# Number Sense Tips <br> \& <br> <br> Problem Solving 

 <br> <br> Problem Solving}

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## Andy Zapata

Azle ISD - 1974 to 2017
$7^{\text {th }}$ grade math, high school math \& Physics teacher
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ETS Physics reader 10 years
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## UIL Elementary \& Junior High Number Sense

Individuals are called upon every day to use their ability to make quick mental calculations to make decisions. The development of such abilities should be an integral part of the math curriculum. Concepts covered include, but are not limited to: addition, subtraction, multiplication, division, proportions, and use of mathematical notation.

Students will be given a 10-minute, fill-in-the-blank test which they must complete without doing calculations on paper or on a calculator. Erasures and mark-outs are not permitted.

Every tenth problem is an estimation problem with an integral answer an a $5 \%$ range of the answer.

## Elementary Problem Guidelines

## Problem 1-20

1. Addition, subtraction, multiplication, \& division of whole numbers
2. Recognizing place value
3. Rounding off whole numbers
4. Multiplication short-cuts
5. Remainder type problems
6. Even \& odd number type problems
7. Expanded notation
8. Sums of whole numbers with regrouping
9. Differences of whole numbers with regrouping
10. Roman numerals/Arabic numbers conversion

## Elementary Problem Guidelines

## Problems 21-40

1. Addition/subtraction of fractions with common denominators
2. Addition, subtraction, multiplication, \& division of decimal fractions
3. Comparing decimal fractions
4. Conversion problems (either way): fraction/decimal, percent/fraction, percent/decimal
5. Order of operations
6. More multiplication short-cuts
7. Ratio/proportion
8. Consumer type problems

## Elementary Problem Guidelines

## Problems 21-40 (continued)

9. Problems about prime numbers
10. Greatest common divisor (GCD) \& least common multiple (LCM)
11. Conversion problems (either way): length, measurements, time

Problems 41-60

1. Addition, subtraction, multiplication \& division of fractions and mixed numbers
2. Substitution problems
3. Perimeter/area of: square, rectangle, triangle
4. Radius/diameter of a circle
5. Powers \& roots of numbers

## Elementary Problem Guidelines

## Problems 41-60 (continued)

6. Solving simple equations
7. Sequences
8. Sets
9. Word problems
10. Volume of cube/rectangular box
11. Right triangle problems
12. More multiplication short-cuts
13. Base systems conversions

## Elementary Problem Guidelines

## Problems 61-80

1. Addition, subtraction, multiplication \& division of integers
2. Inverses
3. Basic geometry facts
4. More area problems
5. Squaring two-digit numbers
6. More multiplication short-cuts
7. More powers and roots of numbers
8. More consumer type problems
9. Inequalities
10. Probability
11. More area problems: parallelogram, rhombus, trapezoid

## Elementary Problem Guidelines

Problems 61-80 (continued)
12. Coordinate geometry - number line
13. More percent type problems

## Junior Problem Guidelines

## Problems 1-20

1. Addition, subtraction, multiplication \& division of whole numbers, fractions, and decimals
2. Order of operations
3. Use of the distributive property
4. Comparison of fractions \& decimals
5. Multiplication short-cuts
6. Squaring numbers
7. Roman numerals/Arabic numbers
8. Mean, median, mode
9. Sums of whole numbers of multiple terms of a finite series

## Junior Problem Guidelines

## Problems 21-40

1. Addition, subtraction, multiplication \& division of mixed numbers and integers
2. More multiplication short-cuts
3. Percent problems
4. Conversion problems (either way):

English/metric, length, area, capacity, time
5. Consumer type problems
6. Substitution problems
7. Solving simple equations
8. Square roots/cube roots
9. Greatest common divisor (GCD) \& least common multiple (LCM)

## Junior Problem Guidelines

## Problems 21 - 40 (continued

10. Number theory - prime numbers and divisors
11. Perimeter/area of: square, rectangle, circle
12. Ratio/proportion
13. Inverses
14. Multiplication of 101, 111

Problems 41-60

1. Sets
2. Word problems
3. Pythagorean theorem
4. Sequences

## Junior Problem Guidelines

## Problems 41-60 (continued)

5. Volume/surface area of rectangular solid/cube
6. Base systems: conversions and basic operations
7. Area of: parallelogram, rhombus, trapezoid, circle
8. Solving inequalities
9. Basic geometry facts
10. Remainder problems

