DISPLACEMENT vs. DISTANCE TRAVELED

If a body with position function s (t) moves along a coordinate line without changing direction, we can calculate the total distance it travels from t = a to t = b. If the body changes direction one or more times during the trip, then we need to integrate the body's speed |v(t)| to find the total distance traveled.

FACT: Distance traveled = $\int_{a}^{b} |v(t)| dt$

FACT: Displacement = $\int_{a}^{b} v(t) dt$

EXAMPLE 1: Find the total distance traveled by a body and the body's displacement for a body whose velocity is $v(t) = 6 \sin 3t$ on the time interval $0 \le t \le \pi/2$.

SOLUTION: To find the distance traveled, we need to find the values of t where the function changes direction. To do this, set v(t) = 0 and solve for t.

 $6\sin 3t = 0 \quad \rightarrow 3t = 0 \quad \rightarrow \quad t = 0$ $\rightarrow \quad 3t = \pi \quad \rightarrow \quad t = \frac{\pi}{3}$ $\rightarrow \quad 3t = 2\pi \quad \rightarrow \quad t = \frac{2\pi}{3}$

t = 0 is the starting point, and t = $2\pi/3$ is not in the time interval. Now to determine which direction the body is going on the time intervals (0, $\pi/3$) and ($\pi/3$, $\pi/2$).

$$(0, \pi/3)$$
 v $(\pi/4) > 0$

$$(\pi/3, \pi/2)$$
 v $(5\pi/12) < 0$

So to find the total distance traveled, I will have two integrals. One will have to be taken times a -1 to make it positive.

Distance traveled =
$$\int_{0}^{\frac{\pi}{3}} 6\sin 3t \, dt - \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 6\sin 3t \, dt = -2\cos 3t \left| \begin{array}{c} \frac{\pi}{3} \\ +2\cos 3t \\ 0 \end{array} \right|_{0}^{\frac{\pi}{2}} = 6$$

Displacement will be the integral from 0 to $\pi/2$.

Displacement =
$$\int_{0}^{\frac{\pi}{2}} 6\sin 3t \, dt = -2\cos 3t \Big|_{0}^{\frac{\pi}{2}} = 2$$

EXAMPLE 2: Find the total distance traveled by a body and the body's displacement for a body whose velocity is v(t) = 49 - 9.8t on the time interval $0 \le t \le 10$.

SOLUTION: Again, we need to determine if the body changes direction during its travels.

$$v(t) = 0 \rightarrow 49 - 9.8t = 0 \rightarrow 9.8t = 49 \rightarrow t = 5$$

- (0, 5) v (4) > 0
- (5, 10) v (6) < 0

On the interval where the velocity is negative, I will have to multiply the integral by a -1 to make the distance positive.

Distance traveled =
$$\int_0^5 (49 - 9.8t) dt - \int_5^{10} (49 - 9.8) dt = 49t - \frac{9.8}{2}t^2 \Big|_0^5 - \left(49t - \frac{9.8}{2}t^2\right)\Big|_5^{10} = 245$$

Displacement = $\int_0^{10} (49 - 9.8t) dt = 49t - \frac{9.8}{2}t^2 \Big|_0^{10} = 0$

EXAMPLE 3: Find the total distance traveled by a body and the body's displacement for a body whose velocity is $v(t) = 6t^2 - 18t + 12 = 6(t - 1)(t - 2)$ on the time interval $0 \le t \le 3$.

SOLUTION: Just like the previous examples, we have to determine where the velocity changes direction, but it is easier this time since we are given the factored form the velocity. The velocity changes direction at t = 1 and t = 2.

- (0, 1) v (0.5) > 0
- (1, 2) v (1.5) < 0
- (2, 3) v (2.5) > 0

Distance traveled =
$$\int_{0}^{1} (6t^{2} - 18t + 12) dt - \int_{1}^{2} (6t^{2} - 18t + 12) dt$$

+ $\int_{2}^{3} (6t^{2} - 18t + 12) dt = \frac{6t^{3}}{3} - \frac{18t^{2}}{2} + 12t \Big|_{0}^{1} - \left(\frac{6t^{3}}{3} - \frac{18t^{2}}{2} + 12t\right)\Big|_{1}^{2}$
+ $\frac{6t^{3}}{3} - \frac{18t^{2}}{2} + 12t \Big|_{2}^{3} = 11$
Displaceme nt = $\int_{0}^{3} (6t^{2} - 18t + 12) dt = \frac{6t^{3}}{3} - \frac{18t^{2}}{2} + 12t \Big|_{0}^{3} = 9$

Did you notice that the displacement is always going to be less than or equal to the distance traveled? This fact can be used to determine if you have made a mistake in your work. Work through the examples presented to you in this set of supplemental notes.