Oscillations

- Simple Harmonic Motion (SHM)
- Position, Velocity, Acceleration
- SHM Forces
- SHM Energy
- Period of oscillation
- Damping and Resonance

Revision problem

Please try problem #31 on page 480

A pendulum clock keeps time by the swinging of a uniform solid rod…

Simple Harmonic Motion

- Pendulums
- Waves, tides
- Springs

Simple Harmonic Motion

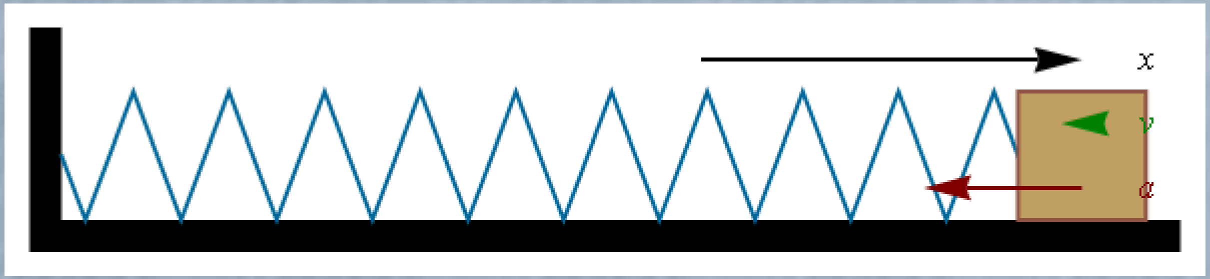
Requires a force to return the system back toward equilibrium

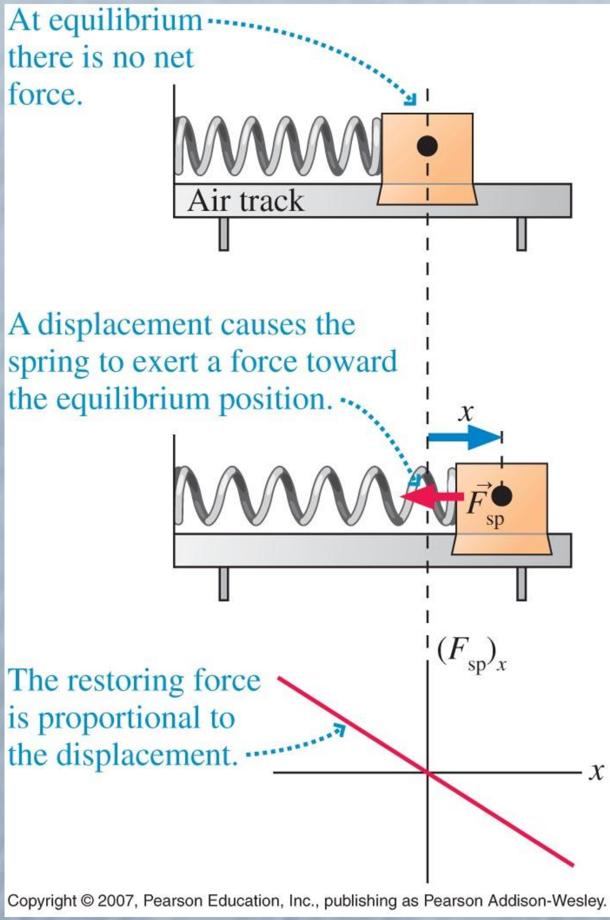
- Spring – Hooke’s Law
- Pendulum and waves and tides – gravity

Oscillation about an equilibrium position with a linear restoring force is always **simple harmonic motion** (SHM)

