# Computer Vision and Measurements in Aerospace Applications


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# Objective

To build a unified theoretical framework for quantitative image-based measurements of morphology and motion fields of deformable bodies like fluids

- ***Geometric Structures:***

  ***Points, Curves, Surfaces***

- ***Motion Fields:***

  ***Points, Curves, Surfaces (Geometric Flow)***

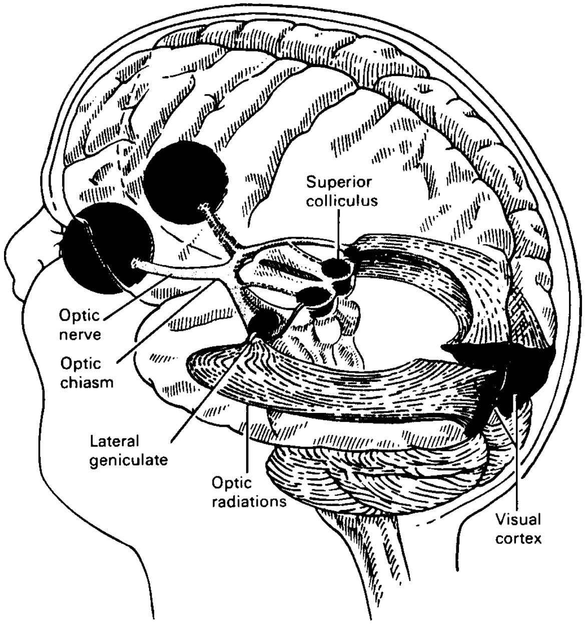
  ***Complex Continuous Patterns***

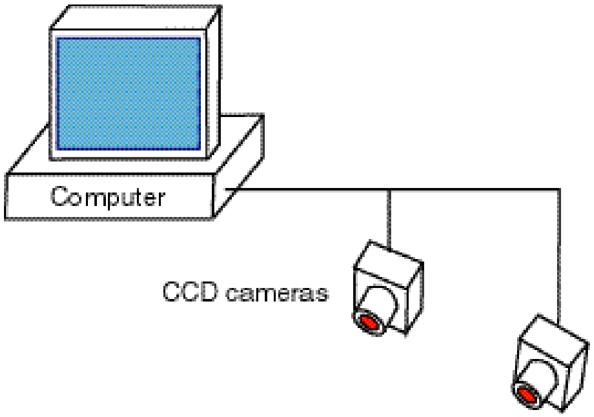
# Background

- ***Photogrammetry***
- ***Computer Vision***

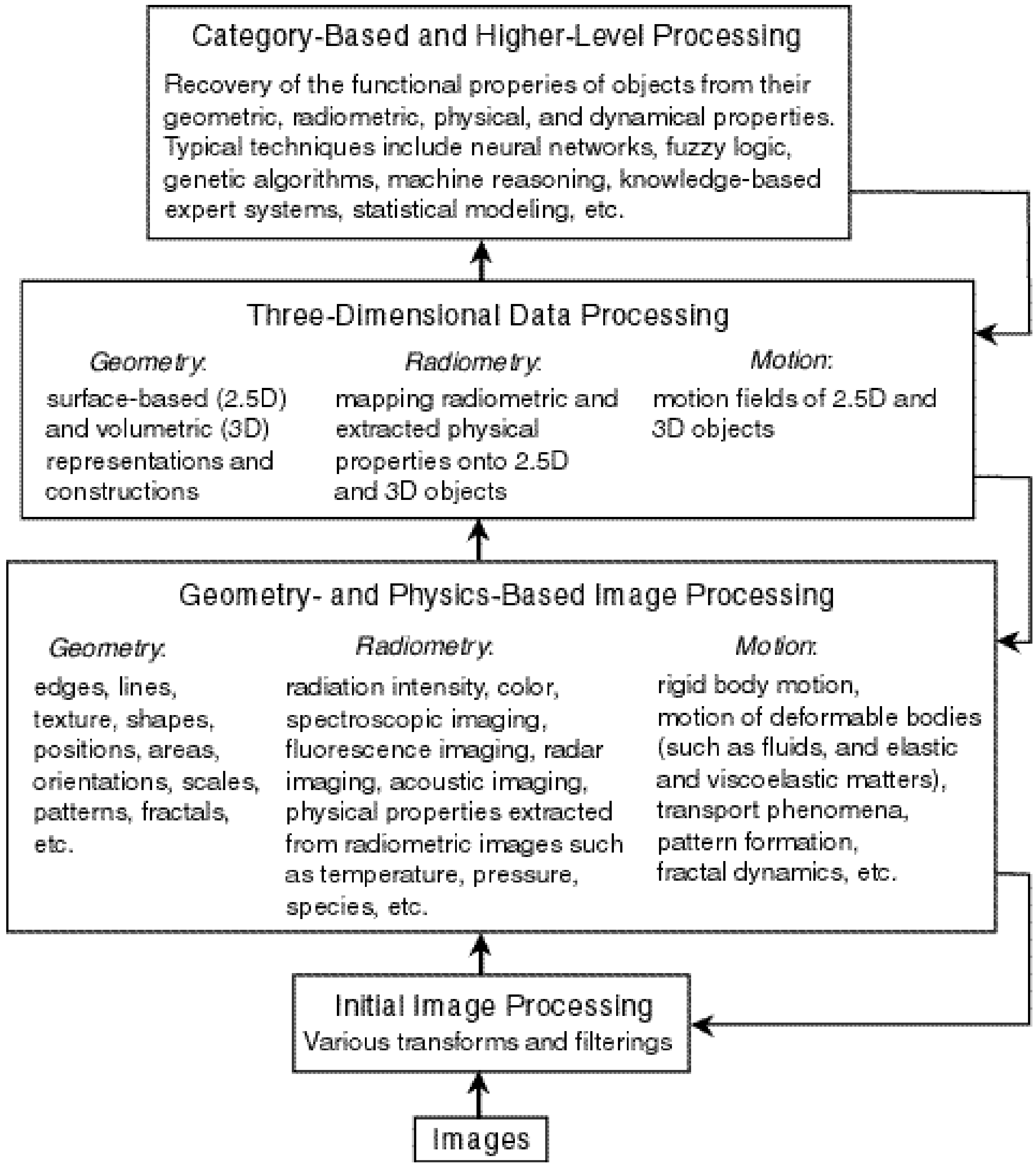
***Human Vision*** ⟷ ***Computer Vision***

