Relative Motion

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UBC Physics 2006

What is Relative Motion?

- First of all the physics concept involved is KINEMATICS (the study of motion of objects - the relation among displacement, velocity, acceleration, and time)
- The motion (or way of moving) of an object viewed by an observer

What is a scalar?

- A quantity specified by magnitude (size) only.
  - Examples
    - Speed (4m/s)
    - Time (20s)
    - Distance (100m)
    - Mass (80kg)
    - Volume ($80kg/m^3$)

What is a Vector?

- A quantity described by both magnitude (size) and direction.
  - Examples
    - Velocity (4m/s north)
    - Displacement (100m south)
    - Acceleration ($4m/s^2$ east)

The motion of an object has these 3 vector components

Kinematics

- Displacement ($\vec{d}$) is a vector quantity and has magnitude and direction
- Velocity ($\vec{v}$) is a vector
- Acceleration ($\vec{a}$) is a vector
- Time (t) is a scalar (only magnitude)

Determining Velocity

- Choose a Frame of Reference

  An arbitrary "place" from which an observer makes measurements

- Relative Velocity

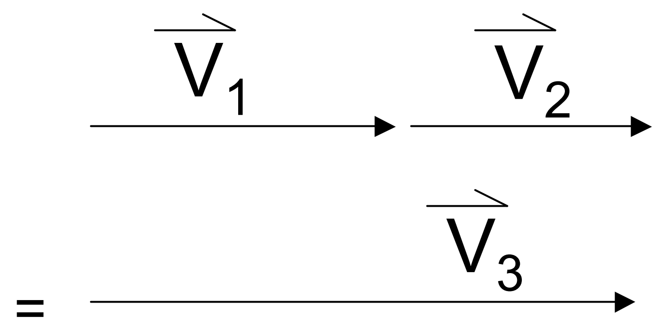
  The velocity an object appears to have to an observer who is moving with a different object

  Let's look at an example with vector addition in 1 dimension……but first a note on vectors

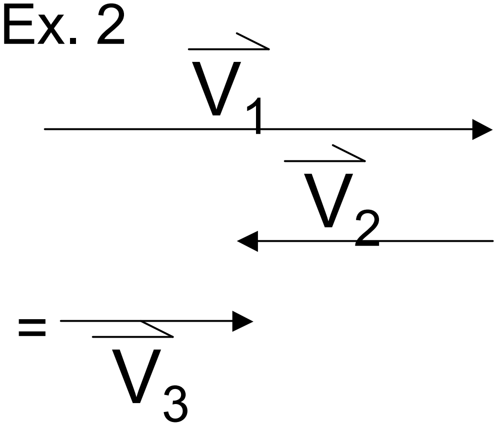
A note on adding Vectors

$$\vec{V}_1 + \vec{V}_2 = \vec{V}_3$$

Ex. 1



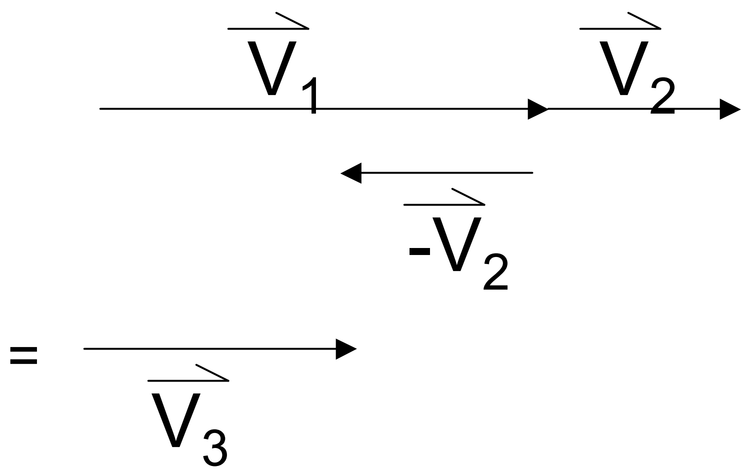
Ex. 2



A note on subtracting Vectors

The difference of two vectors is the sum of the first vector and the negative of the second vector

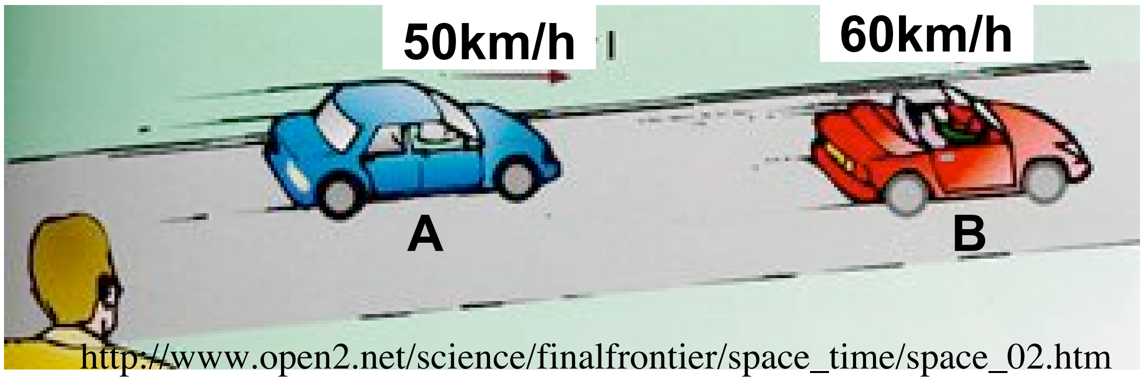
$$\vec{V}_1 + (-\vec{V}_2) = \vec{V}_3$$



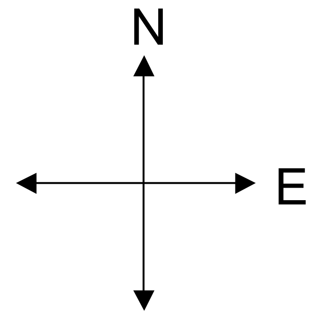
Example 1 - 1 Dimension

- Car A travels on a street with velocity 50km/h east,while Car B travels down the same street with velocity 60km/h east. Both velocities are relative to a stationary observer on earth.
- A person standing on the sidewalk observes the 2 cars driving on the street.

