## To Instructors --

## Homework Review of the Math Needed for Chemistry

For instructors in college General Chemistry, GOB, AP Chemistry, and all courses aimed at preparing students for college chemistry courses, this packet provides over 60 pages of homework assignments (with quizzes) that review the math students need in chemistry.
In General, GOB, Engineering, and AP Chemistry, these assignments save class time by flipping a part of the review of pre-requisite math to study time.
In AP, the assignment may be used as part of a "summer packet."
In HS Honors or college Preparatory Chemistry, the lessons can be used as homework to reinforce math and metric topics when they are covered in lecture.
The lessons are available both online and as paper copies for students who have limited access to computers and/or the internet.

## Quick Preview

To see all pages of student assignments in this packet, see
https://ChemReview.Net/MathOfChemPages.PDF

## Summary of Activities

The packet contains 9 topic sections:

## 1. Why Review Math?

For instructors, topic 1 is a two-page summary of why cognitive experts say a brief review of the math of chemistry, both with and without a calculator, will improve student success in solving calculations and conceptual understanding.
2. Assessment

Topic 2 is a 15 -minute quiz on fundamentals in arithmetic, decimal equivalents, exponential calculations, simplifying fractions, solving algebraic equations, and logarithms. Results can be used to identify math gaps and plan review.
3. Math and Metric Review

Topic 3 contains 9 self-study lessons on mental arithmetic, calculations involving exponential notation, solving exponential notation with a calculator, estimating answers to calculations without a calculator, metric fundamentals, making chemistry flashcards, and calculations that mix numbers, exponential terms, and units.

## 4. Additional Mental Math Review

In Topic 4 are 8 pages of mental math worksheets with rules, strategies and practice in converting fractions to decimal equivalents, simplifying exponentials and fractions, and fractions, estimates, and exponentials.

## 5. Improving Dimensional Analysis

To assist students in solving dimensional analysis calculations with more success, three PDF packets include 42 pages of student in-class and homework assignments. Strategies include how, by adding a few steps before multiplying conversion factors, solving problems is simplified.

## 6. Acid-Base Math Review

For later in the course, these 2 pages provide review of rules and practice in the math of solving for $\left[\mathrm{H}^{+}\right]$in strong acid and base solutions - both with a calculator and by mental math.

## 7. Prep for Electrochemistry

The first half of this 5 page assignment may be used during the introduction to redox reactions. All 5 pages may be used as homework in Gen and AP chem to prepare students in the initial vocabulary and calculations in electrochemistry.

## 8. Other Math Review Resources.

A listing of additional available math review lessons, by topic.

## 9. How the Student Brain Learns Chemistry

For instructors, three pages summarizing cognitive research on how we can help students prepare for careers in the sciences.

Editable quizzes on all of the student lessons are available to instructors - included with the topic activities below.

## Updates

Please download on occasion an updated (and sometimes corrected...) version of this document, which may include new material, by clicking on https://www.ChemReview.Net/MathOfChem.pdf

Additional ideas for assignments in Gen/GOB/AP/Prep/Honors chemistry are posted at www.ChemReview.Net/blog .

Feedback, corrections, and suggestions on this packet are most appreciated. Hope this helps!
-- Eric Nelson, retired instructor, EANelson (at) ChemReview.Net, @RNelson696
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## Detail: Math Review Topics

## Topic 1

## Why Review Math - and Why Mental Math?

Instructors may ask why chemistry should include math review, and why the review should include mental math. What follows are the personal views and research of the packet author.

## Why Review Math?

Calculations are central in the sciences and engineering. Chemistry is where students first encounter math applied to measurements (numbers with units attached). Unfortunately, over the past 20 years, state K-12 math standards in most states have de-emphasized teaching the topics in math needed for scientific calculations (see References 1 and 2 below).

This has put chemistry instructors in a difficult position. We want to focus on chemistry, but given the states' de-emphasis of computation, unless we review the math topics needed for chemistry, many students will struggle to solve calculations they are expected to solve in science courses and careers.

The intention of this packet is to help students and instructors by reviewing math needed for chemistry as homework, to limit the impact of math review on class time.

## Why Review Mental Math?

In the lessons in this packet, math is reviewed both with and without a calculator. Why should students be asked to know mental math?

Scientists who study how the brain solves problems cite many reasons, including:

- For students to learn concepts with numeric components, students learn more quickly if examples and sample problems are contrived to have very simple numbers that can solved by mental math. The reason involves freeing slots in working memory (which is where the brain solves problems) to process conceptual linkages.
- A key finding of recent cognitive research: Working memory has essentially infinite room for facts and procedures that quickly be recalled from long-term memory, but very limited ability to hold and process facts and procedures not well-memorized.
- Working memory's limits mean that if math fundamentals can be recalled "with automaticity" (quickly and accurately), working memory has more space to note concepts and contexts, which deepens understanding.

Cognitive scientist Daniel Willingham suggests that instructors
"[E]xplain to students that automaticity in facts is important because it frees their minds to think about concepts." ${ }^{5}$

- In addition, if fundamentals are not well-memorized, working memory tends to overload during problem solving, and confusion tends to result.

There are also practical reasons to ask students to practice mental math:

- If students can solve a complex fraction simplification or exponential notation calculation that has simple numbers without a calculator, they are better able to remember the sequence of calculator buttons to press when the numbers and/or operations are complex, and
- If students cannot estimate to check an answer calculated by technology, they are less likely to pass testing required for many scientific careers. As one example, the Medical College Admissions Test (MCAT), which 60,000 undergraduates take each year, has several sections of calculations with simple numbers - and the test is "no calculator."

In general in health care and engineering, to be able to estimate to check a calculator answer is a frequently evaluated skill.

- Chemistry is a quantitative science. Math is an essential foundation for chemistry. If students cannot estimate an answer to a chemistry calculation, do they know chemistry?

For mental math to help students, after years of calculator overuse, many in the current generation need to "refresh their memory" of math fact fundamentals.

