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Additional Vocabulary Support

Solving Systems by Graphing

Complete the vocabulary chart by filling in the missing information.

Word or Word Phrase	Definition	Picture or Example
consistent	A system of equations that has at least one solution is <i>consistent</i> .	$y = -x + 1$ $y = x - 3$ $(2, -1)$
dependent	A consistent system that is <i>dependent</i> has infinitely many solutions.	1. $-6y = 6 + x$ $\frac{1}{6}x + y = -1$
inconsistent	2. A system of equations that has no solution is <i>inconsistent</i> .	$y = 3x - 1$ $y = 3x + 2$
independent	A consistent system that is <i>independent</i> has exactly one solution.	3. $y = 2x + 2$ $y = x - 3$ $(-5, -8)$ is the solution.
solution of a system of linear equations	4. Any ordered pair that makes all of the equations in a system true is a <i>solution of a system of linear equations</i> .	$y = 2x + 2$ $y = x - 3$ $(-5, -8)$
system of linear equations	Two or more linear equations form a <i>system of linear equations</i> .	5. $y = x$ $y = x + 5$

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Think About a Plan

Solving Systems by Graphing

Cell Phone Plans A cell phone provider offers plan 1 that costs \$40 per month plus \$.20 per text message sent or received. A comparable plan 2 costs \$60 per month but offers unlimited text messaging.

- How many text messages would you have to send or receive in order for the plans to cost the same each month?
- If you send and receive an average of 50 text messages each month, which plan would you choose? Why?

Know

- What equations can you write to model the situation?



Cell phone plan #2 cost per month $y = 60$

Cell phone plan #1 cost per month $y = 0.20x + 40$

- How will graphing the equations help you find the answers?

The intersection of the graphs is the point at which the costs of the two plans are equal, based on the number of text messages.

Need

- How will you find the best plan?

Graph the two equations. Use the graph to find which plan is cheaper if the number of text messages is 50.

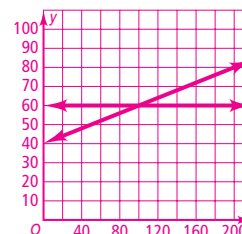
Plan

- What are the equations that represent the two plans? $y = 60$ and $y = 0.20x + 40$

- Graph your equations.

- Where will the solution be on the graph?

The graphs intersect at (100, 60). When the number of text messages is 100, the costs of the two plans are equal.



- What is the solution?

If the number of text messages is 50, choose plan 1, because the cost is lower.

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Practice

Form G

Solving Systems by Graphing

Solve each system by graphing. Check your solution.

- | | | | | | |
|---------------------------------------------|-----------------------------------------------------|------------------------------------------------------|----------|--------------------------------------------|---------|
| 1. $y = -x + 3$
$y = 4x - 2$ | (1, 2) | 2. $y = \frac{1}{2}x - 2$
$y = -3x + 5$ | (2, -1) | 3. $y = \frac{3}{2}x + 6$
$x + y = 1$ | (-2, 3) |
| 4. $y = -5x$
$y = x - 6$ | (1, -5) | 5. $-3x + y = 5$
$y = -7$ | (-4, -7) | 6. $y = -4x - 6$
$y = x + 9$ | (-3, 6) |
| 7. $y = \frac{3}{4}x - 5$
$3x - 4y = 20$ | same graph
means
infinitely
many solutions | 8. $y = \frac{4}{3}x - 3$
$y = -\frac{2}{3}x + 3$ | (3, 1) | 9. $y = -\frac{2}{5}x - 2$
$y = -x - 5$ | (-5, 0) |

10. **Reasoning** Can there be more than one point of intersection between the graphs of two linear equations? Why or why not?
Unless the graphs of two linear equations coincide, there can be only one point of intersection, because two lines can intersect in at most one point.
11. **Reasoning** If the graphs of the equations in a system of linear equations coincide with each other, what does that tell you about the solution of the system? Explain. **If the graphs of two linear equations coincide, then there are infinitely many solutions to the system because every solution of one equation is also a solution of the other equation.**
12. **Writing** Explain the method used to graph a line using the slope and y -intercept. **First use the y -intercept to plot a point on the y -axis. From that point, move one unit to the right and move vertically the value of the slope to plot a second point. Then connect the two points.**
13. **Reasoning** If the ordered pair $(3, -2)$ satisfies one of the two linear equations in a system, how can you tell whether the point satisfies the other equation of the system? Explain. **Substitute 3 for x and -2 for y into the other equation. If the resulting equation is true, $(3, -2)$ is a solution to the equation.**
14. **Writing** If the graphs of two lines in a system do not intersect at any point, what can you conclude about the solution of the system? Why? Explain.
If the lines do not intersect, there is no solution to the system because no ordered pair satisfies both equations.
15. **Reasoning** Without graphing, decide whether the following system of linear equations has one solution, infinitely many solutions or no solution. Explain.
 $y = 3x - 5$
 $6x = 2y + 10$
The system has infinitely many solutions because when you rewrite the second equation in slope-intercept form, it is identical to the first equation.
16. Five years from now, a father's age will be three times his son's age, and 5 years ago, he was seven times as old as his son was. What are their present ages? **father is 40; son is 10**

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Practice (continued)

Form G

Solving Systems by Graphing

17. The denominator of a fraction is greater than its numerator by 9. If 7 is subtracted from both its numerator and denominator, the new fraction equals $\frac{2}{3}$. What is the original fraction? **$\frac{25}{34}$**
18. The sum of the distances two hikers walked is 53 mi, and the difference is 25 mi. What are the distances? **39 mi; 14 mi**
19. The result of dividing a two-digit number by the number with its digits reversed is $\frac{7}{4}$. If the sum of the digits is 12, what is the number? **84**

Solve each system by graphing. Tell whether the system has *one solution*, *infinitely many solutions*, or *no solution*.

20. $y = 3x + 5$
 $x + y = -3$

$(-2, -1)$; one solution

21. $y = 2x + 1$
 $y = -4x + 7$

$(1, 3)$; one solution

22. $2x + y = 8$
 $y = \frac{1}{2}x + \frac{1}{2}$

$(3, 2)$; one solution

23. $y = -2x + 1$
 $y = -\frac{2}{3}x + 5$

$(-3, 7)$; one solution

24. $y = -3x + 2$
 $3x + y = 1$

no solution

25. $y = 5x - 15$
 $y = \frac{3}{4}x + 2$

$(4, 5)$; one solution

26. $y = \frac{1}{2}x - 6$
 $y = -\frac{1}{4}x$

$(8, -2)$; one solution

27. $y = 6x + 4$
 $-2 + y = 6x$

no solution

28. $y = -x - 7$
 $y = 2x + 5$

$(-4, -3)$; one solution

29. $18x - 3y = 21$
 $-y = -6x + 7$

infinitely many solutions

30. $y = 5x - 6$
 $x + y = -6$

$(0, -6)$; one solution

31. $y = -\frac{3}{2}x - 3$
 $y = \frac{1}{4}x + 4$

$(-4, 3)$ one solution

32. The measure of one of the angles of a triangle is 35. The sum of the measures of the other two angles is 145 and the difference between their measures is 15. What are the measures of the unknown angles? **80° and 65°**

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Practice

Form K

Solving Systems by Graphing

Solve each system by graphing. Check your solution.

1. $y = x - 4$ $(0, -4)$
 $y = 3x - 4$

2. $y = -2x + 1$ $(1, -1)$
 $y = x - 2$

3. $y = -3x + 3$ $(2, -3)$
 $y = 2x - 7$

4. $y = x + 3$ $(-1, 2)$
 $y = -4x - 2$

5. $y = -3x + 2$ $(1, -1)$
 $y = 2x - 3$

6. $y = 4x - 11$ $(3, 1)$
 $y = -2x + 7$

7. **Reasoning** If the graphs of two linear equations in a system do not intersect each other, what does that tell you about the solution of the system? Explain.

If the lines are parallel and do not intersect, then there is no solution to the system of equations.

8. **Writing** Describe how to determine the solution of a system of two linear equations by graphing.

Graph both lines on the same coordinate plane to determine where they intersect. The point of intersection is the solution for the system of equations.

9. **Reasoning** Can you determine whether a system of two linear equations has one solution, an infinite number of solutions, or no solution by simply examining the equations without graphing the lines? Explain.

Yes, first solve for y to change the equations to slope-intercept form. If the equations can be simplified to be identical, the lines coincide resulting in an infinite number of solutions. If the slopes of the lines are the same but the y -intercepts are different, the lines are parallel resulting in no solutions. Otherwise, the lines intersect at one point resulting in one solution.

10. **Reasoning** Without graphing, decide whether the following system of linear equations has *one solution*, *infinitely many solutions*, or *no solution*. Explain.

$$8x = 2y - 16$$

$$y = 4x$$

The slopes of the lines are equal but the y -intercepts are not. Therefore, the lines are parallel and the system has no solution.

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Practice (continued)

Form K

Solving Systems by Graphing

11. Right now Seth's age is $\frac{4}{5}$ the age of his brother Eric. Twenty-one years ago, Eric was twice as old as Seth. What are their ages now?

Eric = 35; Seth = 28

12. The sum of two numbers is 62, and their difference is 8. What are the numbers?

35, 27

13. One of the measures of the angles of a triangle is 25° . If the sum of the measures of the other two angles is 155° and the difference between their measures is 5° , what are the measures of the unknown angles?

80° and 75°

Solve each system by graphing. Tell whether the system has *one solution*, *infinitely many solutions*, or *no solution*.

14. $y = -5x + 1$
 $y = -3x - 1$

(1, -4); one solution

15. $y = 2x + 4$
 $y = \frac{1}{3}x - 1$

(-3, -2); one solution

16. $5x + y = -5$
 $10x + 2y - 10 = 0$

no solution

17. $y = 2x - 4$
 $y = \frac{3}{5}x + 3$

(5, 6); one solution

18. $3x - y = -2$
 $y = -\frac{1}{2}x + 9$

(2, 8); one solution

19. $y + 2x = 7$
 $2y - 1 = -4x + 13$

infinitely many solutions

20. **Writing** If two equations represent the same line, what can you conclude about the solution of the equations? Why? Explain.

If the lines coincide, there are an infinite number of solutions that satisfy the system of equations. The equations are identical which means any solution that satisfies one equation will satisfy the other equation.

6-1 Standardized Test Prep

Solving Systems by Graphing

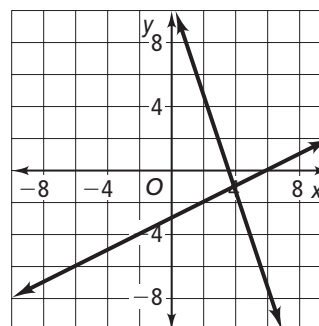
Multiple Choice

For Exercises 1–4, choose the correct letter.

1. Which best describes a system of equations that has no solution? **D**
- | | |
|-----------------------------------|---------------------------------|
| A. consistent, independent | C. consistent, dependent |
| B. inconsistent, dependent | D. inconsistent |

2. How many solutions does this system have? $2x + y = 3$
 $6x = 9 - 3y$ **H**
- F.** 1 **G.** none **H.** infinite **I.** 2

3. What is the approximate solution of the linear system represented by the graph at the right? **D**
- A.** (4, -3) **B.** (6, -1) **C.** (-1, 4) **D.** (4, -1)



4. Which cannot describe a system of linear equations? **G**
- | | |
|---------------------------------|--------------------------------|
| F. no solution | H. infinite solutions |
| G. exactly two solutions | I. exactly one solution |

Extended Response

5. A farmer feeds his cows 200 pounds of feed each day and has 700 pounds of feed in his barn. Another farmer feeds his cows 350 pounds of feed each day and has 1000 pounds of feed in his barn.
- a. In how many days will the two farmers have the same amount of feed left?

2 days

- b. Does your answer make sense? Explain.

The answer makes sense because in two days, both farmers will have 300 lb of feed left.

- c. How would your answer change if both farmers got an additional 1000 pounds of feed?

The graphs would be 1000 units higher, but they would still intersect at $x = 2$.

The answer would be the same: 2 days.

[2] All three parts are answered correctly. Explanations are clear.

[1] Some parts are answered correctly, but explanations may not be complete.

[0] No parts answered correctly.

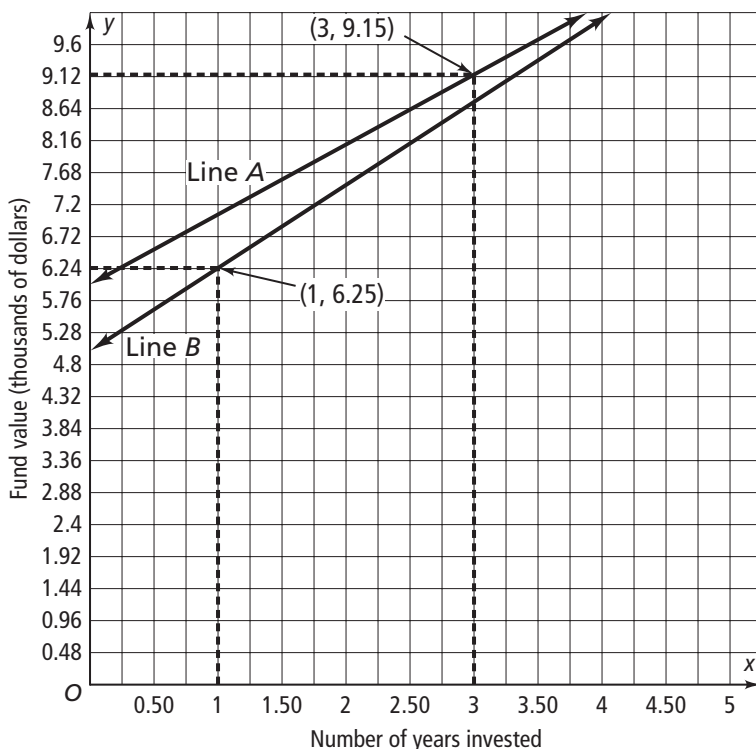
6-1 Enrichment

Solving Systems by Graphing

Using graphs to solve systems of linear equations can be extended to predicting future events by finding the equations of the graphs in slope-intercept form.

Below are models for two investments made over time. Fund A has an initial investment in a mutual fund of \$6,000. Fund B has an initial investment of \$5,000 in a risky mutual fund. By finding the linear equations for A and B, you can predict the value of the investments in future years.

By using the y -intercept and a second point shown for each line, you can find the slope and write each line in slope-intercept form. Fund A is $y = 1.05x + 6$, and fund B is $y = 1.25x + 5$. Knowing this information, you can predict the value of each fund in a particular number of years.



Using the information above, answer the questions below.

1. What will be the value of fund A after 10 years? **\$16,500**
2. What will be the value of fund B after 10 years? **\$17,500**
3. When will the two funds have the same value? **in 5 years (\$11,250)**
4. What would be the combined value of the two funds after 15 years? **\$45,500**

6-1

Reteaching

Solving Systems by Graphing

Graphing is useful for solving a system of equations. Graph both equations and look for a point of intersection, which is the solution of that system. If there is no point of intersection, there is no solution.

Problem

What is the solution to the system? Solve by graphing. Check.

$$x + y = 4$$

$$2x - y = 2$$

Solution

$$y = -x + 4$$

$$y = 2x - 2$$

$$y = -x + 4$$

$$0 = -x + 4$$

$$x = 4$$

$$y = 2x - 2$$

$$0 = 2(x) - 2$$

$$2 = 2x, x = 1$$

Put both equations into y -intercept form, $y = mx + b$.

The first equation has a y -intercept of $(0, 4)$.

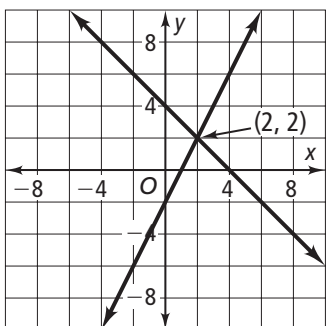
Find a second point by substituting in 0 for y and solve for x .

You have a second point $(4, 0)$, which is the x -intercept.

The second equation has a y -intercept of $(0, -2)$.

Find a second point by substituting in 0 for y and solve for x .

You have a second point for the second line, $(1, 0)$.



Plot both sets of points and draw both lines. The lines appear to intersect $(2, 2)$, so $(2, 2)$ is the solution.

Check

If you substitute in the point $(2, 2)$, for x and y in your original equations, you can double-check your answer.

$$x + y = 4 \quad 2 + 2 \stackrel{?}{=} 4, \quad 4 = 4 \checkmark$$

$$2x - y = 2 \quad 2(2) - 2 \stackrel{?}{=} 2, \quad 2 = 2 \checkmark$$

6-1 **Reteaching** (continued)

Solving Systems by Graphing

If the equations represent the same line, there is an infinite number of solutions, the coordinates of any of the points on the line.

Problem

What is the solution to the system? Solve by graphing. Check.

$$2x - 3y = 6$$

$$4x - 6y = 18$$

Solution

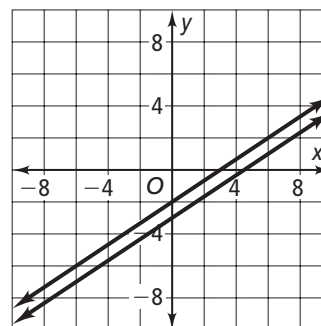
What do you notice about these equations? Using the y -intercepts and solving for the x -intercepts, graph both lines using both sets of points.

$$y = \frac{2}{3}x - 2$$

$$y = \frac{2}{3}x - 3$$

Graph equation 1 by finding two points: $(0, -2)$ and $(3, 0)$. Graph equation 2 by finding two points $(0, -3)$ and $(4.5, 0)$.

Is there a solution? Do the lines ever intersect? Lines with the same slope are parallel. Therefore, there is no solution to this system of equations.



Exercises

Solve each system of equations by graphing. Check.

1. $2x = 2 - 9y$

$$21y = 4 - 6x$$

$$\left(-\frac{1}{2}, \frac{1}{3}\right)$$

2. $2x = 3 - y$

$$y = 4x - 12$$

$$\left(\frac{5}{2}, -2\right)$$

3. $y = 1.5x + 4$

$$0.5x + y = -2$$

$$\left(-3, -\frac{1}{2}\right)$$

4. $6y = 2x - 14$

$$x - 7 = 3y$$

infinitely many solutions

5. $3y = -6x - 3$

$$y = 2x - 1$$

$$(0, -1)$$

6. $2x = 3y - 12$

$$\frac{1}{3}x = 4y + 5$$

$$(-9, -2)$$

7. $2x + 3y = 11$

$$x - y = -7$$

$$(-2, 5)$$

8. $3y = 3x - 6$

$$y = x - 2$$

infinitely many solutions

9. $y = \frac{1}{2}x + 9$

$$2y - x = 1$$

no solution

6-2 Additional Vocabulary Support

Solving Systems Using Substitution

What is the solution of the system? Use substitution. $y = 2x$
 $x + y = -24$

Because $y = 2x$, you can substitute $2x$ for y in $x + y = -24$.

$$x + y = -24$$

Write the second equation.

$$x + 2x = -24$$

Substitute $2x$ for y .

$$3x = -24$$

Simplify.

$$x = -8$$

Divide each side by 3.