Example 1 - 1 Dimension

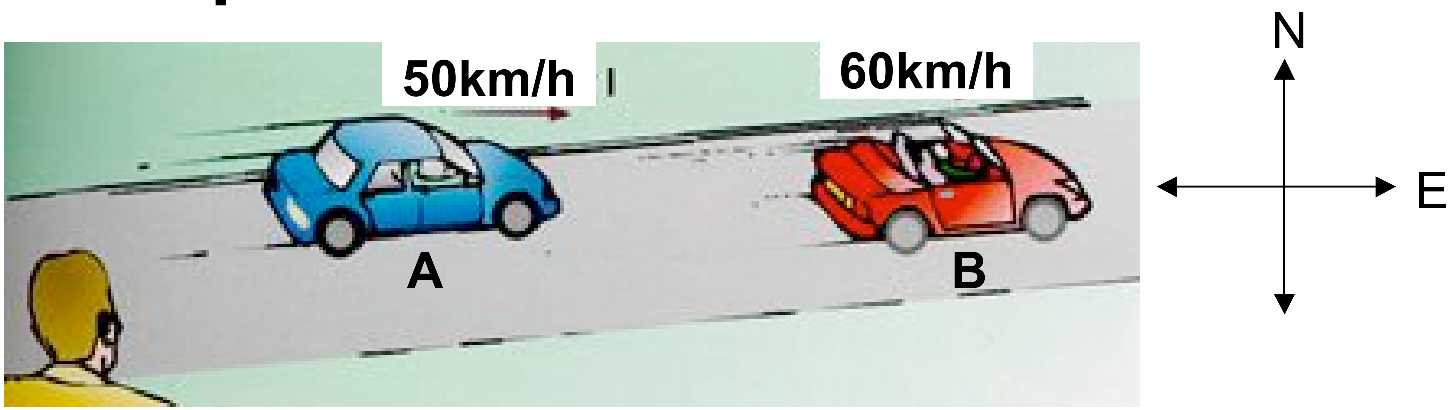


http://www.open2.net/science/finalfrontier/space_time/space_02.htm



- What are the velocities of the 3 objects (Car A, Car B, person on the sidewalk) from:
  - Car A’s frame of reference
  - Car B’s frame of reference
  - The persons frame of reference

Example 1 - 1 Dimension



Let's start with the person on the sidewalk

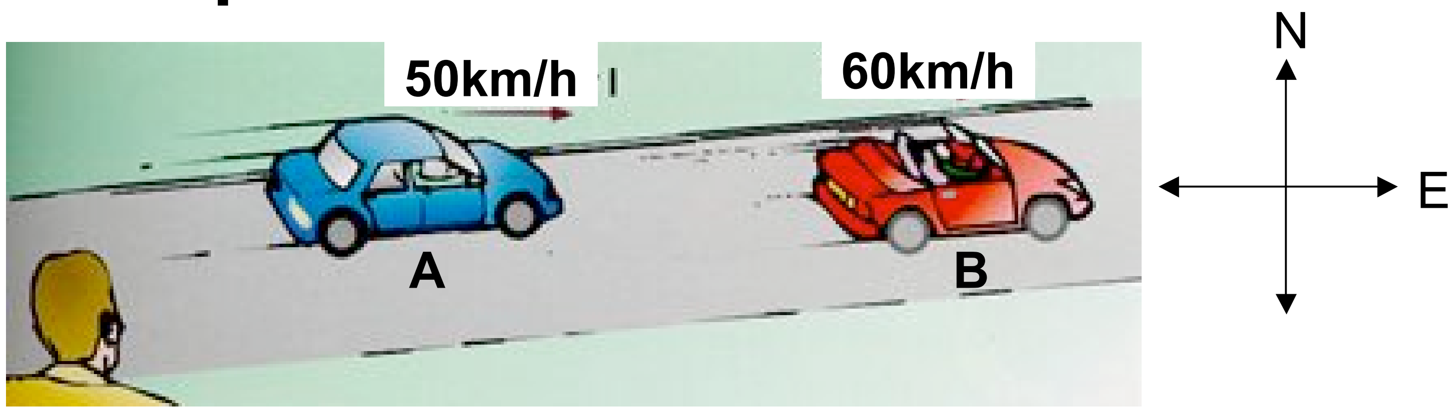
The velocity of the person is…..

