Bond Math 2

The Economics of Bonds

Fall 2011
Webinar Series
October 7, 2011
10:00 AM - 11:15 AM PT

CDIAC provides information, education and technical assistance on public debt and investments to local public agencies and other public finance professionals.

Bond Math 2

The Economics of Bonds

Housekeeping

- Feedback Button
- Questions and Answers
- Polling Questions

Bond Math 2

The Economics of Bonds

Piecing Together Bond Finances

- **Introduction of Speakers**

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- *Over 27 years of experience in Municipal Finance*
- *Has been involved in all aspects of the business across the country, as both an underwriter and financial advisor*
- *Currently assists clients in the structuring and financing of complex airport projects*

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- *Over 30 years of experience in Municipal finance*
- *Has provided financial advice on over 100 financings totaling over $15 billion for airport clients.*
- *Current airport clients include Chicago, New York, Washington, Tampa, San Jose, Oakland, Salt Lake, Reno, Columbus, Providence, Ft. Myers, Pittsburgh, Memphis and many others*

Piecing Together Bond Finance

Presented by Kenneth D. Fullerton & Robert G. Friar, Jr.

The PFM Group

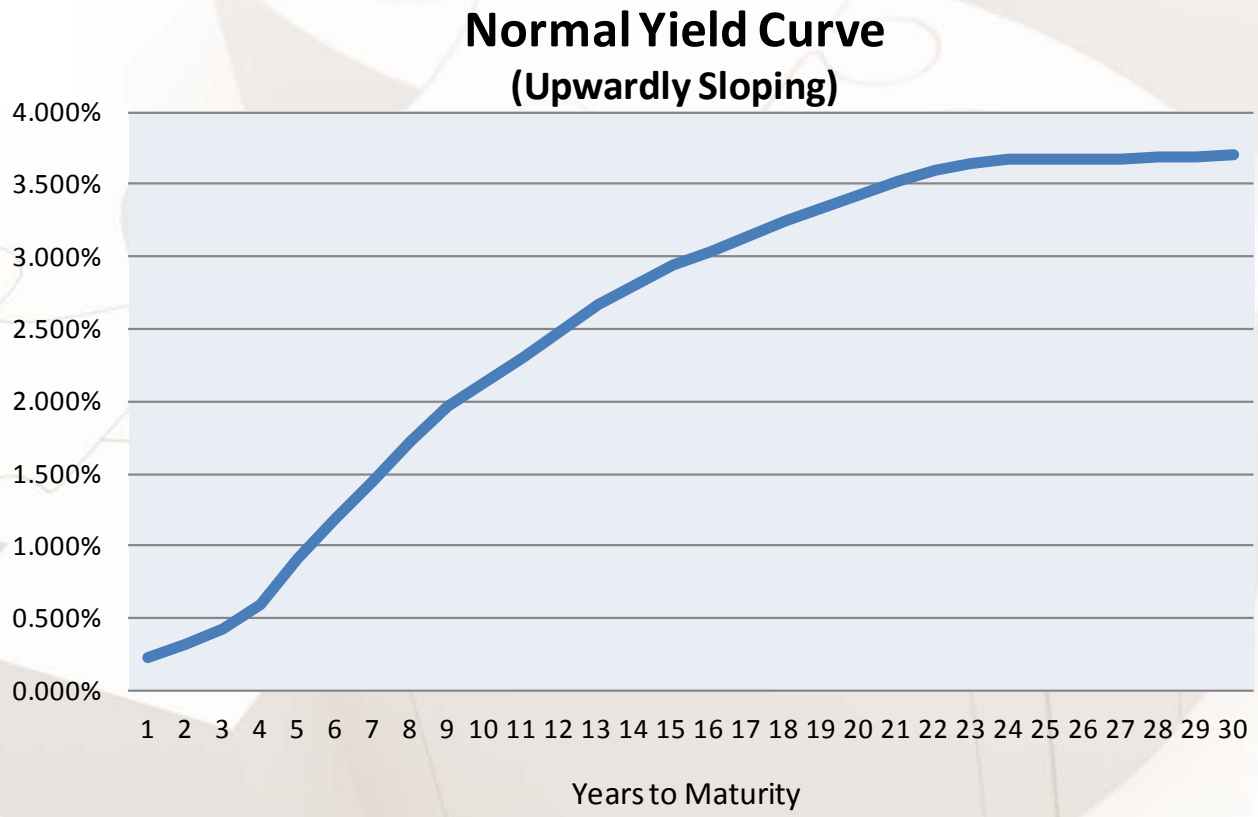
Topics

1. The Yield Curve
   What it is and why it matters
2. Bond Pricing
   Par, premium and discount bonds
3. Other Types of Bonds
   Capital appreciation and zero coupon bonds
4. Bond Redemptions and Accrued Interest
   How bonds are redeemed
5. Spreadsheet Formulas
   Built in functions for doing bond calculations
6. Conclusion