Studies have shown that brief review of pre-requisite math and mental math as a part of chemistry measurably improves student achievement (see Reference 3 and 4).

## More on the Science of Learning

Knowing the unexpected ways that science has found the brain works, we can design better lessons. For more on the science of learning, see Topic 8 - How the Student Brain Learns Chemistry -in this packet.

## References:

1. See Addressing Math Deficits With Cognitive Science (2017) at https://confchem.ccce.divched.org/content/2017fallconfchemp8 with a summary at: J. Chem. Educ., 2018, 95 (8), pp 1440-1442
2. See "Cognitive Science and the Common Core Math Standards" at http://nonpartisaneducation.org/Review/Articles/v13n3.pdf
3. Craig, P. R. Building Student Confidence with Chemistry Computation. J. Chem. Educ. 2018, 95 (8), 1434-1435; and at https://confchem.ccce.divched.org/content/2017fallconfchemp5
4. Penn, L. S. Estimation - An Empowering Skill for Students in Chemistry and Chemical Engineering. J. Chem. Educ. 2018, 95 (8), 1426-1427; and at https://confchem.ccce.divched.org/2017fallconfchemp1
5. Willingham, D. T. Is It True That Some People Just Can't Do Math? Am. Educ. 2009, 33 (4), 1419. At http://www.aft.org/pdfs/americaneducator/winter2009/willingham.pdf

## Topic 2

## Math Assessment

## To the Instructor

Topic 2 is a 15 -minute quiz on math needed for chemistry. Given at the start of the course, the quiz will identify, for sections and/or individual students, areas of math in which your particular group of students would benefit from review.

This quiz has been found to identify who will do well in college first-year chemistry (see discussion below). It will identify topics where your students have gaps in preparation that, if you fill them in with some math review, in class or by homework, will result in their doing better in college chemistry. They need to do well in college chem to succeed in STEM majors.

The quiz is posted at https://www.ChemReview.Net/MathAssessment.PDF . A version in MSWord you can edit is at https://www.ChemReview.Net/MathAssessment.docx .

## Calculator or no Calculator?

The quiz is labeled "no calculator." You can instead allow a calculator.
BUT -- what you want to know is what this quiz measures: Can they do problems with simple arithmetic without the calculator? They need to know the math operations with simple numbers because

- If they can't use rules of math to solve simple-number problems, when they need to know different types of calculator operations with a mix of numbers and exponents, they will have real difficulty remembering the order in which to press the buttons,
- When the calculations get complex, they are more likely to get the answer number right if they do the numbers on the calculator but exponential terms separately, by mental math,
- In the upper level classes you are preparing them for, they will be tested on whether they can estimate an answer without a calculator, as a check on their calculator use, and
- On future exams like the MCAT (Medical College Admissions Test) which many in chem hope to take and do well on, there are many calculations, and calculators are not allowed.


## Results

Does this short quiz predict what topics they need math review on to do well in chemistry? Yes.
This quiz was given by faculty in sections of college general chemistry at multiple four-year colleges. In peer-reviewed papers, those faculty reported that when given "no calculator," the quiz score for each student was highly correlated to subsequent grades in General Chemistry.

When the quiz was given "calculator allowed," higher quiz scores were correlated with lower grades in the course.

These findings are consistent with cognitive research on the importance of "automaticity in fundamentals."

To read the papers, see

1. "MUST-Know Pilot-Math Preparation Study from Texas" at https://confchem.ccce.divched.org/sites/confchem.ccce.divched.org/files/2017FallConfChemP2_0.pdf
2. "Impact of basic arithmetic skills on success in first-semester general chemistry"
in Chemistry Education Research and Practice, 2020, 21, 51-61 at https://pubs.rsc.org/en/content/articlehtml/2020/rp/c9rp00077a
3. "Impact of arithmetic automaticity on students' success in second-semester general chemistry" in Chemistry Education Research and Practice, 2020, 21, 1028-1041 at https://pubs.rsc.org/en/content/articlehtml/2020/rp/dOrp00006j

## Follow-Up

Most of the quiz questions have two components: Knowledge of the procedure steps and ability to do simple mental arithmetic. You may want to analyze:

- On question one: What percentage of your students could correctly multiply two digits times two digits without a calculator?
- On questions 2-8, what was the percentage correct for each of these questions? For problems where a number of students had difficulty, did the difficulty appear to be with the rules of the math operation, or the mental arithmetic, or both?

If mental arithmetic (recall of math facts) is a problem, there are two possibilities. One is that students once knew their math facts well, but have forgotten them due to calculator use. Science tells us that in this case of forgetting, the information is still stored in memory, but the neural "wiring" to reach it needs to be re-grown by practice. Forgotten memories will "refresh" (re-wire) relatively quickly with practice in recall.
The second possibility is that students were never required to "memorize their times tables." In some states prior to 2014, state K-12 math standards required teachers to have students use calculators to do arithmetic starting for third grade state testing, rather than memorize facts. Since 2014, Common Core-type standards have restored a partial emphasis on computational fluency, but this means in many states, current students at some point went from one set of standards to another in the middle of their K-8 schooling. Those students may need substantial practice to gain the quick, automatic recall of math facts that science says they need during scientific problem solving. (On state math standards, see references 1 and 2 for Topic 1 of this packet)
If the quiz and subsequent exercises indicate that some students need help with mental arithmetic, the activities in this packet provide options for additional practice.

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## Topic 3

## Math and Metric Review

Posted online for student use are 50 pages of self-study tutorials ( 9 lessons in 2 chapters) that help students review (or learn) math and metric fundamentals that are a foundation for chemistry. The tutorials may be viewed and downloaded at:
https://www.ChemReview.Net/ExpoMetric.PDF
The tutorial lessons, individually or in groups, can be assigned as homework at the start of General or AP Chemistry, or as part of an AP summer packet. In other courses. the homework can be assigned at any point when students are asked to solve calculations involving

- exponential and scientific notation,
- estimation,
- metric calculations,
- mental math, and
- calculations that mix numbers, exponentials, and units.