The velocity of Car A is…………

The velocity of Car B is…………

Remember, we need a magnitude and direction

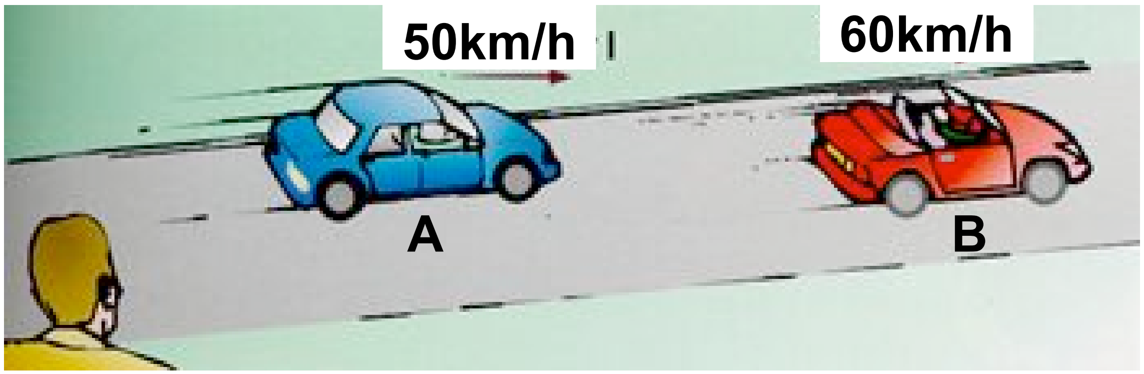
Example 1 - 1 Dimension

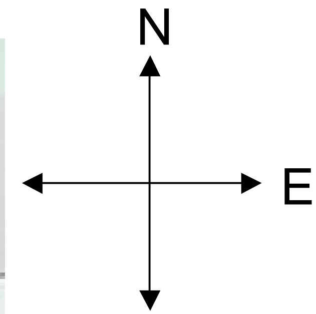


Let's start with the person on the sidewalk

Observer	Person's velocity	Car A (blue) velocity	Car B (red) velocity
Person	0km/h, East	50km/h, East	60km/h, East

Example 1 - 1 Dimension





Next, let's find the velocities from Car A's frame of reference

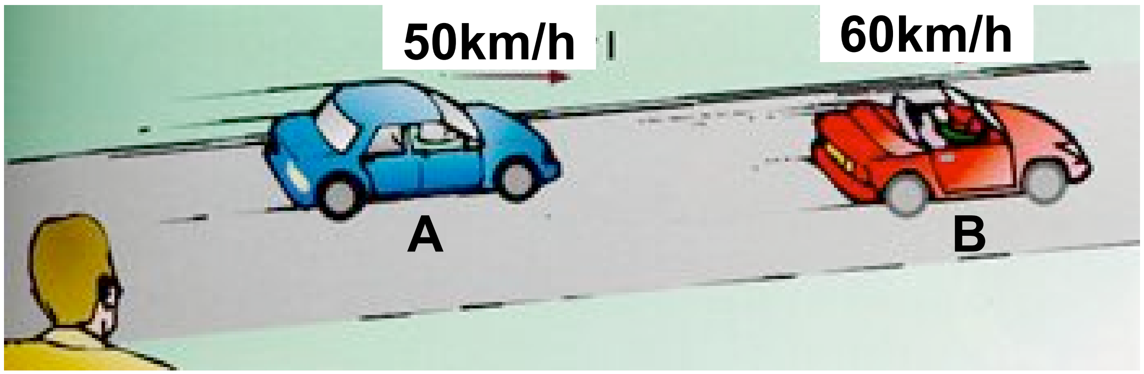
The velocity of the person is…..

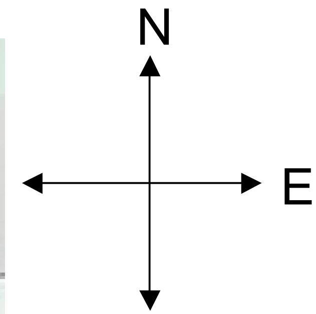
The velocity of Car A is…………

The velocity of Car B is…………

Do we need to add or subtract the

Example 1 - 1 Dimension

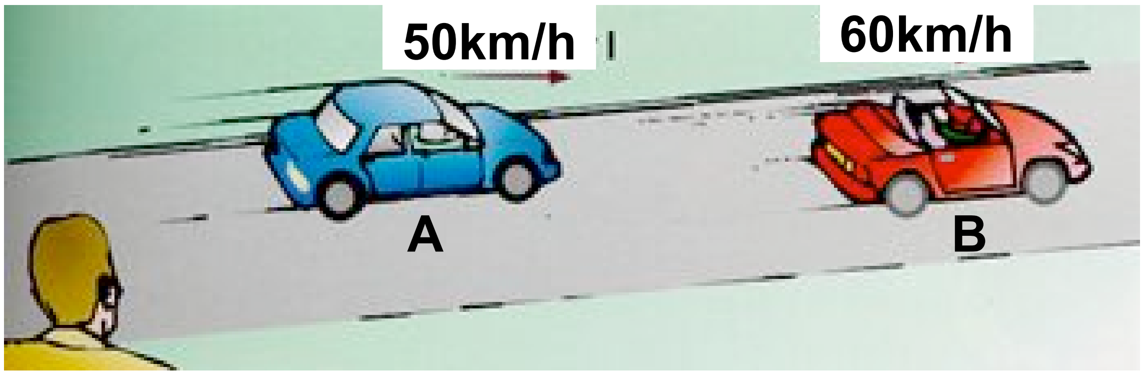


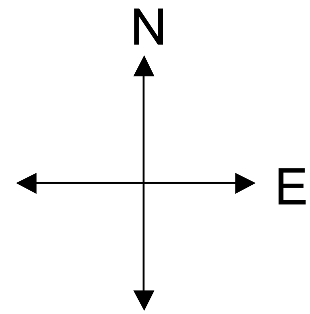


Car A's frame of reference

Observer	Person's velocity	Car A (blue) velocity	Car B (red) velocity
Car A	50km/h, West	0km/h, East	10km/h, East

