

Sample problems from Chapter 10.1

$$FV = Pmt \frac{\left(1 + \frac{r}{n}\right)^{nt} - 1}{\frac{r}{n}}$$

This is the annuities sinking funds formula. This formula is used in most cases for annuities. The payments for this formula are made at the **end** of a period. Your book likes to use tables which are not a real world application. Again, DO NOT USE the charts in the book! This will work for the problems they give you but on tests I will give you rates that are not in the book. So learn to use the formulas! When doing an example from the book, you may be a few cents from the answer in the book which is fine. If you are off by dollars you have done something wrong.

$$FV = Pmt \frac{\left(1 + \frac{r}{n}\right)^{nt} - 1}{\frac{r}{n}}$$

Variables	What they mean.
FV	Future Value , money in the account at the end of a time period or in the future
Pmt	Payment , the amount that is being deposited
r	Rate , this is the interest rate (written as a decimal)
n	Compounding Periods , number of times the account will compound in one year
t	Time , the number of <u>YEARS</u> the account is active

Example 1 (pg 415)

$$a) FV = 800 \frac{\left(1 + \frac{0.04}{4}\right)^{4(8)} - 1}{\frac{0.04}{4}}$$

Enter in your calculator (I am using a TI-30X for this....some will be different keystrokes):
 $800((1+.04/4)^{(4*8)}-1)/(.04/4)$

FV = \$29995.25 in the account in ten years (book has an error so answer is different)

$$b) FV = 800 \frac{\left(1 + \frac{0.08}{4}\right)^{4(8)} - 1}{\frac{0.08}{4}}$$

Calculator: $800((1+.08/4)^{(4*8)}-1)/(.08/4)$

FV = \$35381.62 in the account in ten years

c) $35381.62 - 29995.25 = 5386.37$

Example 2 (pg 416)

$$FV = 600 \frac{\left(1 + \frac{0.06}{2}\right)^{2(17)} - 1}{\frac{0.06}{2}}$$

Calculator: $600((1+.06/2)^{(2*17)}-1)/(.06/2)$

FV = \$34638.11 is in the account after 17 years.

To figure the interest accrued in the account we think of taking that \$600 and putting it in a jar or under the mattress every 6 months, that amount would be what we have without interest.
(34 * 600)

$34638.11 - 20400 = \$14238.11$ of interest over the 17 years.

ANNUITY DUE

This is the annuity due formula. In any problems that you see “**payment at the beginning**” of some time period, this is the formula to use. All the variables have the same meaning as the original annuity formula above.

$$FV = Pmt \frac{\left(1 + \frac{r}{n}\right)^{nt+1} - 1}{\frac{r}{n}} - Pmt$$

Example 3 (pg 416)

$$FV = 500 \frac{\left(1 + \frac{0.08}{4}\right)^{4(7)+1} - 1}{\frac{0.08}{4}} - 500$$

Calculator: $500((1+.08/4)^{(4*7+1)}-1)/(0.08/4) - 500$

FV = \$18896.12 in the account in 7 years

Now for interest, we go back to putting money under the mattress.... $500 * 28 = 14000$

Interest accrued from the account $18896.12 - 14000 = \$4896.12$

Example 4 (pg 419)

$$a) FV = 2000 \frac{\left(1 + \frac{0.06}{1}\right)^{1(33)} - 1}{\frac{0.06}{1}}$$

Calculator: $2000((1+.06/1)^{(1*33)}-1)/(0.06/1)$

FV = \$194686.33

$$b) FV = 2000 \frac{\left(1 + \frac{0.10}{1}\right)^{1(33)} - 1}{\frac{0.10}{1}}$$

Calculator: $2000((1+.10/1)^{(1*33)}-1)/(0.10/1)$

FV = \$444503.09

Example Test Question

I will invest \$500 per quarter for my retirement at 7.3% compounding quarterly for 32 years. I have a choice of making that payment of \$500 at the beginning or the end of the quarter (regular annuity or annuity due). In which account will I have more money and by how much? Which account will earn the most interest and by how much?

$$\text{Regular Annuity} \rightarrow FV = 500 \frac{\left(1 + \frac{0.073}{4}\right)^{4(32)} - 1}{\frac{0.073}{4}}$$

Calculator: $500((1+.073/4)^{4*32}-1)/(.073/4)$

FV = \$249981.20

Interest = $249981.20 - (128*500) = \185981.20

$$\text{Annuity Due} \rightarrow FV = 500 \frac{\left(1 + \frac{0.073}{4}\right)^{4(32)+1} - 1}{\frac{0.073}{4}} - 500$$

Calculator: $500((1+.073/4)^{4*32+1}-1)/(.073/4) - 500$

FV = \$254543.36

Interest = $254543.36 - (128*500) = \190543.36

Most money and interest are from the annuity due. By paying your payment at the beginning of the quarter instead of the end of the quarter I will make an extra $(254543.36 - 249981.20)$ \$4562.16. I make an extra $(190543.36 - 185981.20)$ \$4562.16 in interest. This is the same amount! The only difference in these accounts is the way the interest accumulates over time so that will be the difference and the advantage to using an annuity due rather than a regular annuity.