1. The Yield Curve

Definition: A curve on a graph in which the yield of fixed-interest securities is plotted against the length of time they have to run to maturity.

- Under normal conditions, interest rates on bonds with shorter maturities are lower than bonds with longer maturities.



The Yield Curve
(continued)

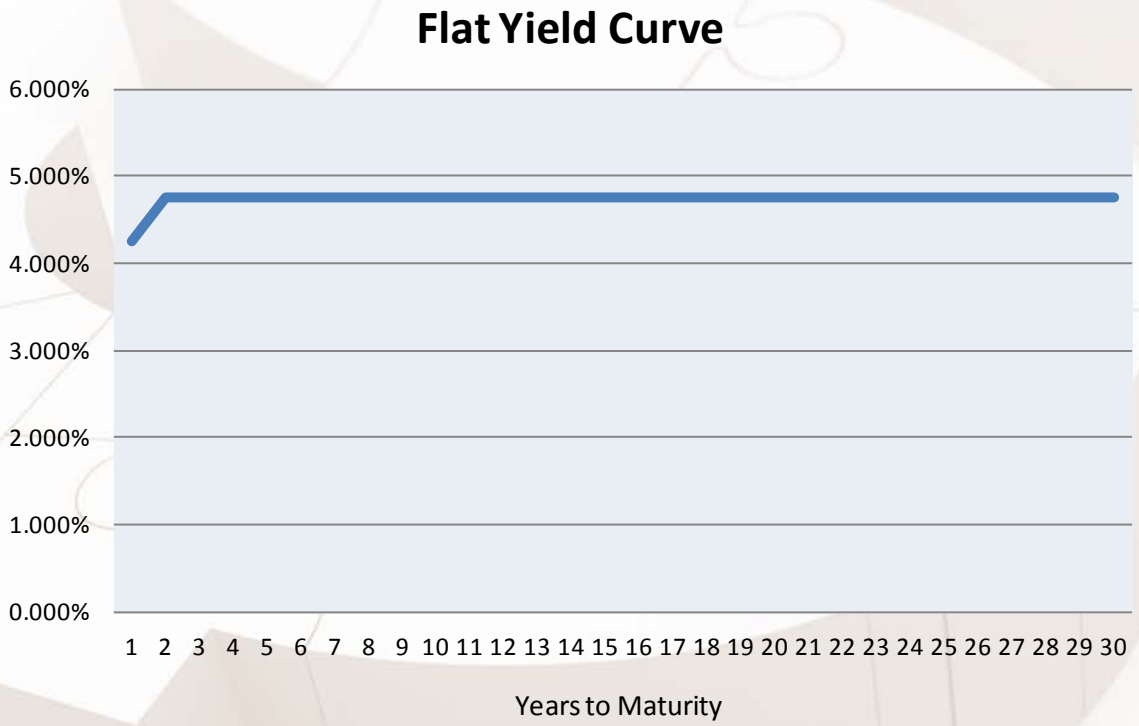
There are a number of theories that attempt to explain why rates tend to be higher on longer maturities than on shorter ones (an "upwardly sloping" yield curve). Two of these are:

- Liquidity Premium: All things being equal, investors face greater uncertainty holding longer term bonds than shorter term bonds. Many more unpredictable things are likely to happen in the next ten years than in the next two and investors demand higher rates to compensate them for this risk.
- Market Expectations: This theory says that investors, in general, expect interest rates in the future will be higher than they are today. They therefore demand an interest rate on longer maturities that would give them the same expected total return had they alternatively invested in a shorter maturity and then reinvested at the higher future rate that they are expecting.