## Problems 61-80

1. Repeating decimals
2. More number theory
3. Powers of numbers

## Junior Problem Guidelines

## Problems 61-80 (continued)

4. Volume of: circular cylinder, cone, sphere
5. Sequences \& series
6. Factorial
7. Coordinate geometry
8. Probability/odds
9. More percent type problems : Advanced
10. More remainder type problems
11. More multiplication short-cuts

## Sample Problems

(1) $25 \times 8=$ $8 \div 4=2$
$2 \times 100=$

200

## Sample Problems

(2) $75 \times 23=$

$$
75=\frac{300}{4}
$$

$23 \div 4=5.75$
$5.75 \times 300$
1725

## Sample Problems

## (3) $\mathrm{XXIX}=$ Arabic Number

$M=1000 ; D=500 ; C=100 ; L=50 ; \quad X=10 ; \quad V=5 ; \quad I=1$


## Sample Problems

$$
\text { (4) } \begin{gathered}
\frac{11}{8}-\frac{5}{8}= \\
\frac{6}{8} \\
\frac{6}{8} \div \frac{2}{2} \\
\frac{3}{4}
\end{gathered}
$$

## Sample Problems

(5) $24 \times 26=$

Since $26=25+1$ and

$$
24=25-1
$$

$$
24 \times 26=(25+1)(25-1)
$$

$$
24 \times 26=25^{2}-1^{2}
$$

$$
24 \times 26=625-1
$$

## Sample Problems

(6) $24 \times 26=\ldots \quad$ (Another Way)

Since ten's digits are the same
And one's digits add up to 10
Multiply units digits $\rightarrow 4 \times 6=24$ - write this down _ 24

Multiply one of the tens digit by the other increased by one $2 \times(2+1)=6-$ write this down for the finished answer.

## Sample Problems

(7) $24 \times 26=\ldots \quad$ (Still Another Way)

Multiply $4 \times 6=24$ - write down 4 and keep 2 in your memory.

Multiply $(4 \times 2)+(2 \times 6)+2=22-$ write down 2 and keep

Multiply $(2 \times 2)+2=6-$ write this down for the finished answer (FIRST)

## Sample Problems

(8) $.121212 \ldots=$
(fraction)

$$
\begin{aligned}
& .121212 \ldots=\frac{12}{99} \\
& \frac{12}{99} \div \frac{3}{3} \\
& \frac{4}{33}
\end{aligned}
$$

## Sample Problems

(9) $1+2+3+\ldots+9=$

$$
\begin{aligned}
& 1+2+3+\ldots+n=\frac{n(n+1)}{2} \\
& \text { SUM }=\frac{9(9+1)}{2} \quad 45
\end{aligned}
$$

Note: You should know the formulas for sums of odd and even integers also!

## Sample Problems

(10) $14+17+20+23+26=$

For sums of equally spaced numbers, multiply the median of the numbers by the number of terms.

$$
\text { SUM }=20 \times 5
$$

## Sample Problems

(11) $21+24+27+30=$ $\qquad$
For sums of equally spaced numbers, multiply the median of the numbers by the number of terms.

$$
\text { SUM }=25 \frac{1}{2} \times 4
$$

102

## Sample Problems

(12) $4 \frac{1}{3} \times 4 \frac{2}{3}=\ldots$ (mixed number)
$\frac{1}{3} \times \frac{2}{3}=\frac{2}{9}$ Write this down....................
$4 \times(4+1)=20$ Write this down $-\cdots . . . . . .$.
$20 \frac{2}{9}$

## Sample Problems

(13) $6 \frac{1}{3} \times 3 \frac{1}{3}=\ldots$ (mixed number)
$\frac{1}{3} \times \frac{1}{3}=\frac{1}{9}$ Write this down-.................
$6 \times 3+(6+3) \frac{1}{3}=21-$ Write
this down

$$
21 \frac{1}{9}
$$

## Sample Problems

(14) $16 \%$ of 36 is $8 \%$ of

In equation form looks like this:
$16 \% \times 36=8 \% \times ?$

Solving for ? $\rightarrow \quad \frac{16 \%}{8 \%} \times 36=$ ?

$$
\frac{2}{1} \times 36=?
$$

72

## Sample Problems

(15) 5 base $8+7$ base $8=\ldots$ base 8

$$
5+7=12
$$

$12 \div 8=1$ remainder 4

Write down 4 and "carry" 1-----

## Sample Problems

(16) The radius of a circle with an area of $16 \pi$ is

$$
\begin{aligned}
& A=\pi r^{2} \\
& r=\sqrt{\frac{16 \pi}{\pi}}
\end{aligned}
$$