Substitute -8 for x in either equation and solve for y .

$$y = 2x$$

Write either equation.

$$y = 2(-8) = -16$$

Substitute -8 for x and solve.

The solution is $(-8, -16)$. Check by substituting $(-8, -16)$ into each equation.

Check	$y = 2x$	$x + y = -24$
	$-16 \stackrel{?}{=} 2(-8)$	$-8 + (-16) \stackrel{?}{=} -24$
	$-16 = -16 \checkmark$	$-24 = -24 \checkmark$

Exercise

What is the solution of the system? Use substitution. $y = 3x$
 $x + y = -36$

Because $y = 3x$, you can substitute $3x$ for y in $x + y = -36$.

$$x + y = -36$$

Write the second equation.

$$x + 3x = -36$$

Substitute $3x$ for y .

$$4x = -36$$

Simplify.

$$x = -9$$

Divide each side by 4.

Substitute -9 for x in either equation and solve for y .

$$y = 3x$$

Write either equation.

$$y = 3(-9) = -27$$

Substitute -9 for x and solve.

The solution is $(-9, -27)$. Check by substituting $(-9, -27)$ into each equation.

Check	$y = 3x$	$x + y = -36$
	$-27 \stackrel{?}{=} 3(\underline{-9})$	$\underline{-9} + \underline{-27} \stackrel{?}{=} -36$
	$-27 = \underline{-27} \checkmark$	$\underline{-36} = -36 \checkmark$

6-2

Think About a Plan

Solving Systems Using Substitution

Art An artist is going to sell two sizes of prints at an art fair. The artist will charge \$20 for a small print and \$45 for a large print. The artist would like to sell twice as many small prints as large prints. The booth the artist is renting for the day costs \$510. How many of each size print must the artist sell in order to break even at the fair?

Understanding the Problem

1. How much will the artist spend to rent a booth? \$510
2. What do you know about selling prices of the prints? small print: \$20; large print: \$45
3. What do you know about the number of prints the artist would like to sell? The artist wants to sell twice as many small prints as large prints.
4. What is the problem asking you to determine? how many of each size print the artist must sell to break even

Planning the Solution

5. What variables are needed? s = number of small prints sold; d = number of large prints sold.
6. What equation can be used to determine the number of prints that the artist would like to sell based on size? $s = 2d$
7. What equation can be used to determine how many prints the artist has to sell to break even? $20s + 45d = 510$

Getting an Answer

8. What is the solution to the system of equations?
The artist must sell 6 large prints and 12 small prints to break even.

6-2

Practice

Form G

Solving Systems Using Substitution

Solve each system by substitution. Check your solution.

1. $x = y$ **(1, 1)**
 $x + 2y = 3$
2. $y = -x + 4$ **(1, 3)**
 $y = 3x$
3. $y = 2x - 10$ **(4, -2)**
 $2y = x - 8$
4. $2y = x + 1$ **(-3, -1)**
 $-2x - y = 7$
5. $x + 2y = 14$ **(6, 4)**
 $y = 3x - 14$
6. $2x - 3y = 13$ **(5, -1)**
 $y = \frac{1}{2}x - \frac{7}{2}$
7. $-3x - 2y = 5.5$ **(-4.5, 4)**
 $x + 3y = 7.5$
8. $6x - 4y = 54$ **(7, -3)**
 $-9x + 2y = -69$
9. $y = \frac{-x}{2} - 4$ **(6, -7)**
 $-2x - y = -5$
10. **Writing** How do you know that substitution gives the answer to a system of equations? Explain.
You can verify your answer by substituting the x- and y- values into the original equations.
11. **Reasoning** With the substitution method, which variable should you solve for first? Explain.
You should solve for a variable that already has a coefficient of 1 or -1.
12. **Writing** How can you use substitution method to solve a system of equations that does not have a variable with a coefficient of 1 or -1?
You can solve for a variable by isolating that term and then dividing by the coefficient.
13. **Writing** When solving the system of equations $\begin{matrix} 6y + 2x = 3 \\ 2x + y = 8 \end{matrix}$ using substitution, which variable will you solve for and which equation will you use to substitute into? **You would solve the second equation for y and then substitute back into the first equation.**
14. **Reasoning** Can you tell that there is no solution for a system by just looking at the equations? Explain and give an example. **If all variables of one equation are multiples of the other, then there is no solution to the system.**
15. If the difference in the side lengths of two squares is 10, and the sum of the side lengths is 18, what are the side lengths? **14 and 4**
16. A shopper purchased 8 T-Shirts and 5 pairs pants for \$220. The next day, he purchased 5 T-shirts and 1 pair of pants for \$112. How much does each T-shirt and each pair of pants cost? **t-shirts: \$20; pants: \$12**

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Practice (continued)

Form G

Solving Systems Using Substitution

17. A student bought 1 box of crayons and 5 reams of paper for \$54. She bought 5 boxes of crayons and 3 reams of paper for \$50. What is the cost of each box of crayons and each ream of paper?

crayons: \$4; paper: \$10

18. Suppose you got 8 mangoes and 3 apples for \$18 and 3 mangoes and 5 apples for \$14.50. How much does each mango and each apple cost?

mango: \$1.50; apple: \$2.00

19. A shopper purchased 4 tables and 2 chairs for \$200 and 2 tables and 7 chairs for \$400. What is the cost of each table and each chair?

tables: \$25; chairs: \$50

20. If the length of the rectangle is twice the width, and the perimeter of the rectangle is 30 cm, what is length and width of the rectangle?

10 cm; 5 cm

21. The population of a city is 2,500. If the number of males is 240 more than the number of females, how many males and females are there in the city?

1130 females and 1370 males

Solve each system by substitution. Tell whether the system has *one solution*, *infinitely many solutions*, or *no solution*.

22. $7x + 2y = -13$

$-3x - 8y = -23$

$(-3, 4)$; one solution

23. $x - 9y = -10$

$6x + y = -5$

$(-1, 1)$; one solution

24. $x = \frac{y}{4} + 1$

$y = 4x - 5$

no solution

25. $x - 2y - 1 = 0$

$y - 5x + 14 = 0$

$(3, 1)$; one solution

26. $y = -8x - 37$

$x + 3y = 4$

$(-5, 3)$; one solution

27. $3x + 6y = 18$

$3y = -\frac{3}{2}x + 9$

infinitely many solutions

28. $5x - 9y = 29$

$12x + y = 47$

$(4, -1)$; one solution

29. $2x = 3y - 9$

$-3x + y = 10$

$(-3, 1)$; one solution

30. $5y = 7x + 22$

$x = -6y + 17$

$(-1, 3)$; one solution

31. $x = 6y + 16$

$9x - 2y = -12$

$(-2, -3)$; one solution

32. $4x - y - 4 = 0$

$3x + 2y - 14 = 0$

$(2, 4)$; one solution

33. $x + 3y = -5$

$-2x - y = 5$

$(-2, -1)$; one solution

6-2

Practice

Form K

Solving Systems Using Substitution

Solve each system using substitution. Check your solution.

1. $x = y$
 $x - 3y = 4$ **$(-2, -2)$**

2. $y = -2x + 5$
 $3y = -x - 5$ **$(4, -3)$**

3. $4y = 5x - 1$
 $3x - 2y = 1$ **$(1, 1)$**

4. $4x - y = -11$
 $y = -\frac{1}{2}x + 2$ **$(-2, 3)$**

5. $2x + 3y = 12$
 $x - 2y = -4.5$ **$(1.5, 3)$**

6. $y = \frac{-x}{4} + 4$
 $x + 2y = 6$ **$(-4, 5)$**

7. **Writing** Explain how a solution found using substitution can be checked.

Substitute the ordered pair into both of the original equations.

8. **Writing** With the substitution method, explain how you find the value of the second variable once you have determined the value of one of the variables.

Substitute the value of one variable into one of the original equations and solve for the other variable.

9. **Reasoning** For the system of equations $x - 2y = -5$
 $2x - 3y = -3$, which variable will

you solve for first? Once you have solved for the first variable, which equation will you use to substitute into? Explain. Solve the system of equations.

Answers may vary. For example, you would solve for x in the top equation first because its coefficient is 1. Then substitute the value of x into the bottom equation and solve for y since the coefficient of x is already 1. The solution is $(9, 7)$.

10. If the difference of two numbers is 43 and the sum of the numbers is 13, what are the numbers? **28 and -15**

11. David earns \$1.50 per hour more than Peter. Together, they earn \$940 if they both work 40 hours in a week. How much money per hour do David and Peter earn?

David earns \$12.50 per hour and Peter earns \$11 per hour.

6-2

Practice (continued)

Form K

Solving Systems Using Substitution

12. The fifth-grade teachers took their classes on a field trip to a museum. The first group had 25 students and two teachers and cost \$97.50. The second group had 32 students and three teachers and cost \$127. What is the cost per student and per teacher?

\$3.50 per student; \$5 per teacher

13. Diana purchased 6 pounds of strawberries and 4 pounds of apples for \$18.90. Then she realized that this was not enough and purchased 3 more pounds of each fruit for \$10.74. What was the cost per pound for each type of fruit?

\$2.29 per lb for strawberries; \$1.29 per lb for apples

14. If the width of the rectangle is three times the length and the perimeter of the rectangle is 72 ft, what are the length and width of the rectangle?

$l = 9$ ft; $w = 27$ ft

15. There are 785 students in the senior class. If there are 77 more females in the class than males, how many male and female seniors are there in the class?

354 males; 431 females

Solve each system by substitution. Tell whether the system has *one solution*, *infinitely many solutions*, or *no solution*.

16. $6x - 3y = 15$
 $y = 2x - 5$

infinitely many solutions

17. $4x + y = -2$
 $-3x - y = 0$

one solution; $(-2, 6)$

18. $5x + 2y = 6$
 $3y = 2x + 9$

one solution; $(0, 3)$

19. $2x - 6y = 12$
 $3y = x + 6$

no solution

20. $4x + y = 0$
 $2x - y = -12$

one solution; $(-2, 8)$

21. $4x + 2y = 7$
 $y = -2x + 3.5$

infinitely many solutions

22. $y = 5x - 1$
 $5x + y = 1$

one solution; $(\frac{1}{5}, 0)$

23. $y = 3x - 6$
 $-3x + y = 6$

no solution

6-2 Standardized Test Prep

Solving Systems Using Substitution

Gridded Response

Solve each exercise and enter your answer on the grid provided.

- For the following system of equations, what is the x -value of the solution? **0**
 $-x + 2y = 6$
 $6y = x + 18$

- The sum of the measures of angle X and angle Y is 90. If the measure of angle X is 30 less than twice the measure of angle Y , what is the measure of angle X ? **50**

- One number is 4 less than 3 times a second number. If 3 more than two times the first number is decreased by 2 times the second number, the result is 11. Use the substitution method. What is the first number? **8**

- An investor bought 3 shares of stock A and 2 shares of stock B for a total of \$41. Stock A costs \$2.00 more per share than stock B. What is the cost of a share of stock A in dollars? **9**

- Solve the following system of equations using substitution. What is the value of y ? **25**
 $2x + 3y = 105$
 $x + 2y = 65$

1.	2.	3.	4.	5.

6-2

Enrichment

Solving Systems Using Substitution

You can use technology (for example, a graphing calculator) to solve systems of equations, whether the solution needs to be exact or approximate.

Problem

At your fundraiser, you served an all-you-can-eat barbeque. You served 210 people and raised \$930. If the amount for each adult was \$6 and for each child was \$3, the equation for money raised is $6x + 3y = 930$. The equation for the total number of adult and child dinners that were served is $x + y = 210$.

Solve the system $\begin{cases} 6x + 3y = 930 \\ x + y = 210 \end{cases}$ using substitution.

$$x = 210 - y \quad \text{Write the second equation in terms of } x \text{ and substitute into the first equation.}$$

$$6(210 - y) + 3y = 930 \quad \text{Simplify.}$$

$$330 = 3y$$

$$110 = y$$

Substitute the value of y into the second equation and you find that $x = 100$. Double check both answers into the first equation with a graphing calculator.

$$6(100) + 3(110) \stackrel{?}{=} 930$$

$$930 = 930 \checkmark$$

You could also graph this system of equations on your graphing calculator. How many adult and child dinners were served? You should have gotten approximately (100, 110) as you did using substitution.

Exercises

1. **Writing** What are the disadvantages and advantages of using one method over the other for the problem above and in general?

Substitution lets you calculate an exact answer, but graphing is quicker.

Solve for x and y using substitution. Check your answer using a graphing calculator.

2. $x + 2y + 3z = 8$ **(4, -1)**

$$y + 2z = 3$$

$$z = 2$$

3. $x + y = a$ **(a - 3b, 3b)**

$$2y = 6b$$

4. $2x - y + 4z = 9$ **($\frac{2}{3}, 12\frac{1}{3}$)**

$$x + y - 3z = -2$$

$$z = 5$$

5. $x - y - d = 0$ **(-d, -2d)**

$$6x - 4y = 2d$$

6. $2x - 2y = 2d$

$$x + y - d = 5$$

$$\left(\frac{5}{2} + d, \frac{5}{2}\right)$$

7. $2x + y + 4z = 18$ **(2, 2)**

$$y + z = 5$$

$$z = 3$$

6-2

Reteaching

Solving Systems Using Substitution

You can solve a system of equations by substituting an equivalent expression for one variable.

Problem

Solve and check the following system:

$$x + 2y = 4$$

$$2x - y = 3$$

Solution

$$x + 2y = 4$$

$$x = 4 - 2y$$

$$2(4 - 2y) - y = 3$$

$$8 - 4y - y = 3$$

$$8 - 5y = 3$$

$$8 - 8 - 5y = 3 - 8$$

$$-5y = -5$$

$$y = 1$$

$$x + 2(1) = 4$$

$$x + 2 - 2 = 4 - 2$$

$$x = 2$$

The first equation is easiest to solve in terms of one variable.

Get x to one side by subtracting $2y$.

Substitute $4 - 2y$ for x in the second equation.

Distribute.

Simplify.

Subtract 8 from both sides.

Divide both sides by -5 .

You have the solution for y . Solve for x .

Substitute in 1 for y in the first equation.

Subtract 2 from both sides.

The solution is $(2, 1)$.

Check Substitute your solution into either of the given linear equations.

$$x + 2y = 4$$

$$2 + 2(1) \stackrel{?}{=} 4$$

$$4 = 4 \checkmark$$

Substitute $(2, 1)$ into the first equation.

You check the second equation.

Exercises

Solve each system using substitution. Check your answer.

1. $x + y = 3$ **(1, 2)**

$$2x - y = 0$$

2. $x - 3y = -14$ **(4, 6)**

$$x - y = -2$$

3. $2x - 2y = 10$ **infinitely many solutions**

$$x - y = 5$$

4. $4x + y = 8$ **$(\frac{11}{7}, \frac{12}{7})$**

$$x + 2y = 5$$

6-2 **Reteaching** (continued)

Solving Systems Using Substitution

Problem

Solve and check the following system:

$$\frac{x}{2} - 3y = 10$$

$$3x + 4y = -6$$

Solve

$$\begin{aligned} \frac{x}{2} - 3y &= 10 \\ \frac{x}{2} &= 10 + 3y \\ x &= 20 + 6y \\ 3x + 4y &= -6 \\ 3(20 + 6y) + 4y &= -6 \\ 60 + 22y &= -6 \\ 22y &= -66, y = -3 \\ \frac{x}{2} - 3(-3) &= 10 \\ \frac{x}{2} + 9 &= 10 \\ x &= 2 \end{aligned}$$

First, isolate x in the first equation.

Add $3y$ to both sides and simplify.

Multiply by 2 on both sides.

Substitute $20 + 6y$ for x in second equation.

Simplify.

Subtract 60 from both sides.

Divide by 22 to solve for y .

Substitute -3 in the first equation.

Simplify.

Solve for x .

The solution is $(2, -3)$.

Check

$$\begin{aligned} 3(2) + 4(-3) &\stackrel{?}{=} -6 \\ -6 &= -6 \checkmark \end{aligned}$$

Now you check the first equation.

Exercises

Solve each system using substitution. Check your answer.

5. $\begin{cases} -2x + y = 8 \\ 3x + y = -2 \end{cases}$ **$(-2, 4)$**

6. $\begin{cases} 3x - 4y = 8 \\ 2x + y = 9 \end{cases}$ **$(4, 1)$**

7. $\begin{cases} 3x + 2y = 25 \\ 2x + 3y = -6 \end{cases}$ **$(17\frac{2}{5}, -13\frac{3}{5})$**

8. $\begin{cases} 6x - 5y = 3 \\ x - 9y = 25 \end{cases}$ **$(-2, -3)$**

6-3 Additional Vocabulary Support

Solving Systems Using Elimination

Tony is trying to find the solution of the system using elimination.

$$2x - 4y = 12$$

$$3x + 4y = 48$$

He wrote these steps to solve the problem on note cards, but they got mixed up.

The note cards contain the following text:

- Solve for x .
- Eliminate one variable. Since the sum of the coefficients of y is 0, add the equations to eliminate y .
- Simplify.
- Substitute 12 for x to solve for the eliminated variable.
- Solve for y .
- Since $x = 12$ and $y = 3$, the solution is $(12, 3)$.

Use the note cards to complete the steps below.

1. First,

eliminate one variable. Since the sum of the coefficients of y is 0, add the equations to eliminate y .

2. Second,

solve for x .

3. Third,

substitute 12 for x to solve for the eliminated variable.

4. Then,

simplify.

5. Next,

solve for y .

6. Finally,

since $x = 12$ and $y = 3$, the solution is $(12, 3)$.

6-3

Think About a Plan

Solving Systems Using Elimination

Nutrition Half a pepperoni pizza plus three fourths of a ham-and-pineapple pizza contains 765 Calories. One fourth of a pepperoni pizza plus a whole ham-and-pineapple pizza contains 745 Calories. How many Calories are in a whole pepperoni pizza? How many Calories are in a whole ham-and-pineapple pizza?

Know

1. What equation will represent the 765 Calories combination of pizza? $\frac{1}{2}x + \frac{3}{4}y = 765$
2. What equation will represent the 745 Calories combination of pizza? $\frac{1}{4}x + y = 745$

Need

3. What possible methods can you use to solve the system of equations?

You can solve the system by graphing, by substitution, or by elimination.