The Yield Curve

(continued)

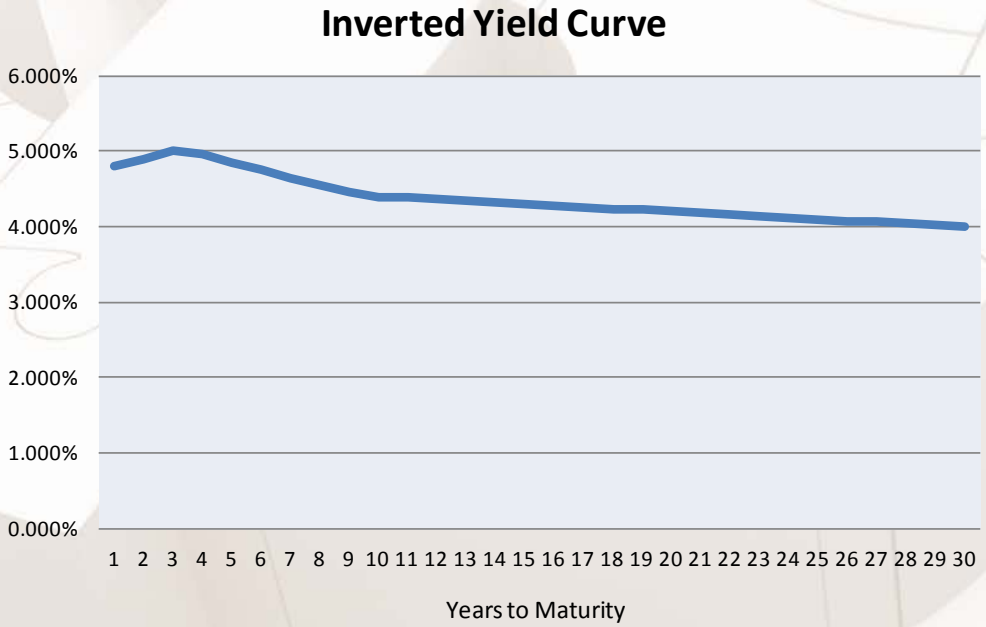
- The yield curve can, and does, change over time as economic conditions change. It can flatten so that short term rates and long term rates are the same.



The Yield Curve

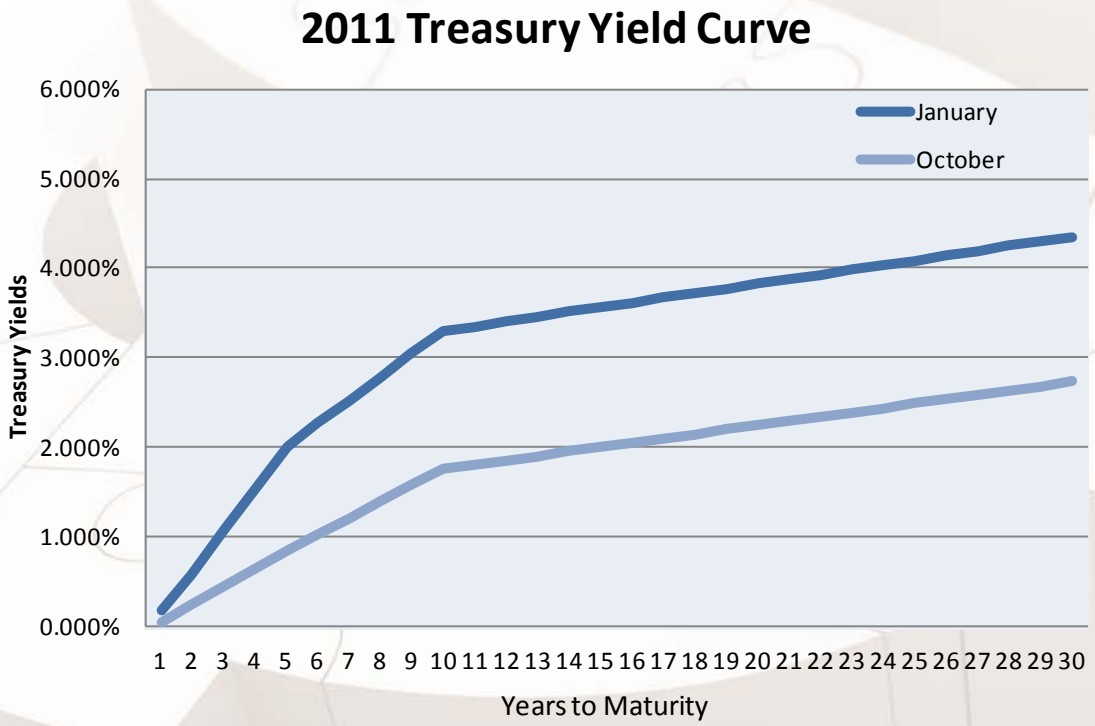
(continued)

The yield curve can even become inverted where long term rates are lower than short term rates. An inverted yield curve is considered to be a negative economic indicator. Inversion indicates that investors believe interest rates will be lower in the future and that often means that economic activity is at least likely to slow and may even mean that a recession is coming. A tightening of short term interest rates by the Federal Reserve in order to slow the economy or to reign in inflation can lead to an inverted yield curve.



