## Chapter 21

## Investment

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## Learning Objectives:

In this chapter, we study

- how firms determine investment in physical capital.
- an important tool for analyzing any kind of investment: the arbitrage equation.
- financial investment and a basic theory of prices in the stock market.

An investment in knowledge always pays the best interest.

- Benjamin Franklin

There is no finer investment for any community than putting milk into babies.
— Winston Churchill

The four most dangerous words in investing are "This time it's different."

- Sir John Templeton


## 1. Introduction

"Invest" is a word that is used frequently in economics, as the quotations above suggest. One can invest in developing new ideas, as suggested by Benjamin Franklin. One can invest in $h u$ man capital, as suggested by Winston Churchill. Or one can invest in financial assets, perhaps the most common use of this word in the business world.

Interestingly, none of these uses conveys the most common meaning of "investment" in macroeconomics: investment in the national income accounting sense. In this context, investment refers to the accumulation of physical capital - roads, houses, computers, and machine tools. Nevertheless, each of these uses of the word "invest" captures something essential: it is by investing that our actions today influence our opportunities in the future.

There are two main reasons to study physical investment more closely. The first is evident in the recent financial crisis: investment fluctuates much more than consumption and falls disproportionately during recessions. Starting from its peak in 2006 at more than 17.5 percent of GDP, investment fell to just over 11 percent of GDP in mid 2009. The second reason for studying investment was already highlighted earlier: it is the key economic link between the present and the future. Broadly construed, investment in physical capital, human capital, and ideas lies at the heart of economic growth.

In this chapter, we begin by studying the economic forces that determine investment in physical capital. We begin with a narrow microeconomic question: how do firms make invest-
ment decisions? We show how these micro decisions aggregate up to determine the evolution of an economy's capital stock and even the value of its stock market.

To study the determination of investment, we introduce a very important tool in economic analysis: the arbitrage equation. This equation turns out to apply to investment in its many different contexts, not just for physical capital. An arbitrage equation can be used to study investment in human capital, new ideas, or even in financial assets.

We illustrate this point in the second half of the chapter by studying financial investment and the stock market. This provides us with a nice opportunity to see in detail what determines the price of a financial asset like a share of stock and what is meant by the notion of "efficient markets" in finance.

Finally, we end the chapter with a closer look at two components of physical investment residential investment and inventory investment - which both have played important roles in the recent recession. Not surprisingly, the arbitrage equation proves useful in these applications as well.

## 2. How do firms make investment decisions?

Should Amazon.com build a new distribution center? Should Nordstrom open another store in Boston? Should Gino's East install a new pizza oven at its famous restaurant in Chicago? Each of these questions is fundamentally about how much a business should invest.

We studied this question in its simplest form back in Chapter 4. There, we wrote down a profit maximization problem for a firm that was choosing how much capital to install and how many workers to hire. The answer turned out to be quite straightforward (see pp. 69-71): $a$ business should keep investing in physical capital until the marginal product of capital (MPK) falls to equal the interest rate. Recall that a crucial part of this argument was that capital runs into diminishing returns. When the firm has very little capital, the marginal product of capital is high. When Gino's has only a single pizza oven, adding one more oven has a high return and substantially expands its ability to make pizzas. However, as we add more and more pizza ovens, the gain in production gets smaller and smaller: there are not enough chefs to keep the ovens busy and the store itself may be too small to handle more ovens. Maximizing profits requires Gino's to continue adding pizza ovens until the last dollar spent on ovens raises revenues by an amount equal to the interest rate.

Letting $r$ denote this interest rate, this reasoning can be expressed as a simple equation: a
firm should invest until

$$
\begin{equation*}
M P K=r . \tag{21.1}
\end{equation*}
$$

### 2.1. Reasoning with an Arbitrage Equation

Profit maximization problems can occasionally become quite complicated and typically require calculus to analyze and solve. Fortunately there is an elegant shortcut, a very beautiful approach that applies to many investment problems - for capital, for ideas, and for financial investments. This approach uses what is called an arbitrage equation.

To see this in the simplest setting, return to Gino and his pizza ovens. Suppose instead of solving the full profit maximization problem involving the choice of capital, labor, and other inputs, we instead consider a slightly different problem.

Suppose the price of pizza ovens is $p_{k}$ (short for the "price of capital"). Gino's East can do two things with its cash on hand: it can put the money in the bank and earn the interest rate $r$, or it can buy pizza ovens. If the business is maximizing profits, then at the margin, both options should yield the same profit - if buying pizza ovens earned a higher profit, then Gino's should do more of that (and vice versa).

Expressed mathematically, this means

$$
\underbrace{r p_{k}}_{\begin{array}{c}
\text { return from }  \tag{21.2}\\
\text { bank account }
\end{array}}=\underbrace{M P K+\Delta p_{k}}_{\begin{array}{c}
\text { return from } \\
\text { pizza oven }
\end{array}} .
$$

where $\Delta p_{k}$ denotes the change in the price of pizza ovens: $\Delta p_{k}=p_{k, t+1}-p_{k, t}$. This equation is an arbitrage equation. Arbitrage equations consider two possible ways of investing money. They then take advantage of a powerful insight: if an investor is maximizing profits, then the two investments must yield the same return. Why? Well, if one alternative yielded a higher return, then the investor could not be maximizing profits. Taking a little money from the lower return activity and switching it to the higher return activity would make even more profit. So at a profit-maximizing position, active investments must yield the same return.

Consider equation (21.2) in the context of our pizza example. On the left side is the return from taking $p_{k}$ dollars and putting it into the bank for a year. This return is simply the interest earned on that sum. On the right side is the return from taking $p_{k}$ dollars and investing in the oven. Gino buys the oven and earns the marginal product of capital, MPK. At the end of the year, Gino sells the oven. In addition to the marginal product, Gino also makes a capital gain or loss on the oven, depending on whether the price went up or down.

To connect this result back to Chapter 4, suppose we are considering investing an extra dollar - that is, let's normalize the initial price $p_{k, t}=1$. In this case, we can rearrange equation (21.2) as

$$
\begin{equation*}
M P K=r-\frac{\Delta p_{k}}{p_{k}} . \tag{21.3}
\end{equation*}
$$

This expression says that Gino's should invest in pizza ovens until the marginal product of capital falls to equal the difference between the interest rate and the growth rate of the price of ovens. Notice that if the price of ovens is constant (so $\Delta p_{k}=0$ ), then this is exactly the same as the solution we obtained in Chapter 4, repeated above in equation (21.1).

### 2.2. The User Cost of Capital

The result in equation (21.2) is even richer, however, because it applies even if the price of ovens is changing over time. This is worth considering in more detail. To begin, let's note an important definition: the growth rate of a price, $\Delta p_{k} / p_{k}$, is often called the capital gain. (Or, if we know the growth rate is negative, this can be called the capital loss.)