*Analogy*

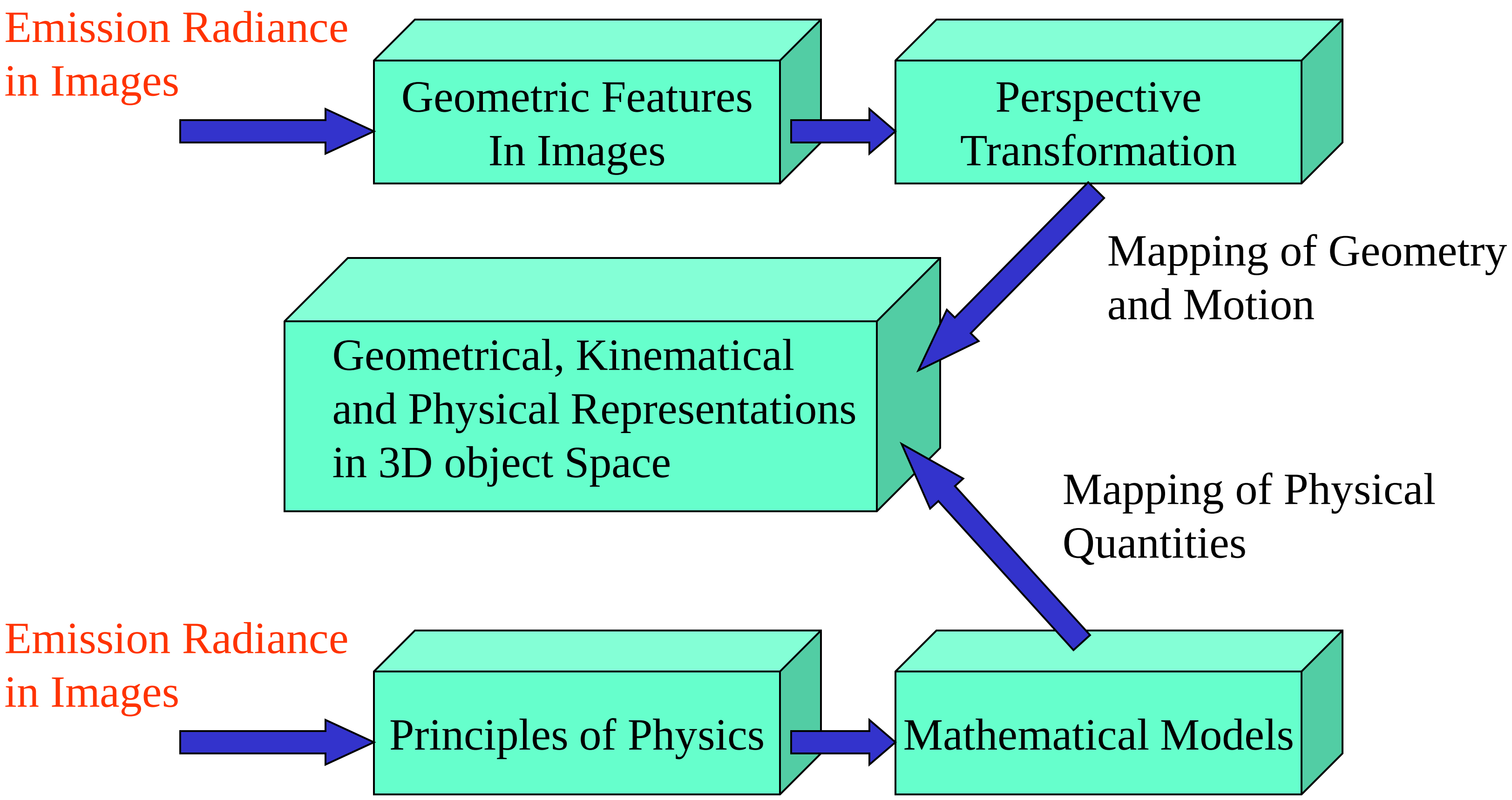


# Optical Info Processing

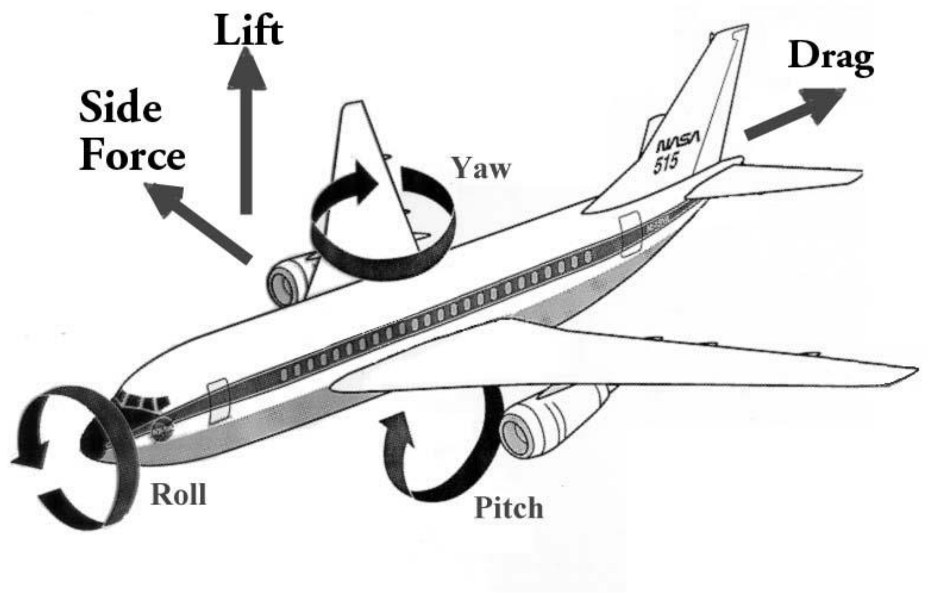


# Geometric and Physical-Based Processing



# Important Quantities in Aerodynamics

*(1) Pressure (2) Temperature (3) Skin Friction*
*(4) Velocity Field (5) Attitude and Kinematics*
*(6) Shape*




**Molecular Sensors!**

# Needs for New Methods

*Quantitative, Dynamic, Universal, Applicable to Complex Patterns & Motions of Deformable Bodies*

# Combination of Approaches

- *Perspective Geometry*
- *Differential Geometry*
- *Continuum Kinematics & Dynamics*
- *Radiometry*

# Relevant Topics

*(1) Perspective Projection Transformation*

*(2) Projective Developable Conical Surface*

*(3) Perspective Projection under Surface Constraint*

*(4) Perspective Projection of Motion Field Constrained on Surface*

*(5) The Correspondence Problem*

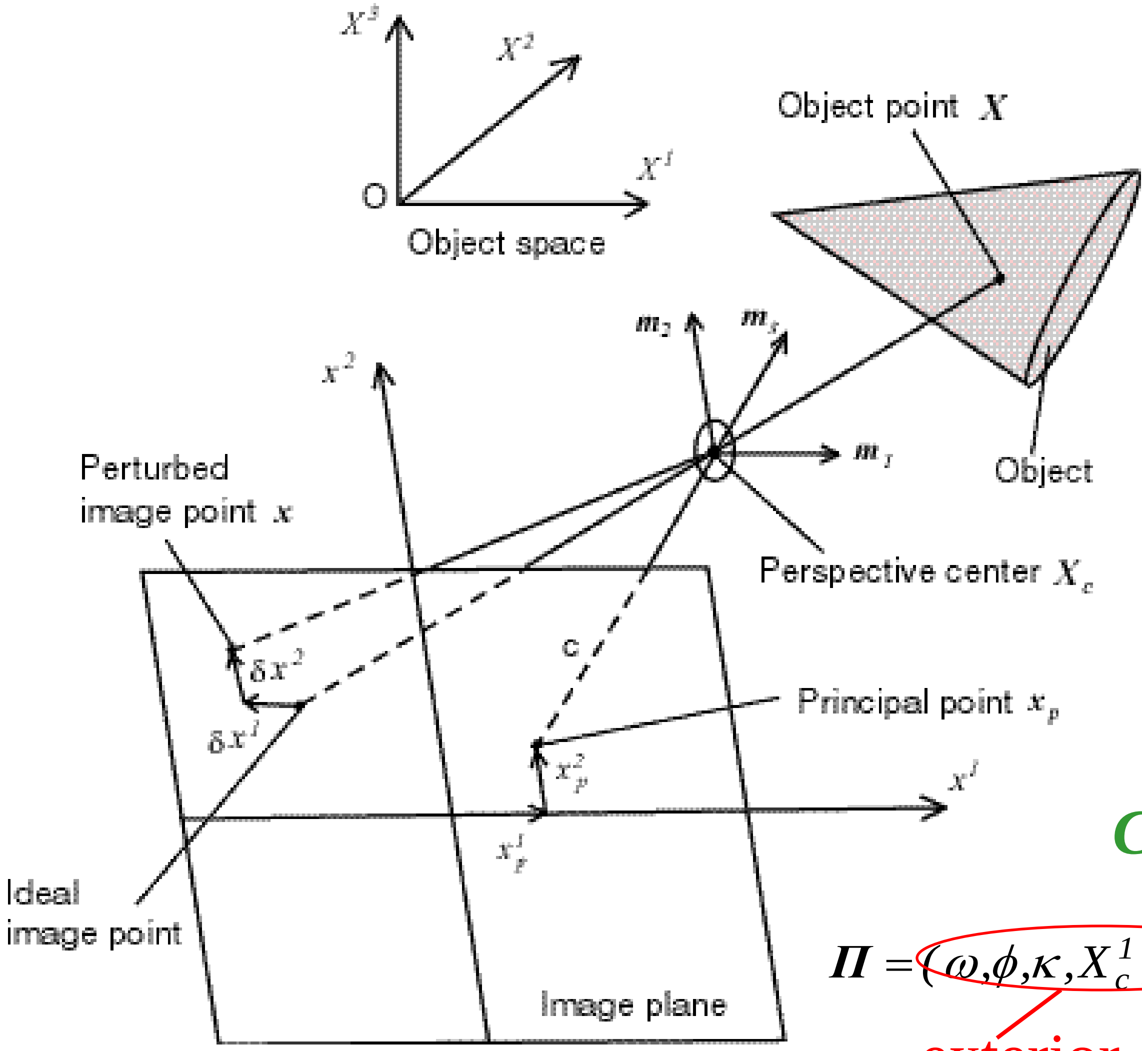
*(6) Composite Image Space and Object Space*

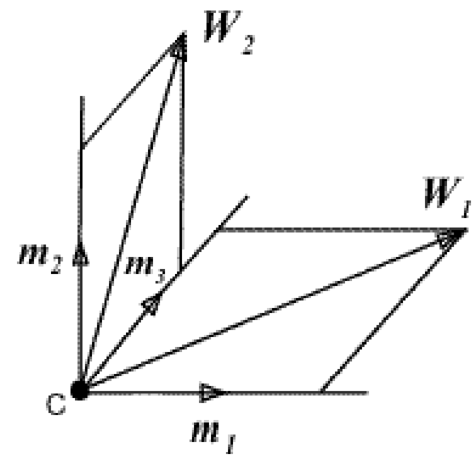
*(7) Perspective Invariants of 3D Curve*