The Yield Curve
(continued)

- So far in 2011 the US Treasury yield curve has flattened considerably:



2. Bond Pricing

Par, Discount and Premium Bonds

- The **yield** on a bond is a measure of the return, or earnings, realized by holding the bond. The yield is determined by the market, the overall supply and demand for the bond. As we saw in the previous section the yield will usually vary depending on the final maturity of the bond.
- The **price** to be paid for a bond also varies based on the **coupon** that is attached to the bond. The coupon is the actual rate of interest that is paid to the bond investor. A 5% coupon on a $100 bond pays $5/year.
- **Par Bond**
- A **par bond** is a bond that can be purchased at 100% of its underlying value. A bond that has a principal amount of $100 at maturity will sell for $100 right now if its coupon is the same as the current market yield.

  Of course, market yields change all the time so a bond that is trading at par today may not be trading at par tomorrow.

Bond Pricing

Par, Discount and Premium Bonds

- One of the more confusing aspects of bonds is the relationship between price and yield:
  - If you own a bond and the **market yield rises** the **price of the bond falls**.
  - If you own a bond and the **market yield falls** the **price of the bond rises**.

This is a table that shows what happens to the price of a bond with a 5% coupon as market yields change

Years to Maturity	Coupon	Market Yield	Price	
5	5.00%	4.00%	104.491	PREMIUM
5	5.00%	4.25%	103.347	
5	5.00%	4.50%	102.217	
5	5.00%	4.75%	101.101	
5	5.00%	5.00%	100.000	PAR
5	5.00%	5.25%	98.913	DISCOUNT
5	5.00%	5.50%	97.840	
5	5.00%	5.75%	96.781	
5	5.00%	6.00%	95.735	

Bond Pricing
(continued)

Discount Bond

If market yields rise to 6%, the bond with a 5% coupon is now paying a rate that is less valuable in the market. The price of this less valuable bond will therefore fall and it will trade at a **discount** to par, say 95% of its original par value if it is five years until the bond matures. By trading at a discount this bond will have an overall yield of 6% (the rate demanded by the market): 5% from the coupon and an additional 1%/year from the discount as the price of the bond will slowly approach 100% of its par value at maturity.

One reason an investor might prefer a discount bond: Call protection. In a market environment where interest rates are falling it will take longer for a bond with a lower coupon to be profitably called by the issuer of the bond when compared to a higher coupon bond.

Bond Pricing
(continued)

Premium Bond

If market yields fall to 4%, the bond with a 5% coupon is now paying a rate the is more valuable in the market. The price of this more valuable bond will therefore rise and it will trade at a **premium** to par, say 105% of its original par value if it is five years until the bond matures. By trading at a premium this bond will have an overall yield of 4% (the rate demanded by the market): 5% from the coupon and -1%/year from the premium as the price will slowly depreciate to 100% of its par value at maturity.

One reason an investor might prefer a premium bond: Less volatility. If interest rates rise after you purchase the bond the value of the bond will fall…but not as quickly as a lower coupon bond would. This is because more of the value of the bond is in the stream of early coupon payments to be received in the near future rather than the final maturity amount. The present value of payments to be received earlier (in this case, the coupon payments) doesn't change as much as payments received later (like the principal amount at maturity) for a given change in interest rates.

3. Other Types of Bonds

Capital Appreciation and Zero Coupon Bonds

- Most fixed rate bonds pay semi-annual interest at the same rate over their entire life…the coupon rate on the bond. This type of bond is called a **"Current Interest Bond."**
- Some bonds don't pay any interest until maturity. Since they don't have any coupon payments investors buy them at a discount to their par value. For instance, in a market where one year bonds yield 5.00%, a bond that matures in one year but has no coupon payments would sell for 95.238% of its par value. (5% of 95.238 is 4.762 so that the investor will be earning 5.00% on the $95.238 he or she spends to buy the bond, getting $100 back a year from now).
- Bonds that only pay interest at maturity are called **"Capital Appreciation"** or **"Zero Coupon"** bonds. Mathematically, they are identical. The difference is that the initial purchase price is considered to be the principal amount for a Capital Appreciation Bond (or "CAB") while the value at maturity is considered to be the principal amount for a Zero Coupon Bond. In either case the value of the bond increases over time until maturity.