Plan

4. How can you solve the system of equations by elimination?

You can multiply one equation by a constant and then add the revised equation to the other original equation. If the constant is chosen carefully, one variable will cancel out and you can solve the resulting equation for the other variable. Then you can substitute that value into one of the original equations to solve for the other variable.

5. How can you eliminate one of the variables to solve the system of equations?

You can eliminate the variable x by multiplying the second equation by -2 and adding the two equations together.

6. Solve the system of equations.

660; 580

7. What is the solution of the system?

(660, 580)

8. How many Calories are in each kind of pizza?

pepperoni: 660 calories; ham-and-pineapple: 580 calories

6-3

Practice

Form G

Solving Systems Using Elimination

Solve each system using elimination.

1. $x + y = 2$ **(3, -1)**
 $x - y = 4$

2. $x + 2y = 3$ **(5, -1)**
 $x - y = 6$

3. $2x - y = 4$ **(-2, -8)**
 $3x - y = 2$

4. $x - 2y = -2$ **(-4, -1)**
 $-x + y = 3$

5. $-x - 3y = -3$ **(2, $\frac{1}{3}$)**
 $2x + 3y = 5$

6. $x + 2y = -4$ **(8, -6)**
 $x + y = 2$

7. $3x - 2y = 8$
 $2x - 2y = 5$ **(3, $\frac{1}{2}$)**

8. $x - 2y = 3$ **($\frac{1}{5}, -\frac{7}{5}$)**
 $3x - y = 2$

9. $2x - 4y = -6$ **(1, 2)**
 $x - y = -1$

10. **Writing** For the system $\begin{cases} 3x - 5y = 9 \\ 2x + y = 3 \end{cases}$, which variable should you eliminate first and why? How will you eliminate that variable?

You should eliminate the y first because you only need to multiply one equation by a constant. You would multiply the second equation by 5 and then add the equations together to eliminate y .

11. **Open-Ended** If you do not have equal coefficients for both variables, can you still use the elimination method? Explain.

You can use the elimination method by multiplying one or both equations by a constant.

12. In a class, 45 students take the SAT exam. The number of boys is 8 more than the number of girls.

- a. Write a system that models the above situation. **$x + y = 45$, $x - y = 8$**
 b. Do you need to multiply any of the equations by a constant? If so, which equation and what is the constant? **You do not need to multiply either equation by a constant.**

13. **Open-Ended** Write a system for which using the elimination method to solve the system is easier than the substitution method. Explain. **Check student's work.**

14. **Error Analysis** A student solved a system of linear equations using the elimination method as follows. Describe and correct the error made by the student.

$3x - 5y = 4$	$6x - 10y = 8$	Multiply equation 1 by 2.
$-2x + 3y = 2$	$\underline{-6x + 3y = 6}$	Multiply equation 2 by 3.
	$-7y = 14$	Add the equations.
	$y = -2$	Divide by -7 .

When the student multiplied the second equation by 3, the student forgot to multiply the y -term. Correct answer: $(-22, -14)$

6-3

Practice (continued)

Form G

Solving Systems Using Elimination

15. A farm raises a total of 220 chickens and pigs. The number of legs of the stock in the farm totals 520. How many chickens and pigs are at the farm?

180 chickens and 40 pigs

16. You drive a car that runs on ethanol and gas. You have a 20-gallon tank to fill and you can buy fuel that is either 25 percent ethanol or 85 percent ethanol. How much of each type of fuel should you buy to fill your tank so that it is 50 percent ethanol?

$8\frac{1}{3}$ gal of 85% ethanol; $11\frac{2}{3}$ gal of 25% ethanol

17. Your math test has 38 questions and is worth 200 points. The test consists of multiple-choice questions worth 4 points each and open-ended questions worth 20 points each. How many of each type of question are there?

35 multiple choice; 3 open-ended

18. A student bought 3 boxes of pencils and 2 boxes of pens for \$6. He then bought 2 boxes of pencils and 4 boxes of pens for \$8. Find the cost of each box of pencils and each box of pens.

pencils: \$1.00; pens: \$1.50

Solve each system using elimination. Tell whether the system has *one solution*, *infinitely many solutions*, or *no solution*.

19. $x - 3y = -7$
 $2x = 6y - 14$

infinitely many solutions

20. $3x - 5y = -2$
 $x + 3y = 4$

(1, 1); one solution

21. $x + 2y = 6$
 $2x - 4y = -12$

(0, 3); one solution

22. $5x + y = 15$
 $3y = -15x + 6$

no solution

23. $3x = 4y - 5$
 $12y = 9x + 15$

infinitely many solutions

24. $3x - y = -2$
 $-2x + 2y = 8$

(1, 5); one solution

25. $x + 2y = -4$
 $-3x + 2y = 4$

(-2, -1); one solution

26. $x + y = -2$
 $-x - y = 4$

no solution

27. $3x - 2y = -3$
 $6y = 9x + 9$

infinitely many solutions

28. $-4x - 3y = 5$
 $3x - 2y = -8$

(-2, 1); one solution

29. $x - 3y = 1$
 $2x + 2y = 10$

(4, 1); one solution

30. $-4x - 2y = 20$
 $2x + y = 19$

no solution

31. How is the multiplication or division property of equality used in the elimination method? Are the properties always needed? Explain.

The multiplication or division property of equality is used when you multiply or divide an equation by a constant to get another true equation. It is not necessary when a variable already has opposite coefficients.

6-3

Practice

Form K

Solving Systems Using Elimination

Solve each system using elimination.

1. $x + y = 7$ (5, 2)
 $x - y = 3$

2. $2x + y = -5$ (-3, 1)
 $3x - y = -10$

3. $x + 3y = 4$ (4, 0)
 $-x + 2y = -4$

4. $2x + 3y = -12$ (-3, -2)
 $-2x + y = 4$

5. $x - 3y = 27$ (6, -7)
 $3x - 3y = 39$

6. $4x + 2y = 2$ (3, -5)
 $3x + y = 4$

7. **Writing** Solve the system $\begin{cases} 3x + y = 5 \\ -2x - y = -5 \end{cases}$ using elimination. Explain how you can check the solution both algebraically and graphically.

(0, 5); You can check the solution algebraically by substituting $x = 0$ and $y = 5$ into both of the original equations. You can check the solution graphically by seeing if the two lines intersect at (0, 5).

8. **Open-Ended** Write a system of equations that can be solved using elimination without multiplication.

Answers may vary. For example,

$x + y = 3$

$x - y = 1$

9. There are 72 members of the show choir. There are 6 more boys than girls in the choir.

a. Write the model of a system for the above situation. $b + g = 72$
 $b - g = 6$

b. Do you need to multiply any of the equations by a constant before solving by elimination? Explain.

No, the g -variable can be eliminated without multiplying either equation.

10. **Writing** Explain the process you use to determine which variable is the best variable to eliminate in a system of two equations in two variables.

First check to see if either variable can be eliminated simply by adding or subtracting the two original equations. If not, see if either variable can be eliminated by just multiplying one equation by a constant and adding the equations together. If this is not possible, then multiply both equations by a constant such that one of the variables will be eliminated when the resulting equations are added.

6-3

Practice (continued)

Form K

Solving Systems Using Elimination

11. The sum of two numbers is 19, and their difference is 55. What are the two numbers? **-18 and 37**
12. For the fundraiser, Will sold 225 candy bars. He earns \$1 for each almond candy bar he sells and \$0.75 for each caramel candy bar he sells. If he earned a total of \$187.50, how many of each type of candy bar did he sell for the fundraiser? **150 caramel and 75 almond**
13. There were 155 people at the basketball game. Tickets for the game are \$2.50 for students and \$4 for adults. If the total money received for admission was \$492.50, how many students and adults attended the game?
85 students and 70 adults
14. Jocelyn has \$1.95 in her pocket made up of 27 nickels and dimes. How many of each type of coin does she have? **12 dimes and 15 nickels**

Solve each system using elimination. Tell whether the system has *one solution*, *infinitely many solutions*, or *no solution*.

15. $x - 2y = -1$
 $2x + y = 4$
one solution; $(1\frac{2}{5}, 1\frac{1}{5})$
16. $x + 3y = 4$
 $2x - 6y = 8$
one solution; (4, 0)
17. $y = -\frac{1}{2}x - 3$
 $x + 2y = -6$
infinitely many solutions
18. $6x - 3y = -18$
 $-2x + 4y = 18$
one solution; (-1, 4)
19. $2x - 8y = -16$
 $y = \frac{1}{4}x - 2$
no solution
20. $3x - y = -1$
 $y = 3x - 5$
no solution
21. $2x - y = 3$
 $5x + 2y = 30$
one solution; (4, 5)
22. $12x - 8y = 18$
 $6x = 4y + 9$
infinitely many solutions

6-3

Standardized Test Prep

Solving Systems Using Elimination

Multiple Choice

For Exercises 1–5, choose the correct letter.

1. What is the solution for the following system of equations? $\begin{matrix} 5x + 7y = 3 \\ 2x + 3y = 1 \end{matrix}$ **C**
- A. $(-2, 1)$ B. $(1, -2)$ C. $(2, -1)$ D. $(-1, 2)$
2. The perimeter of a rectangle is 24 in. and its length l is 3 times the width w . What is the length and width (l, w) of the rectangle? **I**
- F. $(3, 9)$ G. $(14.4, 4.8)$ H. $(12, 4)$ I. $(9, 3)$
3. What do both dependent and inconsistent systems have in common? **C**
- A. no solution C. same slope
B. same y -intercept D. same slope and y -intercept
4. Two linear equations have the same y -intercept and different slopes. How would you classify the system? **F**
- F. consistent, independent H. dependent, inconsistent
G. consistent, dependent I. independent, inconsistent
5. What is the solution of the following system of equations? **B**
- $$\begin{matrix} 5x + 7y = 3 \\ 2x = -3y + 1 \end{matrix}$$
- A. $(11, 17)$ B. $(2, -1)$ C. $(11.5, 8)$ D. $(-2, 1)$

Short Response

6. A hotel is offering the two weekend specials described below.

Plan 1: 3 nights and 4 meals for \$233

Plan 2: 3 nights and 3 meals for \$226.50.

For accounting purposes, the hotel will record income for the stay and the meals separately. So the cost per night for each special must be the same, and the cost per meal for each special must be the same.

- a. What is a system of equations for the situation? $\begin{matrix} 3x + 4y = 233 \\ -3x + 3y = 226.5 \end{matrix}$
- b. Solve the system. What is the cost per night and the cost per meal?

cost per night: \$69.00; cost per meal: \$6.50

[2] Both parts answered correctly**[1] Some parts answered correctly****[0] Neither part answered correctly**

6-3 Enrichment

Solving Systems Using Elimination

Problem

Solve the absolute value system of equations $\begin{cases} |x| = y - 2 \\ 2x - y = 0 \end{cases}$.

$$\begin{aligned} |x| &= 2 - y \\ x = y - 2 \text{ or } x &= 2 - y \end{aligned}$$

$$\begin{aligned} x = y - 2 \quad x &= 2 - y \\ 2x - y = 0 \text{ or } &2x - y = 0 \end{aligned}$$

$$-2x + 2y = 4$$

$$\begin{array}{r} 2x - y = 0 \\ \hline y = 4 \end{array}$$

$$x = 4 - 2 = 2$$

First isolate the absolute value expression and rewrite as a compound expression.

Now there are two systems of linear equations to solve.

Solve the first system. In first equation, add y to both sides then multiply equation by -2 . Add equations.

Substitute 4 for y in first equation. Solve for x .

The solution for the first system is $(2, 4)$

$$-2x - 2y = -4$$

$$\begin{array}{r} 2x - y = 0 \\ \hline -3y = -4 \\ y = \frac{4}{3} \end{array}$$

$$x = 2 - \frac{4}{3} = \frac{2}{3}$$

Solve the second system. In first equation, add y to both sides, then multiply equation by -2 . Add equations and solve for y .

Substitute $\frac{4}{3}$ for y in first equation. Solve for x .

The solution for the second system is $\left(\frac{2}{3}, \frac{4}{3}\right)$. Check both solutions.

Exercises

Solve each system of equations and check your answers.

1. $\begin{cases} |x| = 3 + y \\ x + 4y = 6 \end{cases}$ $\left(\frac{33}{5}, \frac{3}{5}\right)$ and $(-6, 3)$

2. $\begin{cases} -4y + 2x = 8 \\ 2y + 6 = |x| \end{cases}$ $(-1, -\frac{5}{2})$

3. $\begin{cases} 18y + 15 = |3x| \\ 6y - x = 12 \end{cases}$ $\left(-\frac{17}{2}, \frac{7}{12}\right)$

4. $\begin{cases} |x - 2| + y = 2 \\ 2x + 2y = 16 \end{cases}$ **no solution**

6-3 Reteaching

Solving Systems Using Elimination

Elimination is one way to solve a system of equations. Think about what the word “eliminate” means. You can eliminate either variable, whichever is easiest.

Problem

Solve and check the following system of linear equations.

$$\begin{aligned} 4x - 3y &= -4 \\ 2x + 3y &= 34 \end{aligned}$$

Solution The equations are already arranged so that like terms are in columns.

Notice how the coefficients of the y -variables have the opposite sign and the same value.

$$\begin{array}{r} 4x - 3y = -4 \\ 2x + 3y = 34 \\ \hline 6x = 30 \\ x = 5 \\ 4(5) - 3y = -4 \\ 20 - 3y = -4 \\ -3y = -24 \\ y = 8 \end{array}$$

Add the equations to eliminate y .

Divide both sides by 6 to solve for x .

Substitute 5 for x in one of the original equations and solve for y .

The solution is $(5, 8)$.

Check

$$\begin{aligned} 4x - 3y &= -4 \\ 4(5) - 3(8) &\stackrel{?}{=} -4 \\ 20 - 24 &\stackrel{?}{=} -4 \\ -4 &= -4 \checkmark \end{aligned}$$

Substitute your solution into both of the original equations to check.

You can check the other equation.

Exercises

Solve and check each system.

1. $3x + y = 3$ **$(0, 3)$**
 $-3x + y = 3$

2. $6x - 3y = -14$ **$(\frac{2}{3}, 6)$**
 $6x - y = -2$

3. $3x - 2y = 10$ **$(2, -2)$**
 $x - 2y = 6$

4. $4x + y = 8$ **$(1, 4)$**
 $x + y = 5$

6-3 **Reteaching** (continued)

Solving Systems Using Elimination

If none of the variables has the same coefficient, you have to multiply before you eliminate.

Problem

Solve the following system of linear equations.

$$\begin{aligned} -2x + 3y &= -1 \\ 5x + 4y &= 6 \end{aligned}$$

Solution

$$\begin{aligned} 5(-2x - 3y) &= (-1)5 \\ 2(5x + 4y) &= (6)2 \end{aligned}$$

Multiply the first equation by 5 (all terms, both sides) and the second equation by 2. You can eliminate the x variable when you add the equations together.

$$\begin{array}{r} -10x - 15y = -5 \\ 10x + 8y = 12 \\ \hline -7y = 7 \end{array}$$

Distribute, simplify and add.

$$y = -1$$

Divide both sides by 7.

$$5x + 4(-1) = 6$$

Substitute -1 in for y in the second equation to find the value of x .

$$5x - 4 = 6$$

Simplify.

$$5x = 10$$

Add 4 to both sides.

$$x = 2$$

Divide by 5 to solve for x .

The solution is $(2, -1)$.

Check $-2x + 3y = -1$

Substitute your solution into both original equations.

$$-2(2) - 3(-1) \stackrel{?}{=} -1$$

$$-1 = -1 \checkmark$$

You can check the other equation.

Exercises

Solve and check each system.

5. $\begin{aligned} x - 3y &= -3 \quad (9, 4) \\ -2x + 7y &= 10 \end{aligned}$

6. $\begin{aligned} -2x - 6y &= 0 \quad (-6, 2) \\ 3x + 11y &= 4 \end{aligned}$

7. $\begin{aligned} 3x + 10y &= 5 \quad (1, \frac{1}{5}) \\ 7x + 20y &= 11 \end{aligned}$

8. $\begin{aligned} 4x + y &= 8 \quad (1, 4) \\ x + y &= 5 \end{aligned}$

6-4 Additional Vocabulary Support

Applications of Linear Systems

Use the list below to complete the diagram.

use when you want a visual display of the equations	use when it is easy to solve for one of the variables	use when you want an estimation of the solution
use when one equation is already solved for one of the variables	use when the coefficients of one variable are the same or opposites	use when it is not convenient to use graphing or substitution

Choosing a Method for Solving Linear Systems

Graphing

use when you want a visual display of the equations

use when you want an estimation

Substitution

use when one equation is already solved for one of the variables

use when it is easy to solve for one of the variables

Elimination

use when the coefficients of one variable are the same or opposites

use when it is not convenient to use graphing or substitution

6-4**Think About a Plan**

Applications of Linear Systems

Chemistry In a chemistry lab, you have two vinegars. One is 5% acetic acid, and one is 6.5% acetic acid. You want to make 200 mL of a vinegar with 6% acetic acid. How many milliliters of each vinegar do you need to mix together?

Know

1. What types of vinegar do you have available?
5% acetic acid and 6.5% acetic acid
2. What amount of mixed vinegar do you need?
200 mL
3. What percentage of acetic acid do you want in the mixed vinegar?
6%

Need

4. What do you need to find to solve the problem?
the number of mL of each type of vinegar

Plan

5. How will you define the two variables for this problem?
 x = amount of 5% vinegar; y = amount of 6.5% vinegar
6. What is an equation for the total amount of vinegar you want to make?
 $x + y = 200$
7. What is an equation for the acetic acid content?
 $0.05x + 0.065y = 0.06(200)$
8. What method will you use to solve?
Answers may vary. Sample: I will use substitution, because there are variable terms with coefficients of 1.
9. What is the solution of the system of equations?
 $(66\frac{2}{3}, 133\frac{1}{3})$
10. How much of each type of vinegar should you mix together?
You need $66\frac{2}{3}$ mL of 5% vinegar and $133\frac{1}{3}$ mL of 6.5% vinegar.