*(8) Reflection and Shape Recovery*

*(9) Motion Equations of Image Intensity*

# Perspective Projection Transformation




## *Camera parameters*

$$\Pi = (\omega, \phi, \kappa, X_c^1, X_c^2, X_c^3, c, x_p^1, x_p^2, K_1, K_2, P_1, P_2)$$

exterior interior distortion

# Formulations of Perspective Transformation

## *(1) Collinearity Equations*

$$x^1 - x_p^1 + \delta x^1 = -c\,\frac{\mathbf{m}_1^T(\mathbf{X}-\mathbf{X}_c)}{\mathbf{m}_3^T(\mathbf{X}-\mathbf{X}_c)}$$

$$x^2 - x_p^2 + \delta x^2 = -c\,\frac{\mathbf{m}_2^T(\mathbf{X}-\mathbf{X}_c)}{\mathbf{m}_3^T(\mathbf{X}-\mathbf{X}_c)}$$

## *(2) Homogenous Coordinate Form*

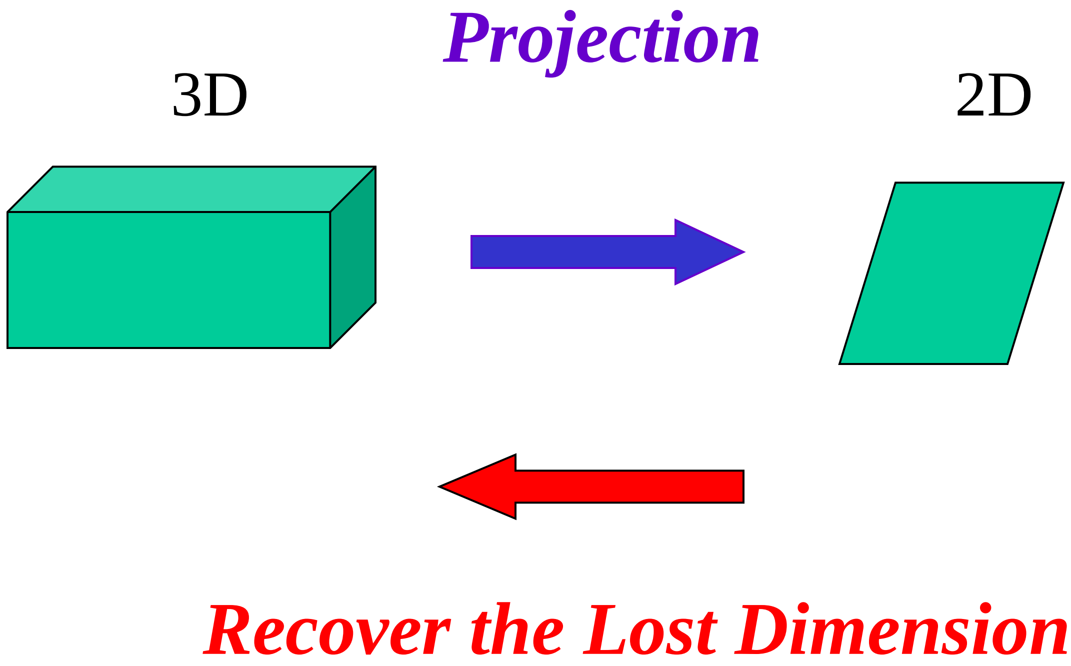
$$\mathbf{x}_h = \lambda\,\mathbf{P}_h\,\mathbf{X}_h$$

## *(3) W-Vector Form*

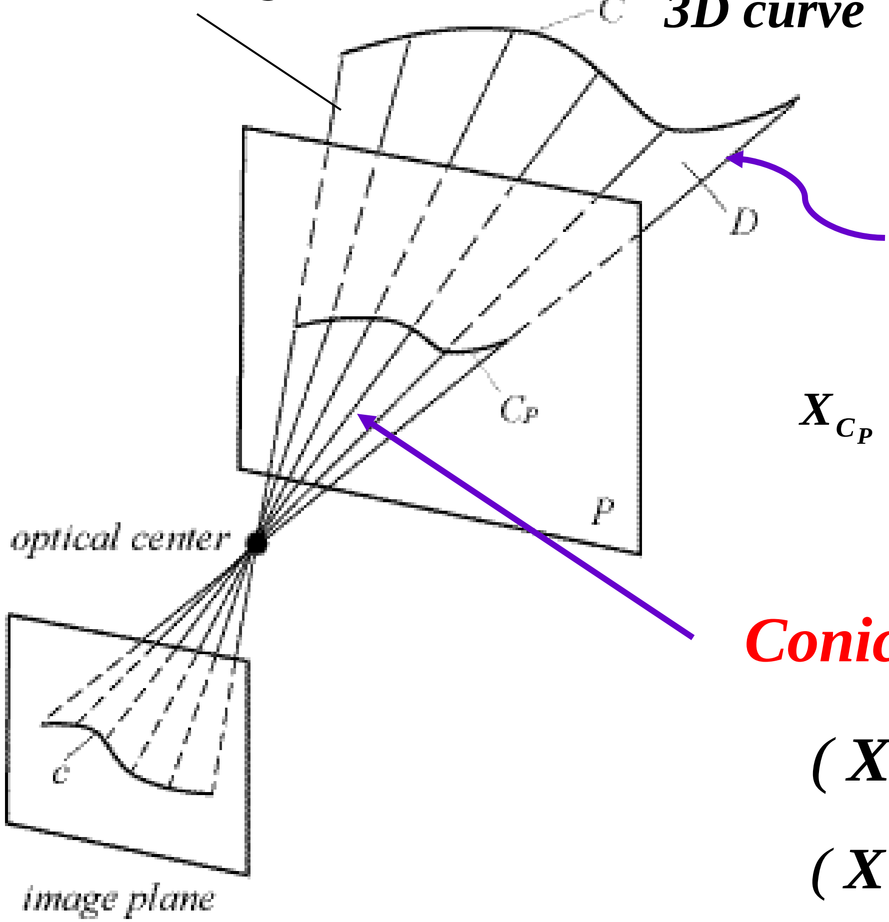
$$\mathbf{W}_1^T(\mathbf{X}-\mathbf{X}_c) = 0$$

$$\mathbf{W}_2^T(\mathbf{X}-\mathbf{X}_c) = 0$$

# Geometric Image Measurements



# Perspective Developable Conical Surface




***Ray Equation***

$$X_{C_P} - X_c = \lambda^{-1}(\overline{P}\, x_{h0} + \int_0^s \overline{P}_{32}\, t\, ds)$$