Why might the price of ovens - or the price of physical capital more generally - be changing? There are at least two reasons. First, suppose the oven depreciates during use. Maybe we start the year with an oven worth $\$ 10,000$, but by the end of the year it is only worth $\$ 8,000$. In this case, we would expect the price to decline by $20 \%$, a capital loss, so $\Delta p_{k} / p_{k}=-.20$. In fact, it is common to put this depreciation term in explicitly (as we did in Chapter 5 in the Solow model, for example). In this case, we would add the depreciation rate, $\bar{d}$, to the right side equivalent to subtracting a negative price change. Then we'd reinterpret the price as the price of a unit of capital in its original condition, not having been used.

This leads to the second reason why prices might change: think about what happens over time to the price of electronics. Because of rapid progress in the electronics industry, the price of many of these goods is declining over time - consider cell phones, computers, or television sets, but the same would be true of many machine tools. Calculators that cost a hundred dollars fifty years ago can be had for a few dollars today. Technological change is one reason why the price of capital goods might change.

The price of structures like factories or retail stores, in contrast, usually go up over time. An important reason for this is that the land they are built on is becoming increasingly scarce, driving up its price.

Taking these considerations into account, we might include depreciation explicitly in equa-
tion (21.3) and write it as

$$
\begin{equation*}
M P K=\underbrace{r+\bar{d}-\frac{\Delta p_{k}}{p_{k}}}_{\text {user cost of capital }} \tag{21.4}
\end{equation*}
$$

The result from the simple framework in Chapter 4 was that firms should invest until the marginal product of capital falls to equal the interest rate. The more sophisticated framework here shows how to generalize this result. Now, the lesson is for firms to invest until the marginal product of capital falls to equal the user cost of capital. The user cost of capital is the total cost to the firm of using one more unit of capital. In the framework of Chapter 4, this was just the interest rate - the cost of financial funds to the firm. Now, though, we see that the user cost also includes the depreciation rate and any capital gain or loss associated with a change in the price of capital. A firm should invest in capital until the value of the extra output that capital produces falls to equal the user cost. ${ }^{1}$

This condition is shown graphically in Figure 21.1. The marginal product of capital is plotted against the amount of capital owned by the firm. This marginal product falls as the capital stock rises, reflecting the diminishing marginal product of capital. The desired amount of capital for the firm to own occurs at the intersection of the marginal product with the user cost of capital. At this point, the extra output produced by one additional unit of capital is precisely enough to cover the extra cost of owning a unit of capital, the user cost.

### 2.3. Example: Investment and the Corporate Income Tax

To see how this arbitrage approach and the user cost of capital are useful, let's work through an exercise. Suppose the economy starts with no taxes and then introduces a corporate income tax. As just one example, large corporations in the United States pay a federal tax rate of 35 percent on the income they earn. How does this affect a firm's desired capital stock and how much it invests?

To answer this question, we need to determine how a tax on corporate income affects the user cost of capital. For this, the arbitrage argument proves quite helpful, as shown in the following equation:

$$
\begin{equation*}
\underbrace{r p_{k}}_{\text {cost of funds }}=(1-\underbrace{\tau) M P K-\bar{d} p_{k}+\Delta p_{k} . . . ~}_{\text {return from investing in capital }} \tag{21.5}
\end{equation*}
$$

[^0]Figure 21.1: How much should a firm invest?


Note: A firm should invest until the marginal product of capital falls to equal the user cost of capital. This implies a desired level of the capital stock, $K^{\text {desired }}$.

The cost of funds to the firm is the interest rate $r$ multiplied by the price of a unit of capital. What is the benefit to the firm of investing the funds in capital? The firm earns the marginal product of capital, MPK, but then has to pay taxes on these additional earnings. Net of taxes, this means the firm gets $(1-\tau) M P K$. The firm then loses some of the capital in depreciation and finally suffers a capital gain or loss. ${ }^{2}$ Together, these terms make up the return to investing in a unit of capital. If the firm is maximizing profits, the cost of investing equals the benefit, as shown in the equation above. ${ }^{3}$

Let's normalize the initial price of capital to one, $p_{k}=1$, so we're considering a dollar's worth of new investment. We can then rewrite the arbitrage equation above as an expression that equates the marginal product of capital to the user cost of capital, and this will tell us what the user cost is for this problem:

$$
\begin{equation*}
M P K=\underbrace{\frac{r+\bar{d}-\frac{\Delta p_{k}}{p_{k}}}{1-\tau}}_{\text {user cost of capital }} . \tag{21.6}
\end{equation*}
$$

Essentially, a corporate income tax raises the user cost of capital. For example, suppose that in the absence of taxes, the user cost of capital is 10 percent (e.g. a 2 percent real interest rate, an 8 percent depreciation rate, and no capital gain). Then a corporate tax rate of 50 percent $(\tau=.50)$ would double the user cost to 20 percent $(1 /(1-.5)=2)$. Because the extra output produced by a unit of capital is taxed, the marginal product of capital must be that much higher in order for the investment to be profitable. As shown in Figure 21.2, a higher user cost reduces the total amount of physical capital that the firm wishes to utilize.

[^1]Figure 21.2: An Increase in the Corporate Income Tax


Note: An increase in the corporate income tax from zero to some positive rate like 35 percent raises the user cost of capital; see equation (21.6). This requires the marginal product of capital to rise, which means the firm desires a smaller amount of capital.

## - Case Study: Corporate Income Taxes across Countries -

Corporate tax rates vary across countries, and this implies variation in the user cost of capital and therefore in investment and the capital stock. How large is this variation?

Table 21.1 helps answer this question. It shows data from the OECD on corporate income tax rates across a number of countries (including taxes at both the "federal" and "state" levels). The variation in corporate tax rates is quite remarkable, ranging from rates in excess of 39 percent in Japan and the United States down to rates of 12 and 15 percent in Ireland and Iceland.

Interestingly, the implied variation in the user cost of capital is significantly less, ranging from about 10 percent in Japan and the United States down to 7 percent in Ireland. These differences in user costs only take into account differences in corporate tax rates, so other differences among countries (say in other aspects of the tax system or in risk or attitudes toward risk) could swamp the effects of corporate taxes.

## __ End of Case Study ___

### 2.4. From Desired Capital to Investment

The condition that firms invest until the marginal product of capital falls to equal the user cost pins down the desired capital stock, for example as was shown back in Figure 21.1. The reason is that the marginal product of capital is a decreasing function of the capital stock itself, holding other inputs constant. For instance, with the Cobb-Douglas production function considered in Chapter 4, $Y=\bar{A} K^{1 / 3} L^{2 / 3}$, the marginal product of capital is given by ${ }^{4}$

$$
\begin{equation*}
M P K=\frac{1}{3} \cdot \frac{Y}{K}=\frac{1}{3} \cdot \bar{A}\left(\frac{L}{K}\right)^{2 / 3} . \tag{21.7}
\end{equation*}
$$

Clearly, the marginal product of capital declines as capital increases - this is the diminishing returns to capital.