6-4**Practice**

Form G

Applications of Linear Systems

Solve each word problem

1. You have \$6000 to invest in two stock funds. The first fund pays 5% annual interest and the second account pays 9% annual interest. If after a year you have made \$380 in interest, how much money did you invest in each account?
\$4000 at 5%; \$2000 at 9%
2. During a sale at the local department store, you buy three sweatshirts and two pairs of sweatpants for \$85.50. Later you return to the same store and buy three more sweatshirts and four more pairs of sweatpants for \$123. What is the sale price of each sweatshirt and each pair of sweatpants?
sweatshirt: \$16; sweatpants: \$18.75
3. The sum of two numbers is 27. The larger number is 3 more than the smaller number. What are the two numbers?
(15, 12)
4. One plane at 520 feet is ascending at a rate of 40 feet per minute, while another plane at 3800 feet is descending at a rate of 120 feet per minute. How long will it take the two planes to be at the same altitude?
20.5 min
5. The perimeter of a rectangle is 24 in. and its length is 3 times its width. What are the length and the width of the rectangle?
length: 9 in.; width: 3 in.
6. You are getting ready to move and have asked some friends to help. For lunch, you buy the following sandwiches at the local deli for \$30: six tuna sandwiches and six turkey sandwiches. Later at night, everyone is hungry again and you buy four tuna sandwiches and eight turkey sandwiches for \$30.60. What is the price of each sandwich?
tuna: \$2.35; turkey: \$2.65
7. You have a cable plan that costs \$39 a month for a basic plan plus one movie channel. Your friend has the same basic plan plus two movie channels for \$45.50. What is the basic plan charge that you both pay?
\$32.50
8. At an all-you-can-eat barbeque fundraiser that you are sponsoring, adults pay \$6 for a dinner and children pay \$4 for a dinner. 212 people attend and you raise \$1128. What is the total number of adults and the total number of children attending?
 - a. What is a system of equations that you can use to solve this problem? $x + y = 212$
 $6x + 4y = 1128$
 - b. What method would you use to solve the system? Why?
I'd use substitution because in the first equation the coefficients are already 1.

6-4

Practice (continued)

Form G

Applications of Linear Systems

Solve each system. Explain why you chose the method you used.

9. $2y = x + 1$ $(-3, -1)$; substitution
 $-2x - y = 7$ because there was a coefficient of 1
10. $6x - 4y = 54$ $(7, -3)$; elimination
 $-9x + 2y = -69$ because no coefficients were 1 or -1 .
11. $3x - 2y = 8$ $(3, \frac{1}{2})$; elimination because
 $2x - 2y = 5$ the y -coefficients are the same.
12. $2x - y = 4$ $(-2, -8)$; elimination
 $3x - y = 2$ because the y -coefficients were the same.
13. $2x - 3y = 13$ $(5, -1)$; substitution
 $y = \frac{1}{2}x - \frac{7}{2}$ because y is isolated in the second equation.
14. $-x - 3y = -3$ $(2, \frac{1}{3})$; elimination because
 $2x + 3y = 5$ the y -coefficients are opposites

15. **Open-Ended** What are three differences between an inconsistent system and a consistent and independent system? Explain.

Answers may vary. Sample: There is one unique solution to an independent system, but no solution to an inconsistent system and infinitely many solutions to a dependent system. The graphs of the equations of an independent system have different slopes, but the slopes are the same for the lines of an inconsistent or dependent system. When solving a system algebraically, you get a specific value for each variable if the system is independent, but you get a false statement for an inconsistent system and a statement that is always true for a dependent system.

16. **Reasoning** One number is 4 less than 3 times a second number. If 3 more than two times the first number is decreased by two times the second, the result is 11. What are both numbers?

(8, 4)

17. **Error Analysis** In Exercise 16, what kind of errors are likely to occur when solving the problem?

Answers may vary. Sample: You might misinterpret the description. For instance, you might think "4 less than 3 times a number" means $4 - 3y$ instead of $3y - 4$.

18. A plane leaves Chicago and flies 750 miles to New York. If it takes 2.5 hours to get to New York flying against the wind, but only 2 hours to fly back to Chicago, what is the plane's rate of speed and what is the wind speed?

wind: 37.5 mi/h; plane: 337.5 mi/h

19. A coin bank has 250 coins, dimes and quarters, worth \$39.25. How many of each type of coin are there?

155 dimes; 95 quarters

20. In 4 years, a mother will be 5 times as old as her daughter. At present, the mother is 9 times as old as the daughter. How old are the mother and the daughter today?

mother: 36; daughter: 4

6-4**Practice**

Form K

Applications of Linear Systems

Solve each word problem.

1. The concession stand is selling hot dogs and hamburgers during a game. At halftime, they sold a total of 78 hot dogs and hamburgers and brought in \$105.50. How many of each item did they sell if hamburgers sold for \$1.50 and hot dogs sold for \$1.25?
32 hamburgers and 46 hot dogs

2. The sum of two numbers is 67. The smaller number is 3 less than the larger number. What are the two numbers?
35 and 32

3. There are two different jobs Jordan is considering. The first job will pay her \$4200 per month plus an annual bonus of \$4500. The second job pays \$3100 per month plus \$600 per month toward her rent and an annual bonus of \$500. Which job should she take?
the first job

4. The perimeter of a rectangle is 66 cm and its width is half its length. What are the length and the width of the rectangle?
length = 22 cm; width = 11 cm

5. A chemist is mixing one solution that is 32% sodium and another solution that is 12% sodium. How many liters of each type should the chemist use to produce 50 liters of the solution that is 20% sodium?
20 L of the 32% solution and 30 L of the 12% solution

6-4

Practice (continued)

Form K

Applications of Linear Systems

6. A community sponsored a charity square dance where admission was \$3 for adults and \$1.50 for children. If 168 people attended the dance and the money raised was \$432, how many adults and how many children attended the dance?
- What are the two systems of equations that you could write to solve this problem?
 $a + c = 168$
 $3a + 1.5c = 432$
 - What method would you use to solve the system? Why?
Answers may vary. For example, you would use elimination by multiplying the top equation by -1.5 to eliminate c . This involves fewer steps to solve than substitution or graphing.
 - How many adults and how many children attended the dance?
120 adults, 48 children

Solve each system. Explain why you chose the method you used.

- $3y = 4x + 1$
 $8x - 2y = 10$
(2, 3); Answers may vary. For example, I used elimination because substitution would involve the use of fractions.
 - $-2y = -4x - 2$
 $3x + 2y = 9$
(1, 3); Answers may vary. For example, I used substitution because y can be easily isolated without involving fractions.
 - $3x - 3y = -3$
 $-2x - 3y = 17$
(-4, -3); Answers may vary. For example, I used elimination because the y -variable is simply eliminated by multiplying either equation by -1 .
 - $x - 2y = 9$
 $x + 3y = -1$
(5, -2); Answers may vary. For example, I used elimination because the x -variable is simply eliminated by multiplying either equation by -1 .
11. **Open-Ended** Write a system of equations for which you would use substitution to solve.
Answers may vary. For example,
 $2x + 3y = 8$
 $y = x + 1$
12. A student invested \$5000 in two different savings accounts. The first account pays an annual interest rate of 3%. The second account pays an annual interest rate of 4%. At the end of one year, she had earned \$185 in interest. How much money did she invest in each account?
\$1500 in the 3% account and \$3500 in the 4% account

6-4 Standardized Test Prep

Applications of Linear Systems

Multiple Choice

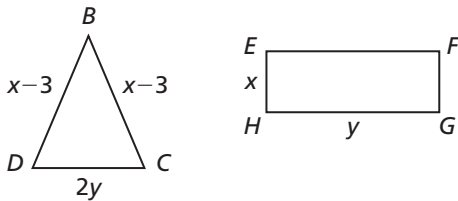
For Exercises 1–5, choose the correct letter.

- You solved a linear system with two equations and two variables and got the equation $-6 = -6$. How many solutions does the system of equations have? **B**
 - no solution
 - infinitely many solutions
 - exactly 1 solution
 - 2 solutions
- The sum of two numbers is 12. The difference of the same two numbers is 4. What is the larger of the two numbers? **I**
 - 4
 - 7
 - 5
 - 8
- You solved a linear system and got the equation $-6 = 0$. How many solutions does the system of equations have? **A**
 - no solution
 - infinitely many solutions
 - exactly 1 solution
 - 2 solutions
- What is the solution of the system of equations? **H**

$$\begin{aligned} -y + 3x &= 6 \\ y &= -6x + 12 \end{aligned}$$
 - $(-2, 0)$
 - $(2, 0)$
 - $(0, -2)$
 - $(0, 2)$
- A kayaker paddles upstream for 1.5 hours, then turns his kayak around and returns to his tent in 1 hour. He travels 3 miles each way. What is the rate of the river's current? **A**
 - 0.5 mi/h
 - 2 mi/h
 - 1 mi/h
 - 1.5 mi/h

Short Response

- Rectangle $EFGH$ has a perimeter of 24 inches, and triangle BCD has a perimeter of 18 inches.



- What is a system of equations for the perimeters of the figures?

$$\begin{aligned} 2x + 2y &= 24 \\ 2(x - 3) + 2y &= 18 \end{aligned}$$
- Without solving, what method you would use to solve the system? Explain.
I would use elimination because the y -coefficients are the same.

[2] Both parts answered correctly. Explanation is clear.
[1] One part answered correctly, or explanation is not clear or complete.
[0] No parts answered correctly.

6-4 Enrichment

Applications of Linear Systems

The general form of the linear equation with three variables is $Ax + By + Cz = D$ where A, B, C and D are real numbers and do not equal 0.

Example

Solve the system of equations below.

$$x + 2y - 3z = 1 \quad \text{Equation (1)}$$

$$2x - 3y + 5z = 11 \quad \text{Equation (2)}$$

$$x - y + 4z = 14 \quad \text{Equation (3)}$$

Solution

$$\begin{array}{r} x + 2y - 3z = 1 \\ \underline{x - y + 4z = 14} \end{array} \quad \text{Subtract equation (3) from equation (1).}$$

$$3y - 7z = -13 \quad \text{Equation (4)}$$

$$\begin{array}{r} 2x - 2y + 8z = 28 \\ \underline{2x - 3y + 5z = 11} \end{array} \quad \begin{array}{l} \text{Multiply equation (3) by 2 and subtract} \\ \text{equation (2).} \end{array}$$

$$y + 3z = 17 \quad \text{Equation (5)}$$

$$\begin{array}{r} 3y + 9z = 51 \\ \underline{3y - 7z = -13} \end{array} \quad \text{Multiply equation (5) by 3 and subtract} \\ \text{equation (4).}$$

$$\begin{array}{r} 16z = 64 \\ z = 4 \end{array} \quad \text{Solve for } z.$$

$$y + 3(4) = 17 \quad \text{Substitute 4 for } z \text{ in equation (5) and solve for } y.$$

$$y = 5$$

$$\begin{array}{r} x + 2y - 3z = 1 \\ x + 2(5) - 3(4) = 1 \\ x = 3 \end{array} \quad \text{Substitute 5 for } y \text{ and 4 for } z \text{ in equation (1) and} \\ \text{solve for } x.$$

The solution of the given system is $x = 3, y = 5$ and $z = 4$.

Practice

Solve each system of equations.

$$\begin{array}{l} 1. \quad 3x - y + 2z = 4 \quad \left(\frac{31}{35}, \frac{9}{35}, \frac{4}{5}\right) \\ \quad \quad x + 2y - 3z = -1 \\ \quad \quad 5x + 3y + z = 6 \end{array}$$

$$\begin{array}{l} 2. \quad -2x + 5y - 3z = 7 \quad (3, 2, -1) \\ \quad \quad 4x - 3y + 2z = 4 \\ \quad \quad -3x - y - 4z = -7 \end{array}$$

6-4

Reteaching

Applications of Linear Systems

You can solve systems of linear equations by graphing, substitution, or elimination. Deciding which method to use depends on the exactness needed and the form of the equations.

Problem

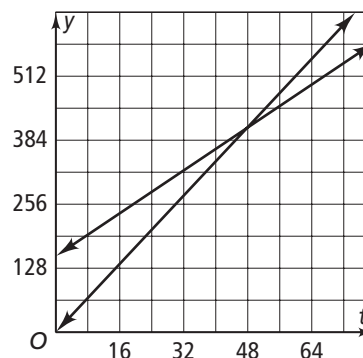
You just bought a coffee shop for \$153,600. The prior owner had an average monthly revenue of \$8600 and an average monthly cost of \$5400. If your monthly costs and revenues remain the same, how long will it take you to break even?

Write equations for revenue and costs, including the price you paid for the shop, after t months. Then solve the system by graphing.

$$y = 8600t \quad \text{Equation for revenue}$$

$$y = 5400t + 153,600 \quad \text{Equation for cost}$$

It appears that the point of intersection is where t is equal to 48 months. Substitute $t = 48$ into either equation to find the other coordinate (y), which is 412.8. Therefore, your breakeven point is after you have run the shop for 48 months, at which point your revenue and cost are the same: \$412,800.

**Problem**

A perfume is made from t ounces of 15% scented Thalia and b ounces of 40% Thalia. You want to make 60 oz of a perfume that has a 25% blend of the Thalia. How many ounces of each concentration of Thalia are needed to get 60 oz of perfume that is 25% strength of Thalia?

Write your systems of equations: $60(0.25) = 0.15t + 0.4b$
 $60 = t + b$

Solve the system by using substitution:

$$60(0.25) = 0.15t + 0.4b \quad \text{Solve the second equation for } t \text{ and substitute in the first equation.}$$

$$15 = 0.15(60 - b) + 0.4b \quad \text{Substitute } 60 - b \text{ for } t \text{ in the first equation.}$$

$$15 = 9 - 0.15b + 0.4b \quad \text{Distributive property}$$

$$24 = b \quad \text{Solve for } b.$$

Substitute 24 for b in second equation to find that $t = 36$. The answer is (36, 24). The blend requires 36 oz of the 15% perfume and 24 oz of the 25% perfume.

6-4**Reteaching** (continued)

Applications of Linear Systems

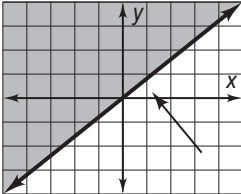
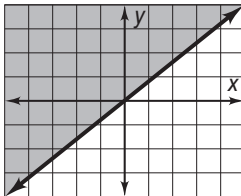


Exercises

1. You have a coin bank that has 275 dimes and quarters that total \$51.50. How many of each type of coin do you have in the bank?
115 dimes; 160 quarters
2. **Open-Ended** Write a break-even problem and use a system of linear equations to solve it.
Check students' work.
3. You earn a fixed salary working as a sales clerk making \$11 per hour. You get a weekly bonus of \$100. Your expenses are \$60 per week for groceries and \$200 per week for rent and utilities. How many hours do you have to work in order to break even?
about 14.5 h
4. **Reasoning** Find A and B so that the system below has the solution $(1, -1)$.
 $Ax + 2By = 0$
 $2Ax - 4By = 16$
 $A = 4$; $B = 2$
5. You own an ice cream shop. Your total cost for 12 double cones is \$24 and you sell them for \$2.50 each. How many cones do you have to sell to break even?
10 ice cream cones
6. **Multi-Step** A skin care cream is made with vitamin C. How many ounces of a 30% vitamin C solution should be mixed with a 10% vitamin C solution to make 50 ounces of a 25% vitamin C solution?
 - Define the variables.
 - Make a table or drawing to help organize the information.**37.5 oz of 30% solution; 12.5 oz of 10% solution**
7. Your hot-air balloon is rising at the rate of 4 feet per second. Another aircraft nearby is at 7452 feet and is losing altitude at the rate of 30 feet per second. In how many seconds will your hot-air balloon be at the same altitude as the other aircraft?
about 219 s

6-5 Additional Vocabulary Support

Linear Inequalities

Complete the vocabulary chart by filling in the missing information.

Word or Word Phrase	Definition	Picture or Example
boundary line	A <i>boundary line</i> is a line that separates the graph into regions.	
half-plane	1. A <i>half-plane</i> is the part of the graph on one side of a boundary line.	
horizontal line	A <i>horizontal line</i> goes across.	2. 
linear inequality	3. A <i>linear inequality</i> in two variables can be formed by replacing the equals sign in a linear equation with an inequality symbol.	$y < x + 5$
solution of an inequality	A <i>solution of an inequality</i> in two variables is an ordered pair that makes the inequality true.	4. $y \geq x + 5$ $(4, 1)$
vertical line	5. A <i>vertical line</i> goes up and down.	

6-5 Think About a Plan

Linear Inequalities

Employment A student with two summer jobs earns \$10 per hour at a café and \$8 per hour at a market. The student would like to earn at least \$800 per month.

- Write and graph an inequality to represent the situation.
- The student works at the market for 60 h per month and can work at most 90 h per month. Can the student earn at least \$800 each month? Explain how you can use your graph to determine this.

Understanding the Problem

1. What do you know about the student's hourly rates?

The student makes \$10/h at the café and \$8/h at the market.

2. What do you know about how much the student would like to earn each month?

The student would like to earn at least \$800 a month.

3. What do you know about the number of hours the student can work each month?

The student can work 60-90 hours per month.

Planning the Solution

4. What inequality represents the number of hours that the student can work each month? $c + m \leq 90$

5. What inequality represents the amount that the student can earn each month?

$10c + 8m \geq 800$

Getting an Answer

6. How can you use these two inequalities to find out if the student working 60 hours a month at the market can make \$800 per month?

Solve the first inequality for one of the variables and substitute it into the second inequality.

7. How can you determine the number of hours that the student should work each month? What are the number of hours the student should work at the market and at the cafe to make at least \$800 per month?

The student would have to work 32 h at the cafe to make more than \$800. Because the student can only work a maximum of 30 h at the café, the student cannot make \$800 a month.

6-5

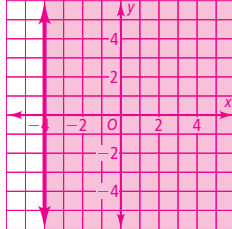
Practice

Form G

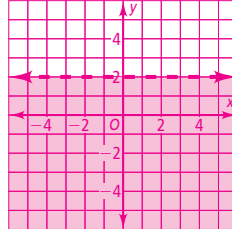
Linear Inequalities

Graph each linear inequality.

