**Solution #3, CSE 191**

Fall, 2014

1. **Page 53, Prob 14.**
   Determine the truth value of each of these statements if the domain consists of all real numbers.

   (a) $\exists x\,(x^3 = -1)$

   (b) $\exists x\,(x^4 < x^2)$

   (c) $\forall x\,\left((-x)^2 = x^2\right)$

   (d) $\forall x\,(2x > x)$

   ***Solution:***

   (a) There exists $x_0 = -1$ s.t. $x_0^3 = -1$. Therefore, the statement is **TRUE**.

   (b) When $-1 < x_0 < 1$, we have $x_0^4 < x_0^2$. Therefore, the statement is **TRUE**.

   (c) $(-x)^2 = (-1)^2\,x^2 = x^2$. Therefore, the statement is **TRUE**.

   (d) When $x_0 \leq 0$, we have $2x_0 \leq x_0$. Therefore, the statement is **FALSE**.

2. **Page 78, Prob 4:**
   What rule of inference is used in each of these arguments?

   (a) Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials.

   (b) It is either hotter than 100 degrees today or the pollution is dangerous. It is less than 100 degrees outside today. Therefore, the pollution is dangerous.

   (c) Linda is an excellent swimmer. If Linda is an excellent swimmer, then she can work as a lifeguard. Therefore, Linda can work as a lifeguard.

   (d) Steve will work at a computer company this summer. Therefore, this summer Steve will work at a computer company or he will be a beach bum.

   ***Solution:***

   (a) Let $p$ be the proposition "Kangaroos live in Australia." and $q$ be the proposition "Kangaroos are marsupials."

   $$\begin{array}{ll} p \wedge q & \\ \hline q & \text{Simplification} \end{array}$$

   (b) Let $p$ be the proposition "Hotter than 100 degrees today." and $q$ be the proposition "Pollution is dangerous."

   $$\begin{array}{ll} \neg p & \\ p \vee q & \\ \hline q & \text{Disjunctive syllogism} \end{array}$$

   (c) Let $p$ be the proposition "Linda is an excellent swimmer." and $q$ be the proposition "Can work as a lifeguard."

   $$\begin{array}{ll} p & \\ p \rightarrow q & \\ \hline q & \text{Modus ponens} \end{array}$$

(d) Let $p$ be the proposition "Steve will work at a computer company this summer." and $q$ be the proposition "Steve will be a beach bum."

$$\begin{array}{ll} p & \\ \hline p \vee q & \text{Addition} \end{array}$$

3. **Page 78, Prob 6.**

Use rules of inference to show that the hypotheses If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on, If the sailing race is held, then the trophy will be awarded, and The trophy was not awarded imply the conclusion It rained.

***Solution:*** Let

- $p$ be the proposition "It rains."
- $q$ be the proposition "It is foggy."
- $r$ be the proposition "The sailing race will be held."
- $s$ be the proposition "The lifesaving demonstration will go on."
- $u$ be the proposition "The trophy will be awarded."

| **Step** | **Reason** |
|---|---|
| 1. $\neg u$ | Premise |
| 2. $r \to u$ | Premise |
| 3. $\neg r$ | Modus tonens from (1) and (2) |
| 4. $\neg r \vee \neg s$ | Addition from (3) |
| 5. $(\neg p \vee \neg q) \to (r \wedge s)$ | Premise |
| 6. $\neg(r \wedge s)$ | logically equivalent to (4) |
| 7. $\neg(\neg p \vee \neg q)$ | Modus tonens from (5) and (6) |
| 8. $p \wedge q$ | logically equivalent to (7) |
| 9. $p$ | Simplification from (8) |

4. **Page 78, Prob 9** (a)

For the premises given, provide a valid argument for the conclusion: "It rained on Thursday".

***Solution:*** Let

- $O(x)$ be "I take the day $x$ off."
- $R(x)$ be "It rains on day $x$."
- $S(x)$ be "It snows on day $x$."

Premises:

- "If I take the day off, it either rains or snows." leads to $\forall x(O(x) \to R(x) \vee S(x))$
- "I took Tue off or I took Thr off" leads to $O(\text{Tue}) \vee O(\text{Thr})$
- "It was sunny on Tuesday" leads to $\neg R(\text{Tue}) \wedge \neg S(\text{Tue})$.
- "It did not snow on Thr" leads to $\neg S(\text{Thr})$

Conclusion: "It rained on Thursday" leads to $R(\text{Thr})$

| Step | Reason |
|---|---|
| 1. $\forall x\,(O\,(x) \to R\,(x) \vee S\,(x))$ | Premise |
| 2. $O\,(\text{Tue}) \vee O\,(\text{Thr})$ | Premise |
| 3. $\neg R\,(\text{Tue}) \wedge \neg S\,(\text{Tue})$ | Premise |
| 4. $\neg S\,(\text{Thr})$ | Premise |
| 5. $\neg\,(R\,(\text{Tue}) \vee S\,(\text{Tue}))$ | De Morgan's laws on (3) |
| 6. $O\,(\text{Tue}) \to R\,(\text{Tue}) \vee S\,(\text{Tue})$ | Universal instantiation from (1) |
| 7. $\neg O\,(\text{Tue})$ | MT from (5) and (6) |
| 8. $O\,(\text{Thr})$ | DS from (2) and (7) |
| 9. $O\,(\text{Thr}) \to R\,(\text{Thr}) \vee S\,(\text{Thr})$ | Universal instantiation from (1) |
| 10. $R\,(\text{Thr}) \vee S\,(\text{Thr})$ | MP from (8) and (9) |
| 11. $R\,(\text{Thr})$ | DS from (4) and (10) |

Therefore, "It rained on Thursday".