Other Types of Bonds

Capital Appreciation and Zero Coupon Bonds

The table below shows two different debt service payment streams for a $10 million bond issued with a 5% yield. The first is issued as a current interest bond at par (so the coupon is 5.00%), the second as a zero coupon bond or CAB.

Debt Service Payments on a …

	Current Interest Bond			Zero Coupon Bond or CAB	Accreted Value of Zero Coupon Bond or CAB
Year	Principal	Interest	Total		
0					10,000,000
1		500,000	500,000		10,500,000
2		500,000	500,000		11,025,000
3		500,000	500,000		11,576,250
4		500,000	500,000		12,155,063
5		500,000	500,000		12,762,816
6		500,000	500,000		13,400,956
7		500,000	500,000		14,071,004
8		500,000	500,000		14,774,554
9		500,000	500,000		15,513,282
10	10,000,000	500,000	10,500,000	16,288,946	16,288,946
	10,000,000	5,000,000	15,000,000	16,288,946	

4. Bond Redemptions and Accrued Interest

Serial, Term Bonds and Sinking Fund Payments

Overview

When bonds are initially sold by an issuer they are sold with a coupon rate and are given a maturity date…the date when the bonds are scheduled to be paid off by the issuer of the bonds. There are two types of structures that these bonds can take.

Serial Bonds

These bond are straightforward. They are structured as you would expect a bond to be structured: a known coupon rate and a specific maturity date. When you buy this bond you know what income you expect to receive until the maturity date when your original principal amount will be returned to you (assuming all goes well!).

Bond Redemptions and Accrued Interest (continued)

Term Bonds

These bonds don't pay back the entire principal at maturity. Instead, a portion of the principal is paid *before* the final maturity. These payments are called "sinking fund payments." Bonds are sold as term bonds usually for convenience…it is easier to sell a term bond of $10,000,000 with a final maturity date of 1/1/2015, but with sinking fund payments from 2012 through 2014, than it is to sell four separate bonds with $2,500,000 maturities from 2012 through 2015. Large investors (like bond mutual funds) also like buying larger maturities as they are more liquid and easier to trade. Here is what the structure looks like for this term bond:

Payment Date	Principal Amount	
1/1/2012	2,500,000	Sinking Fund Payment
1/1/2013	2,500,000	Sinking Fund Payment
1/1/2014	2,500,000	Sinking Fund Payment
1/1/2015	2,500,000	Final Maturity
	10,000,000	

Bond Redemptions and Accrued Interest (continued)

Term Bonds (continued)

The owner of a term bond is not sure exactly when the bond, or a portion of the bond, is going to be redeemed. The bonds that are picked to be redeemed on a sinking fund payment date are usually chosen randomly and the owners are informed that a portion of their bond has been picked to be redeemed. In the previous example a holder of a $1,000,000 bond could have all, or some, of the bond "called away" on January 1st of 2012, 2013, or 2014 with any remaining principal repaid on January 1, 2015 (its final maturity date).

Bond Redemptions and Accrued Interest (continued)

Bond Redemptions and Accrued Interest

Bonds are either redeemed at maturity, on a regularly scheduled sinking fund payment date (for term bonds), or on an optional call date. Money for the payment is transferred from the issuer of the bonds to a "paying agent" who handles getting the money to the bond holder in exchange for the bond which is then canceled. At the time the bond is redeemed interest on the bond is also due. This is the **accrued interest** that has accumulated since the last interest payment date on the bonds. Every day that passes interest "accrues" on a bond and is an obligation that the issuer of the bond has to the owner of that bond.

Bond Humor

Q: What is the difference between a bond and a bond trader?

A: A bond matures.

5. Spreadsheet Formulas

- Spreadsheet programs have built in functions for calculating bond prices, bond yields, and other financial values. In Excel these include:

=Price()	Calculates the price of a bond
=Yield()	Calculates the yield on a bond
=IRR()	Calculates the Internal Rate of Return on a bond issue
=PMT()	Calculates the annual payments required in order to pay off a bond issue

The next few pages contain descriptions of these functions and how they are used.

