

## Chapter 5 - Finance

### 5.1 - Compound Interest

**Simple Interest:** Interest earned on the original investment amount only

If  $P$  dollars (called the principal or present value) earns interest at a simple interest rate of  $r$  per year (as a decimal) for  $t$  years, then the interest earned,  $I$ , is given by:

$$I = Prt$$

So, the accumulated amount (or future value),  $A$ , of the investment is equal to

$$A = P + I = P + Prt = P(1 + rt)$$

**Ex:** Find the accumulated amount at the end of 8 months on a \$1200 deposit paying simple interest at a rate of 7% per year. How much interest was earned?

**Ex:** You take out a loan for \$3000 that is accruing simple interest. After 5 months, you owe \$3112.50.

(a) What is the simple interest rate being charged on this loan?

(b) After how long will you owe \$3450?

**Compound Interest:** Interest earned on both the original investment amount plus previously added interest.

Suppose a principal  $P$  earns interest at an annual interest rate of  $r$  per year (as a decimal) and interest is **compounded**  $m$  times a year. Then, after  $t$  years, the accumulated amount or future value,  $A$ , is:

$$A = P \left( 1 + \frac{r}{m} \right)^{mt}$$

Possible Periods of Conversion (Values of  $m$ ):

Annually:	Semi-annually:	Quarterly:
Monthly:	Weekly:	Daily:

### Compound Interest on the Calculator

- Go to FINANCE and select *TVM Solver*.
- Fill in the variables according to the following:  
 $N = mt$  (the total number of conversion (compounding) periods)  
 $I\%$  = the interest rate in % form  
 $PV = P$  (principal / present value)  
 $PMT$  = regular payment amount per period  
 $FV = A$  (accumulated amount / future value)  
 $P/Y$  = the number of payments made per year  
 $C/Y = m$  = the number of conversion periods per year  
 $PMT$ : END BEGIN
- Move your cursor to the variable you are solving for and press ALPHA ENTER and the answer will appear where the cursor is located.

*Important Note:* In the TVM Solver, the values for PV, PMT, and FV will sometimes be negative. This is done to represent the transfer or flow of money. We will usually look at these problems from the standpoint of the investor or borrower.

A negative number represents an **outflow** of money away from the investor or borrower, i.e. when money is leaving your pocket. Use a negative number when:

- Making payments
- Depositing money in a bank

A positive number represents an **inflow** of money to the investor or borrower, i.e. when you put money in your pocket. Use a positive number when:

- You receive a loan from a bank or lender.
- You receive money from a bank account.

**Ex:** How much money would you have after 5 years if you deposited \$500 into an account paying 8% interest per year, compounded quarterly?

N=	PMT=
I%=	FV=
PV=	P/Y=C/Y=

How much total interest would be earned?

**Ex:** How much money should you deposit in an account paying 5% interest per year compounded monthly, so that you'll have \$5000 in 10 years?

N=	PMT=
I%=	FV=
PV=	P/Y=C/Y=

How much total interest will be earned on your money?

**Ex:** How long would it take for a deposit of \$20,000 to grow to \$30,000 at an interest rate of 8.5%/yr compounded semi-annually?

N=	PMT=
I%=	FV=
PV=	P/Y=C/Y=

**Ex:** Suppose that 4 years ago, I invested \$5000 in an account that compounds interest monthly. Right now I have \$8000 in the account. What is the interest rate for this account (rounded to 4 decimal places)?

N=	PMT=
I%=	FV=
PV=	P/Y=C/Y=

What would happen if your money was compounded more frequently than once every day? If your money was compounded an infinite amount of times, would you earn an infinite amount of interest?

**Continuously Compounded Interest:**  $A = Pe^{rt}$

**Ex:** If you invest \$10000 at 9% per year with interest compounded continuously, how much would you have in your account after 5 months?

**Effective Rate of Interest (Effective Annual Yield):** The simple interest rate that would produce the same accumulated amount in one year as the nominal rate compounded  $m$  times a year.

$$r_{eff} = \left(1 + \frac{r}{m}\right)^m - 1$$

The effective interest rate is often used when comparing two accounts that are compounded differently.