5. **Page 78, Prom 9** (b)
   For the premises given, provide a valid argument for the conclusion: "I did not eat spicy food and it did not thunder."

   ***Solution:*** Let

   - $p$ be "I eat spicy foods."
   - $q$ be "I have strange dreams".
   - $r$ be "There is thunder while I sleep."

   Conclusion: $\neg p \wedge \neg r$

| Step | Reason |
|---|---|
| 1. $p \to q$ | Premise |
| 2. $r \to q$ | Premise |
| 3. $\neg q$ | Premise |
| 4. $\neg p$ | MT, (1), (3) |
| 5. $\neg r$ | MT, (2), (3) |
| 6. $\neg p \wedge \neg r$ | Conjunction, (4), (5) |

6. Given the premises:

$$(p \wedge t) \to (r \vee s)\,,\ q \to (u \wedge t)\,,\ u \to p,\ \neg s,\ q$$

   Show a valid argument for the conclusion $r$ by using rules of inferences.

   ***Solution:***

| Step | Reason |
|---|---|
| 1. $q \to (u \wedge t)$ | Premise |
| 2. $q$ | Premise |
| 3. $u \wedge t$ | Modus ponens from (1) and (2) |
| 4. $u$ | Simplification from (3) |
| 5. $t$ | Simplification from (3) |
| 6. $u \to p$ | Premise |
| 7. $p$ | Modus ponens from (4) and (6) |
| 8. $p \wedge t$ | Conjunction from (5) and (7) |
| 9. $(p \wedge t) \to (r \wedge s)$ | Premise |
| 10. $r \vee s$ | Modus from (8) and (9) |
| 11. $\neg s$ | Premise |
| 12. $r$ | Disjunctive syllogism from (10) and (11) |

7. **Page 79, Prob 16**
For each of these arguments determine whether the argument is correct or incorrect and explain why.

(a) Everyone enrolled in the university has lived in a dormitory. Mia has never lived in a dormitory. Therefore, Mia is not enrolled in the university.

(b) A convertible car is fun to drive. Isaac's car is not a convertible. Therefore, Isaac's car is not fun to drive.

(c) Quincy likes all action movies. Quincy likes the movie Eight Men Out. Therefore, Eight Men Out is an action movie.

(d) All lobstermen set at least a dozen traps. Hamilton is a lobsterman. Therefore, Hamilton sets at least a dozen traps.

***Solution:***

(a) Let

- $p(x)$ be the proposition "$x$ is enrolled in the university."
- $q(x)$ be the proposition "$x$ has lived in a dormitory."

| **Step** | **Reason** |
|---|---|
| 1. $\forall x(p(x) \to q(x))$ | Premise |
| 2. $p(Mia) \to q(Mia)$ | Universal instantiation |
| 3. $\neg q(Mia)$ | Premise. |
| 4. $\neg p(Mia)$ | MT, (2), (3). |

Namely, Mia is not enrolled in the university. The argument is valid.

(b) Let

- $P(x)$ be "$x$ is a convertible car."
- $Q(x)$ be "$x$ is fun to drive."

The premises are: $\forall x(P(x) \to Q(x))$, and $\neg P(\text{Isaac's car})$. From these premises, we can not conclude $\neg Q(\text{Isaac's car})$. The argument is invalid.

(c) False, Quincy might like other kinds movies and all action movies.

(d) Let

- $P(x)$ be "$x$ is a lobsterman."
- $Q(x)$ be "$x$ set at least dozen traps."

| **Step** | **Reason** |
|---|---|
| 1. $\forall x(P(x) \to Q(x))$ | Premise |
| 2. $P(Hamilton) \to Q(\text{Hamilton})$ | Universal instantiation |
| 3. $P(\text{Hamilton})$ | Premise |
| 4. $Q(\text{Hamilton})$ | MP, (2), (3) |

So Hamilton set at least a dozen traps. The argument is valid.

8. **Page 80, Prob 30**
Use resolution to show the hypotheses "Allen is a bad boy or Hillary is a good girl" and "Allen is a good boy or David is happy" imply the conclusion "Hillary is a good girl or David is happy."

***Solution:*** Let

- $p$ be the proposition "Allen is a good boy",
- $q$ be the proposition "Hillary is a good girl" and
- $r$ be the proposition "David is happy".

Then, our assumption are $\neg p \vee q$ and $p \vee r$.

| **Step** | **Reason** |
|---|---|
| 1. $\neg p \vee q$ | Premise |
| 2. $p \vee r$ | Premise |
| 3. $q \vee r$ | resolution |

It is a direct application of resolution rule.