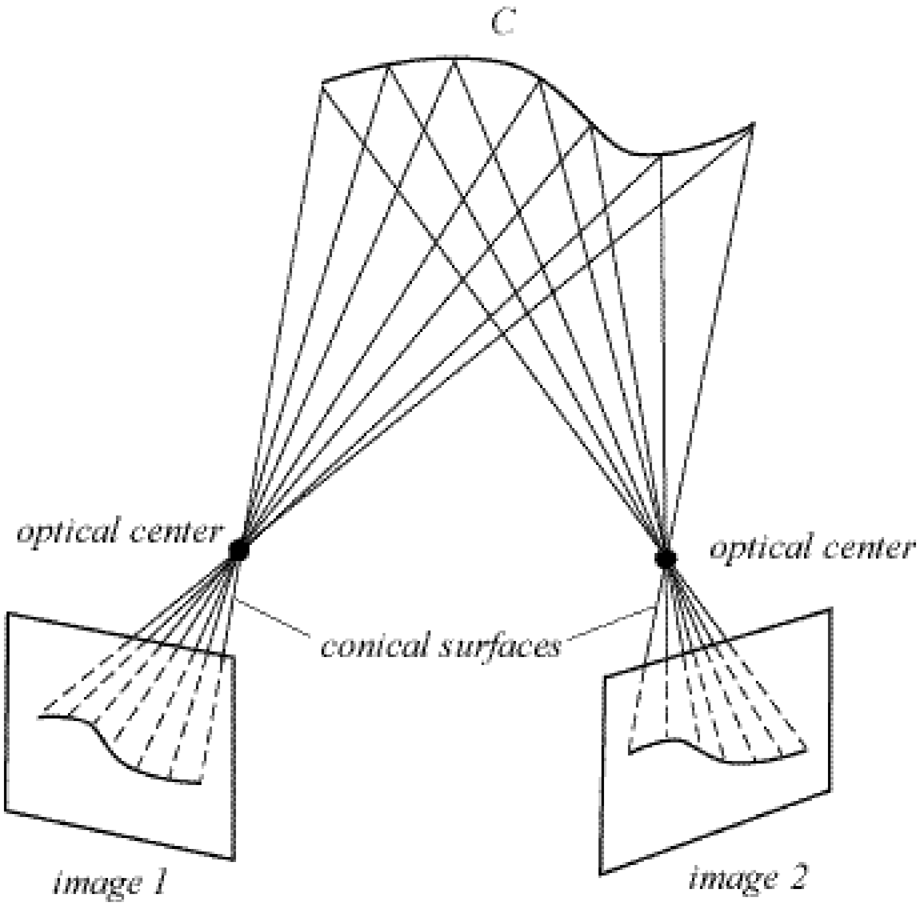
***Conical Surface Equations***

$$(X - X_c) \bullet N_D(s) = 0$$

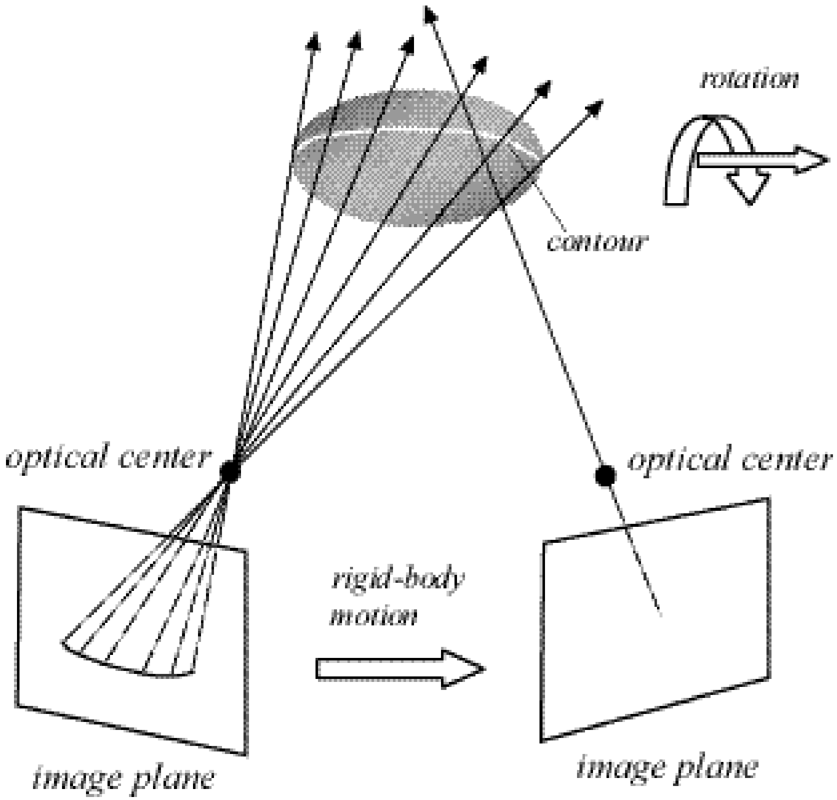
$$(X - X_c) \bullet dN_D(s) / ds = 0$$

# Reconstruction of 3D Curves and Surfaces Using Projective Conical Surfaces

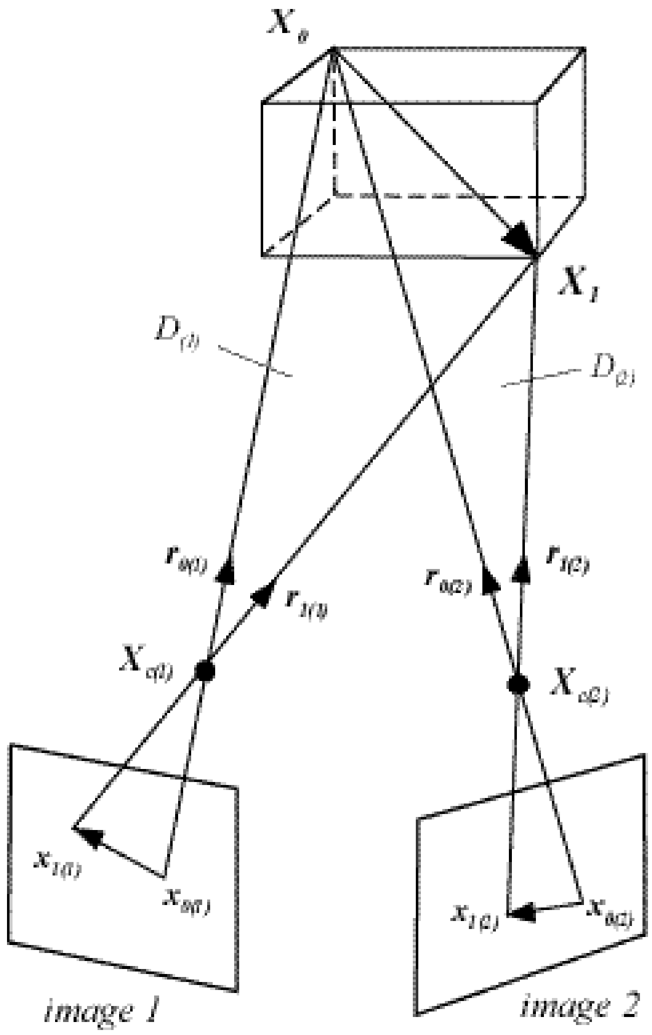
## *3D Space Curve*




## *Surface*

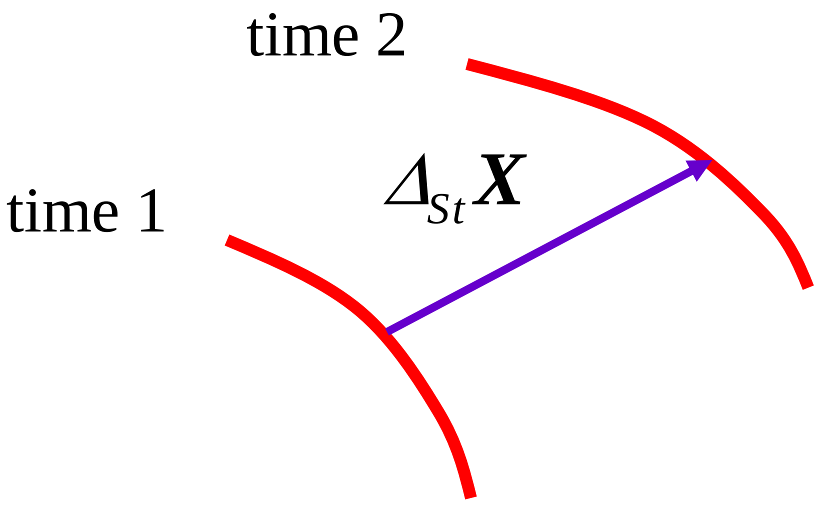


# Reconstruction of 3D Displacement Vectors




Providing a rational and general method for Stereoscopic Particle Image Velocimetry (SPIV) and Scalar Image Velocimetry (SIV)

# Motion Field of 3D Space Curve




***Variational Problem:***

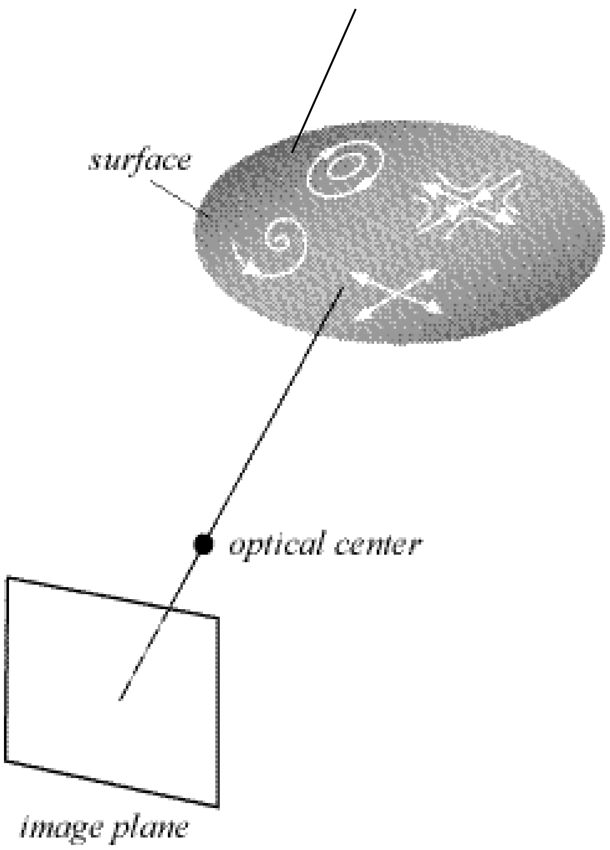
$$\| \Delta_{St}\boldsymbol{X} - \boldsymbol{U}(\boldsymbol{X})\Delta t \| \rightarrow min$$