To connect this expression back to investment, recall the standard capital accumulation equation:

$$
\begin{equation*}
K_{t+1}=I_{t}+(1-\bar{d}) K_{t} . \tag{21.8}
\end{equation*}
$$

This equation, which we studied extensively in Chapter 5, says that the change in the capital

[^2]Table 21.1: Corporate Income Tax Rates and the User Cost of Capital

|  | Corporate <br> Tax Rate <br> (percent) | User Cost <br> of Capital <br> (percent) |
| :--- | :---: | :---: |
| Country | 39.5 | 10.1 |
| Japan | 39.1 | 10.0 |
| United States | 34.4 | 9.3 |
| France | 31.3 | 8.9 |
| Canada | 30.2 | 8.7 |
| Germany | 30.0 | 8.7 |
| Australia | 30.0 | 8.7 |
| Spain | 28.0 | 8.5 |
| United Kingdom | 28.0 | 8.5 |
| Mexico | 27.5 | 8.4 |
| Italy | 26.3 | 8.3 |
| Sweden | 26.0 | 8.2 |
| Finland | 24.2 | 8.0 |
| South Korea | 20.0 | 7.6 |
| Hungary | 20.0 | 7.6 |
| Turkey | 19.0 | 7.5 |
| Slovak Republic | 15.0 | 7.2 |
| Iceland | 12.5 | 7.0 |
| Ireland |  | 8 |

Note: Corporate tax rates (including state taxes) vary substantially across countries. Interestingly, the implied user cost of capital varies less. Source: The OECD Tax Database, Table II.1. The user cost of capital for the United States is assumed to equal 10 percent, and the value in other countries is calculated by scaling this number by $1 /(1-\tau)$. This user cost therefore omits other tax policies (like investment tax credits or depreciation credits) that may be important in practice.
stock is equal to new investment less depreciation. If we replace $K_{t}$ with a firm's initial capital stock and $K_{t+1}$ with the firm's desired capital stock (obtained from, say, Figure 21.1 or 21.1), then this equation tells you how much investment the firm has to undertake over the next year in order to reach the desired level of the capital stock. If the desired capital stock exceeds the current level, then the firm must undertake new investment. If the desired capital stock is less than the current level - as in the corporate income tax example above - then the firm can
invest a small amount and let depreciation bring the capital stock down.
In practice, the capital stock a firm desires may far exceed its current capital stock - think about Amazon.com or Google when they were new firms and were expanding rapidly. It may take several years for a firm to reach its desired capital stock, and the path of investment will need to take into account installation costs and adjustment costs.

That's the intuition for the connection. With a little bit of math, we can make the connection more formally. Let $u c$ denote the user cost of capital (say, the right hand side of equation (21.4)). From equation (21.7), the marginal product of capital is $1 / 3 \cdot Y / K$, so the condition that the marginal product of capital equals the user cost means that

$$
\begin{equation*}
\frac{Y}{K}=3 \cdot u c \tag{21.9}
\end{equation*}
$$

Next, let's rewrite the capital accumulation equation in (21.8) as follows:

$$
\begin{equation*}
\Delta K_{t}=I_{t}-\bar{d} K_{t} \tag{21.10}
\end{equation*}
$$

Dividing both sides of this equation by $K_{t}$ gives

$$
\begin{equation*}
\frac{\Delta K_{t}}{K_{t}}=\frac{I_{t}}{K_{t}}-\bar{d} \tag{21.11}
\end{equation*}
$$

Then multiply and divide the first term on the right side by $Y_{t}$ to get the investment rate:

$$
\begin{equation*}
\frac{\Delta K_{t}}{K_{t}}=\frac{I_{t}}{Y_{t}} \cdot \frac{Y_{t}}{K_{t}}-\bar{d} \tag{21.12}
\end{equation*}
$$

Finally, plug in $3 \cdot u c$ for $Y / K$ and solve for the investment rate to find

$$
\begin{equation*}
\frac{I_{t}}{Y_{t}}=\frac{g_{K t}+\bar{d}}{3 \cdot u c} \tag{21.13}
\end{equation*}
$$

This equation says that the investment rate depends on three main terms: the desired growth rate of the capital stock, $g_{K t} \equiv \Delta K_{t} / K_{t}$, the depreciation rate, and the user cost of capital, $u c$. In particular, the investment rate depends inversely on the user cost: a higher user cost of capital translates into a lower investment rate.

This expression can also be usefully connected to our study of economic growth. In Chapters 5 and 6, we assumed the investment rate was just given by some constant, $\bar{s}$. Equation (21.13) provides the microfoundations for the investment rate. In particular, it shows how higher taxes reduce the investment rate (via the user cost of capital). In fact, combining this result with the

Euler equation from Chapter [Consumption] and the growth models from Chapters 5 and 6, we have a full theory of the key macroeconomic variables in the long run. The growth models of Chapters 5 and 6 pin down the long-run growth rate. The Euler equation for consumption then pins down the long run interest rate (and therefore the user cost of capital). The condition that the marginal product of capital equals the user cost pins down the capital-output ratio. Finally, equation (21.13) pins down the investment rate and therefore the consumption share of GDP as well. Those are all the key endogenous variables in the macroeconomy, at least in a closed economy.

## 3. The Stock Market and Financial Investment

No doubt you have been confused at some point by the terms "capital" and "investment." In macroeconomics, these terms usually refer to the accumulation of physical capital. However, these words also are used frequently in finance: one might speak of financial capital, the capital (net worth) of a financial institution, or making a financial investment in a mutual fund.

While these are different uses of the terms "capital" and "investment," they share a deep linkage, and one way to see that linkage is by considering an arbitrage equation. We've already done so in the previous section for physical capital and physical investment. We turn in this section to the financial concepts. This proves to be quite useful. For example, the arbitrage equation helps us to understand how financial markets price stocks (yet another word with separate but related meanings in macro and in financial economics!).

### 3.1. The Arbitrage Equation and the Price of a Stock

The same arbitrage equation that governs how much a firm invests in physical capital can be used to study financial investments in financial assets like stocks. Suppose a (financial) investor has some extra money to invest. One option is to put the money in a saving account that pays an interest rate $r$. Alternatively, the investor can purchase a stock at price $p_{s}$, hold the stock for a year, and then sell it. This investment has two payoffs. First, the investor receives whatever dividend the stock pays: a dividend is a payment by a firm to its shareholders; think of it as a portion of the accounting profits earned by the firm. Second, when the investor sells the stock at the end of the year, the price may have changed: if the stock price goes up, the investor gets a capital gain, if the stock price goes down, the investor suffers a capital loss.