=Price()

Calculates the price of a bond (given the yield)

The price of a bond, at its most basic, is simply the present value of all the future payments to be received on that bond. The discount rate used to calculate that present value is the yield on the bond. The arguments that the function takes (separated by commas) are:

=Price(delivery date, maturity date, coupon, yield, value at maturity, number of coupon payments/year, day count basis)

delivery date	The date the bond is paid for (also called the "settlement date").
maturity date	The date the final principal payment is received on the bond.
coupon	The periodic interest received on the bonds (5% coupon on a $100 bonds pays $5/year).
yield	The rate used to calculate the present value of the future payments to be received
value at maturity	The value of the bond at maturity (per $100 face value). For municipal bonds you would usually just enter 100.
number of coupon payments/year	For municipal (and the majority of other) bonds you receive semi-annual payments so you enter 2.
day count basis	Different kinds of bonds have different methods for counting the days between dates, whether it is the actual number of days or a simplified way assuming 30 day months. Municipal bonds use the 30 day month/360 day year convention so you enter 0 in this field (although you can leave it blank in which case Excel assumes it is 0).

=Price()

Calculates the price of a bond (given the yield)

- **Example Problem:**

  You are being asked if you are interested in buying a municipal bond. You are not told what the price of the bond is but you are told that the yield is 4.50%, the coupon is 5.00%, that the maturity date is January 1st, 2030 and the delivery date is September 19th, 2011. How can you use this information to calculate the price of the bond?

=Price()
(continued)

- **Example Problem:**

	A	B	C	D	E	F
1	=PRICE()					
2						
3	Formula:	=Price(B7,B8,B9,B10,B11,B12,B13)				
4						
5	Result:	106.179				
6						
7	Delivery (Settlement) Date:	9/19/2011				
8	Maturity Date:	1/1/2030				
9	Coupon:	5.000%				
10	Yield:	4.500%				
11	Value at Maturity:	100				
12	# of Coupon Payments/Year:	2				
13	Day Count Basis:	0				
14						

=Yield()

Calculates the yield on a bond (given the price)

The yield on a bond is the earnings rate you receive if you purchase it. Technically, it is an Internal Rate of Return (IRR): the rate that, when used to present value all the future payments to be received from the bond equals its purchase price. The arguments that the function takes (separated by commas) are:

=Yield (delivery date, maturity date, coupon, price, value at maturity, number of coupon payments/year, day count basis)

delivery date	The date the bond is paid for (also called the "settlement date").
maturity date	The date the final principal payment is received on the bond.
coupon	The periodic interest received on the bonds (5% coupon on a $100 bonds pays $5/year).
price	The price paid for the bond.
value at maturity	The value of the bond at maturity (per $100 face value). For municipal bonds you would usually just enter 100.
number of coupon payments/year	For municipal (and the majority of other) bonds you receive semi-annual payments so you enter 2.
day count basis	Different kinds of bonds have different methods for counting the days between dates, whether it is the actual number of days or a simplified way assuming 30 day months. Municipal bonds use the 30 day month/360 day year convention so you enter 0 in this field (although you can leave it blank in which case Excel assumes it is 0).

=Yield()

Calculates the yield on a bond (given the price)

- **Example Problem:**

This time when you are offered the municipal bond you are told the price but not the yield. You are told that the price of the bond is 106.179, the coupon is 5.00%, the maturity date is January 1st, 2030, and the delivery date is September 19th, 2011. How can you use this information to calculate the yield on the bond?

=Yield()
(continued)

- **Example Problem:**

	A	B	C	D	E	F
1	=YIELD()					
2						
3	Formula:	=Yield(B7,B8,B9,B10,B11,B12,B13)				
4						
5	Result:	4.500%				
6						
7	Delivery (Settlement) Date:	9/19/2011				
8	Maturity Date:	1/1/2030				
9	Coupon:	5.000%				
10	Price:	106.179				
11	Value at Maturity:	100				
12	# of Coupon Payments/Year:	2				
13	Day Count Basis:	0				
14						

=IRR()

Calculates the Internal Rate of Return/True Interest Cost of a bond issue

When a bond issue is sold it is made up of a number of separate bond maturities that are sold at one time. Just as we saw that the yield on a bond is the interest rate that equates the present value of all future payments to be received on a bond to its price, the IRR equates the present value of the payments to be made on ALL the bond maturities issued to the NET price received from the sale of all the separate bonds (net of the costs associated with issuing the bonds). In essence, the IRR is the overall interest rate on the bond issue and in the municipal bond world this is called the True Interest Cost (TIC).

