Chapter 9
Bond and Note Valuation and Related Interest Rate Formulas
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The primary purpose of this document is to show and justify valuation formulas for bills, bonds and notes. The objective here is to understand how prices and/or yields (each determined by the other) are determined in the vast *secondary market* for these securities. The emphasis in this document is on the pricing of U.S. Treasury securities but the general formulas shown apply to equivalent commercial bills, notes, and bonds.

I. Supplementary and Relevant Information

(1) Most bonds and notes (hereafter I will use the term "bonds" to refer to notes and bonds), including all issued by the U.S. Treasury, pay coupon interest either quarterly or semi- rather than annually. U.S. Treasury bonds pay interest *semi-annually*.

(2) Bills (with maturities of one year or less by definition) are discounted (sold at less than par value) and are then redeemed at par value. This is explained in more detail in **Section II** below.

(3) Bonds can be sold in a *secondary market*, just like stocks. Because their coupon interest rate is fixed and may not reflect the yields of equivalent securities at the time they are resold, they will be resold either at a discount (below par) or a premium (above par) so that their effective yields will reflect current market rates at the time they are resold. Generally, if interest rates have risen since the time of issue, the bond will sell at a discount. If interest rates have fallen the bond will sell at a premium. The reason for this will be shown below.

(3) When bonds with at least one coupon payment remaining are resold in the secondary market, interest accrued to the previous owner since the last coupon payment is added to the final price of the security when it is sold. When you buy the bond on the secondary market, if you buy between coupon interest payment dates, which will almost always be the case, you will pay (a) the *ask* price of the bond *and* (b) accrued interest since the last coupon payment. These two values will be summed and reflected in the final price of the bond. It is important to understand that on bond price *quotations*, only the first part, the *ask* price, is included. You, the bond buyer, will then earn the *entire* coupon payment on the bond when it is paid, even if you purchased the bond the day before the payment date.

A detailed example of this complicated point will be provided in section **V.** below.

II. Discounting Bills

By definition, bills, such as the U.S. Treasury Bills that mature in 4, 13, or 26 weeks, mature in less than one year from the date of issue. Therefore they do not pay coupon interest, which is to

say that no separate interest payment is made to the investor. Instead they are sold at **_discount,_** a price that is less than **_par,_** their redemption value. Interest earned for a bill is implicit in the capital gain realized by buying the bill at discount and redeeming it at par.

Although bills (and bonds) are sold in amounts that are multiples of $1,000, their prices are quoted in units of 100, which is called the *par* value. For example, a bill with a redemption value of $10,000 currently quoted at 98.72 (discount) has a current value of $9,872. When the bill is redeemed it will be redeemed at par for $10,000. Therefore, although no interest payment was ever made for this bill, interest is implicit in the capital gain realized.

The formula for determining the *annualized discount yield* of a bill when the discounted price is known is shown here:

1.
$$yield = \frac{100 - price}{price} \times \frac{52}{weeks\ to\ maturity}$$

Using the example above and assuming the bill to be a newly-issued U.S. Treasury Bill with a price of 99.02 and a 13 week maturity, the discount yield will equal

2.
$$yield = \frac{100 - 99.02}{99.02} \times \frac{52}{13} = 0.03959$$

The formula for determining the *price* of a bill with any given number of weeks to maturity (*nwtm*) when the discount yield is known (and confirming equation **2**) is

3.
$$price = \frac{100}{\left(yield \times \dfrac{nwtm}{52}\right) + 1} = \frac{100}{\left(0.03959 \times \dfrac{13}{52}\right) + 1} = 99.02$$

The time-adjustment coefficient in the formulas above assume that the bill in question has a duration expressed in weeks. If the duration is expressed in months, then the coefficient is equal to the number of months duration divided by 12 and if expressed in days, the coefficient is equal to the number of days duration divided by 365. For example, a bill maturing in 24 days priced at 99.745 would be have a yield of

4.
$$yield = \frac{100 - 99.745}{99.745} \times \frac{365}{24} = 0.0389$$

III. Compounding Basics

In order to understand the complicated bond valuation formula, we will build the explanation in steps. We will start by reviewing the basic formulas for calculating compound interest. This will be easy to do in a question and answer format.