Importantly, let's suppose both investments are perfectly safe. That is, the investor knows what the dividend and capital gain will be; these are not uncertain. Obviously, this is an im-
portant simplifying assumption that does not hold in many cases in practice. In fact, much of the contribution of financial economics is in understanding risk and figuring out how that risk affects the prices of assets like stocks and bonds. For our brief excursion into finance, however, we will have to abstract from this important issue. ${ }^{5}$ The simple model that results still conveys many useful insights about the pricing of stocks.

What is the arbitrage equation in this case? Someone investing $p_{s}$ dollars in either the bank account or the stock must get the same financial return. This means

$$
\underbrace{r p_{s}}_{\begin{array}{c}
\text { return in }  \tag{21.14}\\
\text { bank account }
\end{array}}=\underbrace{\text { Dividend }+\Delta p_{s} .}_{\begin{array}{c}
\text { return from } \\
\text { the stock }
\end{array}}
$$

The return from holding the stock is the sum of the dividend it pays and the capital gain or loss. Notice that the dividend here plays the same role that the marginal product of capital played in the physical investment application: it is the key return if there is no change in the price of the asset.

In the application to physical capital, we used the arbitrage equation to determine the desired capital stock and the amount of investment. For the application to financial investment, we will use the equation differently - to tell us what determines prices in the stock market.

Dividing both sides of this arbitrage equation by the price of the stock leads to one of the most common ways to write the arbitrage equation:


On the left is the percentage return in the bank account. On the right is the percentage return to buying the stock. The stock return is the sum of the dividend return (the dividend-price ratio) and the capital gain or loss in percentage terms.

Consider some simple numbers. Suppose the bank account pays an interest rate of 3 percent. Then the stock must also pay a return of 3 percent. For example, it could be that the dividend yield is 2 percent and the capital gain is 1 percent, producing a 3 percent return for the stock.

[^3]Why does the stock have to yield a return of 3 percent? Think about what would happen if this were not true. If the stock paid more, everyone would invest in the stock and noone would invest in the bank account. The initial purchase price of the stock would rise, and this would reduce its return (via both the dividend yield and the capital gain term). If the stock paid less, then everyone would put their money in the bank, noone would demand the stock, and its price would fall. This lower price would raise the return. The only way there is no arbitrage opportunity is if these two investments have the same return.

The key step at this point is to solve the arbitrage equation in (21.15) for the price of the stock. We do this by grouping the bank return and the capital gain term together and then inverting both sides of the equation. ${ }^{6}$ This gives

$$
\begin{equation*}
p_{s}=\frac{\text { Dividend }}{r-\frac{\Delta p_{s}}{p_{s}}}=\frac{\text { Dividend }}{\text { Interest rate }- \text { Capital gain }} \tag{21.16}
\end{equation*}
$$

This last equation is truly beautiful. It essentially says that the price of a stock will equal the the present discounted value of the dividends that the stock will pay. To see this, suppose first that the capital gain term is zero and the dividend is constant. For example, a stock may pay a dividend of 10 dollars per share each year. Then the price of the stock is just $10 / r$, which turns out to be the present discounted value of 10 dollars paid forever (starting a year from now). ${ }^{7}$ If the interest rate is 5 percent, the price of the stock is $10 / .05=200$ dollars per share.

What happens if the dividend is not constant? Suppose the dividend starts out at 10 dollars per share but then grows over time at a constant rate $g$. (For example, $g=.02$ would be 2 percent growth.) Now look back at the equation for the stock price in (21.16) and notice something important: if the dividend is growing at a constant rate, then the growth rate of the stock price must equal the growth rate of the dividend. That is, the capital gain term $\Delta p_{s} / p_{s}$ is also equal to the growth rate of the dividend, $g .{ }^{8}$ This means that when dividend growth is constant, the stock price equals the current dividend divided by $r-g$. Notice what is going on here: when you discount a flow that is constant, you divided by $r$. When you discount a flow that is growing, you simply reduce the interest rate by the growth rate itself.

So now go back to our numerical example. Suppose the initial dividend is 10 dollars, the

[^4]Figure 21.3: The Price-Earnings Ratio in the Stock Market


Note: Whenever the ratio of stock prices to company earnings gets too far away from its mean, it tends to revert back. Notice that this measure reached its two highest peaks in 1929 and 2000. Source: Robert Shiller, http://www.econ.yale.edu/~shiller/data.htm
interest rate is 5 percent, and the growth rate of the dividend is 2 percent. What is the price of the stock? The answer is $10 /(.05-.02)=10 / .03 \approx 333$ dollars.

### 3.2. P/E Ratios and Bubbles?

Equation (21.16) tells us how stock prices should be determined according to a simple model. We can compare actual stock prices to the prices implied by our model. These could disagree either because the simple model is wrong - for example because it does not take sufficient account of risk - or if stock prices themselves are wrong - for example, if there is a "bubble" in the stock market.

In fact, we've already seen a common way of making this comparison. Recall that in Chapter 14 (page 369), we studied a diagram of the price-earnings ratio for the stock market as a whole in an effort to gauge if there might be bubbles in stock prices. With our simple model, we can now understand this graph - which we revisit here as Figure 21.3.

To understand this graph, return to equation (21.16), and divide both sides by the total earnings (earnings are essentially just accounting profits) of all companies in the S\&P 500:

$$
\begin{equation*}
\frac{p_{s}}{\text { Earnings }}=\frac{\text { Dividend / Earnings }}{\text { Interest rate }- \text { Capital gain }} . \tag{21.17}
\end{equation*}
$$

What this equation says then, is that the price-earnings ratio should equal the dividendearnings ratio divided by $r-g$. Now take a look at Figure 21.3 to see what this ratio looks like in the data. What we see is that this ratio is quite volatile. In the early 1980s, this ratio was under 10. But then it rose sharply over the next 20 years, peaking in 2001 at more than 40 . Following the dot-com crash and the financial crisis, the ratio fell sharply to below 15 .

How can we understand these movements? According to the simple model, the price-earnings ratio should be relatively stable to the extent that (a) the dividend-earnings ratio is stable and (b) the difference between the real interest rate and the growth rate of dividends is stable. One view is that the sharp run-up in stock prices at the end of the 1990s reflects a bubble in stock prices. But an alternative view - and one that many observers were highlighting at the time as part of the "new economy" - is that the difference between the interest rate and the growth rate of dividends declined, justifying an increase in the P/E ratio.

What this analysis emphasizes is that it's quite easy to draw a line at the average P/E ratio of 16.3 in Figure 21.3 and call any departures "bubbles." And there is certainly some merit to this approach. However, a more careful economic analysis of this ratio helps you to understand how and why the ratio might change for solid economic reasons. At any point in time, it may be difficult to know for sure if these fundamentals have changed or not, and this makes identifying potential "bubbles" a tricky enterprise.