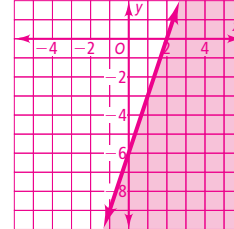
1. $x \geq -4$



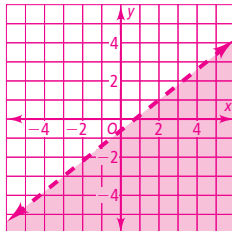
2. $y < 2$



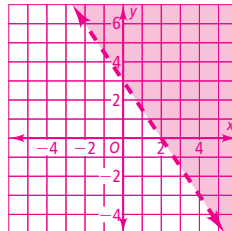
3. $3x - y \geq 6$



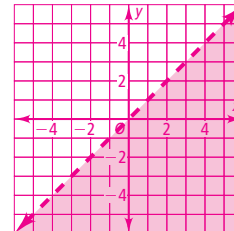
4. $-4x + 5y < -3$



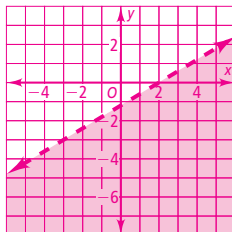
5. $3x + 2y > 6$



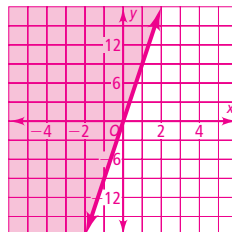
6. $y < x$



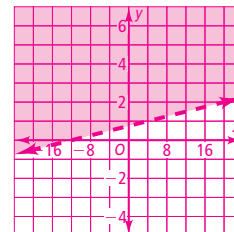
7. $3x - 5y > 6$



8. $x \leq \frac{y}{9}$



9. $\frac{x}{4} < 4y - 3$

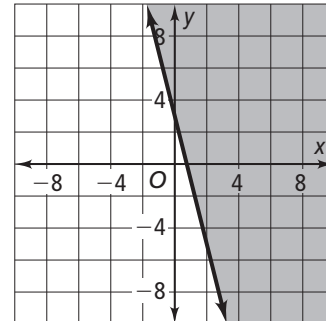


10. **Error Analysis** A student graphed $y \leq -4x + 3$ as shown. Describe and correct the student's error.

The student shaded above the line when the student should have shaded below the line.

11. **Writing** How do you decide which half-plane to shade when graphing an inequality? Explain.

If the inequality is of the form $y > mx + b$ or $y \geq mx + b$, shade above the line. Otherwise, shade below the line.



6-5

Practice (continued)

Form G

Linear Inequalities

Determine whether the ordered pair is a solution of the linear inequality.

12. $7x + 2y > -5$, $(-1, 1)$

not a solution

14. $y + 2x > 5$, $(4, 1)$

solution

16. $y < x + 4$, $(-9, -5)$

not a solution

18. $x - \frac{1}{2}y > 3$, $(9, 12)$

not a solution

13. $x - y \leq 3$, $(2, -1)$

solution

15. $x + 4y \leq -2$, $(-8, -2)$

solution

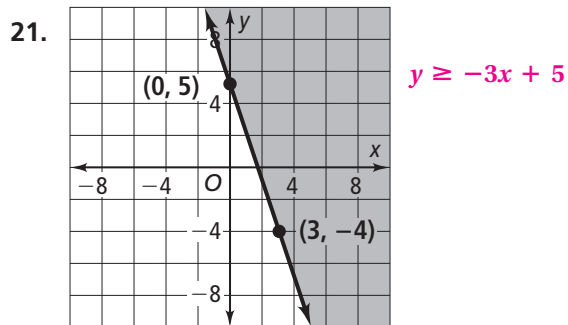
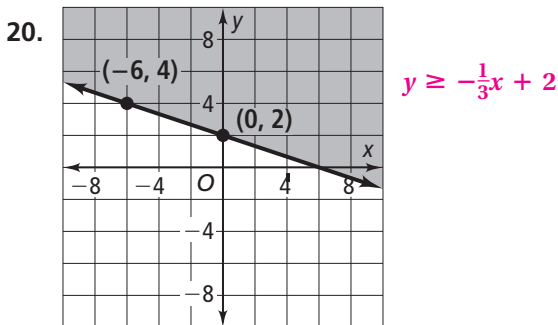
17. $y < 3x + 2$, $(3, 10)$

solution

19. $0.3x - 2.4y > 0.9$, $(8, 0.5)$

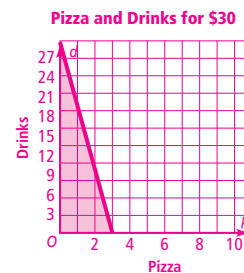
solution

Write an inequality that represents each graph.



22. You and some friends have \$30. You want to order large pizzas (p) that are \$9 each and drinks (d) that cost \$1 each. Write and graph an inequality that shows how many pizzas and drinks can you order?

$d \leq -9p + 30$



23. Tickets to a play cost \$5 at the door and \$4 in advance. The theater club wants to raise at least \$400 from the play. Write and graph an inequality for the number of tickets the theater club needs to sell. If the club sells 40 tickets in advance, how many do they need to sell at the door to reach their goal?

$5x + 4y \geq 400$; 48



24. **Reasoning** Two students did a problem as above, but one used x for the first variable and y for the second variable and the other student used x for the second variable and y for the first variable. How did their answers differ and which one, if either, was incorrect?

The students' answers are reversed. Neither one is incorrect if the variables are defined correctly.

6-5

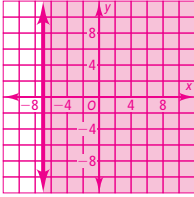
Practice

Form K

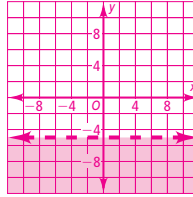
Linear Inequalities

Graph each linear inequality.

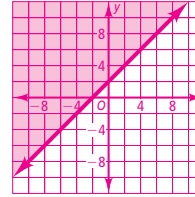
1. $x \geq -7$



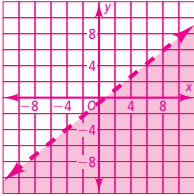
2. $y < -5$



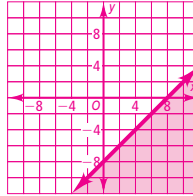
3. $-x + y \geq 2$



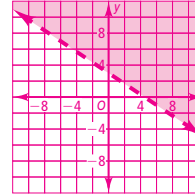
4. $-4x + 5y < -3$



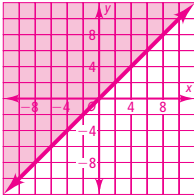
5. $x - y \geq 8$



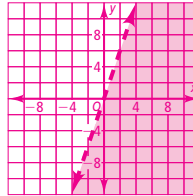
6. $2x + 3y > 9$



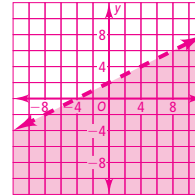
7. $y \geq x$



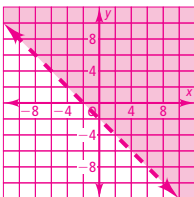
8. $3x > y$



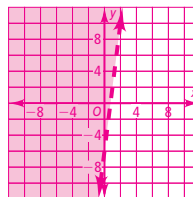
9. $x - 2y > -4$



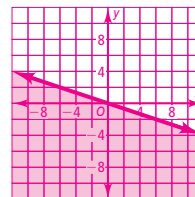
10. $5x + 5y > -10$



11. $4x - \frac{1}{2}y < 3$



12. $x \leq -3y$



6-5

Practice (continued)

Form K

Linear Inequalities

13. Writing How can you check to see that you have shaded the correct half of the coordinate plane after graphing a linear inequality? Explain.

Choose an ordered pair that is clearly in the shaded portion and substitute it into the inequality to see if the inequality is true for this ordered pair.

Determine whether the ordered pair is a solution of the linear inequality.

14. $4x + 3y > -2$; $(-3, -1)$ **no**

15. $x + y > -3$; $(-2, 2)$ **yes**

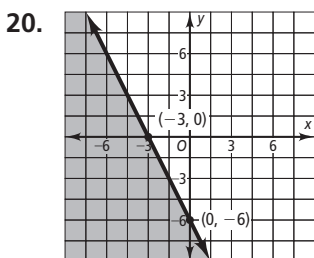
16. $y - 4x \leq 0$; $(1, 4)$ **yes**

17. $2x - 4y > 5$; $(5, -1)$ **yes**

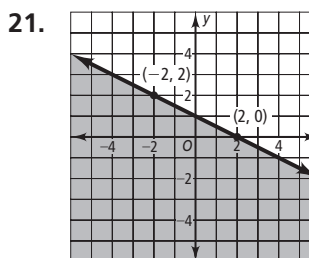
18. $y \leq 2x - 3$; $(-1, -4)$ **no**

19. $y < -3x + 1$; $(3, 5)$ **no**

Write a linear inequality that represents each graph.



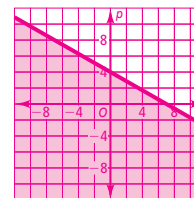
$y \leq -2x - 6$



$y \leq -\frac{1}{2}x + 1$

22. A friend has \$75 to buy some new shirts and pants. Each shirt s costs \$11. Each pair of pants p costs \$19.

a. Write and graph an inequality that shows how many shirts and pants your friend can buy. **$11s + 19p \leq 75$**



b. Which side of the boundary line should you shade?
below the line

c. What inequality symbol did you use? Explain.
 \leq ; She can only spend \$75 or less.

23. Admission to the movie theater costs \$7.50 for adults and \$3.50 for students. The theater must bring in at least \$200 per movie. Write an inequality for the number of tickets the theater needs to sell to make a profit. If the theater sells 15 adult tickets, how many student tickets do they need to sell to make a profit? **$7.5a + 3.5s \geq 200$; 25**

24. Each child at the birthday party was given \$5 to spend at the arcade on games and rides. Each game costs \$0.25 and each ride costs \$0.50. Write an inequality for the number of games and rides a child can enjoy for \$5. What is the maximum number of games or rides each child can enjoy?

$0.25g + 0.5r \leq 5$; 20 games or 10 rides

6-5 Standardized Test Prep

Linear Inequalities

Multiple Choice

For Exercises 1–5, choose the correct letter.

1. What point on the axes satisfies the inequality $y < x$? **C**
A. (0, 1) B. (-1, 0) C. (1, 0) D. (0, 0)
2. For the graph of the inequality $x - 2y \geq 4$, what is a value of x for a point that is on the boundary line and the axes? **F**
F. 4 G. -2 H. 2 I. -4
3. If $x \geq 0$ and $y \geq 0$, then which quadrant holds the solutions? **C**
A. IV B. III C. I D. II
4. Which is the y -value of a boundary point that is an intersecting point not on the axes for this region: $x \geq 0$, $y \geq 0$, $x \leq 4$ and $y \leq 3$? **I**
F. 4 G. 0 H. 1 I. 3
5. How do you decide where to shade an inequality whose boundary does not go through the origin? **B**
A. For $<$, shade above the boundary.
B. If (0, 0) is a solution, shade where (0, 0) is.
C. For $>$, shade below the boundary.
D. If (0, 0) is a solution, shade the boundary.

Short Response

6. A school fundraiser sells holiday cards and wrapping paper. They are trying to raise at least \$400. They make a profit of \$1.50 on each box of holiday cards and \$1.00 on each pack of wrapping paper.
 - a. What is an inequality for the profit the school wants to make for the fundraiser? **$1.5x + y \geq 400$**
 - b. If the fundraiser sells 100 boxes of cards and 160 packs of wrapping paper, will they reach their goal? Show your work. **No, they will not reach their goal.**

[2] Both parts answered correctly. Explanation is clear.

[1] One part answered correctly, or explanation is not clear or complete.

[0] No parts answered correctly.

6-5 Enrichment

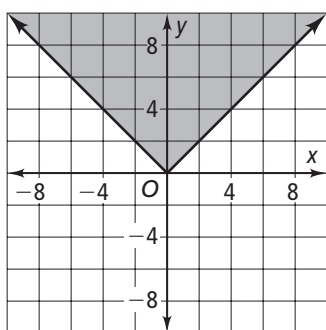
Linear Inequalities

Graphing Absolute Value Inequalities Graphing absolute value inequalities is similar to graphing linear inequalities, but the boundaries are absolute value graphs.

Problem

Graph $y \geq |x|$.

Graph the absolute value equation $y = |x|$ first. Use a solid line because the inequality symbol is \geq .



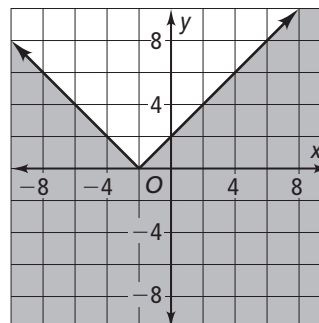
For the inequality part, the sign is \geq . Use an arbitrary point, such as $(0, 4)$ to decide where to shade. You cannot use $(0, 0)$ as this point lies on the boundary. Because $(0, 4)$ is a solution of the inequality, shade the portion above the absolute value graph.

Problem

Graph $y \leq |x + 2|$.

Graph the absolute value equation $y = |x + 2|$. Use a solid line because the inequality symbol is \leq .

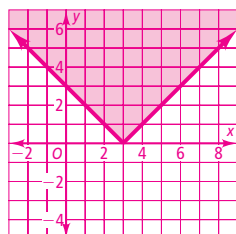
Test $(0, 0)$ to decide where to shade. Because $(0, 0)$ is a solution of the inequality, shade the portion below the absolute value graph.



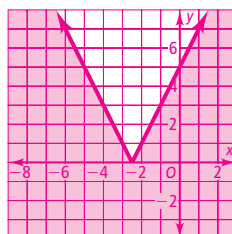
Exercises

Graph the following inequalities.

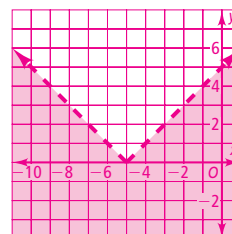
1. $y \geq |x - 3|$



2. $|2x + 5| \geq y$



3. $y < |x + 5|$



6-5 Reteaching

Linear Inequalities

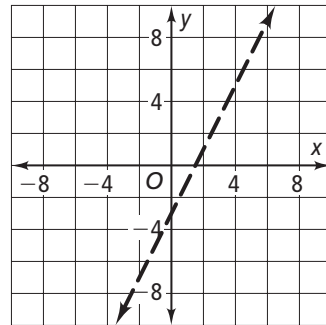
To graph an inequality, graph the line and find the solution region by substituting a test point. The point $(0, 0)$ is a good one unless the line goes through the origin.

Problem

What is the graph of $y > 2x - 3$?

Begin by graphing the line $y = 2x - 3$. Take random values for x , find the corresponding y values, and create a table.

x	$y = 2x - 3$
-2	-7
-1	-5
0	-3
1	-1
2	1



The ordered pairs are $(-2, -7)$, $(-1, -5)$, $(0, -3)$, $(1, -1)$, and $(2, 1)$. You can graph the line using these points. The line should be dashed because the inequality symbol is $>$.

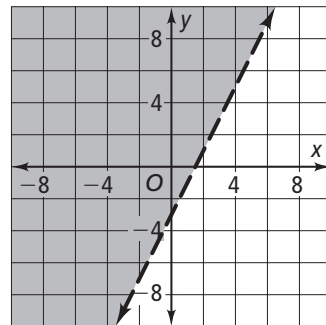
To determine which region to shade, substitute $(0, 0)$ into the inequality to see if it is a solution.

$$y > 2x - 3$$

$$0 \stackrel{?}{>} 2(0) - 3$$

$$0 > -3 \checkmark$$

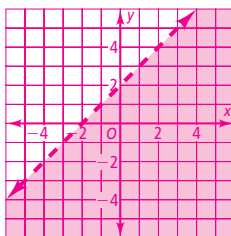
The point $(0, 0)$ satisfies the inequality and is above the line. Therefore, shade the region above the line, which is the solution region.



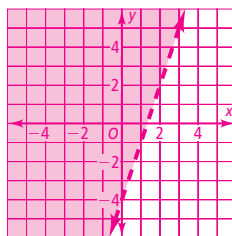
Exercises

Graph each linear inequality.

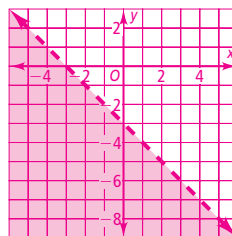
1. $y < x + 2$



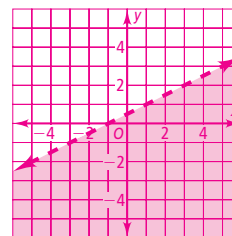
2. $y > 3x - 4$



3. $x + y < -3$



4. $x - 2y > -1$



6-5 **Reteaching** (continued)

Linear Inequalities

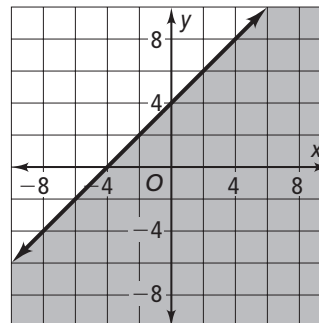
Problem

What is the inequality for the graph shown?

First look for the y -intercept for the boundary line. The y -intercept is the point $(0, 4)$.

Next determine the slope of the boundary line by finding a second point on the line, $(-4, 0)$. Use the slope formula to determine the slope: $\frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 0}{0 - (-4)} = \frac{4}{4} = 1$.

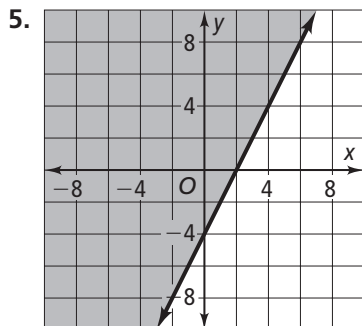
Now you know that the slope is 1 and the y -intercept is 4 and can write an equation for the boundary line $y = x + 4$.



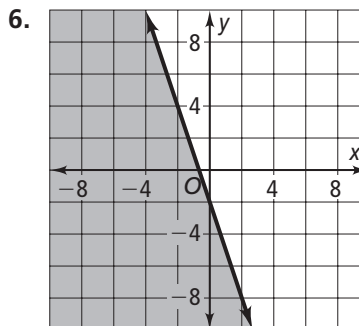
To find the inequality sign, notice that the line is solid. Then note that the shading is below the line, indicating “less than.” The inequality is $y < x + 4$.

Exercises

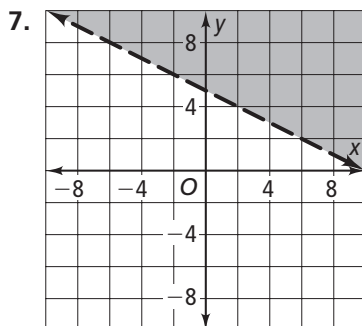
Determine the inequality for each graph shown.