Example 1 - 1 Dimension





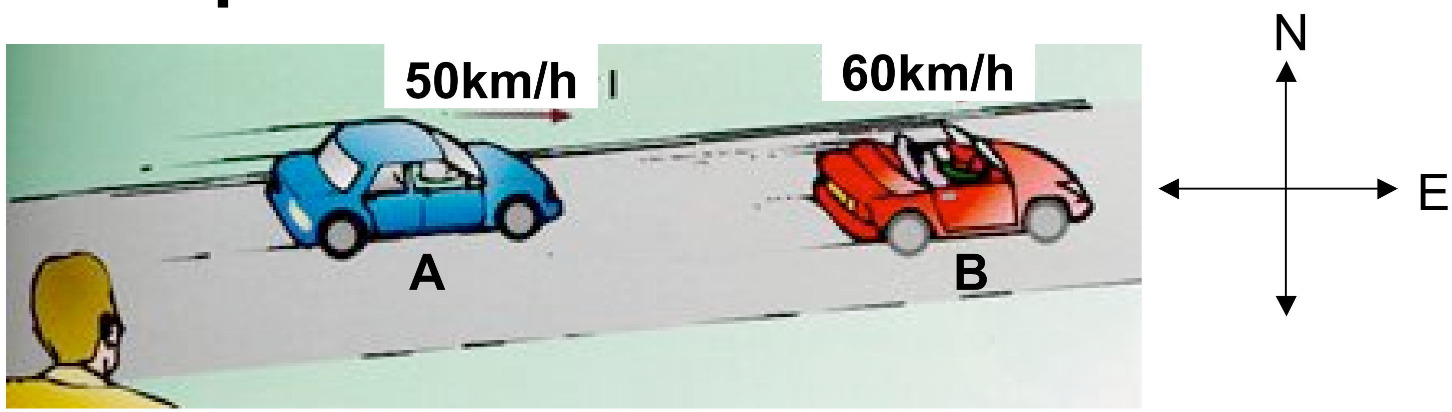
Next, let's find the velocities from Car B's frame of reference

The velocity of the person is…..

The velocity of Car A is…………

The velocity of Car B is…………

Example 1 - 1 Dimension



Car B's frame of reference

Observer	Person's velocity	Car A (blue) velocity	Car B (red) velocity
Car B	60km/h, West	10km/h, West	0km/h, East

First Demonstration

- Let’s simulate a 1-Dimensional River Problem.
  1 - Boat in motion upstream

First Demonstration

- The boat has a velocity of 42cm/s west.
- The river has a velocity of 28cm/s east.
- A person stands on the bank of the river and observes the motion of the boat and river.
  - Let's find the relative velocities of the 3 objects in each observers frame of reference.
  - We will time how long the boat travels. Then we can find how far up the river the boat has traveled relative to person standing on the bank, and relative to its initial position in the river.

Note: the given velocities were calculated from an observer standing on the bank of the river

First Demonstration

- I am going to need some volunteers to help me with the demonstration
    - Timer
    - Someone to start the river and stop the river

First Demonstration

The relative velocities are:

Observer	Person's velocity	Boat's velocity	River's velocity
Person	0m/s	14cm/s west	28cm/s east
Boat	14cm/s east	0m/s	42cm/s east
River (kayak in motion with river)	28cm/s west	42cm/s west	0m/s

First Demonstration

Now let’s find the displacement of the boat relative to the bank and to the river:

- We know that $\vec{v} = \overrightarrow{\Delta d}/\Delta t$
- Solving for $\overrightarrow{\Delta d}$, we get $\overrightarrow{\Delta d}=\vec{v} \times \Delta t$
- Remember $\overrightarrow{\Delta v}$ and $\overrightarrow{\Delta v}$ and vector quantities (magnitude and direction)

Let’s find the displacement of the boat relative to the bank.

First Demonstration

What frame of reference should we use?

What is the velocity of the boat in this frame of reference?

First Demonstration

We should use the person’s frame of reference

First Demonstration

In the person's frame of reference the boat's velocity is14cm/s west

$\overrightarrow{\Delta d} = \overrightarrow{v} \times \Delta t$

First Demonstration

- Now let’s find the displacement of the boat relative to its initial position on the river.
    - What frame of reference do we need to use?
    - What is the velocity of the boat in this frame of reference?

First Demonstration

- We should use the river's frame of reference
  - We can think of this frame of reference as an observer in a kayak floating with the current of the river
- The velocity is 42cm/s west.