Q: If I invest $10 at an annual rate of 8% for one year, what will my investment be worth at the end of the year:

A: $10 \times (1.08) = \$10.80$

Q: What if I leave my interest earned in the account and let it accrue interest for two *more* years? What will my investment be worth at the end of the third year?

A: $10 \times (1.08)^3 = \$12.60$

From this we generalize an interest-compounding formula, assuming a constant interest rate over a number of years and assuming the accrued interest (interest earned over the years) is left in the account.

5. $$X_f = X_p(1+r)^n$$

X_f = future value of investment,
X_p = present value of investment,
r = the annual interest rate, and
n = number of years of the investment.

IV. Present Value Basics

Now let us change the orientation somewhat. In the series above, we wanted to know the *future values* of investments placed today. Suppose we now want to instead know the ***present value*** of some known cash payment that will be paid in the future. Suppose, for example, that someone has given you a contract promising to pay exactly $10,000 in 10 years. What is that contract worth today?

Q: Isn't that contract worth $10,000?

A: No, clearly not. If you were given the option of accepting $10,000 today versus $10,000 in 10 years, would you accept the former or the latter? Clearly you would accept the former, if for no other reason than the fact that you could accept $10,000 today and invest

it at 8% (or the prevailing interest rate) for 10 years and, using formula **(5)** above, end up with $21,590. So $10,000 to be paid 10 years in the future is worth a lot less than $10,000 to be paid today.

Q: So how would I calculate the present value of $10,000 to be paid 10 years from now?

A: Given the prevailing market interest rate on 10 year investments (usually this would be an estimate - let's assume 8% for this example) we have to ask a related question: If I can expect to earn 8% per year over the next 10 years and I want to have $10,000 at the end of that period, how much must I invest today? What initial investment earning 8% compounded annually will leave me with $10,000 after 10 years?

To answer this, all we have to do is look back up to equation **(5)** and see that we can modify that equation solving for the *present value of* X_p using the *future value* X_f over *n* years to get the answer:

$$\textbf{6.} \qquad X_p = \frac{X_f}{(1+r)^n} = \frac{10,000}{(1.08)^{10}} = \$4,631.93$$

This means that $4,631.93 invested at 8% compounded over 10 years will be worth exactly $10,000 at the end of the ten years, making the value financially equal, so a contract offering to pay $10,000 in 10 years would be valued today at $4,631.94 given our assumptions about interest rates.

Q: Please clarify that last point. Why is the interest rate used in the calculation being assumed or estimated?

A: In some cases the interest rate would be known, but because interest rates can change over time, because some investment alternatives are available that offer different rates (e.g. investing in Treasury bills vs. depositing in a bank), and the problem projects into the future, sometimes an interest-rate estimate must be used. In most cases, the "prevailing market rate" on similar or identical alternative investments would be used.

V. An Example of Present Value Applications - The California Lottery

A good example of the present-value application to financial flows over the future can be provided by the California lottery. Winners of the California lottery are not paid one lump sum when they win. They are paid in 20 equal annual payments, the first right after winning the lottery and the last 19 years later. For example, if you win $10 million, you are paid $500 thousand right away and the same amount each year for the next 19 years.

From the discussion just concluded, it is clear that the amount won is not really "worth" $10

million because so much of it is paid in the future and the lottery-winner gets no interest. How much, then, is it worth? What, in other words, is the *present value* of this lottery win?

At first glance this is clearly a more difficult problem than the problem represented in equation **(6)**, which involved the present value of only one payment. In this problem there are 20 payments.