### 3.3. Efficient Markets

This discussion of potential bubbles is related to another fundamental concept in finance, that of "efficient markets." A financial market is said to be informationally efficient if financial prices fully and correctly reflect all available information. When this is the case, it is impossible to make economic profits by trading on the basis of that information.

This is quite a mouthful, so let's stop and consider it more carefully. Suppose the price we are discussing is the price of a share of Google stock. And suppose Google is announcing its 3rd quarter profits next week and experts expect that this quarter was an extremely profitable quarter for Google. If markets are informationally efficient, then this expectation is already incorporated into Google's share price. When the announcement occurs in a week's time, if the announcement is exactly what everyone expected, then Google's stock price will not move that information was already incorporated into the price.

According to this theory, the only thing that moves stock prices is news that was unexpected. But this means that at any point in time, a stock price is equally likely to move up or down. When this is the case, we say the stock price follows a random walk. ${ }^{9}$

Are financial markets informationally efficient? This is one of the fundamental questions in financial economics, as you can imagine. The answer is "almost," and we learn a lot by studying this question closely.

The efficient markets benchmark is an excellent starting point for understanding most financial markets. As one example, consider mutual funds. Such funds are collections of stocks and other financial assets that are held together in a large portfolio, small pieces of which are sold off to individual investors. Some of these mutual funds are "actively managed," meaning they are run by investment managers who are constantly buying and selling financial assets in an effort to deliver the highest possible return in the least risky way. Other mutual funds are "index funds." These funds are essentially managed by a simple computer program that imitates one of the major stock indexes, like the Standard and Poor's 500 index or the Dow Jones Industrial Average. The actively managed funds charge higher fees - the investor must pay for the management skills of the team running the fund. An interesting question is whether or not this active management leads to higher returns. According to the efficient markets theory, one would not expect this to be the case - financial prices already reflect all available information, so the extra effort to "pick" winners and losers in the stock market will be wasted.

In fact, this is what the evidence seems to suggest: by and large, actively managed mutual funds have lower returns than passive index funds. For example, according to Burton Malkiel, for the decade up to 2002, more than three quarters of comparable, actively managed mutual funds failed to perform as well as the S\&P 500 stock market index. Moreover, those that did beat the S\&P cannot sustain their success: the best-performing funds of the 1980s underperformed the S\&P index during the 1990s. ${ }^{10}$

While the efficient markets benchmark is a good starting point for understanding financial markets, however, it is not the final word. If the efficient markets hypothesis is viewed as a shiny new house, departures from this hypothesis can be thought of as dirt and mold that lies hidden in nooks and crannies and closets. Financial economists have explored the house in detail and

[^5]documented with great care where the departures are hidden. For example, a tiny minority of mutual funds seem to beat the S\&P 500 index with more persistance than the efficient markets hypothesis would predict. Financial markets also tend to display substantially more volatility - large swings up or down in prices - than can be justified based on fundamentals. The most extreme version of this, of course, are the "bubbles" that seem to appear from time to time. Explanations of these departures are varied and include "behavioral" elements: economic actors are not always perfectly rational calculating machines. In addition, there may be limits to the extent that skilled investors can bet against a bubble to take advantage of mispricing. This is illustrated by a famous quip that has gathered attention recently, "The market can stay irrational longer than you can stay solvent." ${ }^{11}$

## - Case Study: Tobin's $q$, Physical Capital, and the Stock Market -

James Tobin of Yale University, winner of the Nobel Prize in economics in 1981, developed the implications of a theory like the one we have explored for the relationship between investment and the stock market. In Tobin's approach, the only asset a firm possesses is its capital, so in the simplest case, the stock market value of the firm is the value of its capital stock. Now consider what happens when investmenting in physical capital involves adjustment costs. Replacing a broken-down machine may require stopping the assembly line for some time. Expanding the factory may require a significant amount of installation work. When such adjustment costs are important, the stock market value of the firm - the present discounted value of the profits it will earn today and in the future - can differ from the value of its capital. Why? Imagine that the firm develops a new product that raises its future profitability. With no adjustment costs, the firm should expand immediately. This expansion will reduce profitability because of diminishing returns. It should expand until the marginal product falls to equal the user cost and the value of the firm falls to equal the capital stock. In the presence of adjustment costs, this expansion will occur gradually instead of immediately, and the value of the firm may differ for awhile from the value of its capital. This is the essence of Tobin's argument.

With this motivation, it is relatively easy to understand a key measure, known as Tobin's $q$ :

$$
\begin{equation*}
q=\frac{V}{p_{k} K}=\frac{\text { Stock market value }}{\text { Value of capital }} \tag{21.18}
\end{equation*}
$$

Tobin's $q$ is the ratio of the stock market value of a firm to the value of its capital stock.

[^6]When $q$ is larger than one, the stock market signals that the value of the firm is greater than its capital, and the firm should invest in more capital. Alternatively, when $q$ is less than one, the value of the firm is less than the value of its capital, and one would expect the firm to be "disinvesting." (Why not sell of the pieces of the firm if its capital is greater than its market value?)

This leads to two basic predictions. First, one should expect the value of a firm's $q$ to be close to one, at least apart from any short-run costs to adjusting capital. Second, the value of $q$ should be a useful predictor of firm-level investment.

In practice, the first prediction has significant problems. If capital were the only asset owned by firms, one would expect values of $q$ to be close to one. However, firms also own their brand names (often called "goodwill" capital) and the ideas they have created, some patented, some not. Because these assets also have values that are capitalized into the stock market price, values for $q$ often exceed one.

Empirical evidence on the role of $q$ in investment is somewhat mixed. The most careful studies looking across a large number of firms find that changes in tax policy, which are somewhat exogenous, change investment by affecting $q$ and the user cost of capital. On the other hand, other factors that are not included in the theory so far - such as a firm's cash flow or access to financial markets and bank loans - also seem to play an important role. ${ }^{12}$

## 4. Components of Physical Investment

As discussed in Chapter 2, investment in the national accounts measures long-lasting goods that businesses and households purchase. These goods are traditionally broken down into three basic categories, as shown in Figure 21.4.

- Nonresidential fixed investment: equipment and structures purchased by businesses. In recent years, this component accounted for about two-thirds of investment, about 11 percent of GDP.
- Residential fixed investment: new housing purchased by households.
- Inventory investment: goods that have been produced by firms but that have not been sold. An auto dealer has an inventory of cars on hand to sell to new customers. Inventory

[^7]Figure 21.4: Components of Investment (as a share of GDP)


Note: Investment consists of three main components: nonresidential fixed investment (which includes equipment and structures purchased by businesses), residential investment (housing), and the change in inventories held by businesses. Source: The FRED database.
investment is the change in this stock of goods on hand. As shown in Figure 21.4, inventory investment is usually positive in normal times but turns negative during recessions.