***Physical and Geometric Constraints:***

$$G_i[\boldsymbol{U}(\boldsymbol{X})] = 0 \qquad i = 1,2,\cdots$$

# Perspective Projection under Surface Constraint

$$X^3 = F(X^1, X^2)$$




***One-to-One Differential Relation***

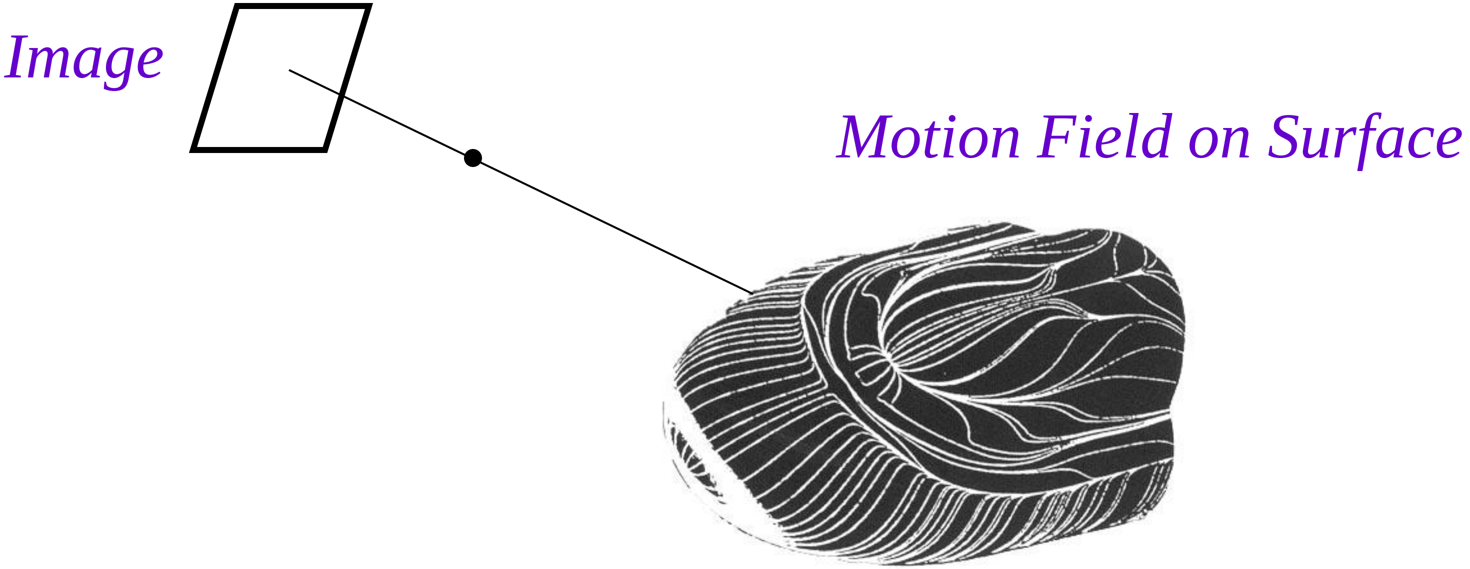
$$\begin{pmatrix} dX^1 \\ dX^2 \end{pmatrix} = \boldsymbol{m}_3^T(\boldsymbol{X}_c - \boldsymbol{f}_s)Q^{-1}\begin{pmatrix} dx^1 \\ dx^2 \end{pmatrix}$$

***Geometric Structures***

$$dS^2 = |d\boldsymbol{X}| = g_{\alpha\beta}\, dx^\alpha dx^\beta$$

***Quantities***: tangents, normal,
length, angle, area
topology

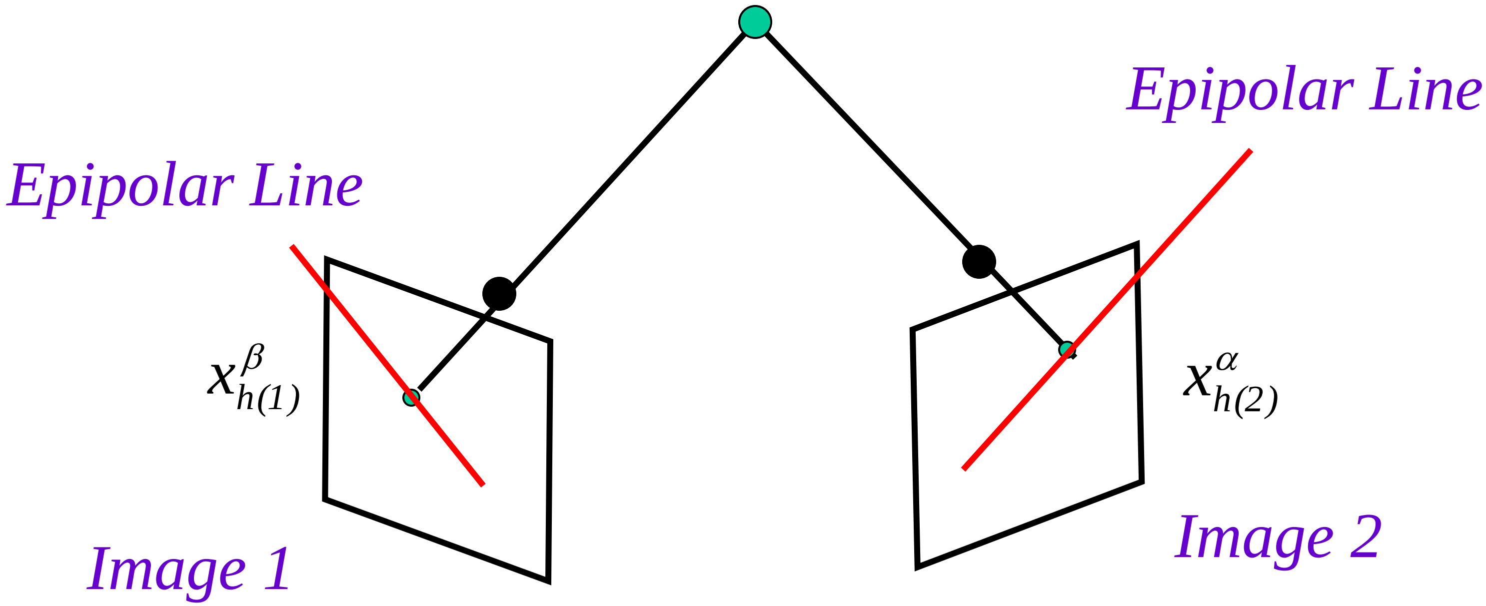
# Surface-Constrained Motion Field




$$\boldsymbol{u} = \frac{d}{dt}\begin{pmatrix} x^1 \\ x^2 \end{pmatrix} = \frac{Q}{\boldsymbol{m}_3^T(\boldsymbol{X}_c - \boldsymbol{f}_S)}\begin{pmatrix} U_1[\,\boldsymbol{f}_S(\boldsymbol{x})] \\ U_2[\,\boldsymbol{f}_S(\boldsymbol{x})] \end{pmatrix}$$

***Optical Flow*** ***Motion Field***

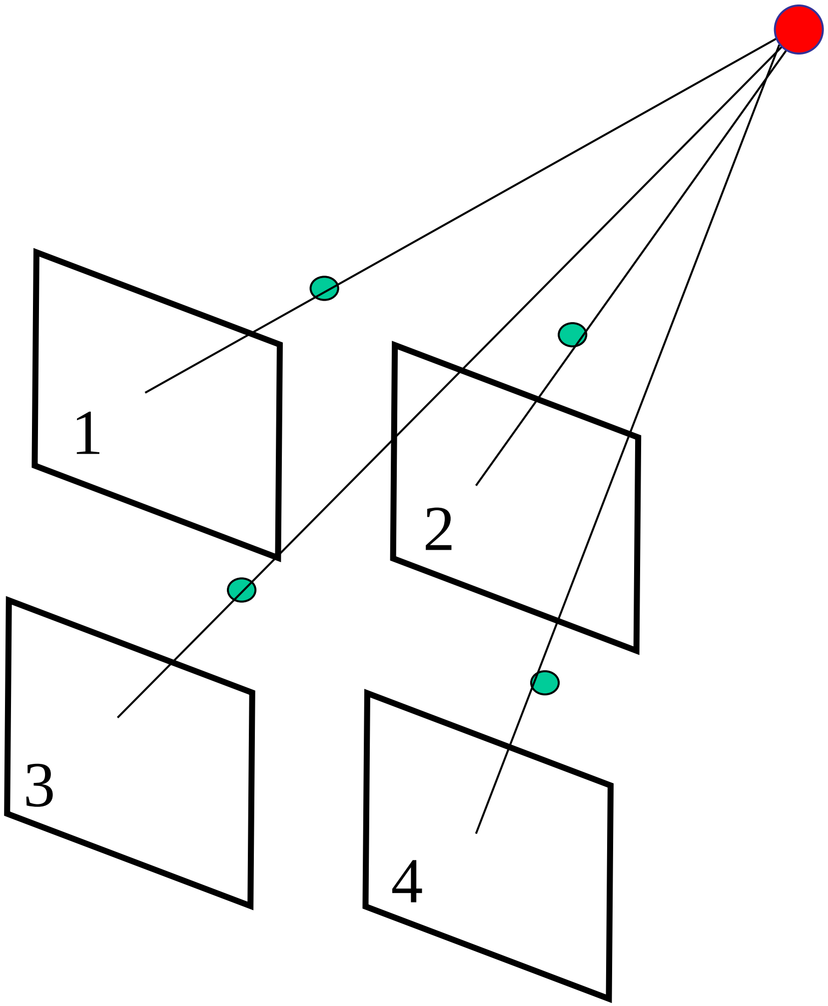
# The Point Correspondence Problem




***Generalized Longuet-Higgins Relation:***

$$(x^{\alpha}_{h(2)} + \delta x^{\alpha}_{h(2)})Q_{\alpha\beta}(x^{\beta}_{h(1)} + \delta x^{\beta}_{h(1)}) = 0$$