The solution here becomes easy to grasp as soon as we realize that when a payment stream includes more than a single payment, the present value of those payments will be equal to *the summed present value* of each individual payment. In the case of our lottery example, the present value of a win is equal to the present value of each of the individual 20 annual payments, summed:

7. $$Value = \frac{500k}{(1.08)^0} + \frac{500k}{(1.08)^1} + \frac{500k}{(1.08)^2} + \cdots + \frac{500k}{(1.08)^{18}} + \frac{500k}{(1.08)^{19}} = \$5,301,800$$

Expressing the same in summation notation:

8. $$Value = \sum_{t=0}^{19} \frac{500k}{(1+r)^t} = \$5,301,800$$

In a few words, the true value of winning \$20 million in the California lottery, if we can assume the prevailing interest rate to be 8%, is \$5,301,800.

VI. Elementary Bond Valuation - a Starting Point

Evaluating a bond in many respects is not much different than giving a value to winning the California lottery. In the eyes of the finance markets a note or bond, whether newly issued or being resold on the secondary markets, is very little more than a future cash-payment stream. For example, a newly-issued 30-year bond paying interest once per year is a promise to pay (a) 30 equal annual interest payments over 30 years, and (b) the bond's par value (100) in exactly 30 years. This is much like the California lottery payment, except the first payment is made at the end of the year instead of immediately, and in the case of the bond, the par value of the bond is redeemed at the end of the 30 years, resulting in one large final payment with a value equal to par.

In is important to understand that in the application below, we are using the formula to evaluate *the value of a bond on secondary market* for a bond that was issued in the past at a time when interest rates were higher or lower.

Suppose we are pricing the hypothetical bond discussed above - a 30-year bond paying interest only once a year. Assume, as always, that the bond has a par value of 100 and a coupon rate of

8% per year. This means, of course, that the bond pays $8 interest per year (per 100). Suppose additionally, that 10 years have elapsed since the original issue of the bond, leaving only 20 years to maturity (exactly). Finally, suppose that market interest yields on equivalent 20-year bonds are now at 10%. What will be the market value of this bond?

Q: Before you answer, let me make sure I have this straight. The bond is being resold with 20 years remaining to maturity, so it must compete with other financial assets that also have ten years of life remaining, including new 20-year notes. Is that correct?

A: Yes. Therefore the effective yield must be competitive with these other financial assets. Since the coupon rate, 8%, is fixed for the life of the bond, this bond must sell at a discount (a price below par) to effectively raise its yield to 10% for the purchaser. And again, what is being valued is a cashflow stream consisting of 20 remaining interest payments and the final redemption value of the bond (100).

Here is the elementary bond formula that will allows us to calculate the value of this bond:

9. $$MV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \ldots + \frac{C}{(1+r)^{n-1}} + \frac{C}{(1+r)^n} + \frac{Par}{(1+r)^n}$$

MV = present market value (asked price).
C = the coupon payment (amount) equal to the *coupon rate* times *par*.
r = prevailing interest rate on equivalent securities.
n = number of years to maturity.

Expressing the same formula in summation notation:

10. $$MV = \sum_{i=1}^{n} \frac{C}{(1+r)^i} + \frac{Par}{(1+r)^n}$$

Finally, to solve for our hypothetical example:

11. $$MV = \sum_{i=1}^{20} \frac{8}{(1.10)^i} + \frac{100}{(1.10)^{20}} = 82.97$$

Because interest rates are higher than at the time this bond was first issues, the bond is trading at a discount for the price of 82.97. This means that if this bond was originally issued in the denomination of $10,000, it is now worth $8,297.