Sample Problems from 10.2

Example 1 (pg 423)

$$\text{a) } FV = 4325 \frac{\left(1 + \frac{0.06}{4}\right)^{4(5)} - 1}{\frac{0.06}{4}}$$

Calculator: $4325((1+.06/4)^{4*5}-1)/(.06/4)$

FV = \$100009.86

b) With this problem, we are discussing a different type of problem and formula. In this case, we are looking for a present value with payments.

$$PV = Pmt \frac{1 - \left(1 + \frac{r}{n}\right)^{-nt}}{\frac{r}{n}}$$

Variables	What they mean.
<i>PV</i>	Present Value , money in the account at the beginning of a time period
<i>Pmt</i>	Payment , the amount that is being deposited
<i>r</i>	Rate , this is the interest rate (written as a decimal)
<i>n</i>	Compounding Periods , number of times the account will compound in one year (if less than one year, the number of times it will compound)
<i>t</i>	Time , the number of <u>YEARS</u> the account is active

$$PV = 4325 \frac{1 - \left(1 + \frac{0.06}{4}\right)^{-4(5)}}{\frac{0.06}{4}}$$

Calculator: 4325(((1-(1+.06/4)^(-4*5)))/(0.06/4)

Watch for the negative on your calculator! There are two negatives on your calculator. One is for subtraction and is in with the other operations. The other is smaller and down by the decimal, this one is for negative numbers. If you get a syntax error with this formula, you probably used the wrong negative.

PV = \$74254.36 would have to be placed in a savings account today to give me \$100009.86 in 5 years.

Example 2 (pg 424)

$$PV = 900 \frac{1 - \left(1 + \frac{0.06}{4}\right)^{-4(12)}}{\frac{0.06}{4}}$$

Calculator: $900((1-(1+.06/4)^{-4*12})/(.06/4))$

PV = \$30638.30

So \$30638.30 will give you the same amount in 12 years as actually paying \$900 each quarter for 12 years.

Interest = $900*48 - 30638.30 = \$12561.70$ in interest would be accumulated over that time

Example 3 (pg 424)

$$PV = 15000 \frac{1 - \left(1 + \frac{.06}{2}\right)^{-2 \cdot 8}}{\frac{.06}{2}}$$

$$PV = 15000 \frac{1 - \left(1 + \frac{0.06}{2}\right)^{-2(8)}}{\frac{0.06}{2}}$$

Calculator:

$15000(1-(1+.06/2)^{-2*8})/(.06/2)$

PV = \$188416.53

This is the amount I would need in an account to pay the employee \$15000 semiannually for 8 years.

Now we need to find the **lump sum** that I put in an account today to have that amount so the company will never have to put any money into this contract again.

Lump sum usually gives you a tip that this will be a onetime payment into an account. Like a savings account if you leave the money and never pay into it.

$$188416.53 = PV \left(1 + \frac{0.06}{2}\right)^{2(5)}$$
$$\frac{188416.53}{\left(1 + \frac{0.06}{2}\right)^{2(5)}} = PV$$

Calculator: $188416.53/((1+.06/2)^{2*5})$

PV = \$140199.60

Example 4 (pg 426)

$$PV = 25000 \frac{1 - \left(1 + \frac{0.08}{1}\right)^{-1(25)}}{\frac{0.08}{1}}$$

Calculator: $25000(1 - (1 + 0.08/1)^{-1*25}) / (0.08/1)$
PV = 266869.40

$$FV = 2000 \frac{\left(1 + \frac{0.08}{1}\right)^{1(33)} - 1}{\frac{0.08}{1}}$$

Calculator: $2000((1 + 0.08/1)^{1*33} - 1) / (0.08/1)$

FV = \$291901.24

So Tish will have enough money to make her retirement plan work.

Example 5 (pg 427)

$$PV = 10000 \frac{1 - \left(1 + \frac{0.08}{4}\right)^{-4(4)}}{\frac{0.08}{4}}$$

Calculator: $10000(1 - (1 + 0.08/4)^{-4*4}) / (0.08/4)$

PV = \$135777.09 + \$80000 down payment is \$215777.09 as a present value for this offer which is more than the \$200000 the other person offered.

Example 6 (pg 427)

$$PV = 20940 \frac{1 - \left(1 + \frac{0.06}{1}\right)^{-1(28)}}{\frac{0.06}{1}}$$

Calculator: $20940(1-(1+.06/1)^{-1*28})/(.06/1)$

PV = \$280725.08 so her social security payments are like having that amount of money in her hands at the beginning of retirement.