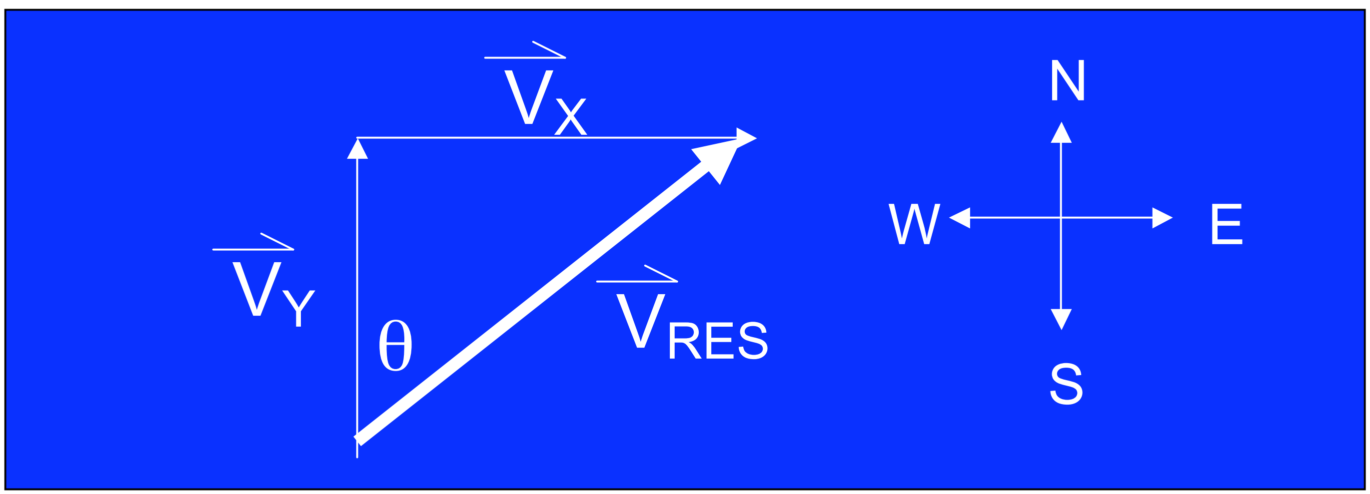
- The displacement is….
- $\Delta\vec{d} = \vec{V} \times \Delta t$
- Compare this value to the measured displacement

Adding another Dimension

- How does this differ from addition of vectors in 1 Dimension?
- Will we have to use Pythagoras and trigonometry?

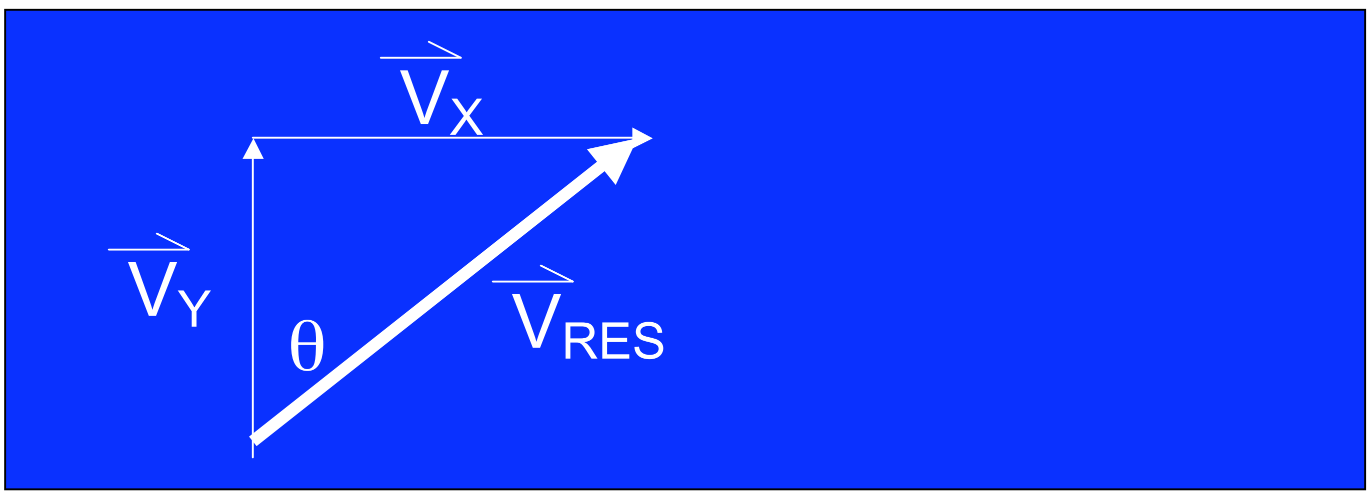
2 Dimensional Relative Motion

- When adding vector components the result is a “Resultant Vector”
- We will call the 2 velocity components $\vec{V}_X$ and $\vec{V}_Y$ and resultant $\vec{V}_{RES}$.