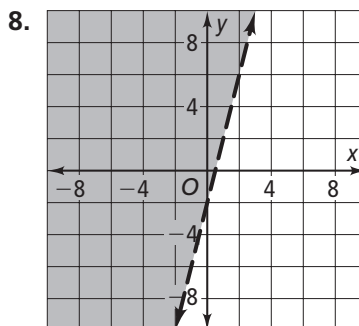
$y \geq 2x - 4$



$y \leq -3x - 2$



$y > -\frac{1}{2}x + 5$



$y > 4x - 2$

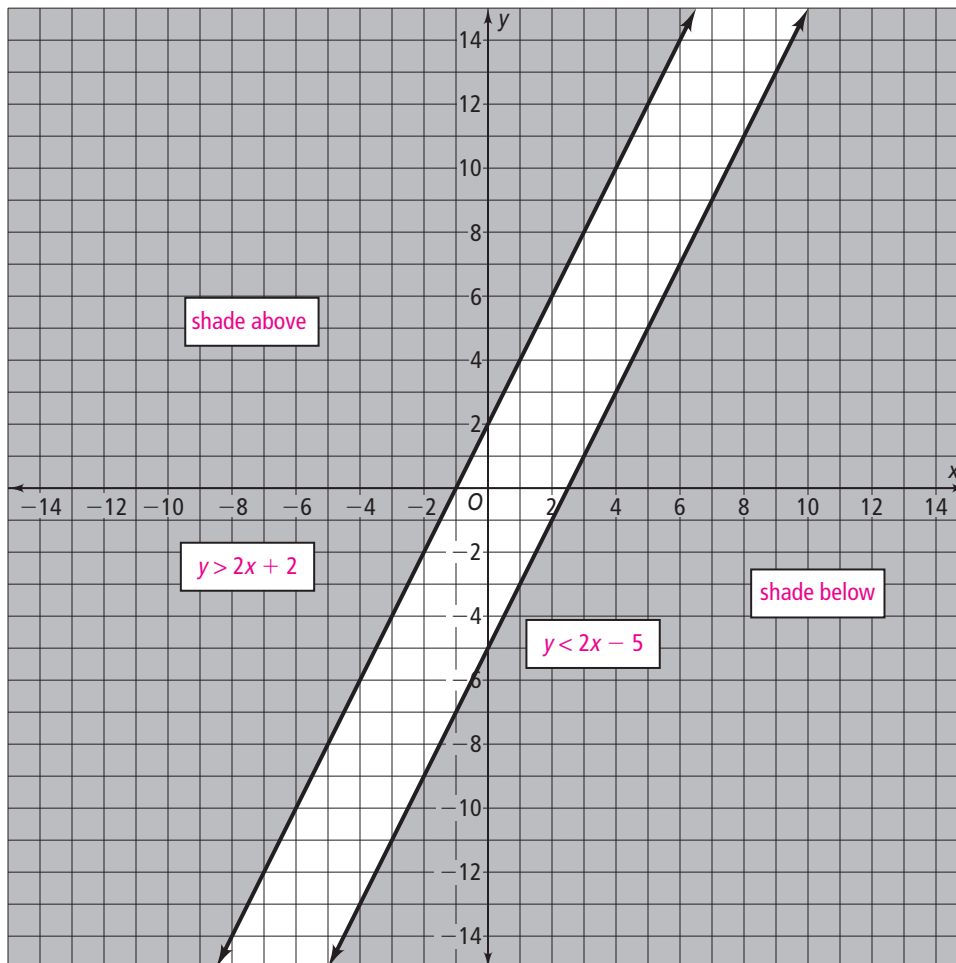
6-6 Additional Vocabulary Support

Systems of Linear Inequalities

Use the list below to complete the diagram.

shade above $y < 2x - 5$ $y > 2x + 2$ shade below

1.



2. What is the solution of the system of inequalities? $y < 2x - 5$
 $y > 2x + 2$

No solution

6-6

Think About a Plan

Systems of Linear Inequalities

Gift Certificates You received a \$100 gift certificate to a clothing store. The store sells T-shirts for \$15 and dress shirts for \$22. You want to spend no more than the amount of the gift certificate. You want to leave at most \$10 of the gift certificate unspent. You need at least one dress shirt. What are all of the possible combinations of T-shirts and dress shirts you could buy?

Understanding the Problem

1. What do you know about the cost of each type of shirt that you want to buy?

t-shirts: \$15; dress shirts: \$22

2. What do you know about how much you can spend? What do you know about how much you want to leave unspent?

You can spend up to \$100. You don't want to spend less than \$90.

3. What do you know about the number of dress shirts that you want to buy?

You want to buy at least one dress shirt.

Planning the Solution

4. What inequality represents the amount of shirts you can buy? **$15t + 22d \leq 100$**

5. What inequality represents the amount of the gift certificate which could be unspent? **$15t + 22d \geq 90$**

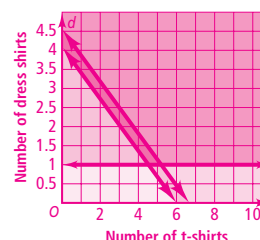
6. What inequality represents the number of dress shirts you want to buy? **$d \geq 1$**

Getting an Answer

7. How can you use these inequalities to find out the number of T-shirts and dress shirts you can buy? **You can graph the inequalities and locate points that satisfy all three inequalities. The coordinates of the points tell you how many of each type of shirt you can buy.**

8. Graph the system of linear inequalities. Point out the region that shows the answer.

9. What combinations of T-shirts and dress shirts are possible? **1 dress shirt and 5 t-shirts or 3 dress shirts and 2 t-shirts**



10. Are there any other possible combinations? Explain.

No. Any other combination would cost more than \$100, less than \$90, or not include a dress shirt.

6-6

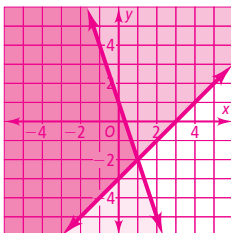
Practice

Form G

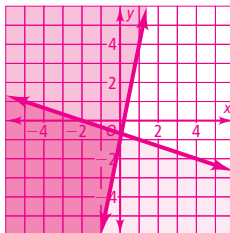
Systems of Linear Inequalities

Solve each system of inequalities by graphing.

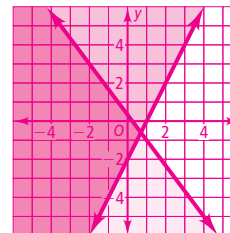
1. $3x + y \leq 1$
 $x - y \leq 3$



2. $5x - y \leq 1$
 $x + 3y \leq -2$



3. $4x + 3y \leq 1$
 $2x - y \leq 2$



4. **Writing** What is the difference between the solution of a system of linear inequalities and the solution of a system of linear equations? Explain.

The graph of a system of linear equations is made up of lines, so the solution, if it exists, is a single point. The graph of a system of linear inequalities is made up of shaded half-planes, so if a solution exists, it is a region where the shaded regions overlap.

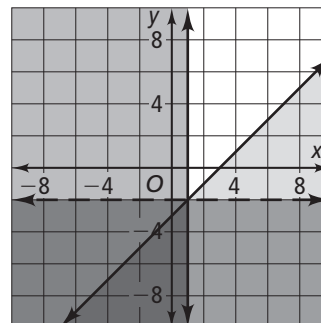
5. **Open-Ended** When can you say that there is no solution for a system of linear inequalities? Explain your answer and show with a system and graph.

Answers may vary. Sample: There is no solution to a system of inequalities when the boundary lines are parallel and the shaded regions do not overlap. Example: $y \leq x - 1$; $y \geq x + 2$; Check students' graphs.

6. **Error Analysis** A student graphs the system below. Describe and correct the student's error.

$x - y \geq 3$
 $y < -2$
 $x \geq 1$

When graphing the inequality $x \geq 1$, the student shaded the wrong half of the plane.



Determine whether the ordered pair is a solution of the given system.

7. (0, 1);
 $1 - x \geq 3y$
 $3y - 1 > 2x$
not a solution

8. (-2, 3);
 $2x + 3y > 2$
 $3x + 5y > 1$
solution

9. (1, 4);
 $2x + y > 3$
 $-3x - y \leq 5$
solution

6-6

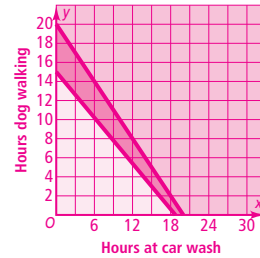
Practice (continued)

Form G

Systems of Linear Inequalities

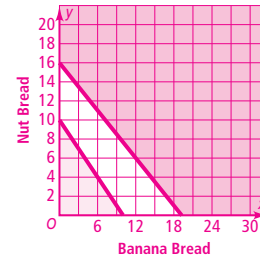
10. Mark is a student, and he can work for at most 20 hours a week. He needs to earn at least \$75 to cover his weekly expenses. His dog-walking job pays \$5 per hour and his job as a car wash attendant pays \$4 per hour. Write a system of inequalities to model the situation, and graph the inequalities.

$$x + y \leq 20, 4x + 5y \geq 75$$



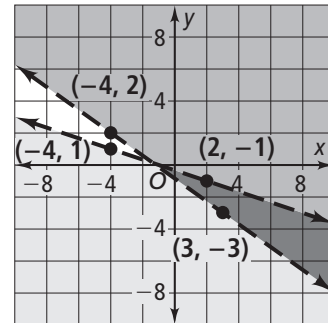
11. Britney wants to bake at most 10 loaves of bread for a bake sale. She wants to make banana bread that sells for \$1.25 each and nut bread that sells for \$1.50 each and make at least \$24 in sales. Write a system of inequalities for the given situation and graph the inequalities.

$$x + y \leq 10, 1.25x + 1.5y \geq 24$$



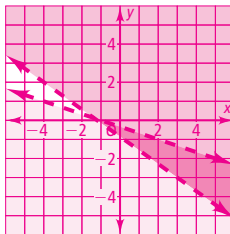
12. Write a system of inequalities for the following graph.

$$y > -\frac{5}{7}x - \frac{6}{7}, y < -\frac{1}{3}x - \frac{1}{3}$$

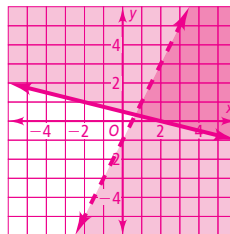


Solve each system of inequalities by graphing.

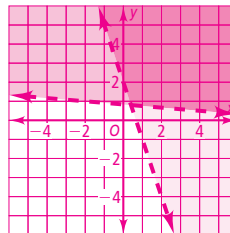
13. $5x + 7y > -6$
 $x + 3y < -1$



14. $x + 4y - 2 \geq 0$
 $2x - y + 1 > 2$



15. $\frac{x}{2} - 5 > -6y$
 $3x + y > 2$



6-6

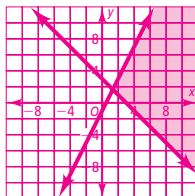
Practice

Form K

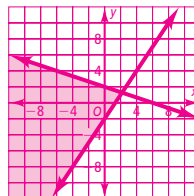
Systems of Linear Inequalities

Solve each system of inequalities by graphing.

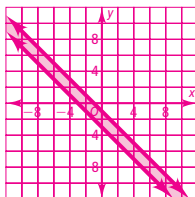
1. $y \leq 2x - 1$
 $y \geq -x + 3$



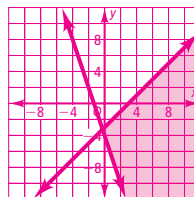
2. $3x - 2y \leq 4$
 $x + 3y \leq 6$



3. $x + y \geq -3$
 $2x + 2y \leq -2$



4. $-y \leq 3x + 4$
 $-3x + 3y \leq -9$

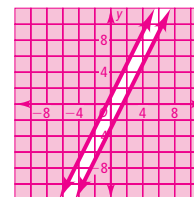


5. **Writing** Describe when you use a solid line or a broken line when graphing inequalities. What does each type of line mean?

You use a solid line for inequalities with \geq and \leq . You use a broken line for inequalities with $>$ and $<$. If a point on the line satisfies the inequality, then you use a solid line. If a point on the line does not satisfy the inequality, then you use a broken line.

6. **Open-Ended** Create a system of inequalities that has no solution. Demonstrate by drawing a graph.

Answers may vary. For example,
 $y \leq 2x - 3$
 $y \geq 2x + 1$



7. The owner of an ice cream stand needs to order waffle cones and sugar cones. There is room to store 10 boxes of cones. Each box of sugar cones costs \$100, and each box of waffle cones costs \$150. He has \$1250 budgeted for the purchase of cones.

- What variables will you use? **s = sugar cones; w = waffle cones**
- How will you decide which inequality signs to use and where to shade?
Both inequalities will be \leq because there must be less than or equal to 10 boxes and less than or equal to a cost of \$1250. The shaded portion will be below both lines.

c. What system of inequalities represents the information?

**$s + w \leq 10$
 $100s + 150w \leq 1250$**

6-6

Practice (continued)

Form K

Systems of Linear Inequalities

Determine whether the ordered pair is a solution of the given system.

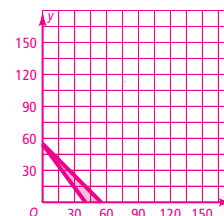
8. $(2, -1)$; $3 - 3y \leq 3y$ **no**
 $3y > 2x + 1$

9. $(-3, -3)$; $5x + 4y > -4$ **no**
 $2x + 3y > 2$

10. A friend makes \$15 per hour at his first job and \$11 per hour at his second job. His goal is to make at least \$600 per week. He does not want to work any more than 55 hours in a week. Write a system of inequalities for the given situation and graph the inequalities.

$$15x + 11y \geq 600$$

$$x + y \leq 55$$



11. For the school fundraiser, a class is selling stationery and greeting cards. The goal for the class is to sell at least 100 items. The school receives \$2.50 for each stationery set that is sold and \$3 for each set of greeting cards that is sold. The goal is to raise at least \$300. Write a system of inequalities for the given situation and graph the inequalities.

$$2.5x + 3y \geq 300$$

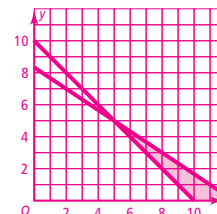
$$x + y \geq 100$$



12. A woman is purchasing fruit for some pies she is making for a party. She wants to purchase at least 10 pounds of strawberries and blueberries. Strawberries are sold for \$2 per pound, and blueberries are sold for \$3 per pound. She does not want to spend more than \$25 total for the fruit. Write a system of inequalities for the given situation and graph the inequalities.

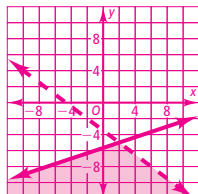
$$x + y \geq 10$$

$$2x + 3y \leq 25$$

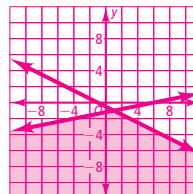


Solve each system of inequalities by graphing.

13. $3x + 4y < -14$
 $x - 3y \geq 17$



14. $x - 5y - 6 \geq 0$
 $2x + 4y + 1 \leq -1$



6-6 Standardized Test Prep

Systems of Linear Inequalities

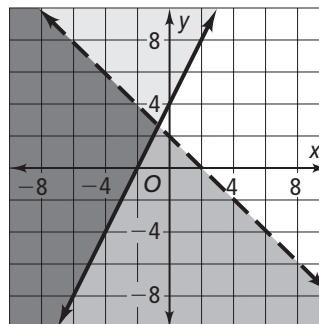
Multiple Choice

For Exercises 1–4, choose the correct letter.

1. You and a friend both would like a salad and a small drink. Between the two of you, you have \$8.00. A salad costs \$2.49 and a small drink is \$.99. Can either of you have a second salad or drink? **C**
- A. yes, 1 salad B. yes, 1 of each C. yes, 1 drink D. no, you cannot

2. Which of the following systems of inequalities represents the graph? **I**

- F. $y > 2x + 4$
 $y \leq -x + 2$
- G. $2x - y \geq 4$
 $y < -x + 2$
- H. $y \geq 2x + 4$
 $-x + y < 2$
- I. $-2x + y \geq 4$
 $x + y < 2$



3. For the graph above, what is the approximate y-value of the point of intersection? **C**
- A. -1 B. 4 C. 3 D. 2
4. A student spends no more than 2 hours on his math and English homework. If math takes about twice as long as English, what is the maximum time that the student can spend on English? **I**
- F. $\frac{1}{3}$ hour G. $\frac{1}{2}$ hour H. 1 hour I. $\frac{2}{3}$ hour

Short Response

5. A young woman wants to make at least \$200 a week and can work no more than 30 hours a week. She works at the library for \$8 an hour and babysits for \$6 an hour.
- a. What system of inequalities shows the possible combination of hours and jobs she can work? **$x + y \leq 30$; $6x + 8y \geq 200$; $x \geq 0$; $y \geq 0$**
- b. Why did you exclude points to the left of the y-axis and below the x-axis?
You cannot work a negative number of hours or earn a negative amount of dollars.

[2] Both parts answered correctly. Explanation is clear.

[1] One part answered correctly, or explanation is not clear or complete.

[0] No parts answered correctly.

6-6 Enrichment

Systems of Linear Inequalities

Linear programming is a method used to find out the maximum or minimum value of a linear expression with constraints (a given set of conditions). The maximum or minimum value will always occur at a vertex of the region determined by the given constraints, called the feasible region.

Problem

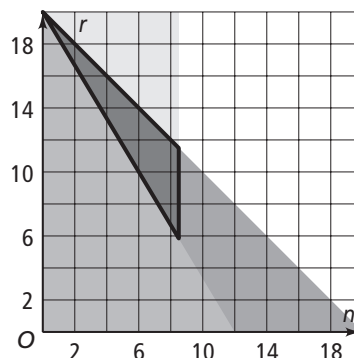
You want to make a health mix of nuts and raisins that provides at least 1800 calories for a hike along Appalachian Trail that has no more than 110 grams of fat and weighs no more than 20 ounces. Let n = nuts, r = raisins, and c = cost.

	cost/oz.	calories	fat(g)
nuts	\$0.50	150	13
raisins	\$0.10	90	0

First find the feasible region, which is the set of points that satisfies the constraints in a linear programming problem. The constraints in this problem are that both n and r are ≥ 0 and the inequalities below.

- $150n + 90r \geq 1800$ 150 calories per oz. nuts, 90 for raisins
- $13n \leq 110$ 13 grams of fat per oz. of nuts, max. fat 110 grams
- $n + r \leq 20$ Number of ounces of nuts & raisins cannot exceed 20 oz.

The triangular shape enclosed by black lines is called the feasible region and contains the possible solutions. In linear programming problems, you want to find more than the feasible region. You want to find the best combination of values to satisfy and minimize or maximize a function, called the optimization equation. For this problem, the optimization equation is $c = .5n + .1r$ to find the minimum cost, subject to the above constraints.



The vertices of the feasible region are substituted into $c = .5n + .1r$ to find the optimal number of nuts and raisins for the least cost. The vertex $\approx (8.5, 6)$ has the optimal values for n and r to meet the constraints and minimize cost. So if you buy 8.5 ounces of nuts and 6 ounces of raisins, you will have a minimal cost of $c = .5n + .1r = .5(8.5) + .1(6) = \4.85 .