**On the calculator...**

1. Go to FINANCE and select *EFF*.
2. Give the arguments as follows:  $EFF(r, m)$  where  $r$  is given in % form

**Ex:** What is the effective annual yield on an account paying 6% interest per year, compounded monthly?

**Ex:** Of the two options below,  
 A: 8% compounded semi-annually  
 B: 7.9% compounded daily

- (a) Which is the better investment?
- (b) Which is the better credit card rate?

**Effective Rate of Interest for Continuously Compounded Interest:**  $r_{eff} = e^r - 1$

**Ex:** What is the effective annual yield on an account paying 6% interest per year, compounded continuously?

## 5.2/5.3 - Annuities, Sinking Funds, and Amortization

**Annuity:** a sequence of payments made at regular time intervals  
In this class, we will assume all payments are equal.

**Ex:**

We will also assume all annuities we are dealing with are

*ordinary, certain, and simple.*

**Ex:** Bob deposits \$60 at the end of each month into a savings account earning interest at the rate of 6% per year compounded monthly.

- (a) How much will he have on deposit in his account at the end of 10 years, assuming he makes no withdrawals during that period?

N=

PMT=

I%=

FV=

PV=

P/Y=C/Y=

- (b) How much interest does Bob earn?

**Ex:** Suppose a person opens up a retirement account in which he/she places \$1000 each quarter into an account that earns interest at a rate of 3.5%/yr compounded quarterly.

(a) How much will be in the account when this person retires in 30 years?

N=	PMT=
I%=	FV=
PV=	P/Y=C/Y=

(b) How much interest is earned in total?

**Sinking Funds** - placing money periodically into an account to accumulate a desired amount at a future date (an account that is set up for a specific purpose at some future date)

**Ex:** Mark's parents anticipate that his first year of college will cost \$12,000. Knowing Mark's first year of college is 10 years away, determine the amount of money they should deposit into an account each year making 7.4% per year compounded annually, if they intend on having the money ready to pay for his first year when he starts college.

N=	PMT=
I%=	FV=
PV=	P/Y=C/Y=

How much would they have to deposit monthly over the same time period to reach the desired goal, if they found an account paying interest at a rate of 7.4%/year compounded monthly?

N=	PMT=
I%=	FV=
PV=	P/Y=C/Y=

**Ex:** A family wants to save up some money to make a \$40,000 down payment on a house in 7 years.

- (a) How much should they deposit each month into an account if the account earns interest at the rate of 8.5%/year compounded monthly?

N=	PMT=
I%=	FV=
PV=	P/Y=C/Y=

- (b) If they can afford to deposit \$400 a month instead, when can they afford the down payment for the house?

N=	PMT=
I%=	FV=
PV=	P/Y=C/Y=

Sometimes it is necessary to determine how much money is needed in an account now so that regular payments can be made in the future.

**Ex:** Suppose you win a lottery worth \$1,000,000 which is paid out with an initial \$40,000 payment and \$40,000 payments for the next 24 years. In order to make these payments to you, how much money must the lottery commission have in an account now if the account earns interest at a rate of 2.5%/yr compounded annually?

N=	PMT=
I%=	FV=
PV=	P/Y=C/Y=

**Amortization:** paying off a debt with regular payments

**Ex:** What monthly payment is required to amortize a loan of \$50,000 over 20 years, if interest at the rate of 8% per year, compounded monthly, is charged on the unpaid balance at the end of each month?

N=	PMT=
I%=	FV=
PV=	P/Y=C/Y=

**Ex:** Sally made a down payment of \$5000 toward the purchase of a new car. To pay the balance, she secured a loan at the rate of 4.9% per year compounded monthly. Under the terms of her finance agreement, she is required to make payments of \$450/month for 48 months.

(a) What is the cash price of the car?

N=	PMT=
I%=	FV=
PV=	P/Y=C/Y=

(b) How much total interest did Sally pay on the loan?



**Ex:** A family secured a 25-year bank loan of \$150,000 to purchase a house. The bank charges interest at a rate of 9% per year, compounded monthly.