## Sample Problems

(17) $12 \div 4 \times 3=$

$$
12 \div 4=3 \quad 3 \times 3 \quad 9
$$

(18) $12+4^{2} \times 3=$

$$
4^{2}=16
$$

$16 \times 3=48$
$12+48$
60

## Sample Problems

(19) $26 \times 86=$

Since one's digits are the same
And ten's digits add up to 10
$6 \times 6=36-$ write this down 36
$(2 \times 8)+6=22-$ write this down for the finished answer

## Sample Problems

(20) $113^{2} \div 4$ has a remainder of

Just look at the last two digits of the number
$13 \div 4 \rightarrow$ remainder $=1$
$1^{2} \div 4=0+$ remainder

## Sample Problems

(21) $\left(13^{2}+11 \times 15\right) \div 7$ has a remainder of
$13 \div 7 \rightarrow$ remainder $=6$
$6^{2} \div 7 \rightarrow$ remainder $=1$
$11 \div 7 \rightarrow$ remainder $=4 \ldots(1+ \pm \times 1) \div 7$
$15 \div 7 \rightarrow$ remainder $=1-\cdots=0+$ remainder

## Sample Problems

(22) How many total subsets can be made of the set $\{A, U, S, T, I, N\}$ ?

The set has 6 elements, so the number of subset is
$2^{6}$

64

## Sample Problems

(23) The area of a rhombus with diagonals 17 and 20 is $\qquad$
Area of a rhombus $=\frac{\left(\text { diagonal }_{1}\right) \times\left(\text { diagonal }_{2}\right)}{2}$

$$
\begin{equation*}
\mathrm{A}_{\text {rhombus }}=\frac{17 \times 20}{2} \tag{64}
\end{equation*}
$$

## Sample Problems

(24) What is the area of a square with diagonal 14 ?

$$
\begin{aligned}
& \text { Area }= \frac{(\text { diagonal })^{2}}{2} \\
& \text { A }=\frac{14^{2}}{2} \quad 98
\end{aligned}
$$

## Sample Problems

(25) What is the length of the side of an equilateral triangle with area $9 \sqrt{3}$ ?

$$
\begin{array}{ll}
\text { Area }=\frac{(\text { side })^{2} \sqrt{3}}{4} & \rightarrow \text { side }=\sqrt{\frac{4(\text { Area })}{\sqrt{3}}} \\
\text { side }=\sqrt{\frac{4(9 \sqrt{3})}{\sqrt{3}}} & 6
\end{array}
$$

## Sample Problems

(26) $91 \times 96=$
$100-91=9$ and $100-96=4$.
Multiply $9 \times 4$ and write down.

Subtract 9 from 96 or 4 from 91 and write down- $-=$
8736

## Sample Problems

(27) $6 \frac{3}{4} \div \frac{1}{4}=$

Recall $\div \frac{1}{4}$ is same as multiplying by 4
Also recall $6 \frac{3}{4}$ is the same as $\left(6+\frac{3}{4}\right)$
$\left(6+\frac{3}{4}\right) \times 4=24+3 \quad 27$

## Sample Problems

(28) $\frac{5}{9}+\frac{9}{5}=\ldots$ (mixed number)

Write down the number 2 for the whole number part of the answer

$$
2 \frac{16}{45}
$$



Square the difference between the numerator and denominator and place this over the product of the numerator and denominator as the fraction part of the answer.

## Sample Problems

(29) 100101110 base $2=\ldots$ base 8

Starting at the right end of the number group the digits into sets of 3 digits.


Convert each of the sets from base 2 to base 8 numbers and write down as final answer.

456

## Sample Problems (Estimation)

*(30) $69+79+199=\square$
350
$330-364$

## Sample Problems (Estimation)

*(31) $624 \times 240=$
Recall $\frac{5}{8}=.625$
$\frac{5000}{8} \times 240=150000$

142272 - 157248

## Sample Problems (Estimation)

*(32) $101^{2}-99^{2}=$

$$
\begin{gathered}
(101-99) \times(101+99) \\
\vdots \\
(2) \times(200) \\
=400 \\
380-420
\end{gathered}
$$

## Sample Problems (Estimation)

*(33) $167 \times 359+33=$

$$
\begin{aligned}
& \frac{1}{6} \approx .167 \\
& \frac{1000}{6} \times 360+0 \\
& =60000
\end{aligned}
$$