- Find the minimum and maximum values of $C = 4x + 5y$ subject to the constraints $x \geq 0, y \geq 0, x + y \leq 6$. **Minimum: 0; Maximum: 30**
- Find the minimum and maximum values of $C = 6x + 7y$ subject to the constraints $x \geq 0, y \geq 0, 4x + 3y \geq 24, x + 3y \geq 15$. **Minimum: 46; Maximum: 90**
- How is the optimization equation used in a linear programming problem and how is the system of constraints used? **The system is used to define the feasible region. The optimization equation is used to evaluate the vertices to see which value combination yields the maximum and/or minimum value.**

6-6 **Reteaching**

Systems of Linear Inequalities

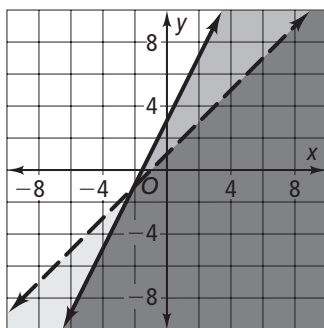
A system of linear inequalities is a set of linear inequalities in the same plane. The solution of the system is the region where the solution regions of the inequalities of the system overlap.

Problem

What is the graph of the system of linear inequalities: $x - y > -1$?
 $y \leq 2x + 3$?

Put the first inequality into slope-intercept form, $y < x + 1$. Use a dashed line since $<$ does not include the points on the boundary line in the solution. Using the point $(0, 0)$, decide where to shade the first inequality. The point $(0, 0)$ makes the inequality true, so shade the region including $(0, 0)$.

Then graph the boundary line of the second inequality, $y \leq 2x + 3$. It is a solid line because of the \leq sign. Use the point $(0, 0)$ to decide where to shade the second inequality. The point $(0, 0)$ makes the second inequality true, so shade the region including $(0, 0)$.



The overlapping region of the 2 inequalities is the solution to the system. It includes the points $(0, 0)$, $(1, 1)$, $(3, 1)$. You can test any point in the region in both equations to see if it makes both equations true. In word problems, the solutions often cannot be negative (cars, tickets sold, etc.). Two requirements are that $x \geq 0$ and $y \geq 0$. Keep this in mind when graphing word problems.

Problem

A cash register has fewer than 200 dimes and quarters worth more than \$39.95. How many of each coin are in the register?

type of coin	quantity	value of coin	value in cents
quarters	q	\$0.25	$25q$
dimes	d	\$0.10	$10d$
TOTAL	200		3995

The system of inequalities that you get from the table is: $q + d < 200$
 $25q + 10d > 3995$

6-6 **Reteaching** (continued)

Systems of Linear Inequalities

Using elimination, solve for q by multiplying all terms in the first equation by -10 and eliminating d : $(q + d < 200)(-10)$.

$$-10q - 10d > -2000$$

$$\underline{25q + 10d > 3995}$$

$$15q > 1995$$

$$q > 133$$

$$q + d < 200$$

$$133 + d < 200, d < 67$$

Now add the 2 systems together to solve for q .

Write first inequality.

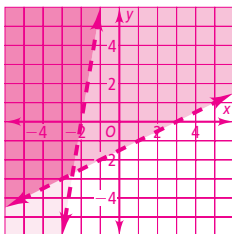
Substitute in 133 for q , subtract 133 from both sides and solve for d .

The register contains at least 133 quarters and no more than 67 dimes.

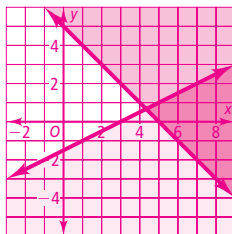
Exercises

Graph the following systems of inequalities.

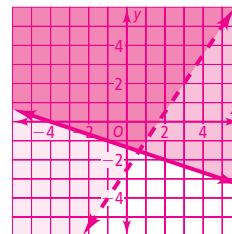
1. $x - 2y < 3$
 $\frac{y}{2} > 3x + 6$



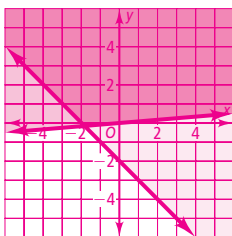
2. $y \geq -x + 5$
 $-x \leq -2y - 3$



3. $x + 3y \geq -4$
 $3x - 2y < 5$



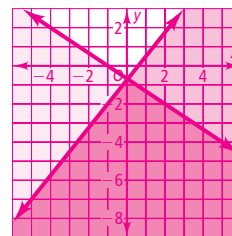
4. $3y \geq \frac{x}{4}$
 $-y \leq x + 2$



5. $2x - y < 1$
 $x + 2y < -4$



6. $5x - 4y \geq 3$
 $2x + 3y \leq -2$



Chapter 6 Quiz 1

Form G

Lessons 6-1 through 6-4

Do you know HOW?

Solve each system by graphing. Tell whether the system has *one solution*, *infinitely many solutions*, or *no solution*.

$$\begin{aligned} 1. \quad & y = 2x + 5 \\ & y - 4x = -2 \\ & \left(3\frac{1}{2}, 12\right); \text{ one} \\ & \text{solution} \end{aligned}$$

$$\begin{aligned} 2. \quad & y - 2x = 1 \\ & y = 2x + 6 \\ & \text{no solution} \end{aligned}$$

$$\begin{aligned} 3. \quad & y + \frac{x}{2} = 8 \\ & 2y = -x + 16 \\ & \text{infinitely many} \\ & \text{solutions} \end{aligned}$$

$$\begin{aligned} 4. \quad & 2x - 3y = 3 \\ & x + 4y = -2 \\ & \left(\frac{6}{11}, -\frac{7}{11}\right); \text{ one} \\ & \text{solution} \end{aligned}$$

Solve each system using substitution.

$$\begin{aligned} 5. \quad & x = y + 2 \\ & 2y = x - 1 \\ & (3, 1) \end{aligned}$$

$$\begin{aligned} 6. \quad & y = 3x + 5 \\ & y = x + 3 \\ & (-1, 2) \end{aligned}$$

$$\begin{aligned} 7. \quad & 3x = y + 2 \\ & -2y = 1 - 3x \\ & (1, 1) \end{aligned}$$

$$\begin{aligned} 8. \quad & x = \frac{y}{2} - 1 \\ & 6x + 5y = 26 \\ & (1, 4) \end{aligned}$$

Solve each system using elimination.

$$\begin{aligned} 9. \quad & 3x + 4y = 31 \\ & 2x - 4y = -6 \\ & (5, 4) \end{aligned}$$

$$\begin{aligned} 10. \quad & 3x + 5y = 54 \\ & 6x + 4y = 72 \\ & (8, 6) \end{aligned}$$

$$\begin{aligned} 11. \quad & -14x + 9y = 46 \\ & 14x - 9y = 102 \\ & \text{no solution} \end{aligned}$$

$$\begin{aligned} 12. \quad & 4x + 3y = 16 \\ & 7x - 5y = 69 \\ & (7, -4) \end{aligned}$$

13. John paid \$34 for two algebra and three geometry books. He paid \$36 for three algebra and two geometry books. What is the cost of each book?

algebra book: \$8; geometry book: \$6

14. The sum of two numbers is 14. If one of the numbers is doubled, the sum will become 22. What are the numbers?

6 and 8

15. Peter invested \$450 in insurance and stocks. He put \$50 more in insurance than stocks. How much did he invest in each?

\$250 in insurance; \$200 in stocks

16. The measure of one of two supplementary angles is three times the measure of the other angle. What are the measures of the angles?

45° and 135°

Do you UNDERSTAND?

17. **Reasoning** If a system of linear equations has no solution, what does that tell you about the slopes and y -intercepts of the graphs of the equations?

The lines have the same slope with different y -intercepts.

18. **Open-Ended** Write a system of linear equations that has no solution and a system of equations that has infinitely many solutions.

Answers may vary. Sample:

no solution: $x + y = 4$ and $x + y = -3$;

infinitely many solutions: $6x + 2y = 5$ and $y = -3x + 2.5$

Chapter 6 Quiz 2

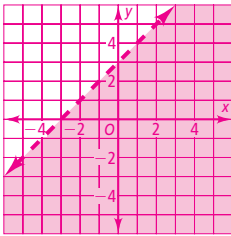
Form G

Lessons 6-5 through 6-6

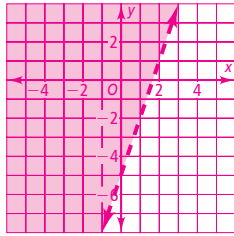
Do you know HOW?

Graph each inequality in the coordinate plane.

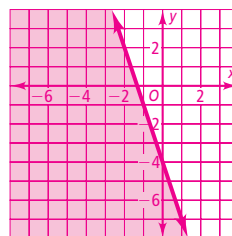
1. $y < x + 3$



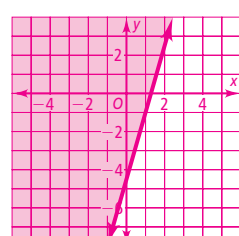
2. $y > 3x - 5$



3. $3x + y \leq -4$



4. $9 \geq 7x - 2y$



Solve each system of inequalities by graphing.

5. $-x + 7y > 16$

$11x - y \leq 12$

(1.3, 2.5)

6. $2x + 5y \leq 6$

$5x - 3y \geq 9$

(2, 0.4)

7. $3y < \frac{x}{3} - 1$

$2y \leq 2x + 1$

(-0.4, -0.4)

8. $3x \geq 5y - 4$

$-2y < 4x - 1$

(-0.3, 0.6)

Do you UNDERSTAND?

9. **Writing** How do you decide what region to shade when graphing a system of linear inequalities? Explain with an example and graph.

Test a point (0, 0) if it is not on the line) to see if it is a solution. If so, shade the half-plane containing the point. If not, shade the other half-plane.

10. **Open-Ended** When do you use a dashed line or a solid line when graphing inequalities? What do you know about the points on the dashed or solid line?

If the inequality is \leq or \geq , use a solid line because the points on the line are solutions to the inequality. If the inequality is $<$ or $>$, use a dashed line because the points on the line are not solutions to the inequality.

11. **Reasoning** When can you not use (0, 0) as a point to determine the area to be shaded? How would you choose a point?

You cannot use (0, 0) when the boundary line passes through the origin. Use any other point that is not on the line, such as a point on an axis.

Chapter 6 Test

Form G

Do you know HOW?

Solve each system by graphing. Tell whether the system has *one solution*, *infinitely many solutions*, or *no solution*.

1. $x - 2y = 3$

$y = -2x + 6$

(3, 0); one solution

2. $x + y = 3$

$3x - 2y = 4$

(2, 1); one solution

3. $2x = -4y + 10$

$6y = -3x + 12$

no solution

Solve each system using substitution.

4. $3x - 5y = -1$

$x - y = -1$

(-2, -1)

5. $x + 2y = -1$

$2x - 3y = 12$

(3, -2)

6. $2x + 3y = 9$

$3x + 4y = 5$

(-21, 17)

7. $7x = 2y + 1$

$4y = -3x + 15$

(1, 3)

8. $x + \frac{y}{2} = 4$

$\frac{x}{3} + 2y = 5$

(3, 2)

9. $\frac{x}{2} + \frac{y}{4} = 3$

$2x - y = 4$

(4, 4)

Solve each system using elimination.

10. $x + y = 4$

$x - y = 6$

(5, -1)

11. $-2x + 3y = 9$

$2x - 2y = -4$

(3, 5)

12. $x + y = 7$

$3x - 2y = 11$

(5, 2)

13. $7x - 8y = 11$

$8x - 7y = 7$

(-1.4, -2.6)

14. $0.4x + 0.3y = 1.7$

$0.7x - 0.2y = 0.8$

(2, 3)

15. $3x - 7y + 10 = 0$

$y - 2x - 3 = 0$

(-1, 1)

Write a system of equations to model each situation. Solve by any method.

16. Ten years from now, A will be twice as old as B. Five years ago, A was three times as old as B. What are the present ages of A and B?

A is 50 and B is 20

17. The ratio of incomes of two persons is 9:7. The difference in their weekly incomes is \$200. What are their weekly incomes?

\$900 and \$700

18. A change purse contains a total of 100 nickels and dimes. The total value of the coins is \$7. How many coins of each type does the purse contain?

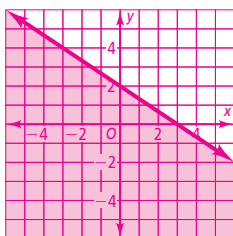
40 dimes and 60 nickels

Chapter 6 Test (continued)

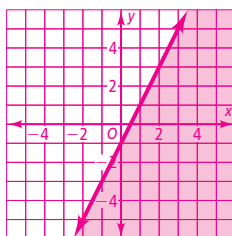
Form G

Graph each inequality in the coordinate plane.

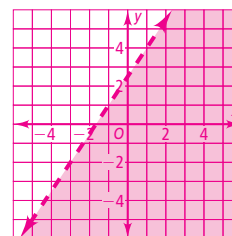
19. $2x + 3y \leq 6$



20. $2x - y \geq 1$

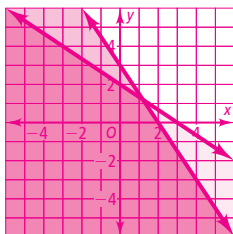


21. $-3x + 2y < 5$

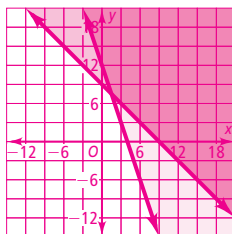


Solve each system of inequalities by graphing.

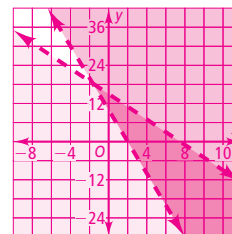
22. $2x + 3y \leq 6$
 $3x + 2y \leq 6$



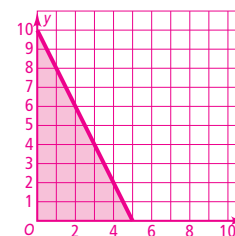
23. $x + y \geq 9$
 $3x + y \geq 12$



24. $5x + y > 10$
 $2x + y < 15$



25. For a party, you can spend no more than \$20 on cakes. Egg cake cost \$4 and cream cake cost \$2. Write the linear inequality that models the situation. Graph the inequality. $4x + 2y \leq 20$



Do you UNDERSTAND?

26. **Open-Ended** Write a system of linear equations that has infinitely many solutions.

Answers may vary. Sample: $y = 2x - 5$; $-4x + 2y = -10$

27. **Error Analysis** A student determined that (1, 1) is one of the solutions of the linear inequality $y \leq 2x - 3$, as shown below. What error did the student make?

$$y \leq 2x - 3$$

$$1 \leq 2(1) - 3$$

$$1 \leq 1$$

When the student simplified the expression $2(1) - 3$, the student got 1 instead of -1 .

Chapter 6 Quiz 1

Form K

Lessons 6-1 through 6-4

Do you know HOW?

Solve each system by graphing. Tell whether the system has *one solution*, *infinitely many solutions*, or *no solution*.

$$\begin{aligned} 1. \quad y &= 3x - 7 \\ y &= 2x - 6 \end{aligned}$$

(1, -4); one solution

$$\begin{aligned} 2. \quad y - 3x &= -2 \\ y &= 3x + 5 \end{aligned}$$

no solution

$$\begin{aligned} 3. \quad y &= -3x + 12 \\ x + \frac{1}{3}y &= 4 \end{aligned}$$

infinitely many solutions

Solve each system using substitution.

$$\begin{aligned} 4. \quad x &= 2y - 2 \\ 3y &= x + 6 \end{aligned}$$

(6, 4)

$$\begin{aligned} 5. \quad y &= 2x + 9 \\ y &= x + 6 \end{aligned}$$

(-3, 3)

$$\begin{aligned} 6. \quad 2x &= y - 4 \\ -3y &= x - 5 \end{aligned}$$

(-1, 2)

Solve each system using elimination.

$$\begin{aligned} 7. \quad 3x + 2y &= -6 \\ -x - 2y &= 10 \end{aligned}$$

(2, -6)

$$\begin{aligned} 8. \quad x + 2y &= 9 \\ 2x - 5y &= -27 \end{aligned}$$

(-1, 5)

$$\begin{aligned} 9. \quad 3x + 5y &= -18 \\ 4x - 10y &= -24 \end{aligned}$$

(-6, 0)

Write a system of equations to model each situation. Solve by any method.

10. A piggy bank contains 100 coins consisting of nickels and dimes. The total value of the coins is \$8.50. How many coins of each type does the bank contain?

30 nickels and 70 dimes**Do you UNDERSTAND?**

11. **Writing** Explain how you know if a system of equations has one solution, no solutions, or infinitely many solutions.

You can determine the number of solutions algebraically by examining the slope and the y-intercept of both equations. If the slopes and the y-intercepts are both the same, the equations are the same and there are infinitely many solutions. If the slopes are the same, but the y-intercepts are different, the lines are parallel and there is no solution. If the slopes are different, the lines intersect at one point resulting in one solution.

12. **Reasoning** If two lines have the same y-intercept but different slopes, what can you conclude about the solution of the system? Explain.

Since the lines have different slopes, they intersect, so the system has one solution. Since they have the same y-intercept, they intersect at their y-intercept. If their y-intercept is b , the solution is $(0, b)$.

Chapter 6 Quiz 2

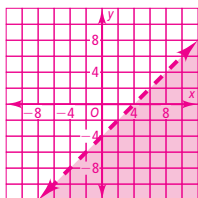
Form K

Lessons 6-5 through 6-6

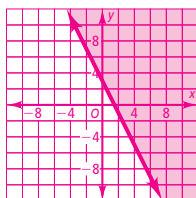
Do you know HOW?

Graph each inequality in the coordinate plane.

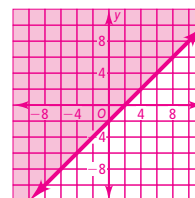
1. $y < x - 4$



2. $y \geq -2x + 3$

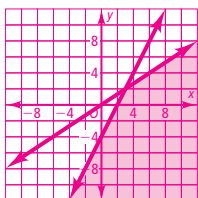


3. $6 \geq 3x - 3y$

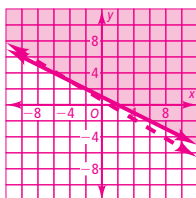


Solve each system of inequalities by graphing.

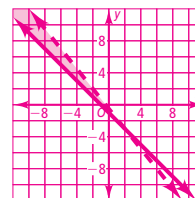
4. $-2x + 3y \leq 0$
 $4x - 2y \geq 8$



5. $-3x - 5y < -3$
 $x + 2y \geq 2$

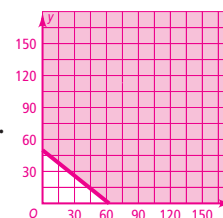


6. $-3x \leq 3y + 3$
 $4x < -5y - 2$



7. Shari is working two jobs to save at least \$500 for her trip. She earns \$8 per hour at the first job and \$10 per hour at her second job. What is the inequality that can help Shari know how many hours she needs to work at each job to save the money? Graph the inequality.