Suggestions for a format to assign the tutorials are included in the PDF at the link above.

## Assignment Length

In Gen Chem/AP, these topics should be review. Students should be able to complete the two chapters in the assignment in a homework week.
For Prep Chem or HS Honors students, lessons can be assigned to reinforce lecture at a more gradual pace.

## Quizzes

To encourage timely homework completion, quizzes are provided on the tutorial content. For access, click www.ChemReview.Net/QuizRequest.html
Quizzes are available for both Chapter 1 only and Chapters 1 and 2 combined.
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## Topic 4

## Additional Mental Math Review

## To the Instructor

Topic 4 is a sequence of five 1-2-page lessons with rules, strategies and practice on

- The standard algorithms for multi-digit multiplication and division,
- Simplifying fractions,
- Converting fractions to decimal equivalents,
- Simplifying exponential terms in fractions, and
- Estimating answers using mental math.

Depending on the class level, you may want to assign selected lessons.
The practice sheets are printed below with suggestions for the instructor with each. The practice sheets for students (without the notes to instructors, are in an 8-page PDF at www.ChemReview.Net/MentalMathPractice.pdf
A 5 minute quiz covering the assignments is included in this packet after the practice sheets.
In General, GOB, or AP Chemistry, you may want to assign selected pages or all 8 pages as one homework assignment.
In college Prep Chem or high school introductory courses, the lessons can be assigned at a gradual pace as reinforcement when teaching the math of calculations.
The problems can be completed "from the screen" without paper copies, but students would need to copy most questions. IF you are able to copy and hand out the pages you assign, the assignment will be easier for students to complete.
A suggested assignment format would be:

## Online Homework:

- Download the 8-page PDF at www.ChemReview.Net/MentalMathPractice.pdf and complete the problems on pages $X$ to $X$.
- Be ready for a quiz - which will be similar to the questions in the lessons - on (date).

Or - If handed out:

- Complete the pages in this packet. Be ready for a quiz - which will be similar to the questions in the lessons - on (date).

Detail for each of the 5 assignments is below.

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## Practice 4A -

## Multi-digit Multiplication and Division

## To the Instructor

Practice 4A and 4B are similar to the coverage of these topics in Lesson 1.2 of the homework tutorials.

In Gen/GOB/AP chem, it may be Practice 4A and 4B can be skipped. What is in Lesson 1.2 will likely suffice.

In HS Honors/Prep Chem, students may benefit from both lecture coverage and homework reinforcement. You may want to use 4A and 4B in class to introduce the topics and provide feedback as students work the problems a few at a time. Some problems can be left to complete as homework, or as a warmup review at the start of the next class.

Practice 4A below reviews the standard algorithms for multiplication and for simple long division. This practice is recommended if mental arithmetic gaps were identified in the fluency quiz.

Using the multiplication "standard algorithm" requires practicing recall of multiplication and addition. The division algorithm requires practicing multiplication, division and subtraction fact recall.

The benefits of requiring students to occasionally perform these types of calculations include

- They are a way to practice the overlearning of math fundamentals science says is essential for student understanding of quantitative concepts;
- Problems like these can be put on the board at the start of a class throughout the term for productive student "refreshing of memory" while you tend to administrative necessities.

The explanation of the two algorithms is the same as that in Lesson 1.2 in the homework tutorials, but the problems have different numbers. This sheet will supply additional practice.

The Common Core and current math standards in most states require teaching both of these standard algorithms, but until 2014 in many states, students were not asked to learn long division. Even now, this may impact your students.

To start, you may want to assign one problem each of multiplication and division, then glance at a few papers to get an idea of what background your students have in these procedures.

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## Practice 4A: Multi-digit Multiplication and Division

To understand the numeric relationships in science, you need to be able to do arithmetic quickly "in your head." One way to keep mental math skills sharp is to occasionally solve simple multi-digit multiplication and division without a calculator. To do this without overwhelming "working memory" (where you think), you need to apply a "standard algorithm."

1. The multiplication algorithm usually taught in the US includes these steps. For $76 \times 42=$ ?

Step 1: ${ }^{\mathbf{1} 76}$
$\times \frac{42}{152}$
Steps 2 and 3: ${ }^{\mathbf{2} 76}$
$\times 42$
152
$\frac{304}{\mathbf{3 1 9 2}} \leftarrow$ (putting a $\mathbf{0}$ after the 4 is an option)

Without a calculator, multiply these:
a. $\quad 95$
$\times 16$
b.
84
$\begin{array}{r}83 \\ \hline\end{array}$
c. $\quad 39$
$\times \underline{62}$
d. 57
$\times \underline{48}$
2. The "long division" algorithm usually taught in US education includes these steps:

For $2048 \div 8=$


Without a calculator, try these "evenly divisible" cases (the answer will be a multi-digit whole number -- no decimals or remainders).
a.
6)516
b. 9$) 2187$
c. 8$) 5560$

Answers: 1a. 1520 1b. 6132 1c. 2418 1d. 2736 2a. 86 2b. 243 2c. 695 (Permission by ChemReview.Net granted for use. See www.ChemReview.Net/blog )

## Practice 4B

## Additional Mental Arithmetic Practice

## To the Instructor

Practice 4B covers numeric simplification operations that are useful both during simple calculations and when estimating to check a calculator answer.
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## Practice 4B: Using Mental Arithmetic to Simplify Fractions

WithOUT a calculator, use your mental arithmetic skills to reduce these fractions to a one or two digit whole number. Show your work on this paper. Check your answers at the bottom.
Each problem may have multiple ways to cancel and solve. Any way to a correct answer works.

