. This is a valid justification for use of the NAEP data. The limitations, however, were realized and candidly discussed. Lack of results from this study regarding factors related to attitude were not surprising. It is unclear why the study considered attitude as anything more than a factor in the achievement analysis. In addition to the low reliability of the attitude scale, supporting reviews and research cited failed to address attitude as a dependent variable. On the other hand, in terms of mathematics achievement, this work illustrated the merit of a coordinated research effort.

These analyses appeared to be another step in an ongoing investigation of the productivity factors most closely related to learning. Certainly the NAEP survey investigated only a small set of the productivity factors identified. Definition and measurement of the constructs also created difficulties. The stability of responses gained from this age group was yet another concern. In light of these limitations, it is especially interesting that every factor selected for analysis was significantly related to achievement, and accounted for 32% of the variance.

The statement made by the authors, "whether or not the variables relate to achievement is an experimental question", strongly suggests that these results will be a link to future model-building and research. Further efforts may be able to increase the consistency across construct definitions and measurement. This consistency is a necessary condition for the identification of educational implications.

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