Our discussion of physical investment at the start of this chapter did not distinguish the components of investment. That discussion applies precisely to nonresidential fixed investment - the equipment and structures purchased by businesses. The two remaining categories of physical investment deserve additional consideration.

### 4.1. Residential Investment

Residential investment is the formal name for the construction of new housing, which is ultimately sold to households. As can be seen in Figure 21.4, the residential component is usually very sensitiv. And as was discussed in Chapter 13, housing investment played a large role in the financial crisis. Before the recent financial crisis, residential investment reached more than 6 percent of GDP. It then fell to less than half this amount during the depths of the recession.

Residential investment can be studied using the tools developed in this chapter. For example, consider the following arbitrage argument. Suppose some investor has $\$ 20,000$ to invest.

She can put this in the bank and earn interest. Or she can use it as the downpayment to purchase a $\$ 100,000$ condominium, rent out the condo for the year, and then sell it the following year and earn the capital gain (adjusted for depreciation). If this transaction involved no risk and ignoring any transactions costs, these two options should yield the same return:


Dividing both sides by the price of the condo, $P^{\text {house }}$, and solving gives

$$
\begin{equation*}
P^{\text {house }}=\frac{\text { Rent }}{r \cdot d p+\bar{d}-\frac{\Delta \text { Phouse }}{P \text { house }}} \tag{21.20}
\end{equation*}
$$

where $d p \equiv \frac{\text { Downpayment }}{P \text { pouse }}$ is the fraction of the purchase price that is put as a down payment, such as 20 percent. Notice that $d p$ is really just the inverse of leverage: you put down 20 thousand dollars and can buy a condo for 100 thousand dollars, so the leverage ratio is 5 and the down payment ratio is $1 / 5=20$ percent.

The equation we've just derived says that the price of a condominium is the present discounted value of the amount you can earn by renting out the condo. In this case, however, the discounting is more complicated than just the interest rate for two reasons. First, notice that the last term in the denominator, $\frac{\Delta P^{\text {house }}}{P \text { house }}$, is the capital gain term - the amount by which the condo price is expected to appreciate. The more you expect condo prices to rise, the higher is the initial price. This is how, for example, a bubble in housing prices can feed on itself. Second, notice the role of $d p$. Suppose that for some reason (it could be innovations in financial markets or it could just be the real estate industry doing something it shouldn't have done) the required downpayment rate on a condo falls from $20 \%$ to $10 \%$ : you can now lever your financial capital at a rate of 10 to 1 instead of 5 to 1 . According to equation (21.20), condo prices will rise as a result of this change. A numerical exercise at the end of this chapter will show you that these effects can be quite large.

What we see here provides insight into housing prices in the last decade. Housing prices can rise because of financial innovations that allow that market to take advantage of higher leverage or they can rise if people believe the rate of increase in housing prices has gone up.

Trying to disentangle whether these changes are occuring for solid economic reasons or are possibly related to a bubble of some kind can be difficult. Notice that this discussion very much parallels our discussion about bubbles and the price-earnings ratio for the stock market
in Section 3.2.

### 4.2. Inventory Investment

Inventories are goods that have been produced but have not yet been sold. Think about the goods on the shelf in a supermarket or the cars on a lot at an auto dealership. As can be seen in Figure 21.4, inventory investment is a small but very cyclical component of investment. In a booming economy, inventory investment is generally positive - firms are producing more goods than they are selling. In a recession, the opposite occurs: firms cut production sharply and run down their inventories.

There are several main motives that govern a firm's holding of inventories. First, firms may wish to engage in production smoothing. It is costly to ramp up production quickly when demand is high, so firms may wish to produce slightly more than they need during bad times and slightly less than they need during good times. Notice that if this force were dominant in the aggregate, then it implies that inventory investment would be counter-cyclical. That is, firms would accumulate inventories during recessions. However, this is not what we see in Figure 21.4 - instead, inventory investment appears to be high when the economy is booming and low during recessions.

A second motive that helps to explain the procyclicality of inventories is the pipeline theory. That is, firms hold inventories as part of the production process itself. Consider the production of a laptop computer. The producer will have a collection of computer screens, flash drives, keyboards, and memory chips on hand - inventories of the laptop components. When demand goes up and the firm needs to ramp up production of laptops, its collection of components will naturally rise as part of the production process. This is one explanation for the procyclical behavior of inventory investment.

Another motive for holding inventories is stockout avoidance. Firms will hold inventories of final goods on hand to make sure they have these goods available when a customer wishes to make a purchase.

## - Case Study: Inventory and Supply Chain Management -

One of the places where information technology has had a substantial impact on business productivity is in the management of inventories, both of final goods and of materials and parts that make up the so-called "supply chain." A famous example is Wal-Mart, the largest public company in the world in terms of revenues and the largest private employer in the United States. Wal-Mart was an early adopter of information technology for managing inventories and
networking suppliers. Computer networks link retail outlets with suppliers to ensure that new goods are ordered precisely when needed. Other techniques such as cross docking also keep inventories low: goods from various suppliers are unloaded at a distribution center and then directly loaded onto different trucks to get the goods to retail with little or no storage in between. Reduced inventories lead to lower costs and higher productivity. ${ }^{13}$

Steven Davis and James Kahn discuss the macroeconomics consequences of these improvements in inventory and supply chain management. For example, they document that the ratio of inventories to sales revenues for durable goods manufacturing fell from more than $60 \%$ during much of the 1970s and early 1980s to less than $50 \%$ in the late 1990s and early 2000s. More controversially, Davis and Kahn argue that improvements in inventory management were an important factor in reducing the volatility of the U.S. macroeconomy in recent decades - the so-called "Great Moderation." ${ }^{14}$
___ End of Case Study ___

## 5. Summary

1. Investments of all kinds - in physical capital, in human capital, in new ideas, and in financial assets - are ways of transferring resources from the present to the future.
2. The arbitrage equation is a fundamental tool in economics for studying investments of all kinds. It says that two investments of equal riskiness must have the same return (otherwise investors would flock to the activity with the higher return). Typically, this equation can be written in a form that says the interest rate (a return in a bank account) is equal to the sum of a "dividend" return and a "capital gain" return, both in percentage terms.
3. Applying the arbitrage argument to physical capital leads to a key result: firms invest in physical capital until the marginal product of capital falls to equal the user cost of capital.
4. The user cost of capital is the total economic cost of using one unit of capital for one period. It typically involves the interest rate, the depreciation rate, and any capital gain or loss associated with a changing price of capital. It can be augmented to include effects from taxation.