=IRR(values, guess)

values	A series of amounts entered in the separate cells of the spreadsheet. There must be at least one negative and one positive value.
guess	A yield that gives the IRR formula a place to start its calculations. If omitted Excel assumes 10% (0.10).

=IRR()
(continued)

Example Problem:

As an example, we will assume a simple bond issue with just two separate maturities. In the spreadsheet, we enter the cash flows to be paid on each bond sold, add up those payments by date paid, add up the total amount received from the sale of the bonds and then calculate the IRR.

Our simplified bond issue is sold on January 1st, 2012. The first bond matures on 1/1/2014, has a coupon of 4.00%, a yield of 4.25% (and therefore a price of 99.525), and a par value of $500,000. The second maturity is on 1/1/2015, has a coupon of 4.75%, a yield of 4.50% (and therefore a price of 100.694), and a par value of $750,000.

The costs associated with issuing the bonds (payments to lawyers, underwriters, and financial advisors) is 1.00% of the total par value. With this information we can calculate the overall True Interest Cost (IRR) of the bond issue.

=IRR() (continued)

- **Example Problem:**

	A	B	C	D	E	F	G	H	I	J	K
1	**=IRR()**										
2											
3	Formula:	=IRR(C37:C43,0.045)*2			(the *2 at the end is added to convert from						
4					a semi-annual to an annual calculation)						
5	Result:	4.836%									
6											
7	**Bond Details…**										
8											
9		1/1/2012	:Settlement Date								
10								Less			
11			Maturity				Total	Costs of Issuance	Net		
12		Amount	Date	Coupon	Yield	Price	Value	(1.00%)	Price		
13											
14	1st Bond Maturity:	500,000	1/1/2014	4.000%	4.250%	99.525	497,627	-5,000	492,627		
15	2nd Bond Maturity:	750,000	1/1/2015	4.750%	4.500%	100.694	755,207	-7,500	747,707		
16											
17		1,250,000					1,252,835	-12,500	1,240,335		
18											
19	**Debt Service Payments…**										
20											Total
21		Payment	1st Bond Maturity				1st Bond Maturity				Cash
22		Date	Principal	Coupon	Interest	Total	Principal	Coupon	Interest	Total	Flow
23											
24		7/1/2012			10,000	10,000			17,813	17,813	27,813
25		1/1/2013			10,000	10,000			17,813	17,813	27,813
26		7/1/2013			10,000	10,000			17,813	17,813	27,813
27		1/1/2014	500,000	4.000%	10,000	510,000			17,813	17,813	527,813
28		7/1/2014			0	0			17,813	17,813	17,813
29		1/1/2015			0	0	750,000	4.750%	17,813	767,813	767,813
30											
31			500,000		40,000	540,000	750,000		106,875	856,875	1,396,875
32											
33	**IRR Calculation…**										
34			Overall								
35		Cash Flow	Cash								
36		Date	Flow								
37		1/1/2012	-1,240,335								
38		7/1/2012	27,813								
39		1/1/2013	27,813								
40		7/1/2013	27,813								
41		1/1/2014	527,813								
42		7/1/2014	17,813								
43		1/1/2015	767,813								

=PMT()

Calculates the annual payments required to pay off a bond issue

The payment function is a handy formula for estimating the annual payments required to pay off a bond issue. It is not as accurate as a complete debt service schedule which incorporates all of the details of the bonds that make up an entire bond issue but is does provide a rather quick, simple and accurate estimate of the annual payments required to pay off a bond issue of a given size with equal annual payments.

=PMT(interest rate, years to maturity, bond issue size, 0, 0)

interest rate	An estimate of the average interest rate of the bond issue.
years to maturity	How long the bond issue will be outstanding.
bond issue size	The total amount of bonds to be issued.