Example Test Question

I am looking ahead to my retirement and want to be able to retire at 70 and hope to live to 95 and make \$3200 a month from an account compounding monthly at 4.5%. I am currently 27 and I am going to deposit \$1000 at the beginning of each quarter until I am 70 in an account that pays 8.5% and is compounded quarterly. Will I have enough to make it happen and by how much am I above or below?

Find the amount I need to support those requirements from age 70 to 95.

$$PV = 3200 \frac{1 - \left(1 + \frac{0.045}{12}\right)^{-12(25)}}{\frac{0.045}{12}}$$

Calculator: $3200(1-(1+.045/12)^{-12*25})/(.045/12)$

PV = \$575713.03 is needed to support me from 70-95 years old.

At depositing 350 per quarter will I have enough?

$$FV = 350 \frac{\left(1 + \frac{0.085}{4}\right)^{4(43)} - 1}{\frac{0.085}{4}}$$

Calculator: $350((1+.085/4)^{(4*43)}-1)/(.085/4)$

FV = \$596479.44 will be in the account at 70 years old.

I will have enough money to pull 3200 out every month. I will have $(596479.44 - 575713.03)$ \$20766.41 extra in the account.

Sample problems from 10.3

We are using the same formulas but now we will be solving for payments instead of a future or present value.

Example 1 (pg 431)

$$a) \quad 16500000 = Pmt \frac{\left(1 + \frac{0.06}{4}\right)^{4(5)} - 1}{\frac{0.06}{4}}$$

so solve for Pmt

$$\frac{16500000}{\frac{\left(1 + \frac{0.06}{4}\right)^{4(5)} - 1}{\frac{0.06}{4}}} = Pmt$$

Calculator: $16500000/(((1+.06/4)^{(4*5)}-1)/(.06/4))$

Pmt = \$713554.64 payment per quarter

b) Interest is like putting the money under your mattress..... $713554.64 * 20$

$$16500000 - (713554.64 * 20) = 2228907.20$$

Our solution is somewhat different from the book. If you notice they say their amount will yield more money than they wanted. Ours would actually yield the money that was required.

Example 2 (pg 432)

$$100000 = Pmt \frac{\left(1 + \frac{0.10}{1}\right)^{1(8)} - 1}{\frac{0.10}{1}}$$

so solve for Pmt

$$\frac{100000}{\frac{\left(1 + \frac{0.10}{1}\right)^{1(8)} - 1}{\frac{0.10}{1}}} = Pmt$$

Calculator: $100000 / (((1 + .10/1)^{(1*8)} - 1) / (.10/1))$

Pmt = \$8744.40 payment per year

Interest = $100000 - (8744.40 * 8) = \30044.80 in interest

Example 3 (pg 432)

The table can be created using the formula from above and following the work that was done in the example.

Example 4 (pg 434)

To find cost in 4 years:

$$FV = 850000 \left(1 + \frac{0.05}{1}\right)^{1(4)}$$

Calculator: $850000(1+.05/1)^{(1*4)}$

FV = \$1033180.31 for the unit in 4 years.

$$1033180.31 = Pmt \frac{\left(1 + \frac{0.08}{4}\right)^{4(4)} - 1}{\frac{0.08}{4}}$$

so solve for Pmt

$$\frac{1033180.31}{\frac{\left(1 + \frac{0.08}{4}\right)^{4(4)} - 1}{\frac{0.08}{4}}} = Pmt$$

Calculator: $1033180.31/(((1+.08/4)^{(4*4)}-1)/(.08/4))$

Pmt = \$55430.25 per quarter for the new unit

Example Test Question

I am setting up a fund for my son to go to college. I figure that he will need \$50,000 by the time he is old enough to go to college. I found an account that pays 5.75% compounded monthly. How much will my monthly payment be to get my son set up for college in 17 years? How much interest will the account accrue?

$$50000 = Pmt \frac{\left(1 + \frac{0.0575}{12}\right)^{12(17)} - 1}{\frac{0.0575}{12}}$$

so solve for Pmt

$$\frac{50000}{\frac{\left(1 + \frac{0.0575}{12}\right)^{12(17)} - 1}{\frac{0.0575}{12}}} = Pmt$$

Calculator: $50000 / (((1 + 0.0575/12)^{(12*17)} - 1) / (0.0575/12))$

Pmt = \$145.06 per month for \$50,000 in 17 years.

Interest = $50000 - (145.06 * 204) = \$20,407.76$ in interest

**After hearing that, I think I am not the best at making payments and usually with a little extra work (and lucky sale) can get good bonuses at work. In the same account as above, how much money would I have to invest in a lump sum to have \$50000 for my child's college?

$$50000 = PV \left(1 + \frac{0.0575}{12}\right)^{12(17)}$$

So I would need \$18856.50 today (present value) to have 50000 in that account in 17 years for my child.