[^8]5. Applying the arbitrage argument to financial investment leads to a simple theory of stock prices: the price of a stock is the present discounted value of dividends. This present value can typically be written as the ratio of the current dividend to the difference between the interest rate and the capital gain.
6. If financial markets are informationally efficient, then stock prices will reflect all publicly available information. Unexpected "news" will change stock prices, making them equally likely to go up or down (apart from a "normal" return). This means that stock prices will follow a random walk.
7. Applied to residential investment (housing), the arbitrage argument says that the price of a house should equal the present discounted value of the amount the house could be rented for, adjusting for leverage.
8. Inventory investment is highly procyclical, rising sharply in booms and falling sharply in recessions. This likely reflects the pipeline story of inventories.

## 6. Key Concepts

arbitrage equation, capital gain, capital loss, dividend, informational efficiency, inventory investment, mutual funds, nonresidential fixed investment, price-earnings ratio, pipeline theory, production smoothing, random walk, residential fixed investment, stockout avoidance, Tobin's $q$, user cost of capital

## 7. Review Questions

1. Why do economists use the terms"investment" and "capital" in very different contexts (physical investment and physical capital versus financial investment and financial capital)?
2. What is the arbitrage equation and why is it useful in studying investment?
3. What is a capital gain and what role does it play in the arbitrage equation?
4. What is the user cost of capital? How is this user cost related to investment in physical capital?
5. When is the value of the stock market equal to the value of the capital stock? How is this related to Tobin's $q$ ?
6. What is a "dividend return" and a "capital gain," and how do these terms enter the arbitrage equation when it is written in percentages?
7. In the simple theory developed in the chapter, why is the stock price equal to the dividend divided by interest rate (net of the capital gain)?
8. What determines the price-earnings ratio for a stock? What does this imply about detecting bubbles in the stock market?
9. What does it mean when economists say that the stock market is, at least to a great extent, "informationally efficient"?
10. How does the arbitrage equation help pin down the price of housing in our simple theory? What role does leverage play?

## 8. Exercises

worked exercise

1. The user cost of capital: Consider the basic formula for the user cost of capital in the presence of a corporate income tax. Suppose the baseline case features an interest rate of 2 percent, a rate of depreciation of 6 percent, a price of capital that rises at 1 percent per year, and a zero corporate tax rate. Starting from the baseline case, what is the user cost of capital after the following changes?
(a) No changes - the baseline case.
(b) The corporate tax rate rises to 35 percent.
(c) The interest rate doubles to 4 percent.
(d) Both (b) and (c).
2. Interest rates and the tax code: An economy begins in steady state with an investment rate of 20 percent, a corporate tax rate of 25 percent, a real interest rate of 2 percent, a depreciation rate of 7 percent, and a price of capital that falls at an annual rate of 2 percent.
(a) What is the user cost of capital?
(b) Suppose the central bank tightens monetary policy, raising the real interest rate from 2 percent to 4 percent. By how much does the user cost of capital rise?
(c) How would your answer have differed if the corporate tax rate had been zero? Explain the effect that taxes have on the extent to which monetary policy affects the user cost of capital (and hence the investment rate).
3. Investment and the corporate income tax: Suppose the user cost of capital in an economy with no corporate income tax is 10 percent.
(a) What is the user cost if the corporate tax rate rises to 20 percent? 30 percent?
(b) Suppose an economy's steady state investment rate is 30 percent when the corporate tax rate is zero. What happens to this investment rate if the corporate tax rate rises to 20 percent? 30 percent?
(c) Are differences in corporate tax rates across countries a plausible explanation for the large variation in investment rates that we see in the data?
4. Investment tax credits and the user cost of capital: Consider the user cost of capital in the presence of taxes, starting with equation (21.5). Suppose the price of capital $p_{k}$ is constant, so there is no capital gain term. What is new, however, is that there is an investment tax credit: rather than costing $p_{k}$, a unit of capital costs $(1-I T C) p_{k}$. That is, the government subsidizes the purchase of new capital, and the amount of the subsidy is given by $I T C$. As just one example, in 1981, the U.S. government created a 10 percent investment tax credit to spur the economy out of its recession, so we might suppose $I T C=.10$.
(a) How does the arbitrage equation change in the presence of the investment tax credit?
(b) What is the user cost of capital in this case?
(c) What happens to the user cost of capital if the investment tax credit is exactly equal to the corporate income tax rate? Why?
5. Total factor productivity and investment: Suppose the TFP parameter, $\bar{A}$, increases permanently.
(a) What happens to the desired capital stock?
(b) What happens to investment?
(c) (Hard) What happens to the investment rate in the long run? Why?
6. Pricing stocks: Suppose the initial dividend paid by a stock is 10 dollars per year. Let the interest rate and the growth rate of dividends be given by the table below.

| Interest <br> rate | Growth rate <br> of dividends | Stock <br> price? |
| :---: | :---: | :---: |
| $4 \%$ | $0 \%$ |  |
| $4 \%$ | $2 \%$ |  |
| $4 \%$ | $3 \%$ |  |
| $4 \%$ | $3.9 \%$ |  |
| $6 \%$ | $0 \%$ |  |
| $6 \%$ | $2 \%$ |  |
| $6 \%$ | $5 \%$ |  |

(a) For each case, compute the value of the stock according to the simple theory developed in the chapter.
(b) What happens as the growth rate of dividends gets closer and closer to the interest rate? Why?
(c) What does this imply about using a plot of the price-earnings ratio in the stock market to identify bubbles or the mispricing of individual stocks?
7. Housing prices: Suppose a condominium can be rented for 1000 dollars a month, it depreciates at 10 percent per year, and the annual interest rate is 5 percent. Let the downpayment rate and the annual growth rate of condominium prices be given by the table below.

| Growth rate <br> of condo prices | Downpayment <br> rate, $d p$ | Price of <br> the condo? |
| :---: | :---: | :---: |
| $0 \%$ | $20 \%$ |  |
| $2 \%$ | $20 \%$ |  |
| $5 \%$ | $20 \%$ |  |
| $10 \%$ | $20 \%$ |  |
| $5 \%$ | $100 \%$ |  |
| $5 \%$ | $10 \%$ |  |
| $5 \%$ | $5 \%$ |  |

(a) For each case, compute the value of the housing price according to the simple theory developed in the chapter.
(b) Based on your results, discuss the sensitivity of condo prices to the expected capital gain.
(c) Based on your results, discuss the sensitivity of condo prices to the downpayment rate.
8. The price of a patent: Let's use the arbitrage equation to the determine the price of a patent in a simple setting. Let $r$ denote the interest rate, let $p_{i}$ denote the price of an "idea" that is under patent, and let Prof denote the extra profit that can be earned by a firm that owns this idea.
(a) Set up the basic arbitrage equation that will ultimately pin down the value of the patent. On the left side show the return from investing $p_{i}$ dollars in a saving account. On the right, show the return from using these funds to purchase the patent.
(b) Solve this equation for the price of the idea.
(c) What is the economic interpretation of this result?