1. Example: $\frac{8 \times 15 \times 3}{72}=\frac{8 \times 15 \times 3}{72}=\frac{8 \times 15 \times \not 2}{72}=\frac{15}{3}=5$
(When doing multiple cancellations, you may want to re-write at some point, as in the "next to last" step above, to keep your progress clear.)

Hint: It usually helps to try to reduce the larger numbers on both the top and bottom first.
2. $\frac{49 \times 2 \times 8}{4 \times 7}=$
3. $\frac{42 \times 20 \times 5}{2 \times 6 \times 7}=$
4. $\frac{63 \times 4 \times 42}{6 \times 7 \times 9}=$
5. $\frac{48 \times 6 \times 11}{4 \times 18}=$
6. $\frac{35 \times 2 \times 8}{40 \times 14}=$
12. Double these: $\begin{array}{lllllllll}42 & 17 & 36 & 45 & 16 & 24 & 32 & 48\end{array}$
13. Cut these values in half: $\begin{array}{llllllll}44 & 98 & 86 & 38 & 46 & 78 & 56\end{array}$

| Answers: | 2.28 | 3. | 50 | 4.28 | 5.44 | 6.1 | 7.24 | 8.7 | 9.21 | 10.6 |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 11. 10 | $12 . ~$ | 24 | 34 | 72 | 90 | 32 | 48 | 64 | 96 |  | 13. 22 | 49 | 43 | 19 |
| 23 | 39 | 28 |  |  |  |  |  |  |  |  |  |  |  |  |

Answers:
12. 84

347290
32486496
13. $22434349 \quad 23 \quad 39 \quad 28$

## Practice 4C: - Converting Fractions to Decimal Equivalents

## To the Instructor

The 2-pages of Practice 4C are printed below. It is suggested that you take a look at those two pages now, then return here.
In Topic 3, tutorial Lesson 1.2, students are asked to commit to memory the decimal equivalents of eight commonly encountered fractions. Practice 4C, 4D, and 4E ask students to apply those values to calculate additional decimal equivalents and solve calculations.
Practice 4C-E work best if students have previously been given a quiz that required memorization of the fundamental decimal equivalents. Students should have these answers very well memorized, using flashcards if necessary (see tutorial Lesson 2.3).
You may want to post 4C Question 1 in some format at the start of class for several days in the first weeks of the semester, to drive these into memory.
If the memorization of the decimal equivalents has not yet been assigned, it is suggested that in class, you ask students to complete worksheet question one, go over the answers and various ways to calculate them, and then assign the remaining problems either in class or as homework.
If Practice 4C is used in-class instead of as homework
In HS Honors/Prep Chem, Practice 4C could be scheduled for 20-30 minutes at the start of a class period. The content could be projected, but it will limit "recopying the question" if the 2 pages are handed out.

Most problems can be solved in multiple ways. In intro classes, this worksheet can be used for the student to try the problem first, then for the instructor to answer questions, provide feedback, and demonstrate multiple ways to solve, especially on the tougher questions.

A possible procedure would be to ask students to answer Questions 1 to 5, allow 5 minutes, and then ask someone how they did 4c. Then ask for someone else to offer a different way (addition 3 times, or multiplication, or adding 0.125 to 0.250 could be shown on the board).
You might then ask they finish Questions 6 and 7, and pick one part of each question to ask for multiple ways to solve that can be written up on the board.

To "distribute" this practice, you may want to have students complete page one on the first day and page 2 the second.
On Question 9, you might ask students to do two parts at a time, ask for how they solved, stop when it seems most are getting it, and ask that they finish the remaining parts for homework.

Some students may have difficulty with the "place value" concepts in $0.040+0.040$ and $5 \times 0.040$. This is math that in some state standards has been "de-emphasized." The fundamental arithmetic may need to be demonstrated a few times -- step by step.

## Follow-Up

Practice 4D and 4E build on Practice 4C. A quiz on 4C to 4E is in Topic 4F.

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## Practice 4C: Converting Fractions to Decimal Equivalents

On this exercise, do NOT use a calculator.

1. Every numeric fraction $\mathrm{Y} / \mathrm{X}$ (if $\mathrm{X} \# 0$ ) has a numeric decimal equivalent. In Lesson 1.2, for the eight fractions encountered most often in scientific calculations, you were asked to commit the decimal equivalents to memory.
From memory, write the decimal equivalents for those fractions.
a. $\frac{1}{2}=0.50$
b. $\frac{1}{3}=$
c. $\frac{1}{4}=$
d. $\frac{1}{5}=$
e. $\frac{2}{3}=$
f. $\frac{3}{4}=$
g. $\frac{1}{8}=$
h. $\frac{3}{2}=$
2. If you know the decimal equivalent for a fraction $1 / X$, you can calculate the value of any decimal equivalent $\mathbf{Y} / \mathrm{X}$ by adding the $1 / \mathrm{X}$ decimal equivalent Y times.
Because $\frac{2}{5}=\frac{1}{5}+\frac{1}{5}=0.2+0.2=\mathbf{0 . 4}$, then (finish) $\frac{3}{5}=$
3. Or, knowing $\mathbf{1} / \mathrm{X}$, you can find a decimal equivalent for $\mathbf{Y} / \mathrm{X}$ using $\mathrm{Y} / \mathrm{X}=\mathbf{Y}(\mathbf{1} / \mathrm{X})$. .