$8x + 10y \geq 500$



Do you UNDERSTAND?

8. **Error Analysis** A student determined that (3, 1) is one of the solutions of the linear inequality $y \geq 3x - 5$, as given below. What error did the student make?

$y \geq 3x - 5$

$3 \geq 3(1) - 5$

$3 \geq -2$

The student switched the x- and y-values when they substituted.

9. **Reasoning** A point lies on the dashed line of the graph of an inequality. Is the point part of the solution? Why or why not?

It is not part of the solution. When graphing inequalities, a dashed line is used for greater than or less than. Points on the boundary line are not part of the solution for greater than or less than inequalities.

Chapter 6 Test

Form K

Do you know HOW?

Solve each system by graphing. Tell whether the system has *one solution*, *infinitely many solutions*, or *no solution*.

1. $y = \frac{1}{2}x + 4$
 $y = -2x - 1$

$(-2, 3)$; one solution

2. $y = x + 2$
 $y = 3x + 6$

$(-2, 0)$; one solution

3. $x + y = 2$
 $x + y = -1$

no solution

Solve each system using substitution.

4. $x + y = 1$
 $2x + 3y = -4$

$(7, -6)$

5. $x - 4y = 11$
 $2y - x = -7$

$(3, -2)$

6. $2x + y = 1$
 $x - 2y = 23$

$(5, -9)$

Solve each system using elimination.

7. $2x + 3y = 10$
 $2x - y = -14$

$(-4, 6)$

8. $x + y = -6$
 $x - y = 6$

$(0, -6)$

9. $3x = -2y - 5$
 $2y = -5x + 5$

$(5, -10)$

Solve each problem.

10. The sum of two numbers is 23. If one of the numbers is halved, the sum will become 17. What are the numbers?

11 and 12

11. The perimeter of a rectangle is 60 cm. The length is four times the width. What are the length and the width of the rectangle?

length = 24 cm; width = 6 cm

Write a system of equations to model each situation. Solve by any method.

12. Sarah is 25 years older than her son Gavin. In ten years, Sarah will be twice Gavin's age. How old are Sarah and Gavin now?

Sarah = 40 yr; Gavin = 15 yr

13. A chemist is mixing a solution that is 2% acid and another solution that is 8% acid. She needs to make 75 mL of a solution that is 5% acid. How much of each solution should she use?

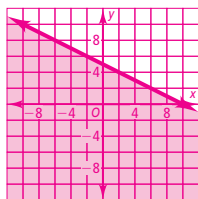
37.5 mL of the 2% solution and 37.5 mL of the 8% solution

Chapter 6 Test (continued)

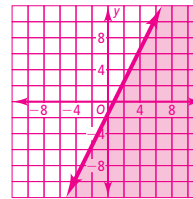
Form K

Graph each inequality in the coordinate plane.

14. $x + 2y \leq 10$

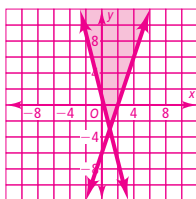


15. $4x - 2y \geq 3$

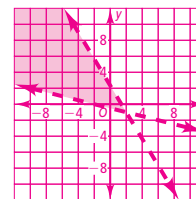


Solve each system of inequalities by graphing.

16. $4x + y \geq 1$
 $3x - y \leq 6$

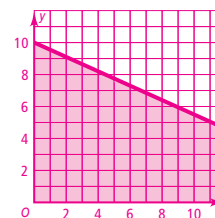


17. $x + 4y > -2$
 $5x + 3y < 7$



18. For a work banquet, Jack can spend no more than \$200 on dessert. Fruit pies cost \$9 each and cakes cost \$20 each. Write the linear inequality that models the situation. Graph the inequality.

$9x + 20y \leq 200$



Do you UNDERSTAND?

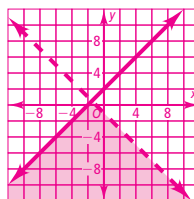
19. **Writing** How do you check to see if an ordered pair satisfies a system of inequalities graphically?

You can plot the point on the graph and see if it lies within the shaded region.

20. **Open-Ended** Write a system of inequalities in which the shaded region is below both lines. Graph the system.

Answers may vary. Sample:

$y \leq x + 2$
 $y < -x - 1$



21. **Open-Ended** Write a system of linear equations that has no solution.

Answers may vary. Sample:

$y = 3x + 5$
 $y = 3x - 1$

Performance Tasks

Chapter 6

Give complete answers. Show all your work.

Task 1

Explain which method (graphing, substitution, or elimination) would be most appropriate for solving the given systems of linear equations. Then solve each system of linear equations using your chosen method.

a. $y = 3x + 5$
 $2y - 6x = 4$

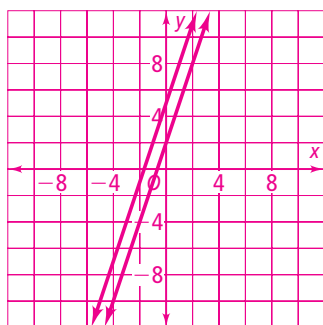
graphing, because one equation is already in slope-intercept form; no solution

b. $9x + 4y = -17$
 $12y = -3 - 3x$

substitution, because the y -term in the second equation is already isolated; $(-2, \frac{1}{4})$

c. $5x - 7y = -21$
 $14y - 5x = 22$

elimination, because of the coefficients of the x -terms are opposites; $(-4, \frac{1}{7})$



- [4] Student shows understanding of the task, completes all portions of the task appropriately with no errors in computation, and fully supports work with appropriate diagram and explanations.
- [3] Student shows understanding of the task, completes all portions of the task appropriately with one error in computation, and supports work with appropriate diagram and explanations.
- [2] Student shows understanding of the task, but makes errors in computation resulting in incorrect answer(s), or needs to explain better.
- [1] Student shows minimal understanding of the task or offers little explanation.
- [0] Student shows no understanding of the task and offers no explanation.

Task 2

Two students disagree about how to solve a problem and have asked you for help. They have 220 ft of ribbon to enclose a rectangular space in the gym for a dance floor. The dance floor has to be at least 70 ft long to accommodate the anticipated number of dancers.

- a. Write a system of inequalities. Then draw a graph that shows all of the possible dimensions of the dance floor.

$2x + 2y \leq 220$ so $y \leq -x + 110$ and $y \geq 70$; $x \geq 0$ and $y \geq 0$

- b. Write two possible solutions to the problem that would be appropriate dimensions for a dance floor. Compare your solutions and determine which would offer more dancing space. Explain your reasoning.

If the dance floor is 70ft by 40ft, the area would be 2800 sq.ft. If it is 90ft by 16ft, the area would be 1350sq. ft, or less than half of the other possibility and it would be too narrow. It would be best if the dance floor is 70ft by 40ft.



- [4] Student shows understanding of the task, completes all portions of the task appropriately, and fully supports work with appropriate diagram and explanations.
- [3] Student shows understanding of the task, completes all portions of the task appropriately, and supports work with appropriate diagram and explanations with a minor error.
- [2] Student shows understanding of the task, but needs to explain better.
- [1] Student shows minimal understanding of the task or offers little explanation.
- [0] Student shows no understanding of the task and offers no explanation.

Performance Tasks (continued)

Chapter 6

Task 3

You and a friend are starting a computer repair business. You estimate that your expenses are \$500 per week.

- a. Select a reasonable average number of dollars you would expect to spend (expense) on replacement parts for each computer you repair. Then decide on an average fee that you will charge for each computer you repair (income). Let x represent the number of computers you repair each week. Write a linear equation that describes your total weekly expenses, and a second linear equation that describes your weekly income.

Check students' work.

- b. Graph the equations that represent your weekly expenses, weekly income, and break-even point on the same coordinate plane.

Check students' work.

- c. Explain why the intersection of the two lines on your graph represents the break-even point. Determine the coordinates of the break-even point for your graph and explain what these numbers indicate about your business.

Check students' work.

[4] Student shows understanding of the task, completes all portions of the task appropriately with no errors in computation, and fully supports work with appropriate diagram and explanations.

[3] Student shows understanding of the task, completes all portions of the task appropriately, with one error in computation, and supports work with appropriate diagram and explanations.

[2] Student shows understanding of the task, but makes errors in computation resulting in incorrect answer(s), or needs to explain better.

[1] Student shows minimal understanding of the task or offers little explanation.

[0] Student shows no understanding of the task and offers no explanation.

Task 4

Write a detailed solution to the system **Write each inequality in slope-intercept form and graph.**

$$x + 3y \leq 9$$

$$y \leq -2x + 6, \text{ where } x \geq 0 \text{ and } y \geq 0.$$

Graph the system to show all possible solutions.

[4] Student shows understanding of the task, completes all portions of the task appropriately, and fully supports work with appropriate diagram and explanations.

[3] Student shows understanding of the task, completes all portions of the task appropriately, and supports work with appropriate diagram and explanations with a minor error.

[2] Student shows understanding of the task, but needs to explain better.

[1] Student shows minimal understanding of the task or offers little explanation.

[0] Student shows no understanding of the task and offers no explanation.

Cumulative Review

Chapters 1–6

Multiple Choice

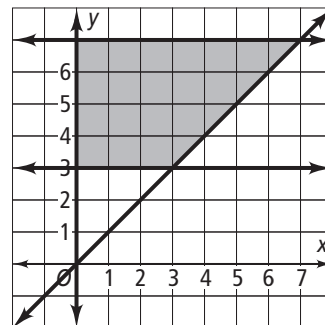
For Exercises 1–10 choose the correct letter.

- A train moving at a constant speed travels 180 mi in 4 h. How far does the train travel in 7 h? **B**
A. 360 mi B. 315 mi C. 280 mi D. 420 mi
- Which is a solution of $-8x + 5 \geq 11$? **I**
F. $-\frac{1}{2}$ G. 3 H. 0.25 I. -1
- Which two quadrants contain all of the solutions to the following system? **A**
 $y > 4x - 2$
 $y > -3x + 5$
A. I and II B. II and III C. III and IV D. I and IV
- Which of the following equations represents a horizontal line through $(5, -3)$? **H**
F. $x = -3$ G. $y = 5$ H. $y = -3$ I. $x = 5$
- The graphs of the two lines $4x = 3y + 23$ and $4y + 3x = -19$ **D**
A. do not intersect. C. intersect at $(-2, 5)$.
B. are identical. D. are perpendicular.
- What is the slope of the line with equation $4x + 2y = 8$? **F**
F. -2 G. -0.5 H. 0.5 I. 2
- Which of the following is the solution of $-4(5 - 2x) = 8$? **A**
A. 3.5 B. 2.5 C. -4 D. 1.5
- To solve the following system of equations by elimination, which operation would you perform first? **H**
 $234x + 65y = 219$
 $1225x + 65y = -427$
F. addition G. subtraction H. multiplication I. division

Cumulative Review (continued)

Chapters 1–6

9. Which of the following points are not solutions of $2y - 5x > 6$? **C**
- I. $(-3, 4)$ II. $(5, -\frac{5}{3})$ III. $(\frac{3}{4}, 5)$ IV. $(2.6, 5)$
- A. I and II B. III only C. II and IV D. III and IV
10. Find the slope of each line.
- a. A line that is parallel to the graph of $y = \frac{1}{2}x + 7$. $\frac{1}{2}$
- b. A line that is perpendicular to the graph of $y = -2x - 3$. $\frac{1}{2}$
11. Find the x -intercept of the graph of each equation.
- a. $3x + 2y = 7$ $\frac{7}{3}$
- b. $2x + 3y = 7$ $\frac{7}{2}$
12. Find the number of solutions to each system.
- a. $4x - y + 1 = 0$ **no solution**
 $4x - y + 3 = 0$
- b. $2x - y + 4 = 0$ **infinitely many solutions**
 $4x - 2y + 8 = 0$
13. **Open-Ended** Write a question that can be solved using a system of linear equations.
Answers may vary. Sample: Six daffodil bulbs and 4 iris bulbs cost \$24. Five daffodil bulbs and 6 iris bulbs cost \$28. How much does each bulb cost?
14. Solve the following system of equations by graphing. **$(-2, 1)$**
- $$y = x + 3$$
- $$y = -2x - 3$$
15. Suppose your office gives you \$200 to buy binders. Small binders cost \$7 each. Large binders cost \$8 each. Write the inequality that describes how many of each kind of binder you can buy. **$7s + 8l \leq 200$**
16. Write the set of inequalities that defines the trapezoid shown at the right. **$y \leq 7, x \geq 0, y \geq 3, y \geq x$**
17. Write an equation of the line with a y -intercept of -4 that is parallel to $6x - 2y = 13$. **$y = 3x - 4$**
18. A company's sales of garden tractors increased about 106% from 2007 to 2008. The number of tractors sold in 2007 was 347. How many tractors were sold in 2008? **about 715**
19. Southside Bowling Alley charges \$3 for the first game and \$.50 for each additional game. Eastside Bowling Alley charges \$1 per game. How many games would you have to bowl to make Southside the less expensive choice? **6 games**



Chapter 6 Project Teacher Notes: Let's Dance

About the Project

Students will assume the roles of student council members planning a dinner dance. They use systems of equations to analyze costs and make decisions about dinner and bands. Then students write reports detailing costs of catering and bands, and make recommendations for ticket prices.

Introducing the Project

- Have students work with partners or in small groups. Have each group brainstorm factors to consider when organizing a school dinner dance, such as the band, dinner, ticket prices, decorations, and venue.
- Ask the groups to consider how the cost of the band and dinner might affect the ticket price. They should understand that the higher the cost of the band and dinner, the higher the ticket cost will need to be.
- Challenge students to research costs to hire a typical band and caterer. Have them create spreadsheets to show their results.

Activity 1: Graphing

Students write linear equations to compare the costs of Bands A and B. Then they graph each equation to find the number of tickets that would need to be sold so that the cost of hiring the two bands would be equal.

Activity 2: Calculating

Students calculate the fixed cost and the cost per person served based on the information supplied.

Activity 3: Writing

Students use the information they previously calculated to write reports recommending a band and a ticket price. In their reports, they list their choices assuming both 200 and 300 attendees.

Activity 4: Graphing

Students use the ticket prices they calculated in Activity 3 to graph a system of linear inequalities that shows the total amount received for the tickets.

Finishing the Project

You may wish to plan a project day on which students share their completed projects. Encourage groups to explain their processes as well as their results. Have students review their project work and update their folders.

Chapter 6 Project: Let's Dance

Beginning the Chapter Project

Suppose you are the student council member that is responsible for planning a student dinner dance. Plans include hiring a band and buying and serving dinner. You want to keep the ticket price as low as possible to encourage student attendance.

As you work through the following activities, you will use systems of equations to analyze costs and make decisions. You will write a report detailing your choice of band, the cost of a catering service, and your ticket price recommendation.

List of Materials

- Calculator
- Graph paper

Activities

Activity 1: Graphing

Band A charges \$600 to play for the evening. Band B charges \$350 plus \$1.25 for each ticket sold.

- Write a linear equation for the cost of each band.
- Graph each equation and find the number of tickets for which the cost of the two bands would be equal.

$y = 600$; $y = 1.25x + 350$; Check students' graphs. The costs will be equal if 200 tickets are sold.

Activity 2: Calculating

A caterer charges a fixed cost for preparing a dinner plus an additional cost for each person served. You know that the cost for 100 students will be \$750 and the cost for 150 students will be \$1050. Find the caterer's fixed cost and the cost per student served. **Fixed cost: \$150; cost per student: \$6**

Activity 3: Writing

Use your information from Activities 1 and 2. Assume that 200 students attend the dance. **Check students' work.**

- Write a report listing which band you would choose and the cost per ticket that you need to charge to cover expenses.
- Repeat the process assuming that 300 students attend.

Chapter 6 Project: Let's Dance (continued)

Activity 4: Graphing

In Activity 3, you found two ticket prices. Each price covers the cost of the dinner dance under certain conditions. Plan for between 200 and 300 people, that is $x > 200$ and $x < 300$. **Check students' work.**

- If your objective is to keep the ticket price as low as possible, even at the risk of not covering your costs, which ticket price would you select? Based on this choice, write a linear equation that gives the total amount collected for ticket sales. Change your equation to an inequality to indicate that this represents the least amount of money you expect to collect from ticket sales.
- If your objective is to be sure that you are able to cover the cost of the dinner dance, which ticket price would you select? Based on this choice, write a linear equation that gives the total amount collect for ticket sales. Change your equation to an inequality to indicate that this represents the greatest amount of money you expect to collect from ticket sales.
- The two inequalities you have written, along with $x > 200$ and $x < 300$, form a system of linear inequalities. Graph this system to show the total amount received from ticket sales.

Finishing the Project

The answer to the four activities should help you complete your project. Your report should include your analysis of the cost for dinner and each band, depending on how many people buy tickets. Include your recommended ticket price and note any conditions under which this ticket price leads to a loss for the event. Illustrate your reasoning with graphs of linear equations and inequalities.

Reflect and Revise

Present your analysis of this data to a small group of classmates. After you have heard their analyses and presented your own, check to see that your work is complete, clear, and convincing. If necessary, make changes to improve your presentation.

Extending the Project

Consider other expenses you could expect to have in planning and holding this dinner dance. Estimate the additional expenses and change your recommended ticket price as necessary.

Chapter 6 Project Manager: Let's Dance

Getting Started

Read the project. As you work on the project, you will need a calculator, materials on which you will record your calculations, and materials to make accurate and attractive graphs. Keep all of your work for the project in a folder.

Checklist

- Activity 1: graphing equations
- Activity 2: finding fixed and per-person costs
- Activity 3: writing a report
- Activity 4: graphing linear inequalities
- cost analysis

Suggestions

- Consider what scales to use for the graph and look for the point of intersection.
- Write two equations and use elimination to solve the system.
- Consider all expenses.
- Check the accuracy of your graph by calculating actual ticket prices.
- Does your analysis show clear and convincing evidence that hiring one of the two bands would be more cost-effective than hiring the other? How might your conclusions change if it is determined that only 100 students will attend the dinner dance? What if 500 students attend?

Scoring Rubric

- 3** The report demonstrates sound reasoning and includes a detailed analysis of the cost for dinner and each band based on the number of tickets sold. The student writes accurate linear equations and inequalities. The graphs are neat and have the appropriate scales and labels. All calculations are correct.
- 2** The report demonstrates sound reasoning but needs more detail. There are minor errors in the linear equations and inequalities. Graphs and calculations are mostly correct.
- 1** The report lacks essential details. Linear equations and inequalities have some problems. Graphs and calculations lack accuracy.
- 0** Major elements of the project are incomplete or missing.

Your Evaluation of Project Evaluate your work, based on the *Scoring Rubric*.

Teacher's Evaluation of the Project