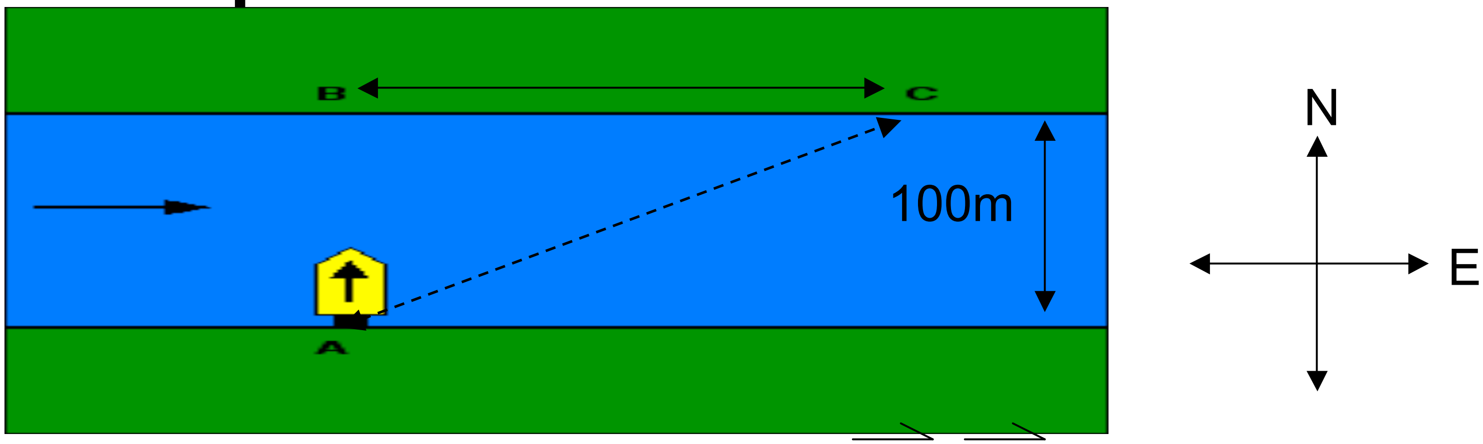


2 Dimensional Relative Motion

- Using Pythagoras we get $\vec{V}_{RES}^2 = \vec{V}_X^2 + \vec{V}_Y^2$
- If we only know one vector component, then we can use trigonometry to solve for the other vector components, if we know the angle θ.



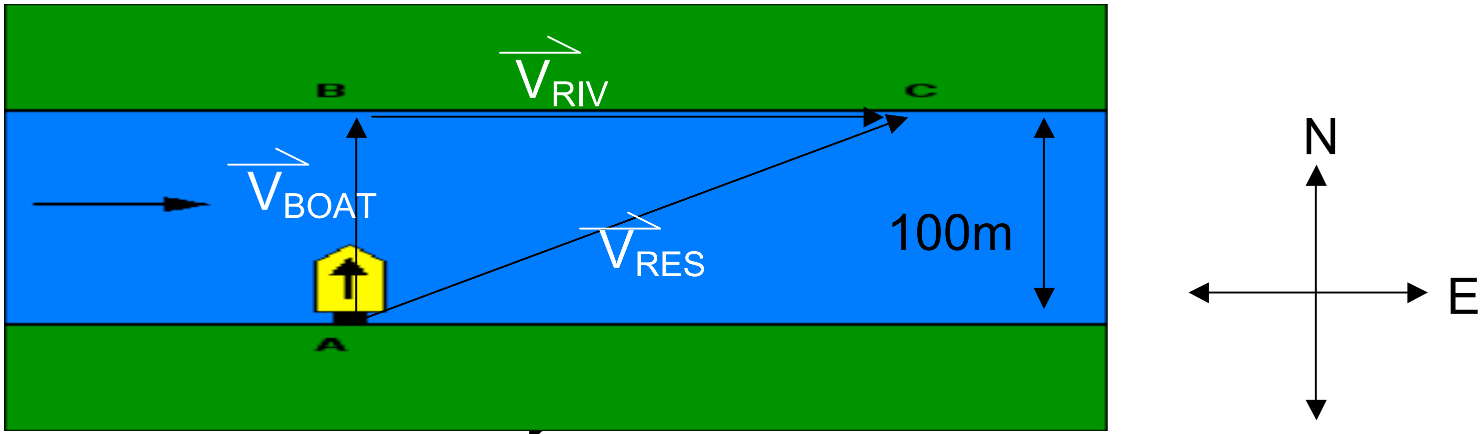
Example 2 - 2 Dimensions



- We have a boat with velocity $\vec{V}y = \vec{V}_{BOAT}$ 4m/s
  - $\vec{V}_{BOAT}$ is in the North direction
- The river has a velocity of 3m/s east
  - The current contributes to the $\vec{V}x$ component of the boat.
  - We can call the $\vec{V}_X$ component $\vec{V}_{RIV}$
  - $\vec{V}_{RES}$ is the resultant of these 2 vectors ($\vec{V}_{RIV}$ and $\vec{V}_{BOAT}$)

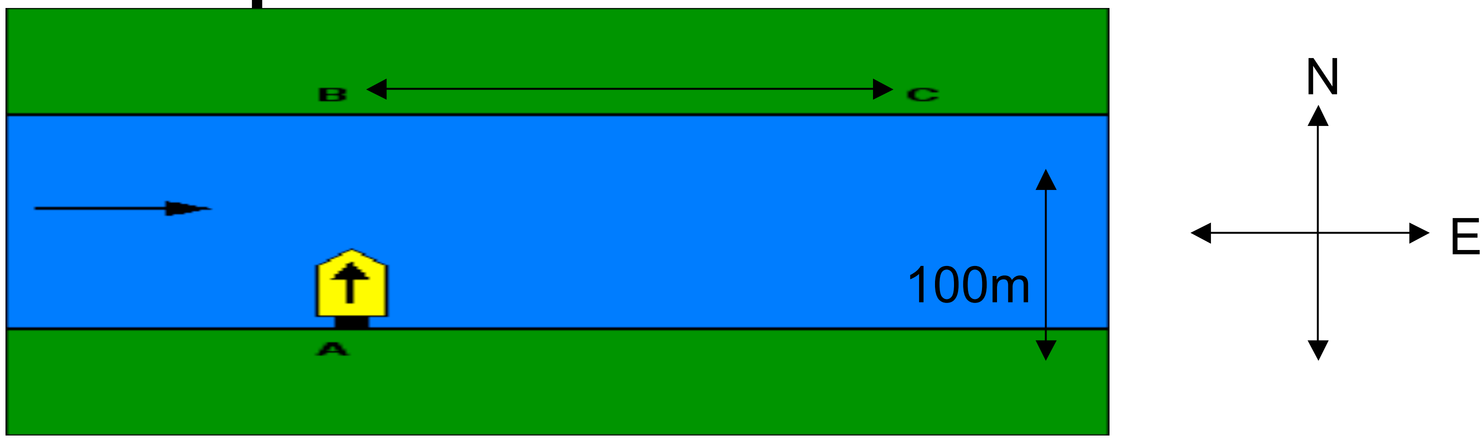
What is the resultant velocity of the boat and what is the displacement down the bank (the distance and direction between points B and C) if the river is 100m wide?