Finish the calculation below by multiplication to find the decimal equivalent.

$$
\frac{4}{5}=4 \times \frac{1}{5}=4 \times=
$$

4. Some decimal equivalents can be solved by reducing the fraction to obtain a $\mathrm{Y} / \mathrm{X}$ that can be recalled from memory or is easier to solve.
Fill in any blanks in these fractions, then write the decimal equivalent in the form 0.XXX
a. $\frac{1}{8}=\mathbf{0}$.
b. $\frac{2}{8}=\underline{1}=$
c. $\frac{3}{8}=\mathbf{0}$.
d. $\frac{4}{8}=\underline{1}=$
5. What is the trend in the Question 4 answers?
6. Using any of the strategies above, convert these fraction to a decimal equivalent value in the format 0.XXX
a. $\frac{5}{8}=$
c. $\frac{7}{8}=$
f. $\frac{6}{8}=$
7. Given that $1 / 25=0.040$, convert these to a decimal equivalent in the format $0 . \mathrm{XXX}$
a. $\frac{2}{25}=$
b. $\frac{3}{25}=$
c. $\frac{5}{25}=$
d. $\frac{9}{25}=$
e. $\frac{12}{25}=$
8. When a non-memorized fraction has an odd number in the numerator, but you know the value for $1 / X$, it often helps to split the numerator into two added parts where the second is $1 / X$. The formula is:

$$
\frac{\mathrm{Y}_{o d d}}{\mathrm{X}}=\frac{\mathrm{Y}-\mathbf{1}}{\mathrm{X}}+\frac{\mathbf{1}}{\mathrm{X}} \quad \begin{gathered}
\mathrm{Y}_{\text {odd }}-1 \text { is even, giving a fraction that can often be reduced } \\
\text { to a fraction with a familiar decimal equivalent. }
\end{gathered}
$$

Follow the steps of this worked example:

$$
\frac{7}{8}=\frac{6}{8}+\frac{1}{8}=\frac{3}{4}+\frac{1}{8}=0.75+0.125=\mathbf{0 . 8 7 5}
$$

On separate paper, convert these to 0.XXXX:
a. If $1 / 12=0.0833$, then $7 / 12=$ ?
b. If $1 / 16=0.0625$, then $5 / 16=$ ?
c. If $1 / 16=0.0625$, then $13 / 16=$ ?
9. If a denominator ends in a single zero before the decimal, the decimal equivalent will be the same as the equivalent for the denominator without the zero, but the equivalent will have its decimal moved one to the left.

But let's explain that with a formula and some examples.

$$
\begin{aligned}
& \text { If } \frac{\mathrm{Y}}{\mathrm{X}}=0 . \mathrm{ABC} \text { then } \frac{\mathrm{Y}}{\mathrm{X} \mathbf{0}}=0.0 \mathrm{ABC} \quad \text { Examples: } \\
& \text { Since } \frac{1}{5}=0.200 \text {, then } \frac{1}{50}=0.020 \quad \text { Since } \frac{3}{4}=0.75, \text { then } \frac{3}{40}=0.075
\end{aligned}
$$

Follow the logic of the math:

$$
\frac{3}{50}=\frac{3}{5} \times \frac{1}{\mathbf{1 0}}=3 \times \frac{1}{5} \times \frac{1}{10}=3 \times 0.200 \times 0.10=0.600 \times 10^{-1}=\mathbf{0 . 0 6 0}
$$

On a separate paper, convert these to a decimal equivalent in the format $0 . \mathrm{XXXX}$
a. $\frac{1}{40}$
b. $\frac{1}{80}$
c. $1 / 20$
d. $13 / 20$
e. 5/80
10. On separate paper, using any strategy you choose, convert these to the form 0.XXX
a. $\frac{1}{20}$
b. $\frac{3}{12}$
c. $\frac{7}{20}$
d. $\frac{9}{72}$
e. $\frac{8}{40}$
f. $\frac{2}{30}$
g. $\frac{3}{50}$
h. $\frac{8}{120}$
Answers: 1b. 0.333 1c. $0.25 \quad$ 1d. $0.20 \quad$ le. $0.667 \quad$ 1f. $0.75 \quad 1$ g. $0.125 \quad 1$ h. $1.5 \quad 2.0 .6$

## Practice 4D: Exponentials and Fractions

## To the Instructor

The practice pages are printed below. Take a look at the pages now, then return here.
Practice 4D is 2-pages combining the decimal equivalent work in Topic 3 with the exponential notation review in tutorial Chapter 1. It is intended to be started in class at any time after both Topic 3 and Chapter 1 have been completed.

These pages can be assigned as homework or completed in class.

## If Used In-Class

You might start by asking students to complete Parts a and bon Questions 1 and 2, demo how you would do one part of each question, and then handle Questions 3 and 4 the same way, leaving un-done parts for homework or for class tomorrow.

How many parts each student will need to complete to become confident will depend on many factors, including the grade in which they were allowed to first use calculators for arithmetic (which will vary by state and district). Student skills in mental math should improve a bit with each passing year, since most states since 2014 have forbidden calculator use until $7^{\text {th }}$ grade.

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## Practice 4D: Fractions and Exponentials

On this sheet, do NOT use a calculator.

1. Convert each term to a whole number power of 10 (such as $10^{-4}$ ), then simplify. Write your final answer as a whole number power of 10 .
a. $\frac{100 \times 0.10}{0.0001 \times 0.010}=\square=$
b. $\frac{1}{1,000 \times 0.010}=\square=$
c. $\frac{10 \times 0.0010}{10,000 \times 0.010}=$
d. $\frac{0.10 \times 0.010}{1,000 \times 100,000}=$
2. Calculate, then convert the final answer to scientific notation with 3 digits in the significand.
a. $\frac{3 \times 10^{-5}}{4 \times 10^{-2}}=$
b. $\frac{2 \times 10^{9}}{8 \times 10^{-2}}=$
c. $\frac{1}{80 \times 10^{-2}}=$
d. $\frac{3 \times 0.0010}{5 \times 0.010}=$
3. Zeros before the decimal but after all other numbers can "cancel on the top and bottom."

Example: $\frac{60 \times 4,000}{800}=\frac{6 \theta \times 4,00 \theta}{8 \theta \theta}=\frac{2400}{8}=\mathbf{3 0 0}$
Below, cancel zeros top and bottom, then simplify using mental arithmetic. Write the answer as a fixed decimal number.
a. $\frac{600 \times 40}{10 \times 30}=$
b. $\frac{7500 \times 90}{30 \times 25}=$
c. $\frac{160 \times 300}{3200 \times 60}=$
4. Fractions with zeros in the denominator before the decimal but after all other numbers can be separated into two parts and converted to exponential notation.