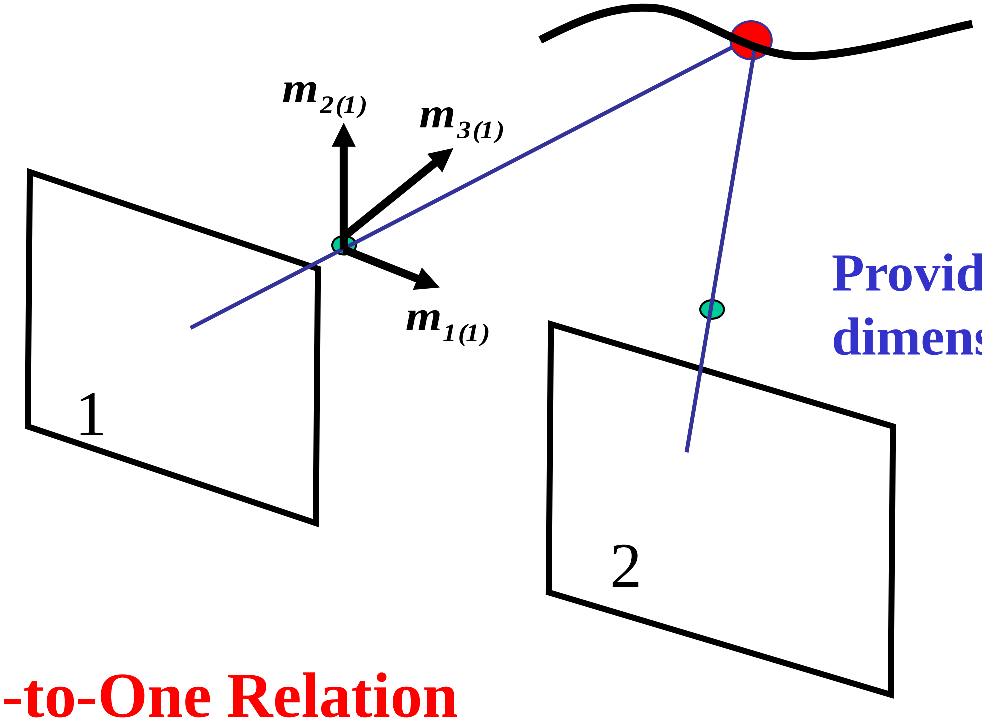
# Determining Point Correspondence




*Four Images or Cameras*

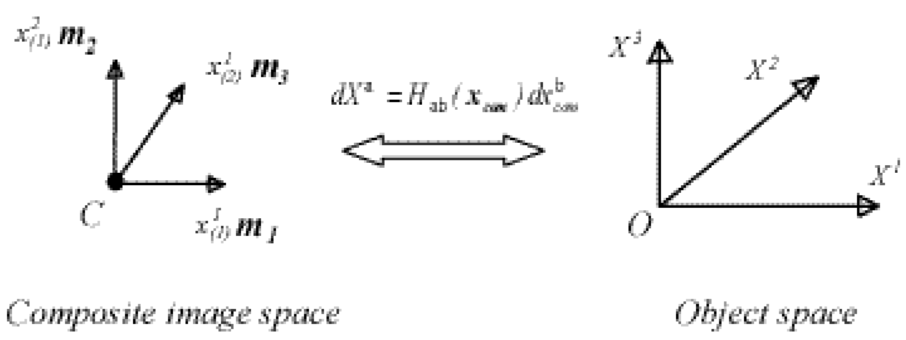
*Six Longuet-Higgins Equations for Six Unknowns*

# Composite Image Space and Object Space



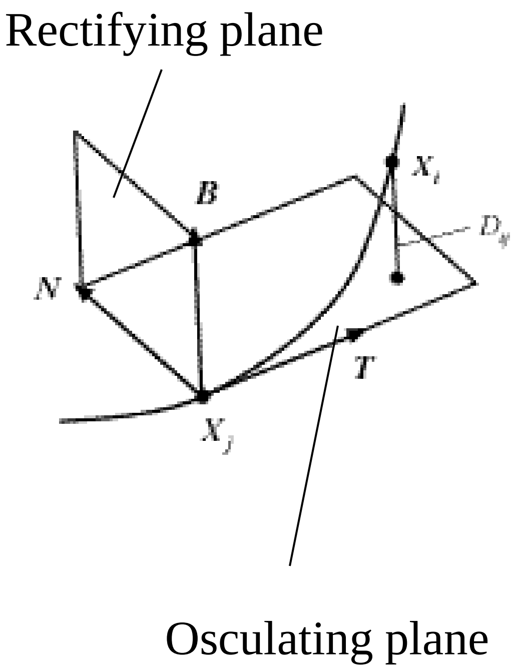

**Provide additional dimensions**

**One-to-One Relation**




***Reconstruct 3D Displacement Vectors from Composite Image Coordinates***

# Perspective Invariants of 3D Curve




**Torsion**

$$\frac{\tau_{im,1}\, d_{12}^2}{\tau_{im,2}\, d_{21}^2} = \frac{\tau_{obj,1}\, D_{12}^2}{\tau_{obj,2}\, D_{21}^2}$$

new

**Curvatures**

$$\frac{\kappa_{im,2}\, d_{12}\, i^2(1,1',2,3)}{\kappa_{im,1}\, d_{21}\, i^2(1,2,2',3)} = \frac{\kappa_{obj,2}\, D_{12}\, I^2(1,1',2,3)}{\kappa_{obj,1}\, D_{21}\, I^2(1,2,2',3)}$$

new

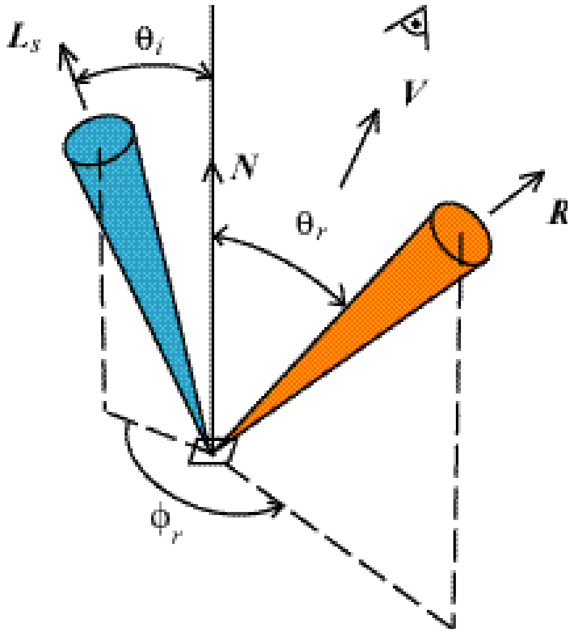
**Distances**

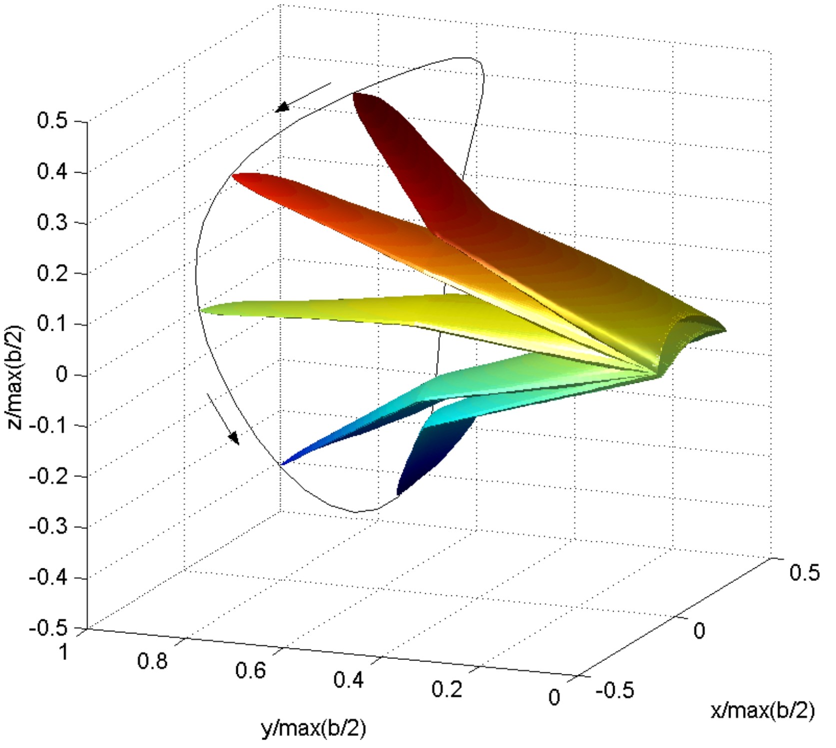
$$\frac{d_{21}\, d_{43}}{d_{41}\, d_{23}} = \frac{D_{21}\, D_{43}}{D_{41}\, D_{23}}$$

