Solving Literal Equations

Literal Equations – Equations with multiple variables where you are asked to solve for just one of the variables. (Usually represent formulas used in the sciences and/or geometry)

To solve literal equations: Use the same process you use to isolate the variable in an algebraic equation with one variable. It's just that you are going to be adding, subtracting, multiplying, and dividing (and sometimes factoring) variables as well as numbers.

CAUTION: BE CAREFUL <u>NOT</u> TO COMBINE UNLIKE TERMS!

Example 1: Solve $E = IR \ \ for \ \ R.$

Goal: Isolate *R* to get *R* = an expression in *E* and *I*

$$E = IR$$

To isolate *R*, divide both sides of the equation by *I*:

$$\frac{E}{I} = \frac{IR}{I}$$

Simplify:

$$\frac{E}{I} = R$$

Solution: $R = \frac{E}{I}$

Example 2: Solve $\frac{d}{t} = r \ \ for \ \ t.$

Goal: Isolate *t* to get *t* = an expression in *d* and *r*

$$\frac{d}{t} = r$$

First multiply both sides of the equation by *t* to clear the fractions:

$$\frac{d}{t}(t) = r(t)$$

Simplify:

$$d = rt$$

To isolate *t*, divide both sides of the equation by *r*:

$$\frac{d}{r} = \frac{rt}{r}$$

Simplify:

$$\frac{d}{r} = t$$

Solution: $t = \frac{d}{r}$

Example 3: Solve $A = \frac{1}{2}h(b_1 + b_2) \ \ for \ \ b_1$

Goal: Isolate b_1 to get b_1 = an expression in *A*, *h*, & b_2 (Note: b_1 and b_2 are two *different* variables.)

First multiply both sides of the equation by 2 to clear the fractions:

$$(2)A = (2)\frac{1}{2}h(b_1 + b_2)$$

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Simplify:

$$2A = h(b_1 + b_2)$$

Distribute *h*:

$$2A = hb_1 + hb_2$$

Next subtract hb_2 from both sides of the equation to get hb_1 alone:

$$\begin{array}{rcl} 2A = & hb_1 + & \cancel{-hb_2} \\ \underline{-hb_2} & & \underline{\cancel{-hb_2}} \end{array}$$

$$2A - hb_2 = hb_1$$

To isolate b_1, divide both sides of the equation by *h*:

$$\frac{2A - hb_2}{h} = \frac{\cancel{h}b_1}{\cancel{h}}$$

Simplify:

$$\frac{2A - hb_2}{h} = b_1$$

Solution: $b_1 = \frac{2A - hb_2}{h}$

Example 4: Solve $I = \frac{PN}{RN+A}$ for N

Goal: Isolate *N* to get *N* = an expression in *I*, *P*, *R*, & *A*:

First multiply both sides of the equation by (*RN+A*) to clear the fractions:

$$(RN + A)I = \frac{PN}{\cancel{RN+A}}(\cancel{RN + A})$$

Simplify:

$$(RN + A)I = PN$$

Distribute *I*:

$$IRN + IA = PN$$

Subtract *IRN* from both sides to get all *N*'s on the same side:

$$\begin{array}{rcl} \cancel{IRN} + & IA = & PN \\ \underline{\cancel{-IRN}} & & \underline{-IRN} \end{array}$$

Note: *PN* & *IRN* are <u>not</u> like terms we cannot combine them!

$$IA = PN - IRN$$

But we <u>can</u> factor out the *N* from each term!

$$IA = N(P - IR)$$

Finally, we can divide both sides by (*P – IR*) to isolate *N*:

$$\frac{IA}{P-IR} = \frac{N(\cancel{P-IR})}{\cancel{P-IR}}$$

Simplify:

$$\frac{IA}{P - IR} = N$$

Solution: $N = \frac{IA}{P-IR}$

Practice Problems

1. Solve $d = rt \ \ for \ \ r$
2. Solve $P = \frac{144p}{y} \ \ for \ \ p$
3. Solve $R = \frac{CS}{d} \ \ for \ \ C$
4. Solve $P = a + b + c \ \ for \ b$
5. Solve $T = m - n \ \ for \ n$
6. Solve $A = \frac{a+b}{2} \ \ for \ b$
7. Solve $V = lwh \ \ for \ w$
8. Solve $m = \frac{y_2 - y_1}{x_2 - x_1} \ \ for \ \ y_2$
9. Solve $ax + by = c \ \ for \ \ y$
10. Solve $A = \frac{a+b+c+d}{4} \ \ for \ \ c$
11. Solve $S = 2(lw + lh + wh) \ for \ \ w$
12. Solve $P = 2(l + w) \ \ for \ \ l$
13. Solve $d = \frac{C}{\pi} \ \ for \ \ \pi$
14. Solve $\frac{1}{f} = \frac{1}{a} + \frac{1}{b} \ \ for \ \ f$
15. Solve $A = p(1 + rt) \ \ for \ \ t$
16. Solve $I = prt \ \ for \ \ r$
17. Solve $ax + b = c \ \ for \ \ a$
18. Solve $S = 2\pi rh \ \ for \ \ h$
19. Solve $A = 2\pi r^2 + 2\pi rh \ \ for \ \ h$
20. Solve $y - y_1 = m(x - x_1) \ \ for \ \ x$
21. Solve $R = \frac{l+3w}{2} \ \ for \ \ w$
22. Solve $ax + by + c = 0 \ \ for \ \ y$
23. Solve $C = \frac{5}{9}(F - 32) \ \ for \ \ F$
24. Solve $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \ \ for \ \ R$
25. Solve $H = \frac{62.4NS}{33{,}000} \ \ for \ \ N$
26. Solve $B = \frac{703w}{h^2} \ \ for \ \ w$
27. Solve $K = \frac{1}{2}mv^2 \ \ for \ \ m$
28. Solve $5t - 2r = 25 \ \ for \ \ t$
29. Solve $S = R - rR \ \ for \ \ R$
30. Solve $V = \frac{1}{3}\pi h^2(3r - h) \ \ for \ \ r$
31. Solve $A = \frac{1}{2}nal \ \ for \ \ n$
32. Solve $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \ \ for \ \ T_1$
33. Solve $F = \frac{gm_1m_2}{d^2} \ \ for \ \ g$
34. Solve $\frac{12ds}{w} = CD \ \ for \ \ w$
35. Solve $A = \frac{1}{2}bh \ \ for \ \ b$
36. Solve $s = r\theta \ \ for \ \ \theta$
37. Solve $h = vt - 16t^2 \ \ for \ \ v$
38. Solve $C = \frac{100B}{L} \ \ for \ \ L$
39. Solve $A = S(1 - DN) \ \ for \ \ N$
40. Solve $D = \frac{11}{5}(P - 15) \ \ for \ \ P$
41. Solve $E = IR \ \ for \ \ I$
42. Solve $E = mc^2 \ \ for \ \ c^2$
43. Solve $F = \frac{lt}{d} \ \ for \ \ l$
44. Solve $A = 2\pi r^2 + 2\pi rh \ \ for \ \ \pi$

Practice Problems Key

1. $r = \frac{d}{t}$
2. $p = \frac{Py}{144}$
3. $C = \frac{Rd}{S}$
4. $b = P - a - c$
5. $n = m - T$
6. $b = 2A - a$
7. $w = \frac{V}{lh}$
8. $y_2 = mx_2 - mx_1 + y_1$
9. $y = \frac{c-ax}{b}$
10. $c = 4A - a - b - d$
11. $w = \frac{S-2lh}{2l+2h}$
12. $l = \frac{P-2w}{2}$
13. $\pi = \frac{C}{d}$
14. $f = \frac{ab}{b+a}$
15. $t = \frac{A-p}{pr}$
16. $r = \frac{I}{pt}$
17. $a = \frac{c-b}{x}$
18. $h = \frac{S}{2\pi r}$
19. $h = \frac{A-2\pi r^2}{2\pi r}$
20. $x = \frac{y-y_1+mx_1}{m}$
21. $w = \frac{2R-l}{3}$
22. $y = \frac{-ax-c}{b}$
23. $F = \frac{9}{5}C + 32$
24. $R = \frac{R_1R_2}{R_2+R_1}$
25. $N = \frac{33{,}000H}{62.4S}$
26. $w = \frac{Bh^2}{703}$
27. $m = \frac{2k}{v^2}$
28. $t = \frac{2}{5}r + 5$
29. $R = \frac{S}{1-r}$
30. $r = \frac{3V+\pi h^3}{3\pi h^2}$
31. $n = \frac{2A}{al}$
32. $T_1 = \frac{T_2P_1V_1}{P_2V_2}$
33. $g = \frac{Fd^2}{m_1m_2}$
34. $w = \frac{12ds}{CD}$
35. $b = \frac{2A}{h}$
36. $\theta = \frac{S}{r}$
37. $v = \frac{h+16t^2}{t}$
38. $L = \frac{100B}{C}$
39. $N = \frac{S-A}{SD}$
40. $P = \frac{5}{11}D + 15$
41. $I = \frac{E}{R}$
42. $c^2 = \frac{E}{m}$
43. $l = \frac{Fd}{t}$
44. $\pi = \frac{A}{2r^2+2rh}$