## 9. Worked Exercises

1. The user cost of capital: Looking back at equation (21.6), the user cost of capital when there is a corporate income tax is

$$
u c=\frac{r+\bar{d}-\frac{\Delta p_{k}}{p_{k}}}{1-\tau}
$$

(a) In the baseline case, this user cost is

$$
u c=\frac{.02+.06-.01}{1-0}=.07,
$$

or 7 percent.
(b) If the corporate tax rate is 35 percent instead of zero, the user cost rises to

$$
u c=\frac{.02+.06-.01}{1-.35} \approx .108,
$$

or about 10.8 percent.
(c) On the other hand, if the tax rate remains zero but the interest rate rises to 4 percent the user cost rises to

$$
u c=\frac{.04+.06-.01}{1-0}=.09,
$$

or 9 percent.
(d) Finally, if we combine these two changes, the user cost is

$$
u c=\frac{.04+.06-.01}{1-.35} \approx .138
$$

or 13.8 percent.
6. Pricing stocks: Looking back at equation (21.16), the formula for pricing stocks in our simple model is

$$
p_{s}=\frac{\text { Dividend }}{r-\frac{\Delta p_{s}}{p_{s}}}
$$

(a) Applying this formula to the numbers in the table leads to the following results (for the first several rows; you can fill in the rest yourself):

| Interest <br> rate | Growth rate <br> of dividends | Stock <br> price? |
| :---: | :---: | :---: |
| $4 \%$ | $0 \%$ | 2.5 |
| $4 \%$ | $2 \%$ | 5 |
| $4 \%$ | $3 \%$ | 10 |
| $4 \%$ | $3.9 \%$ | 100 |
| $6 \%$ | $0 \%$ | $? ?$ |
| $6 \%$ | $2 \%$ | $? ?$ |
| $6 \%$ | $5 \%$ | $? ?$ |

(b) As the growth rate of the dividend rises, the stock price rises. This makes sense: the stock is the present value of dividends, and if dividends are growing faster, then the stock will be worth more. Notice that as the growth rate of the dividend approaches the interest rate, the stock price rises very rapidly, reaching infinity in the limit! The reason is that the discounting and growth terms effectively cancel, so this is analogous to computing the present discounted value of 10 dollars per year, forever, when the interest rate and growth rate are zero - the present value is infinite.
(c) Hint: Take a look back at Section 3.2. and Figure 21.3. Think about what would happen if the capital gain term were to rise as a result of changes in the economy (say, the "new economy" of the late 1990s).


[^0]:    ${ }^{1}$ This approach to investment was developed by Dale Jorgenson, "Capital Theory and Investment Behavior," American Economic Review, vol. 53 (May 1963), pp. 247-259; see also Robert E. Hall and Dale Jorgenson, "Tax Policy and Investment Behavior," American Economic Review, vol. 57 (June 1967), pp. 391-414.

[^1]:    ${ }^{2}$ The careful reader will notice that the depreciation term should really be $\bar{d} \cdot p_{k, t+1}$ — that is, it should be valued at the price at which the capital is sold rather than bought. The approach here is simpler and represents a traditional (and typically very accurate) approximation.
    ${ }^{3}$ The argument here assumes that the firm does not pay any taxes on the capital gain associated with the changing value of the capital. This standard assumption is justified because most capital is held until it depreciates fully and taxes are only paid on capital gains if and when they are realized.

[^2]:    ${ }^{4}$ You may recall that deriving this equation requires some calculus: the marginal product of capital is just the derivative of the production function with respect to $K$. If you are not comfortable with taking the derivative, just take the equation above as a statement of fact.

[^3]:    ${ }^{5}$ It turns out that everything we will say is valid even in the presence of uncertainty as long as one interprets the dividend and capital gain as expected values and as long as investors are risk neutral. What this means, for example, is that investors place equal value on the the following two options: one hundred dollars for certain and a 50/50 chance at getting either two hundred dollars or zero dollars. That is, investors do not care about risk. Of course, this is an unrealistic assumption in many cases.

[^4]:    ${ }^{6}$ To derive the result, collect the interest rate and the capital gain on the same side of the equation to get

    $$
    r-\frac{\Delta p_{s}}{p_{s}}=\frac{\text { Dividend }}{p_{s}} .
    $$

    Then bring $p_{s}$ to the left side and switch the interest rate and capital gain to the denominator on the right side.
    ${ }^{7}$ If you need a reminder of why this is true, take a look back at Worked Exercise 4(c) in Chapter 7 on page 187.
    ${ }^{8}$ To be more precise, the capital gain term equals the dividend growth rate only if the capital gain term is constant as well. One can think of this as holding in a steady state, or in the long run.

[^5]:    ${ }^{9}$ If this discussion sounds somewhat familiar, this is because we said many similar things about consumption under the permanent income hypothesis in Chapter [Cons]. In that case, consumption reflects all information about income and therefore only responds to news and follows a random walk. There is a nice mathematical similarity between consumption and stock prices.
    ${ }^{10}$ The sample considered here is of actively-managed large-cap equity mutual funds, so these are funds with comparable risk to the $\mathrm{S} \& \mathrm{P} 500$. Malkiel is the author of one of the more famous popular books on financial markets, $A$ Random Walk Down Wall Street. The findings reported in this paragraph are taken from an excellent overview of the efficiency of financial markets in Burton G. Malkiel, "The Efficient Market Hypothesis and Its Critics," Journal of Economic Perspectives, vol. 17(Winter 2003), pp. 59-82.

[^6]:    ${ }^{11}$ Further discussion of some of the departures from efficient markets can be found in Robert J. Shiller, "From Efficient Markets Theory to Behavioral Finance," Journal of Economic Perspectives, vol. 17 (Winter 2003), pp. 83-104. The quip is studied formally in a famous paper by Andrei Shleifer and Robert Vishny, "The Limits to Arbitrage," Journal of Finance, vol. 52 (March 1997), pp. 35-55.

[^7]:    ${ }^{12}$ A good overview of this evidence can be found in Kevin A. Hassett and R. Glenn Hubbard, "Tax Policy and Business Investment," in Alan Auerbach and Martin Feldstein, Handbook of Public Economics, vol 3 (2002), chapter 20, pp. 12931343.

[^8]:    ${ }^{13}$ For a general discussion of information technology and business productivity, see Erik Brynjolfsson and Lorin M. Hitt, "Beyond Computation: Information Technology, Organizational Transformation and Business Performance" Journal of Economic Perspectives, vol. 14 (Autumn 2000), pp. 23-48.
    ${ }^{14}$ Steven J. Davis and James A. Kahn, "Interpreting the Great Moderation: Changes in the Volatility of Economic Activity at the Macro and Micro Levels," Journal of Economic Perspectives, vol. 22 (Fall 2008), pp. 155-180.