Example: $\frac{3}{500}=\frac{3}{5 \times 100}=\frac{3}{5} \times \frac{1}{100}=0.60 \times 10^{-2}=6.0 \times 10^{-3}$
Separate these into two parts, then simplify. Write the final answer in scientific notation with two digits in the significand.
a. $\frac{64}{160}=$
b. $\frac{144}{1200}=$
c. $\frac{63}{7,000}=$
5. Convert these to fixed decimal values in the form 0.XXX
a. $\frac{7}{28 \times 2}=$
b. $\frac{70}{2 \times 50}=$
c. $\frac{4 \times 45}{30 \times 9}=$
d. $\frac{400}{20 \times 60}=$
6. Re-write each of the two quantities to scientific notation, multiply or divide, then convert to scientific notation with a 2-digit significand.
a. $\left(7,000 \times 10^{-2}\right)\left(0.080 \times 10^{-11}\right)=$
b. $\left(0.090 \times 10^{9}\right)\left(0.0070 \times 10^{-8}\right)=$
c. $\frac{8 \times 10^{-3}}{200 \times 10^{-2}}=$
d. $\frac{72 \times 10^{-3}}{900 \times 10^{7}}=$------------------$=$
Answers: 1a. $10^{7}$
1b. $10^{-1}$
1c. $10^{-4}$
1d. $10^{-11}$
2 a. $7.50 \times 10^{-4}$
2b. $2.50 \times 10^{10}$
2c. $1.25 \times 10^{0}$
2 d. $6.00 \times 10^{-2}$
3a. 80
3b. $900 \quad 3 \mathrm{c} .0 .25$
4a. $4 \times 10^{-1}$
4b. $1.2 \times 10^{-1}$
4 c. $9.0 \times 10^{-3}$
5a. 0.125
5b. 0.700
5c. 0.667
5d. 0.333
6a. $5.6 \times 10^{-11}$
6b. $6.3 \times 10^{-3}$
6c. $4.0 \times 10^{-3}$
6 d. $8.0 \times 10^{-12}$

## Practice 4E: Fractions, Estimates, and Exponentials

## To the Instructor

Practice 4E can be started in class at any time after both Practice 4D and tutorial Chapter 1 have been completed.

Some students may have difficulty with long division with a decimal point involved. This is a topic that in some state standards omitted between 1990 and 2014.

## If Used In-Class

On this sheet, problems are grouped with several of each type in a row. To cover each type, you might ask students to complete every $2^{\text {nd }}$ or $3^{\text {rd }}$ problem, then assign and go over a similar part in the question if a particular type of problem gives them difficulty.

```
# # # # #
```


## Practice 4E: Fractions, Estimation, and Exponentials

On this sheet, do NOT use a calculator.

1. $1 / 9=0.1111 \ldots 2 / 9=0.22222$ (you get the picture) $8 / 9=0.8888 \ldots$

Convert to the form 0.XXXX :
a. $2 / 90=$
b. $7 / 900=$
2. To simplify fractions composed of fixed decimal numbers, the decimals on the top and bottom can be moved the same number of places - to get an easy division problem or a fraction with a familiar decimal equivalent.

$$
\text { Example: } \frac{0.440}{0.020}=\frac{\begin{array}{c}
\cap \cap \downarrow \\
0.440 \\
0.020 \\
\cup \cup \uparrow
\end{array}}{=\frac{44.0}{2.0}=\mathbf{2 2} \quad \begin{array}{l}
\text { (Move to get } 2 \text { in } \\
\text { the denominator) }
\end{array}}
$$

Convert to either whole numbers or decimal equivalents in the form $0 . \mathrm{XXXX}$ :
a. $\frac{0.025}{0.075}=---=$
b. $\frac{5.6}{0.70}=---$
3. If you see a simple division or fraction that can be gotten by moving decimals different amounts in the numerator and denominator, write the simple numbers, add a power of 10 to each to keep its value the same, and use exponential math to solve.

$$
\text { Example: } \frac{0.0060}{2400}=\frac{6 \times 10^{-3}}{24 \times 10^{2}}=0.25 \times 10^{-5}=2.5 \times 10^{-6}
$$

On separate paper, convert to scientific notation with 2 digits in the significand:
a. 4000
b. $\frac{0.56}{800}$
c. $\frac{60}{1.8 \times 10^{-14}}$
4. Re-write each of the two quantities to scientific notation, multiply or divide, and then convert your final answer to scientific notation with a 2-digit significand.
a. $\frac{200 \times 10^{12}}{0.40 \times 10^{4}}=-$
b. $\frac{0.010 \times 10^{3}}{0.50 \times 10^{-7}}=$-----------------= $=$
c. $\frac{0.10}{0.004}=$ - $=$
d. $\frac{0.020}{5,000}=$-------------------$=$
e. $\frac{0.048}{0.008}=-$
5. Round these value, then estimate to an answer with one non-zero digit.
a. $\frac{183}{21 \times 2}=$
b. $\frac{24}{5 \times 53}=$
c. $\frac{4 \times 27}{525}=$
6. Solve these in any way you choose. Convert your final answer to scientific notation with a 2digit significand.
a. $\frac{0.00062}{0.20}=$
b. $\left(4.5 \times 10^{-4}\right)\left(0.020 \times 10^{7}\right)=$
c. $\frac{96 \times 10^{-3}}{120 \times 10^{-1}}=$
d. $\frac{75 \times 10^{16}}{0.025 \times 10^{-4}}=$
7. Decimal equivalents may also be solved by long division.

$$
5 / 6=?=5 \div 6=\frac{0.833 \ldots}{6) 5.000}=0.833
$$

On separate paper, solve by long division. Round to 3 places past the decimal.
a. $5 / 12$
b. $4 / 11$