## Sample Problems (Estimation)

*(34) $269 \times 3 \frac{5}{9}=$

$$
\begin{aligned}
& 270 \times \frac{32}{9} \\
& 270 \div 9=30 \\
& 30 \times 32=960
\end{aligned}
$$

$$
909-1005
$$

## Sample Problems (Estimation)

*(35) $\sqrt{224} \times \sqrt{325}=$
Recall: $15^{2}=225$ and $18^{2}=324$
$15 \times 18=270$

$$
257-283
$$

## Sample Problems (Estimation)

*(36) $83 \frac{1}{3} \times 2390=$

$$
\text { Recall: } \frac{5}{6}=.83333 \ldots
$$

$$
\frac{500}{6} \times 2400=200000
$$

189209-209125

## Sample Problems (Practice)

(1) $25 \times 32=$ $\qquad$
(2) $1+2+3+\ldots+19=$ $\qquad$
(3) $97 \times 93=$ $\qquad$
(4) What is the area of a square with diagonal 8 ? $\qquad$
(5) $\mathrm{DCLX}=$ $\qquad$ (Arabic Number)
(6) $17 \times 97=$ $\qquad$
(7) $113^{2} \div 9$ has a remainder of
(8) $77 \times 73=$ $\qquad$
*(9) $119 \times 165=$ $\qquad$
*(10) $119 \times 251=$ $\qquad$

## Some Resources

## AMT Test Writing Service

-675 Miller Rd., Azle, TX 76020
-entermeet@gmail.com
-Phone: 817-444-3655
Offers Number Sense: Elements of Number Sense by Jim Cummings. Contains preparatory material for the Number Sense Contest

## Some Resources

## D \& R Enterprises

- 1101 W. Monte Cristo Rd. West, Edinburg, TX 78541
-Phone: 956-383-0372
No Sense in Mathematics (4 ${ }^{\text {th }}$ edition). By Don Skow


## Some Resources

## Hexco, Inc.

-PO Box 199, Hunt, TX 78024-0199
-800/725-2627; Fax: 830-367-3824
-Email: hexco@hexco.com Web site: www.hexco.com
Supplies materials for Dictionary Skills, Number Sense, and Spelling contests. Offers Dictionary Skills and Spelling practice tests. Also available, Spelling Complements for each graded list containing all the dictionary work for the A+ Spelling List and for Word Power, plus audio tapes, spelling software and spelling rules book. For Number Sense, offers software and practice tests.

## Some Resources

## Mental Mathematics for Number Sense

-Frances Walzel
-2023 CR 08, Cameron, TX 76520
-E-Mail: walzel@vvm.com
Offers 103-page booklet of problems, keys and coded pages for elementary and secondary number sense.

## Some Resources

## MRC Jr.

-1024 Scenic Drive, Justin, TX 76247
-Phone: 940-648-8587; Fax: 940-648-8580
-Email: tomcat2243@ev1.net
Offers study materials and tests for Maps, Graphs and Charts, Dictionary Skills, 5/6 and 7/8 Social Studies, Science I, Science II, Number Sense and Mathematics.

## Some Resources

## Dr. Numsen / Doug Ray

- PO Box 312578, New Braunfels, TX 78131
- Phone: (512) 797-2158; Fax: (208) 575-9617
- Email: doug@academicmeet.com
- Web site: www.academicmeet.com

Provides workbooks and practice tests for elementary and junior high Number Sense, Calculator Applications and Mathematics.

## Some Resources

## Leo Ramirez

- 9801 W. Parmer Lane \#2622, Austin, TX 78717
- Phone: (956) 491-3155 (cell)
- Email: toywiz127@aol.com
- Website: http://www.rammaterials.com/

Number Sense, Calculator Applications, Mathematics, and Science Workbooks (including Number Sense: A Starter's Kit, Middle School Magic, Number Sense Magic, Revised Calculator Applications workbook), DVDs and practice tests. Mr. Ramirez is available for writing invitational meet tests and conducting workshops.

## Some Resources

## TMSCA Test Pool

-Texas Math/Sciences Coaches Association
-PO Box 206, Olney Texas, 76374-0206
-(940) 563-1005
-TMSCA.org
Offers study materials for math, number sense, calculator and science contests.

## Some Resources

The handbook, Developing Middle School Number Sense Skills, is available. It is the same edition first published in 1996. Stock \#217. Cost: $\$ 6.00$.

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PO Box 8028
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FAX 512-232-6471