(a) What is their monthly payment?

$N=$	$PMT=$
$I\%=$	$FV=$
$PV=$	$P/Y=C/Y=$

(b) How much total interest will they end up paying?

**Outstanding principal** is how much you still owe at a given point. To find the outstanding principal, find the **present value of the remaining payments**.

(c) What is the outstanding principal after 8 years? In other words, how much do they still owe after 8 years?

$N=$	$PMT=$
$I\%=$	$FV=$
$PV=$	$P/Y=C/Y=$

(d) How much have they actually paid in these 8 years?

Why not subtract this from \$150,000 to find the outstanding principal?

**Equity** in a loan scenario is how much of the item you actually OWN. It is how much principal you have paid on the original loan plus any down payment (what belongs to you). The interest you pay does NOT count towards your equity. At any moment in time, the following is true:

$$\text{Value of Item} = \text{Equity} + \text{Outstanding Principal}$$

(e) What is their equity after 8 years?

**Ex:** Four years ago, Emily got a bank loan for the purchase of a home. The home was worth \$250,000 and she made a 20% down payment. The loan was at 7.5%/year compounded monthly and the term of the loan was 30 years.

- (a) What is Emily's current monthly mortgage payment?

N=	PMT=
I%=	FV=
PV=	P/Y=C/Y=

- (b) After these first four years, Emily decides to refinance her home. What is her outstanding principal at this point? Equity?

N=	PMT=
I%=	FV=
PV=	P/Y=C/Y=

- (c) Emily refinanced her home by securing a new 20-year loan for the outstanding principal at a new rate of 6.75%/year compounded monthly. What will be her new monthly mortgage payment now?

N=	PMT=
I%=	FV=
PV=	P/Y=C/Y=

- (d) How much money will Emily save by refinancing the loan?

**Ex:** John buys a computer with a cash price of \$2500. Assume he makes a down payment of 10% of the cash price and secures financing for the balance at a rate of 12% per year compounded monthly.

(a) What will his monthly payment be if the computer is financed over 2 years?

$$\begin{array}{ll} N= & PMT= \\ I\%= & FV= \\ PV= & P/Y=C/Y= \end{array}$$

(b) How much total interest will John pay in this situation?

Every time you make a payment on a loan, part of this payment is going towards the principal and part of the payment is going towards the interest that has accrued during that period. We can use an amortization table to see what is going on with each payment.

For the example with John above:

Monthly Interest Rate  $\Rightarrow$

End of First Period:

Payment =

Interest Owed =

Principal Paid =

Now Owe (Outstanding Principal) =

Use this method to fill in the first few lines of the amortization table below.

End of Period	Pmts Remaining	Payment	Amount Twds Interest	Amount Twds Principal	Outstanding Principal	Equity
0	24					
1	23					
2	22					
3	21					
4	20					
5	19					

**Ex:** You have a \$2500 credit card bill on a card that charges interest at a rate of 19.8% per year, compounded monthly, on the unpaid balance.

- (a) If you do not make any additional purchases on the card and make a \$42 payment each month, how long will it take you to pay off your bill? How much total interest do you end up paying?

- (b) If you instead plan to pay off this credit card at the end of two years, how much will you have to pay each month? How much of your first payment goes towards interest? How much of your first payment goes towards principal (paying off your debt)?

**Ex (from W/C):** Your original mortgage was a \$96,000, 30-year 9.75%/yr mortgage, where interest was compounded monthly. After 6 years you refinance the remaining principal with a mortgage for 30 years at 6.875%/year compounded monthly. What was your original monthly payment? What is your new monthly payment? How much will you save in interest over the course of the loan by refinancing?

**Ex (from Tan):** The Taylors have purchased a \$270,000 house. They made an initial down payment of \$30,000 and secured a mortgage with interest charged at the rate of 8%/year on the unpaid balance, compounded monthly. If the loan is to be amortized over 30 years, what monthly payment (made at the end of the month) will the Taylors be required to make? What is their equity (disregarding appreciation) after 5 years? After 10 years? After 20 years? How much total interest do they pay over the full life of the loan?