=PMT()

Calculates the annual payments required to pay off a bond issue

Example Problem:

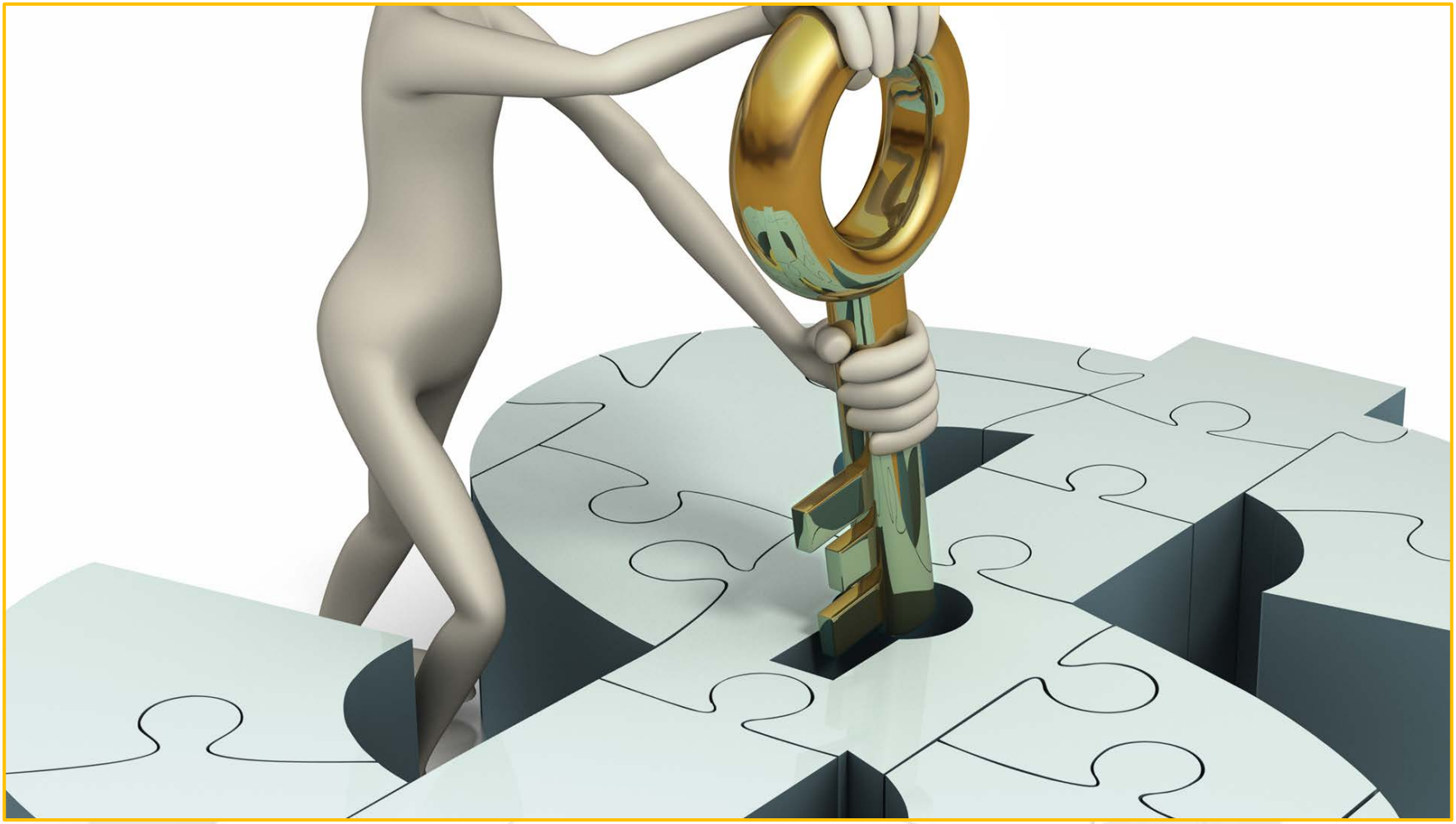
Smallville is considering a proposal to build a new library that costs $10 million. It wants to quickly make a general estimate of what the annual debt service would be on a bond issued to build the library.

We assume the bonds will have a 30 year final maturity, the average bond rate will be 5.00% and the annual payments on the bonds will be level over the life of the bonds.

=PMT()
(continued)

- **Example Problem:**

	A	B	C	D
1	=PMT()			
2				
3	Formula:	=-Pmt(B7,B8,B9,0,0)		
4				
5	Result:	650,514		
6				
7	Interest Rate:	5.000%		
8	Years to Maturity:	30		
9	Bond Issue Size:	10,000,000		



Conclusion

Please contact us if you have any other questions:

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