(Brill, et al. 1992)

**Geometrical Flow Problem**

# Shape and Reflection




$$I(\mathbf{x}) = c_{sys}\rho_a E_a$$
$$+ c_{sys}E_{ls}[\rho_d\, \mathbf{N} \bullet \mathbf{L_s} + \rho_s\, p(a_N \mathbf{N} \bullet \mathbf{V} + a_L \mathbf{L_s} \bullet \mathbf{V})]$$

# Motion Equations of Image Intensity

## *Image Intensity and Radiance*

$$I(\mathbf{x}, t) = c\,L(\mathbf{X}, \mathbf{a}, \mathbf{g}, t)$$

Physical parameters: $\boldsymbol{p} = (p_1, p_2, \cdots, p_N)^T$

Geometric parameters: $\boldsymbol{q} = (q_1, q_2, \cdots, q_M)^T$

Radiance: $L(X, p, q)$

# Generic Motion Equations of Image Intensity

$$\frac{\partial I}{\partial t} + \boldsymbol{u} \bullet \nabla_x I = c_{sys} \left( \frac{\partial L}{\partial t} + \boldsymbol{U} \bullet \nabla_X L + \frac{d\boldsymbol{p}}{dt} \bullet \nabla_p L + \frac{d\boldsymbol{q}}{dt} \bullet \nabla_q L \right)$$

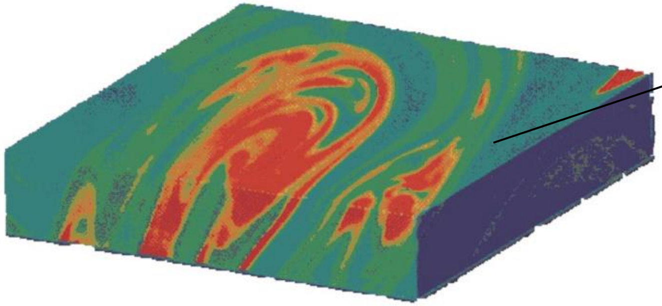
***Optical Flow*** → ***Velocity Field***

Physical parameters: $\boldsymbol{p} = (p_1, p_2, \cdots, p_N)^T$

Geometric parameters: $\boldsymbol{q} = (q_1, q_2, \cdots, q_M)^T$

Radiance: $L(X, p, q)$

# Emitting Passive Scalar Transport



Luminescence

***Governing Equation***

$$\frac{d\psi}{dt} = \frac{\partial \psi}{\partial t} + \boldsymbol{U} \bullet \nabla \psi = D_{\psi} \nabla_{X}^{2} \psi$$

***Radiance***

$$L(\boldsymbol{X}, t) = c_{\psi} \psi(\boldsymbol{X}, t)$$

**Perspective Projection onto Image Plane**

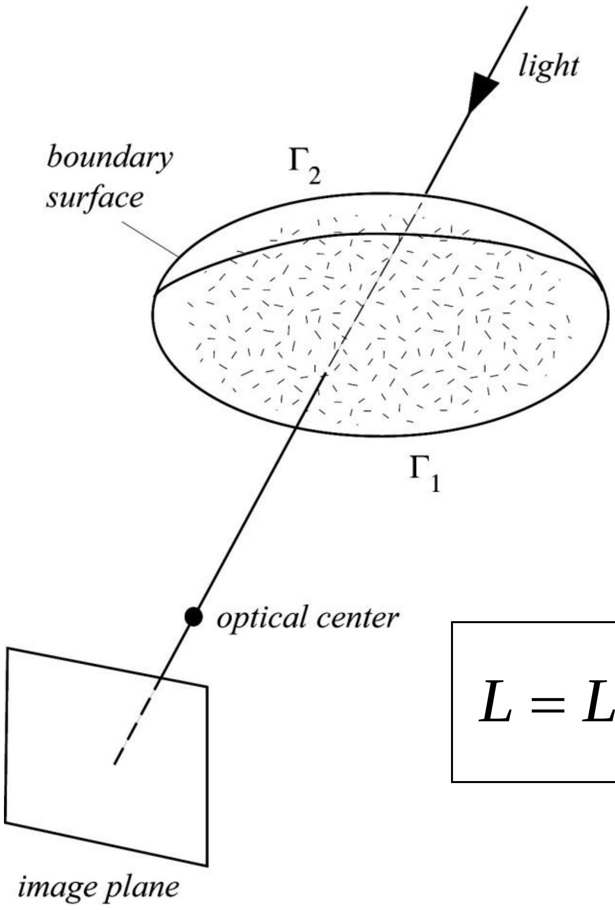
## Motion Equation of Image Intensity for Emitting Passive Scalar Transport

$$\frac{\partial I}{\partial t} + u_\alpha \frac{\partial I}{\partial x^\alpha} = D_\psi \left( h_\lambda \frac{\partial I}{\partial x^\lambda} + h_{\lambda\alpha} \frac{\partial^2 I}{\partial x^\alpha \partial x^\lambda} \right)$$

## Optical Flow and Velocity Field on Surface

$$\boldsymbol{u} = \frac{d}{dt}\begin{pmatrix} x^1 \\ x^2 \end{pmatrix} = \frac{\boldsymbol{G}}{\boldsymbol{m}_3 \bullet (\boldsymbol{X}_c - \boldsymbol{f}_S)} \begin{pmatrix} U_1[\, \boldsymbol{f}_S(\boldsymbol{x})] \\ U_2[\, \boldsymbol{f}_S(\boldsymbol{x})] \end{pmatrix}$$

# Light Transmitting Scalar Transport




## *Governing Equation*

$$\frac{d\psi}{dt} = \frac{\partial\psi}{\partial t} + \boldsymbol{U} \bullet \nabla\psi = D_\psi \nabla_X^2 \psi$$

$$L = L_0 \, exp( -\int_0^s \beta_{ext} \, ds )$$

**Perspective Projection onto Image Plane**

## *Motion Equation of Image Intensity for Light Transmitting Scalar Transport*

$$\frac{\partial I}{\partial t} + u_\beta \frac{\partial I}{\partial x^\beta} = D_\psi \lambda^2 \left( \frac{\partial^2 I}{\partial x^\beta \partial x^\beta} - I^{-1} \frac{\partial I}{\partial x^\beta} \frac{\partial I}{\partial x^\beta} \right)$$

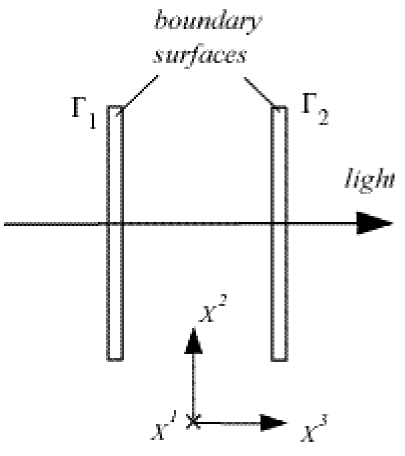
## *Optical Flow and Path-Averaged Velocity*

$$u_\alpha \frac{\partial I}{\partial x^\alpha} = < \boldsymbol{U}_{12} >_\psi \bullet \nabla_{12} I$$

***where***

$$< \boldsymbol{U}_{12} >_\psi = \frac{\int_{\Gamma_1}^{\Gamma_2} \psi \boldsymbol{U}_{12} \, d\overline{X}^3}{\int_{\Gamma_1}^{\Gamma_2} \psi \, d\overline{X}^3}$$

# Schlieren Image of Density-Varying Flows




## *Image Intensity and Density Gradient*

$$\frac{I - I_K}{I_K} = C_{schl} \int_{\Gamma_1}^{\Gamma_2} \frac{\partial \rho}{\partial X^2}\, dX^3$$

## *Motion Equations of Image Intensity*

$$\frac{\partial}{\partial t} \int_{X_0^2}^{X^2} (I - I_K)\, dX^2 + \nabla_{12} \bullet [ < \mathbf{U}_{12} >_\rho \int_{X_0^2}^{X^2} (I - I_K)\, dX^2 ] = 0$$

***where***

$$< \mathbf{U}_{12} >_\rho = \frac{\int_{\Gamma_1}^{\Gamma_2} \rho \mathbf{U}_{12}\, d\overline{X}^3}{\int_{\Gamma_1}^{\Gamma_2} \rho\, d\overline{X}^3} \qquad \nabla_{12} = (\partial / \partial X^1, \partial / \partial X^2)^T$$

# Shadowgraph Image of Density-Varying Flows

## *Image Intensity and Second-Order Density Derivative*

$$\frac{I - I_T}{I_T} = C_{shad} \int_{\Gamma_1}^{\Gamma_2} \nabla_{12}^2 \rho \, dX^3$$

## *Motion Equations of Image Intensity*

$$\frac{\partial}{\partial t} \nabla_{12}^{-2}(I - I_T) + \nabla_{12} \bullet [< \boldsymbol{U}_{12} >_\rho \nabla_{12}^{-2}(I - I_T)] = 0$$