Readers with a good math background will recognize that formulas **(9)** and **(10)** are a geometric

series. Therefore the elementary bond formula can be reduced to a simpler formula that is easier to calculate on a calculator or in a computer program:

$$\textbf{12.} \qquad MV = C \times \left[\frac{1 - \left(\frac{1}{(1+r)^n} \right)}{r} \right] + \left(Par \times \frac{1}{(1+r)^n} \right)$$

The hypothetical problem used in this section would be solved as

$$\textbf{13.} \qquad MV = 8 \times \left[\frac{1 - \left(\frac{1}{1.10^{20}} \right)}{0.10} \right] + \left(100 \times \frac{1}{1.10^{20}} \right) = 82.97$$

The derivation of this formula is shown in the **Appendix A.**

Note that the values for equations **(11)** and **(13)** agree, as they should.

VII. Elementary Bond Valuation - Periodic Interest Payments

The formula above is just a beginning approximation. It does not take into account the facts that bonds usually pay interest either twice per year, quarterly or monthly. For example U.S. Treasury Bonds pay interest twice per year. The formula for periodic payments is a slight modification of formula **(9)** above:

$$\textbf{14.} \qquad MV = \frac{C/m}{\left(1 + r/m\right)} + \frac{C/m}{\left(1 + r/m\right)^2} + \frac{C/m}{\left(1 + r/m\right)^3} + \ldots + \frac{C/m}{\left(1 + r/m\right)^{n-1}} + \frac{C/m}{\left(1 + r/m\right)^n} + \frac{Par}{\left(1 + r\right)^{n/m}}$$

where variables have the same values as above *except*

m = the number of times per year that interest is paid.
n = number of remaining coupon payments rather than number of remaining years.

Note that C still equals the *annual* value of the coupon payment (the *coupon rate* times *par*) and that the final term is unchanged from formula **(9)**, given that n is equal to m times the number of years.

The summation notation version of this equation needs only a small modification of equation **(10)**:

15.
$$MV = \sum_{i=1}^{n} \frac{C/m}{\left(1 + r/m\right)^i} + \frac{Par}{\left(1 + r\right)^{n/m}}$$

Using the same example of a 30-year bond with a coupon rate of 8%, but this time paid semi-annually instead of annually, with 20 years remaining in its life at a time when equivalent bonds are paying 10% (the same assumptions, except for semi-annual interest payments, used to calculate equation **(11)** above):

16.
$$MV = \sum_{i=1}^{40} \frac{8/2}{1.05^{40}} + \frac{100}{1.10^{20}} = 83.50$$

Finally, the reduced-form equation for this geometric series is a slight modification of equation **(12)**:

17.
$$MV = C/m \times \left[\frac{1 - \left(\frac{1}{\left(1 + r/m\right)^n} \right)}{\left(r/m\right)} \right] + \left(Par \times \frac{1}{\left(1 + r\right)^{n/m}} \right)$$

Using the same values used in equation **(16)** applied to equation **(17)** yields

18.
$$MV = 8/2 \times \left[\frac{1 - \left(\frac{1}{1.05^{40}} \right)}{0.05} \right] + \left(100 \times \frac{1}{1.10^{20}} \right) = 83.50$$

Note that the solution values are the same for equations **(16)** and **(18)**.

Again, the derivation of this reduced-form equation is shown in the **Appendix A**.

As expected, the answers obtained from equations **(11)** and **(13)** when compared to equations **(16)** and **(18)** are similar but slightly different because interest is being compounded twice per year in the latter example, making that bond worth more.

VIII. The Complete Bond Valuation Formula

The final formula for bond valuation for sales on the secondary market takes into account the fact that you seldom buy or sell a bond on the exact day a coupon payment is made. Usually the

transaction is made on a day in between coupon payments. This poses two problems: **(1)** the present value calculation must be able to take into account *fractions* of coupon payment periods, and **(2)** since the registered bond owner on the day of a coupon payment always receives the full interest payment for the previous coupon period, some mechanism must be in place to transfer a certain portion of interest accrued since the last coupon payment to the previous owner.

It is the latter condition that complicates the formula the most. As stated in the opening section on **I. Supplementary and Relevant Information** the payment for a bond after purchase will consist of two parts: **(1)** the market value (the *ask* price) of the bond, the value of which we have been calculating all along, and **(2)** accrued interest to the previous owner since the last coupon payment. The latter will be a payment to the prior bond owner for accrued interest since the last coupon payment.