## Answers:

$\begin{array}{llll}\text { 1a. } 0.0222 & \text { 1b. } 0.0078 & \text { 2a. } 25 / 75=1 / 3=0.3333 & \text { 2b. } 56 / 7=8\end{array}$
3a. $4.4 \times 10^{4}$
3b. $7.0 \times 10^{-4}$
3c. $3.3 \times 10^{15}$
4a. $5.0 \times 10^{10}$
4b. $2.0 \times 10^{8}$ 4c. $2.5 \times 10^{1}$

4d. $4.0 \times 10^{-6}$
4e. $6.0 \times 10^{0}$
5a. Between 3 and $6 \quad 5 \mathrm{~b}$. Close to $0.1 \quad 5 \mathrm{c}$. Close to 0.2
6a. $3.1 \times 10^{-3}$
6b. $9.0 \times 10^{1}$
6c. $8.0 \times 10^{-3}$
6 d. $3.0 \times 10^{23}$
7a. 0.417 7b. 0.364

## Topic 4F: Exponentials and Fractions: 5 Minute Quiz

## To the Instructor

Below are 4 quizzes on the previous mental arithmetic homework worksheets. The suggested timing for the quiz would be:

- Assign the worksheets to be completed, gradually or all at once.
- When the last is assigned, announce a quiz on their content will be given in 2-4 days.
An announced quiz tends to encourage homework completion.
The quiz is "no calculator." It should take 5 minutes. It is designed to be easy if students completed the homework.
There are 4 different copies -- with simple but scrambled numbers. All 4 copies may be shuffled and handed out if security is a concern.
You may want to hand out or project one copy as a practice quiz on the day before the real quiz, let them work for 5 minutes, then post answers and solicit how to solve for a few.
The real quiz is intended to be closed notes and given in a class meeting when security can be maintained.

If errors are found, please email: ToTheAuthors (at) ChemReview.Net .

## Answer Key

Quiz 32 in numerator
1a. 0.888 1b. 0.250 1c. 0.240
2a. $10^{3} \quad 2 b .10^{9}$
3a. $4.2 \times 10^{5}$
3 b. $\quad 4.0 \times 10^{-11}$
3c. $5.0 \times 10^{11}$
$3 \mathrm{~d} . \quad 2.2 \times 10^{1}$
3e. $4.0 \times 10^{-3}$
Quiz 16 in numerator
1a. 0.444 1b. 0.125 1c. 0.640
2a. $10^{2} \quad 2$ b. $10^{6}$
3a. $4.2 \times 10^{4}$
3b. $4.0 \times 10^{-8}$
3c. $5.0 \times 10^{11}$
$3 \mathrm{~d} . \quad 5.6 \times 10^{1}$
3e. $4.0 \times 10^{-4}$

## Quiz 28 in numerator

1a. 0.778 1b. 0.150 1c. 0.480
2a. $10^{1}$
2b. $10^{7}$
3a. $2.1 \times 10^{3}$
3b. $2.0 \times 10^{-9}$
3c. $2.0 \times 10^{13}$
3 d. $4.4 \times 10^{1}$
3e. $2.0 \times 10^{-13}$
Quiz 20 in numerator
1a. 0.556 1b. 0.100 1c. 0.320
2a. $10^{4}$
2b. $10^{8}$
3a. $2.1 \times 10^{2}$
3b. $8.0 \times 10^{-10}$
3c. $2.0 \times 10^{11}$
3d. $3.3 \times 10^{1}$
3e. $2.0 \times 10^{-14}$

Name: $\qquad$
$\qquad$

## Quiz On Fractions and Exponential Mental Math

Do not use a calculator. Show your work on this paper.

1. Convert to 0.XXX :
a. $32 / 36=$
b. $\frac{10 \times 15}{12 \times 50}=$
c. $1 / 25=0.04 ; \frac{6}{25}=$
2. Answer as a $10^{x}$
a. $\frac{100 \times 0.10}{0.010}=$
b. $\frac{1}{10^{-8} \times 0.10}=$
3. Write the final answer in scientific notation with two digits in the significand.
a. $\left(0.0060 \times 10^{9}\right)(0.070)=$
b. $\frac{16}{400 \times 10^{9}}=$
c. $\frac{10^{5}}{2.0 \times 10^{-7}}=$
d. $\frac{0.20}{0.009}=$
e $\frac{48 \times 10^{-5}}{120 \times 10^{-3}}=$

Name: $\qquad$
$\qquad$

## Quiz On Fractions and Exponential Mental Math

Do not use a calculator. Show your work on this paper.

1. Convert to 0.XXX :
a. $16 / 36=$
b. $\frac{5 \times 15}{12 \times 50}=$
c. $1 / 25=0.04 ; \frac{16}{25}=$
2. Answer as a $10^{x}$
a. $\frac{10 \times 0.10}{0.010}=$
b. $\frac{1}{10^{-5} \times 0.10}=$
3. Write the final answer in scientific notation with two digits in the significand.
a. $\left(0.0060 \times 10^{8}\right)(0.070)=$
b. $\frac{16}{400 \times 10^{6}}=$
c. $\frac{10^{5}}{2.0 \times 10^{-7}}=$
d. $\frac{0.50}{0.009}=$
e $\frac{48 \times 10^{-5}}{120 \times 10^{-2}}=$

Name: $\qquad$
$\qquad$

## Quiz On Fractions and Exponential Mental Math

Do not use a calculator. Show your work on this paper.

1. Convert to 0.XXX :
a. $28 / 36=$
b. $\frac{6 \times 15}{50 \times 12}=$
c. $1 / 25=0.04 ; \frac{12}{25}=$
2. Answer as a $10^{x}$
a. $\frac{100 \times 0.010}{0.10}=$
b. $\frac{1}{10^{-6} \times 0.10}=$
3. Write the final answer in scientific notation with two digits in the significand.
a. $\left(0.0030 \times 10^{7}\right)(0.070)=$
b. $\frac{16}{800 \times 10^{7}}=$
c. $\frac{10^{5}}{5.0 \times 10^{-9}}=$
d. $\frac{0.40}{0.009}=$
e $\frac{24 \times 10^{-5}}{120 \times 10^{7}}=$

Name: $\qquad$
$\qquad$

## Quiz On Fractions and Exponential Mental Math

Do not use a calculator. Show your work on this paper.