Example 2 - 2 Dimensions



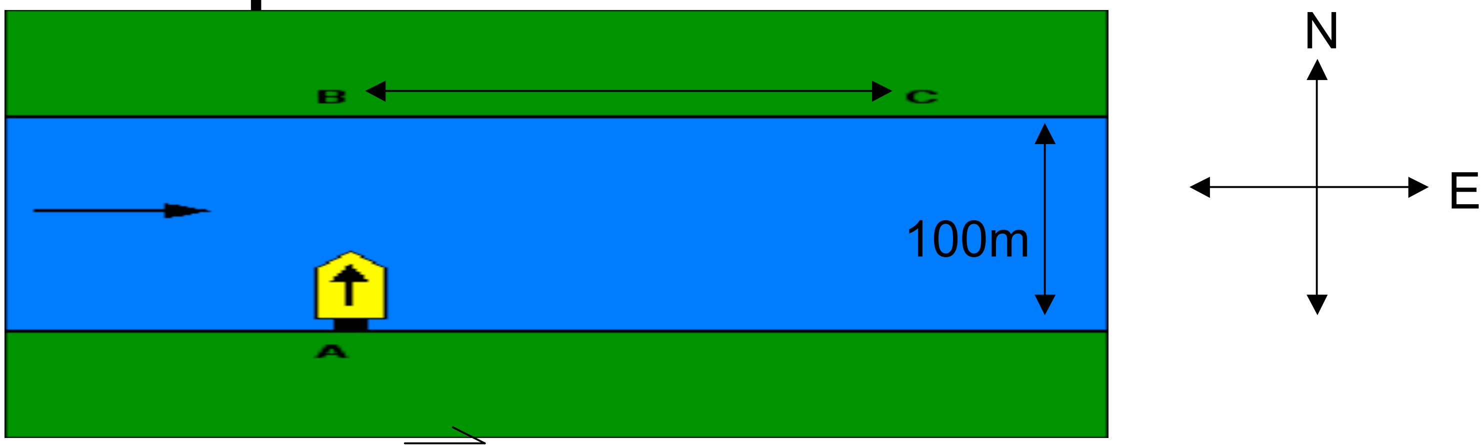
- Let's start by finding $\vec{V}_{RES}$ in the frame of reference of an observer standing on the bank.
  - $\vec{V}_{RES}^2 = (4m/s)^2 + (3m/s)^2$
  - $\vec{V}_{RES} = 5m/s$, in what direction?
  - Using trig. we have $\sin\theta = \vec{V}_{RIV}/\vec{V}_{RES}$
  - $\theta = 36.9^{\circ}$ North of East
- How do we calculate the time it takes to cross the river?
  - We can use the $\vec{V}_{BOAT}$ component (4m/s) and the width of the river (100m). $t = \vec{\Delta d}/\vec{V}_{BOAT}$ ===> t=25s

Example 2 - 2 Dimensions



- Now we need to determine the displacement downstream that the boat traveled as it crossed the river.
  - We know that t=25s, what component of velocity should we use to calculate the displacement of the boat along the bank (from point B to point C)?

Example 2 - 2 Dimensions



- We can use the $\vec{V}_{RIV}$ component of velocity which is in the direction of the current.
  - $\vec{V}_{RIV}$=3m/s, and t=25s
  - $\vec{d}=\vec{V}_{RIV}$ x t
  - $\vec{d}$=3m/s x 25s
  - $\vec{d}$=75m east (this is the distance the boat moved east due to the velocity of the river)

2nd Demonstration

- Let's simulate a boat crossing a river.
  - The boat is aiming directly across the river with a velocity ($\vec{V}_{BOAT}$) of 13cm/s north
  - The velocity of the river ($\vec{V}_{RIV}$) is 28cm/s east
  - The width of the river is 24cm

- What is the resultant velocity, $\vec{V}_{RES}$ relative to an observer at the launch point and what is the distance down river the boat travels

2nd Demonstration

- The most important concept that you should get from this demo is how to find the resultant vector.
  - We should first find the direction of the vector
  - Then we can find the magnitude of the vector
    - If we have time we can find the resultant velocities in the boat's and river's frame of reference
    - But first let's try another demonstration

3rd Demonstration

- This time the driver of the boat wants to land his boat directly across the river
  - What does this tell us?
    - In order to cross the river directly, the boat must be pointed upstream at some angle θ.
  - How do we determine the angle?
    - We know the velocity of the boat and the velocity of the river.
    - We can use trigonometry to determine the angle
    - We know $\vec{V}_{RIV}$= 28cm/s east
    - We know the magnitude of $\vec{V}_{BOAT}$ = 43cm/s
    - The angle θ will tell us the direction of $\vec{V}_{BOAT}$