Most important to understand, the bond price *quotations*, such as those that appear in the *Wall Street Journal*, include only the first part, the *ask* price (or the *bid* price, for those selling the bond - the quoted yield is always calculated from the *ask* price, and neither includes the accrued interest). If you buy a bond on the secondary market, you will pay the quoted *ask* price *plus* accrued interest.

Q: This is confusing. I need an example.

A: OK. Suppose you buy the bond being used for our example on February 8, 2009. Suppose also that the bond makes its coupon payments on March 15 and September 15 of every year and the bond matures on September 15, 2029.

Q: So I'm in between coupon payments. There is a little more than a month before the next. Is that your point?

A: Yes. On settlement date of February 8, it will been 146 days since the last coupon payment. The next will be in only 36 days. Yet on that day you will receive the full coupon payment for half a year.

Q: So technically I owe the previous owner 146 days of accrued interest, which will be included in the final price. Is there a formula for calculating the accrued interest?

A: Yes, and that formula will be included in the final formula for bond valuation. Here is the formula for accrued interest using our example:

19. $$AI = \left(\frac{C}{2} \times \frac{a}{p} \right) = \left(4 \times \frac{146}{182} \right) = \$3.21$$

where

AI = accrued interest

C = the annual coupon payment (*coupon rate* times *par*)
a = accrued days, or the number of days since the last coupon payment
p = the number of days in the full coupon period.

For example, assuming that the bond had an 8% coupon,

At this point it is probably best to show the complete bond valuation formula, then explain it. Remembering that this is the formula for calculating *ask* price, here it is:

20.
$$MV = \frac{C}{m}\sum_{i=1}^{n}\frac{1}{\left(1+\dfrac{r}{m}\right)^{i-a/p}} + \frac{Par}{(1+r)^{(n-1)/m+(p-a)/365}} - \left(\frac{C}{m}\times\frac{a}{p}\right)$$

where

MV = market value, the quoted *ask* price of the bond,
C = the annual coupon payment, equal to the coupon rate times par,
Par = 100
r = the prevailing annual market yield, expressed as *ask* yield or yield-to-maturity,
m = the number of coupon payments per year,
n = the number of remaining coupon payments,
p = the number of days in *this* coupon period (between 181 and 184, use 182 if unknown),
a = the number of days between the last coupon payment and the settlement day.

The first term represents the present value of all remaining coupon payments, the second term represents the present value of the par redemption at the time of maturity, and the third term, which is *subtracted* from the first two, represents the accrued interest that is owed to the prior owner.

Also notice that the exponents in the first two terms are no longer whole numbers, but fractions representing accurately the number of days to each coupon payment and maturity.

Let us apply this formula to our hypothetical bond discussed above. Here is a reminder of the assumptions made:

Purchase date: February 8, 2009 (146 days since last coupon payment, 36 to next),
Next coupon date: March 15, 2009 (the first of 42 coupons remaining),
Redemption date: September 15, 2029 (for par and last coupon),
Coupon rate/amount: 8% yielding $4 per coupon payment,
Present market rate (*ask* yield): 10%.

Knowing that the fraction

$$\frac{a}{p} = \frac{146}{182} = 0.8022$$

here is the calculation for the market value (*ask* price) which would be quoted:

21.
$$MV = \frac{8}{2}\sum_{i=1}^{42}\frac{1}{\left(1+\frac{.10}{2}\right)^{i-0.8022}} + \frac{100}{(1+.10)^{20.5+0.099}} - \left(\frac{8}{2}\times 0.8022\right)$$

Simplifying this we get

22.
$$MV = 4\sum_{i=1}^{42}\frac{1}{(1.05)^{i-0.8022}} + \frac{100}{(1.10)^{20.60}} - (3.21) = 83.31$$

This bond would be quoted with an *ask* price of $83.31. Accrued interest would equal $3.21. To buy this bond would cost **$86.52** per 100 (par), the sum of equations **(22)** and **(19)**. To buy $10,000 "worth" of this bond would cost $8,652 of which $321 is accrued interest.