1. Convert to 0.XXX :
a. $20 / 36=$
b. $\frac{4 \times 15}{12 \times 50}=$
c. $1 / 25=0.04 ; \frac{8}{25}=$
2. Answer as a $10^{x}$
a. $\frac{1000 \times 0.10}{0.010}=$
b. $\frac{1}{10^{-7} \times 0.10}=$
3. Write the final answer in scientific notation with two digits in the significand.
a. $\left(0.0030 \times 10^{6}\right)(0.070)=$
b. $\frac{16}{200 \times 10^{8}}=$
c. $\frac{10^{5}}{5.0 \times 10^{-7}}=$
d. $\frac{0.30}{0.009}=$
e $\frac{24 \times 10^{-5}}{120 \times 10^{8}}=$

## Topic 5

## Improving Dimensional Analysis

Would you like to help students solve calculations with higher rates of success?
Https://www.ChemReview.Net/DHone.pdf and https://ChemReview.Net/DHtwo.pdf include 28 pages of worksheets that help to teach students to organize their data before they begin to "chain conversion factors. The worksheets can be used in-class in "first courses" in chemistry and as a fast review or homework "preparation for lecture" in Gen Chem, GOB, and AP courses.

Topics include word problem strategies, grams, moles, and molecule conversions, stoichiometry, and molarity calculations.

These strategies have been tested - and, as reported in J Chem Ed, raised ACS Gen Chem Examination scores 20 percentiles. Quizzes for instructor use are included.

## Topic 6

## Acid-Base Math Review

Topic 6 is intended for use when you reach solving for $\left[\mathrm{H}^{+}\right]$in strong acid and base solutions. It is intended as a "just-in-time refreshing of memory" of the math for those calculations.

The topic includes 2 pages of math practice and a quiz.
Detail is at https://www.ChemReview.Net/ABMathToInstructors.pdf

## Topic 7

## Preparation for Electrochemistry

Topic 7 is a 5 page assignment designed as homework in General, Engineering, and AP Chem to prepare students in the initial vocabulary and calculations for electrochemistry.

Detail is at www.ChemReview.Net/ElectroPrepToInstructors.pdf

## Topic 8

## Other Math Review Resources

The Math and Metric Review tutorials in Topic 3 are taken from the first two chapters of the textbook Calculations in Chemistry - An Introduction, available from W. W. Norton. The publisher has granted the authors of the text permission to post the first two chapters for free use by students and instructors at all times during the school year.

All college and high school instructors may request a free examination copy of all 24 chapters of the paperback or eBook. To receive a copy, click
http://books.wwnorton.com/books/Calculations-in-Chemistry/ and use the "Request Exam Copy" box near the bottom.

Or you can purchase a used copy. For review or used copies, the authors (who include me) receive no royalties - so I don't receive income from suggesting you read the text.

The textbook includes the following additional lessons which provide additional review of the math topics of chemistry, both with and without a calculator:

## Chapter 3 Significant Figures

Chapter 4 Dimensional Analysis
Chapter 5 Word Problem Strategies
Lesson 12.1 Solving for Ratio Units
Lessons 12.3 Fraction and Percentage Calculations
Lesson 16.3 Cancellation of Complex Units
Lesson 17.1 Choosing Consistent Units
Lesson 17.4 Choosing the Right Equation
Lesson 21.1 Powers and Roots of Exponential Notation
Lesson 22.1 Acid-Base Math -- Review
Lesson 23.1 Base 10 Logarithms
Lesson 24.3 Natural Logarithms
These lessons may provide ideas for math review lessons you can design on your own.
(I was asked on Facebook about how students could use the text from which the Exponential and Metric lessons are taken. Here's what I said:)

I am a co-author, but the book is from WWNorton. They did the typesetting, proofing, eBook programming, orders - the tough and \$\$\$ part.

IF you teach in college or in private schools where students buy their books, they can buy the paperback. It's under $\$ 50$.

For public High Schools that by law must supply books free, a license for the full 24 chapters of the eBook is available from Norton for $\$ 6$ per year if purchased for a 6-year adoption (\$36 per six year license). Details are www.ChemReview.Net/ClassSets.html .

You might say: Schools never have money. Normally, true. But in the recent "Rescue Plan" bill, the feds gave public schools a good chunk of stimulus money to "recover from the pandemic." When that filters down, your admins will tell you: "We have money we need to spend this week." That may get you a class set or 3 .

If $\$ \$ \$$ is limited, what may work is getting enough licenses to use for review homework in AP for six weeks, then have Honors/PreAP use the licenses as online homework reinforcement of lecture the rest of the year.

And that helps for the next six years. Renting an eBook text for $\$ 6$ a year is a good deal. A standard eBook text is $\$ 30$ plus to rent per year.

OR you can get a free copy, type with modifications into 150+ one per day worksheets, and have school pay for xerox copies. It's more work for you and costs school more, but schools are not always rationale places, are they?

In class, can you put up on the screen a few pages for class use? Probably. But check with your school librarian -- the expert on "fair use and copyright."

Hope that helps! -- totally un-biased co-author

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# # # # #
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## Topic 9

## How the Student Brain Learns Chemistry

## For Instructors

A 4 page introduction to scientific research on how the student brain solves chemistry problems - and learns to solve problems - is posted at https://www.ChemReview.Net/Brain101.pdf

This is new science that the author believes will markedly improve the design of your lessons, the success of your students hoping to enter science majors, and your career satisfaction.

> (End of Packet)


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