3rd Demonstration

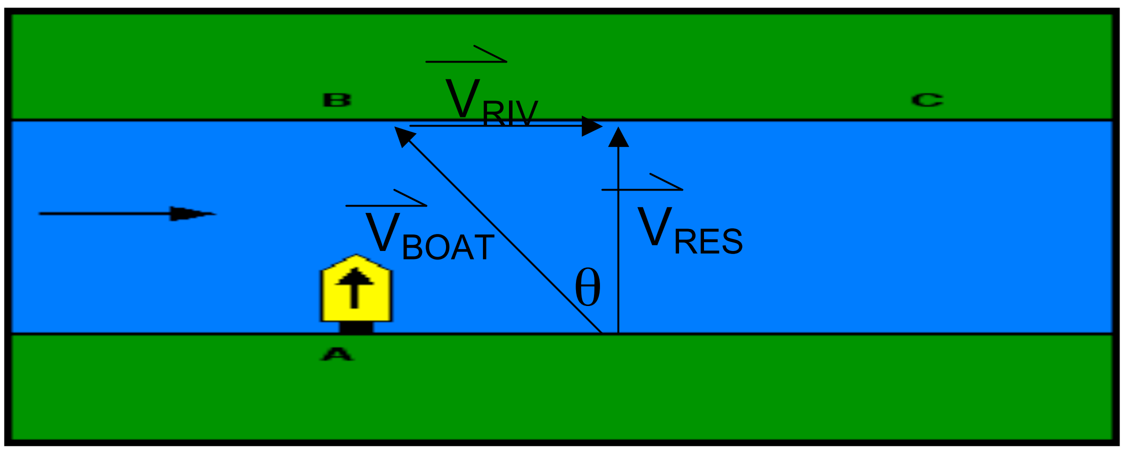
- Now that we know the angle $\theta=40°$ we can find the resultant velocity of the boat
- Let's find the relative resultant velocities of the 3 objects in each frame of reference
  - We will start with an observer standing at the launch point of the boat.

3rd Demonstration

Observer	Person’s Velocity	Boat’s Velocity	River’s Velocity
Person	0cm/s	33cm/s North	28cm/s East
Boat	33cm/s South	0m/s	43cm/s 40° E of S
River	28cm/s West	43cm/s 40° W of N	0m/s

Time for a Quiz

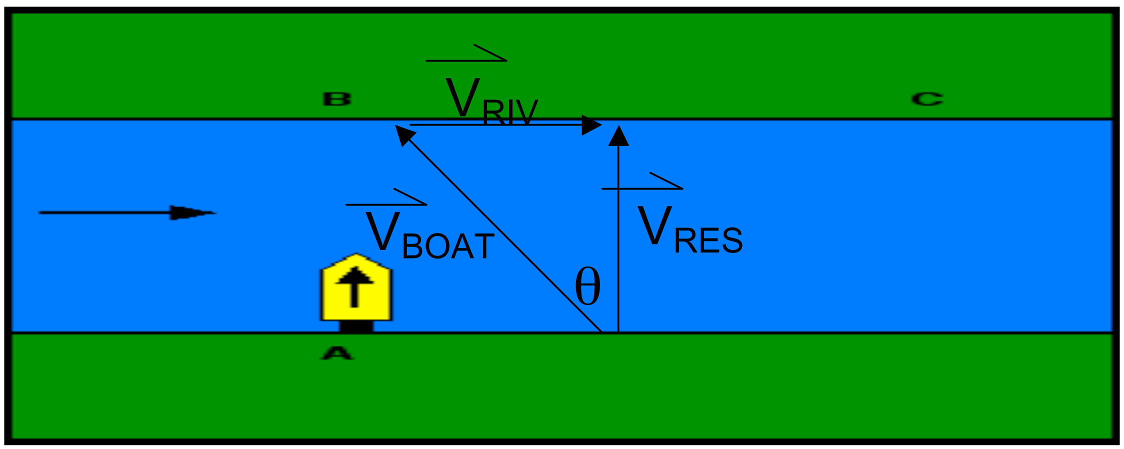
- If we increased the speed of the river will we need to point the boat at a greater or less angle upstream to land directly across the stream?



Time for a Quiz

- If we increased the speed of the river will we need to point the boat at at greater or less angle upstream to land directly across the stream?
  - We will need a greater angle upstream

- How does this change the component of the boat's velocity across the river ($\vec{V}_{RES}$)?



Time for a Quiz

- If we increased the speed of the river will we need to point the boat at at greater or less angle upstream to land directly across the stream?
  - We will need a greater angle upstream
- How does this change the component of the boat’s velocity across the river?
  - The boat’s velocity across the river would decrease and the time to cross the river would increase
- What happens if we increase the angle so the boat travels directly upstream?

Time for a Quiz

- If we increased the speed of the river will we need to point the boat at at greater or less angle upstream to land directly across the stream?
  - We will need a greater angle upstream

- How does this change the component of the boat’s velocity across the river?
  - The boat’s velocity across the river would decrease and the time to cross the river would increase

- What happens if we increase the angle so the boat travels directly upstream?
  - The boat would NEVER get across the river. The boat would not have a velocity component across the river

Time for a Quiz

- Last question.
- If we want the boat to cross the river in the least amount of time, what direction should the boat be pointed?

Time for a Quiz

- Last question.
- If we want the boat to cross the river in the least amount of time, what direction should the boat be pointed?
    - We want the velocity component across the river to be as big in magnitude as possible
    - We should aim our boat perpendicular to the current!

That’s it…