The coupon-payment component is, once again, a geometric series that can be reduced to a much simpler equation that is easier to solve with a calculator, in Excel, or with a computer program. Here is the reduced-form equation:

23.
$$MV = \frac{C}{m}\left[\frac{1-\frac{1}{\left(1+\frac{r}{m}\right)^{n}}}{\frac{\frac{r}{m}}{\left(1+\frac{r}{m}\right)^{a/p}}}\right] + \left(Par \times \frac{1}{(1+r)^{(n-1)/m + (p-a)/365}}\right) - \left(\frac{C}{m}\times\frac{a}{p}\right)$$

Using the same values that were used for the geometric series version of this in equation **(21)**, when applied to the reduced-form equation **(23)**,

24.
$$MV = 4\left[\frac{1-\dfrac{1}{(1.05)^{42}}}{\dfrac{0.05}{(1.05)^{0.8022}}}\right] + \left(100 \times \frac{1}{(1.10)^{20.5+0.099}}\right) - (4 \times 0.8022) = 83.31$$

Derivation of equation **(23)** is shown in **Appendix B.**

Q: Can this formula be used to solve for the market yield (*r*) rather than the market price? Suppose I wanted to pay 86 for this bond. Could I use this formula to calculate the effective market yield *r*?

A: Yes and no. You can't *directly* use this formula to solve for *r* (too many roots!) but you can use this formula and a common iterative technique (requiring a computer program or a dedicated calculator chip) to converge to a solution. Any business calculator, Excel, or computer program that can solve for equation **(20)** or **(23)** above can also, given a market value, solve for *r* as an unknown.

APPENDIX A: Reduced-form Version of the Elementary Bond Formula

After introducing equations **(9)** and **(10)** in the main text, both were identified as a geometric series that can be reduced to a simpler equation, which was introduced at equation **(12)**. Shown below are copies of equations **(9)** and **(12)**.

9.
$$MV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \ldots + \frac{C}{(1+r)^{n-1}} + \frac{C}{(1+r)^n} + \frac{Par}{(1+r)^n}$$

12.
$$MV = C \times \left[\frac{1-\left(\dfrac{1}{(1+r)^n}\right)}{r}\right] + \left(Par \times \frac{1}{(1+r)^n}\right)$$

The purpose of this appendix is to show how equation **(9)** can be converted to equation **(12)**.

If we let *C* represent the value of the coupon payment and *V* represent the present discounted value of the *coupon interest payments only* and substitute *X* for *(1 + r)* then the first half of the bond value formula can be represented as

A1. $\dfrac{1}{X} + \dfrac{1}{X^2} + \cdots + \dfrac{1}{X^n} = V\!\!\Big/\!\!{}_C$

This can be converted to a standard geometric series equation by multiplying both sides times X^n. This will result in

A2. $X^{n-1} + X^{n-2} + \cdots + X + 1 = V\!\!\Big/\!\!{}_C\, X^n$

A standard geometric series of the form above can be reduced to a more elementary equation by multiplying both sides of the equation times $(1 - X)$ which results in

A3. $1 - X^n = V\!\!\Big/\!\!{}_C\left(X^n - X^{n+1}\right)$

To make this more elegant multiply both sides of the equation times $-\dfrac{1}{X^n}$

A4. $1 - \dfrac{1}{X^n} = V\!\!\Big/\!\!{}_C\left(X - 1\right)$

Solving for *MV* results in

A5. $V = C\left(\dfrac{1 - \dfrac{1}{X^n}}{X - 1}\right)$

Substituting *(1 + r)* back into equation **(5)** for *X* yields the front part of the reduced form equation as shown below

A6.
$$V = C\left(\dfrac{1 - \left[\dfrac{1}{(1+r)^n}\right]}{r}\right)$$

Remembering that **C** is equal to the coupon rate times par, if the formula for discounting the present value of the bond at redemption is added to equation **(A6)**, the reduced form equation is identical to equation **(12)** and is complete.