***where*** $\nabla_{12}^2 \phi = I - I_T$ $\xrightarrow{\text{solution}}$ $\nabla_{12}^{-2}(I - I_T)$

# Transmittance Image of Density-Varying Flows

## *Image Intensity and Density*

$$\frac{I - I_T}{I_T} = C_{trans} \int_{\Gamma_1}^{\Gamma_2} \rho \, dX^3$$

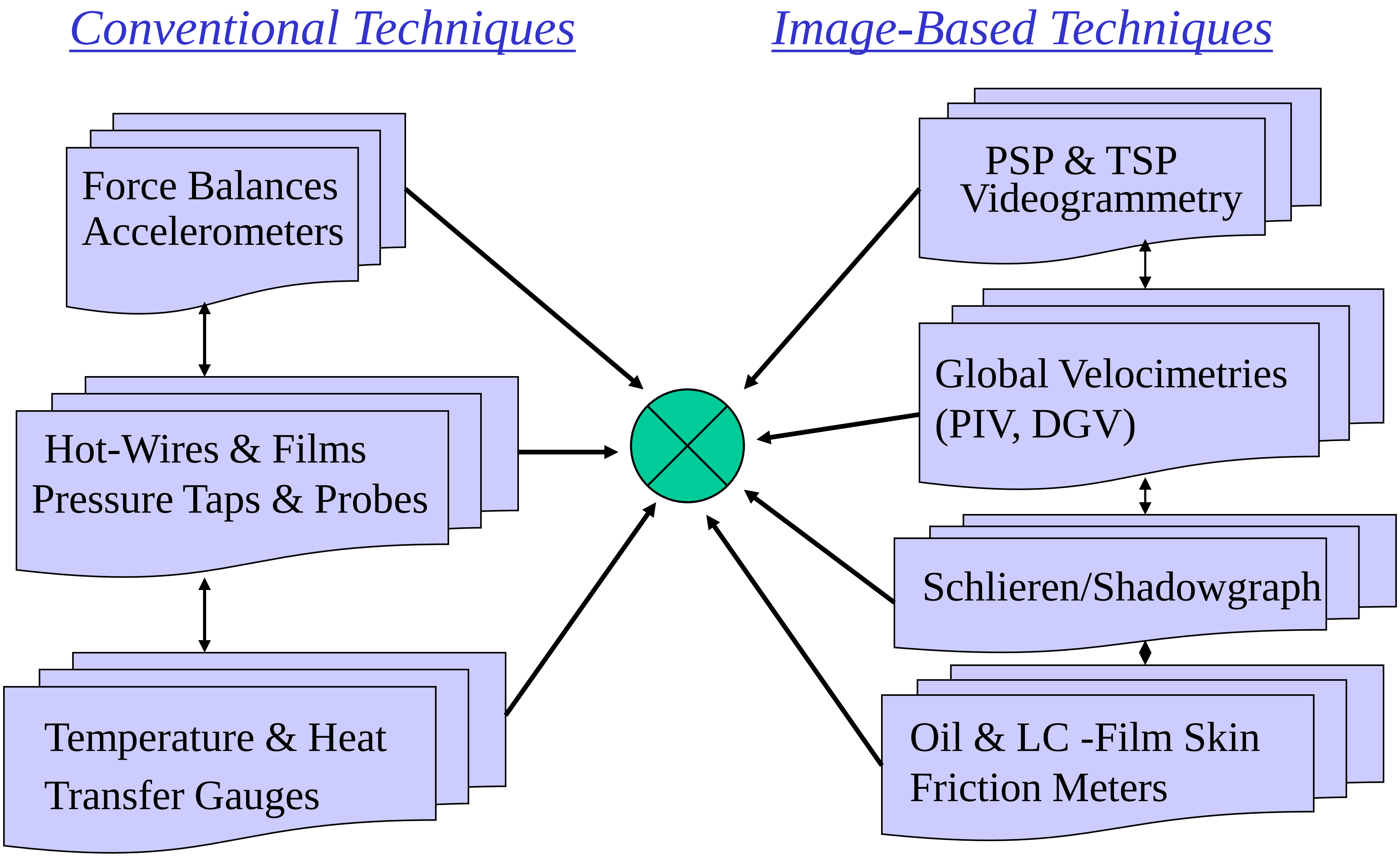
## *Motion Equations of Image Intensity*

$$\frac{\partial}{\partial t}(I - I_T) + \nabla_{12} \bullet [ < \boldsymbol{U}_{12} >_{\rho} (I - I_T)] = 0$$

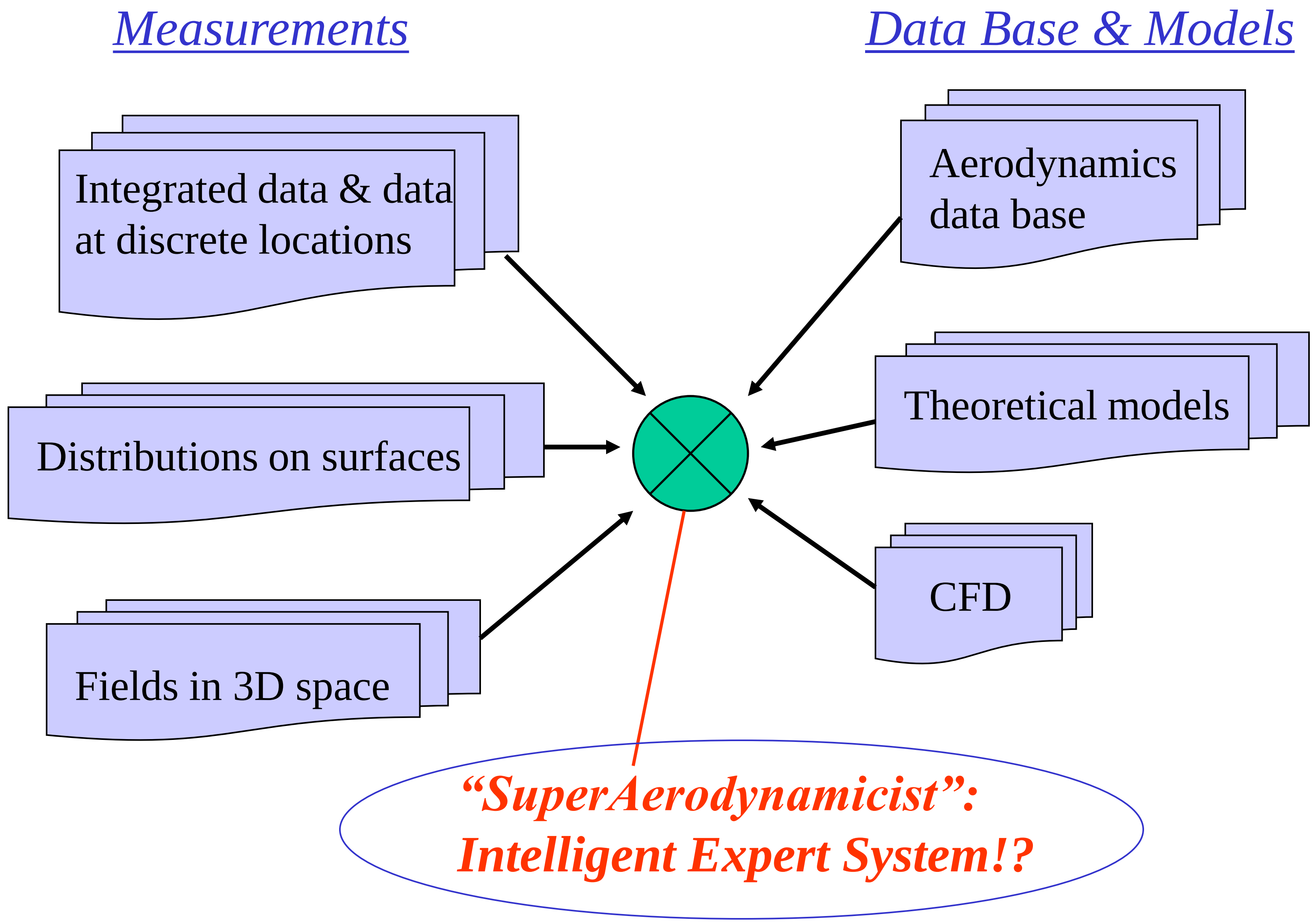
# Typical Applications

- Aerodynamic measurements: pressure and temperature sensitive paints, videogrammetric attitude measurement, stereoscopic PIV, laser-tagging technique, schlieren, shadow and transmittance imaging, oil-film/liquid-crystal skin friction measurements.
- Metrology and kinematics of large inflatable space structure.

# Unification of Measurement Systems



# Data Fusion and Understanding



# Conclusions

*We will see unified image-based instrumentation providing non-contact, global measurements of important physical, geometric and dynamical quantities in wind tunnel testing.*

**Ideal Aerodynamics Lab**