APPENDIX B: Reduced-form Version of the Complex Bond Formula

Equation **(20)**, reproduced below, includes a component, the present value calculation of the coupon interest payments, that can be reduced to a simpler equation **(23)**, also reproduced below.

20.
$$MV = \frac{C}{m}\sum_{i=1}^{n}\frac{1}{\left(1+\dfrac{r}{m}\right)^{i-a/p}} + \frac{Par}{(1+r)^{(n-1)/m + (p-a)/365}} - \left(\frac{C}{m}\times\frac{a}{p}\right)$$

23.
$$MV = \frac{C}{m}\left[\frac{1 - \dfrac{1}{\left(1+r/m\right)^n}}{\dfrac{r/m}{\left(1+r/m\right)^{a/p}}}\right] + \left(Par\times\frac{1}{(1+r)^{(n-1)/m + (p-a)/365}}\right) - \left(\frac{C}{m}\times\frac{a}{p}\right)$$

As in **Appendix A**, define *V* to represent the present discounted value of the coupon interest payments only, let *X* equal $(1 + r/m)$, let λ equal a/p, and let β equal Vm/C. Given these assumptions, the present value of the coupon interest payment from equation **(20)** can be written

B1.
$$\frac{1}{X^{1-\lambda}} + \frac{1}{X^{2-\lambda}} + \frac{1}{X^{3-\lambda}} + \cdots + \frac{1}{X^{n-\lambda}} = \beta$$

Multiply both sides of the equation times $X^{n-\lambda}$, which gives us our standard geometric series,

B2. $\qquad X^n + \cdots + X^2 + X + 1 = \beta X^{n-\lambda}$

To simplify, multiply both sides of the equation times $(1 - X)$

B3. $\qquad 1 - X^n = \beta\left(X^{n-\lambda} - X^{n+1-\lambda}\right)$

To further simplify, multiply both sides of the equation times $- \frac{1}{X^n}$ (the last two steps could have been done in one step, but it is easier to see how we are getting to our results by doing it in two steps)

B4. $\qquad 1 - \left(\dfrac{1}{X^n}\right) = \beta\left(X^{1-\lambda} - X^{-\lambda}\right) = \beta X^{-\lambda}\left(X - 1\right)$

At this point it is useful to substitute $\left(1 + \frac{r}{m}\right)$ for X and $\frac{Vm}{C}$ to see where this is going

B5. $\qquad 1 - \dfrac{1}{\left(1 + \frac{r}{m}\right)^n} = \left(\frac{Vm}{C}\right)\left(1 + \frac{r}{m}\right)^{-a/p} \times \frac{r}{m}$

Rearranging and solving for V (the present discounted value of the coupon payments alone) yields

B6. $\qquad V = \dfrac{C}{m}\left[\dfrac{1 - \dfrac{1}{\left(1 + \frac{r}{m}\right)^n}}{\dfrac{\frac{r}{m}}{\left(1 + \frac{r}{m}\right)^{a/p}}}\right]$

If we now add the present discounted value of the redemption value and the subtraction for accrued interest, equation **B6** becomes equation **(23)** above, which, despite its visual complexity, is easy to program into Excel, a computer program, or a calculator.

The elegance of equation **B6** is easier to see if we assume only one coupon interest payment per

year, which implies that $m = 1$ and that n is equal to the number of remaining years in the life of the bond. Equation **B6** becomes

B7.
$$V = C\left[\frac{1 - \dfrac{1}{(1+r)^n}}{\dfrac{r}{(1+r)^{a/p}}}\right]$$