Springs

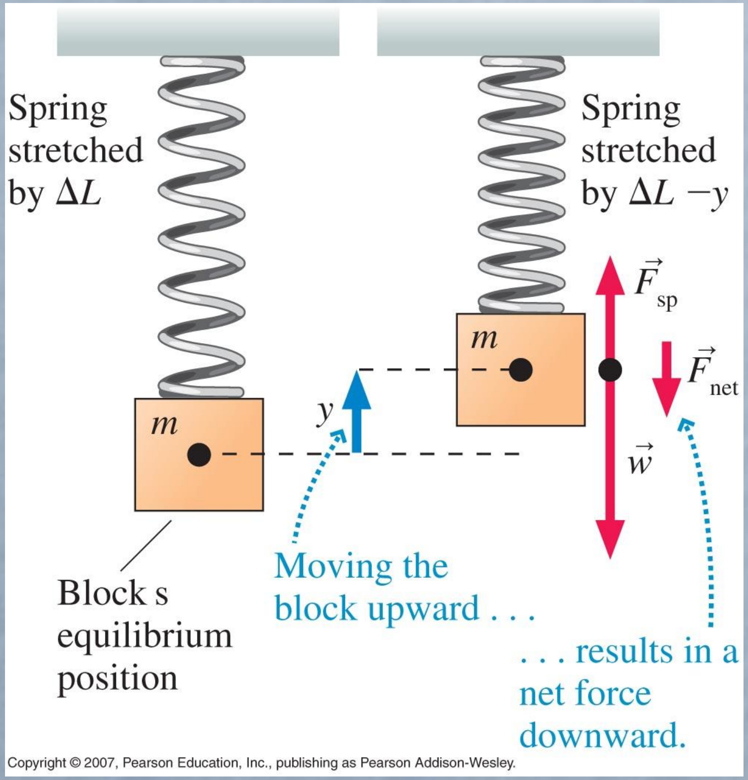
Hooke's Law F=-kx

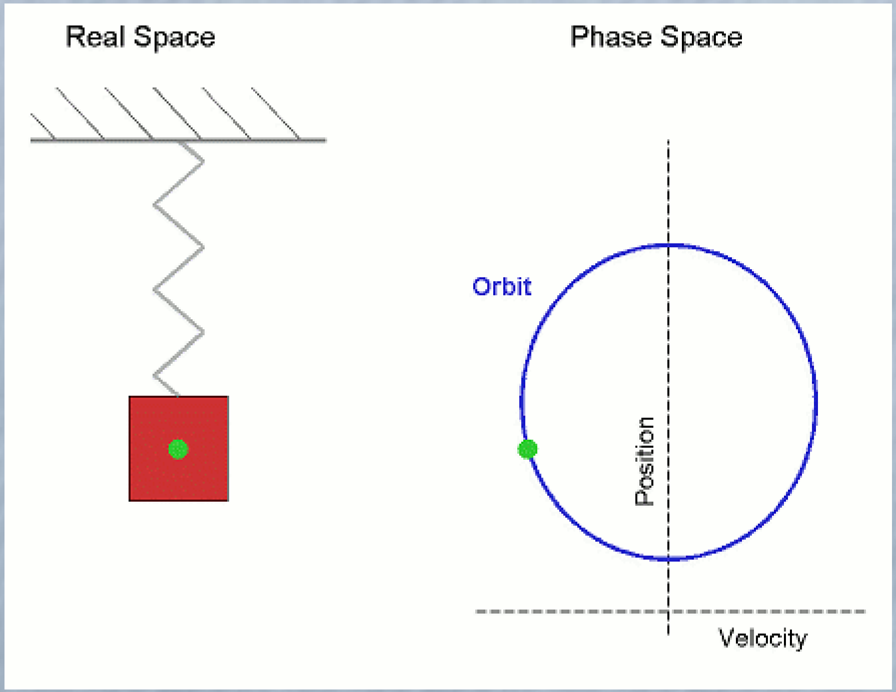




Springs

Hooke's Law F=-kx

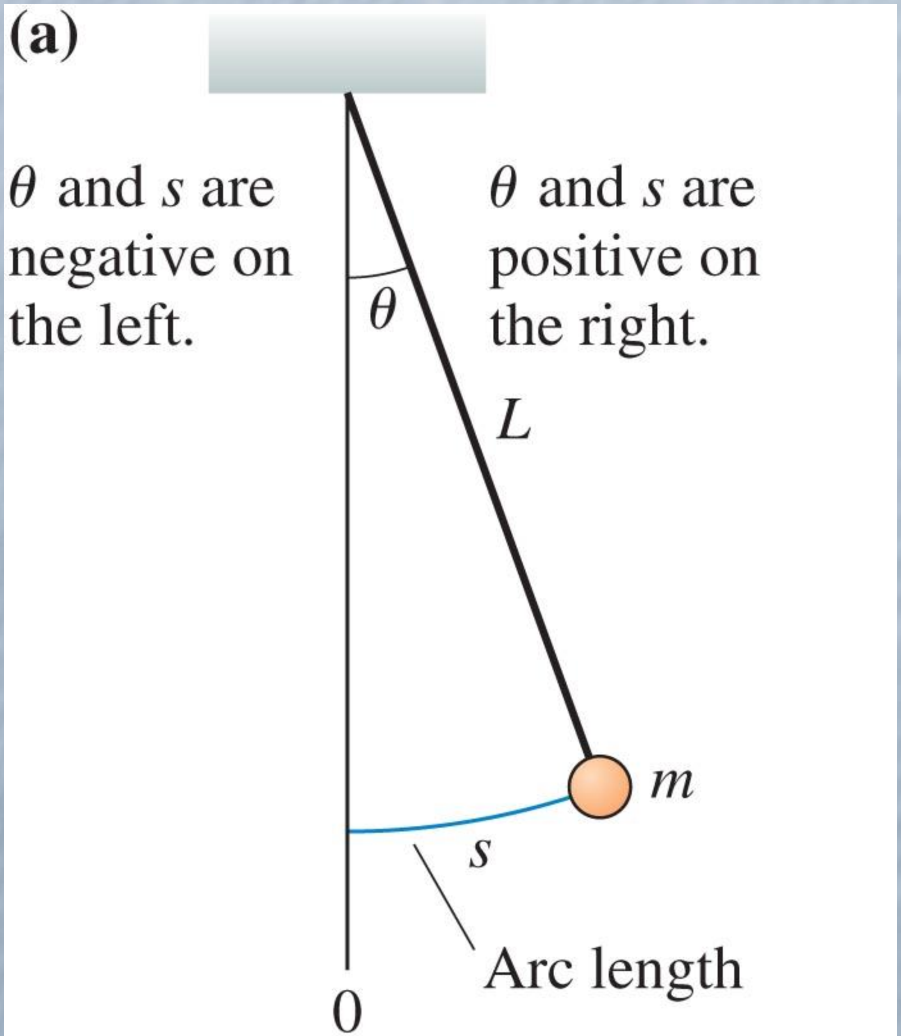




Pendulum

For a small angle, the force is proportional to angle of deflection, θ.

$$F_{return} = mg\sin\theta$$

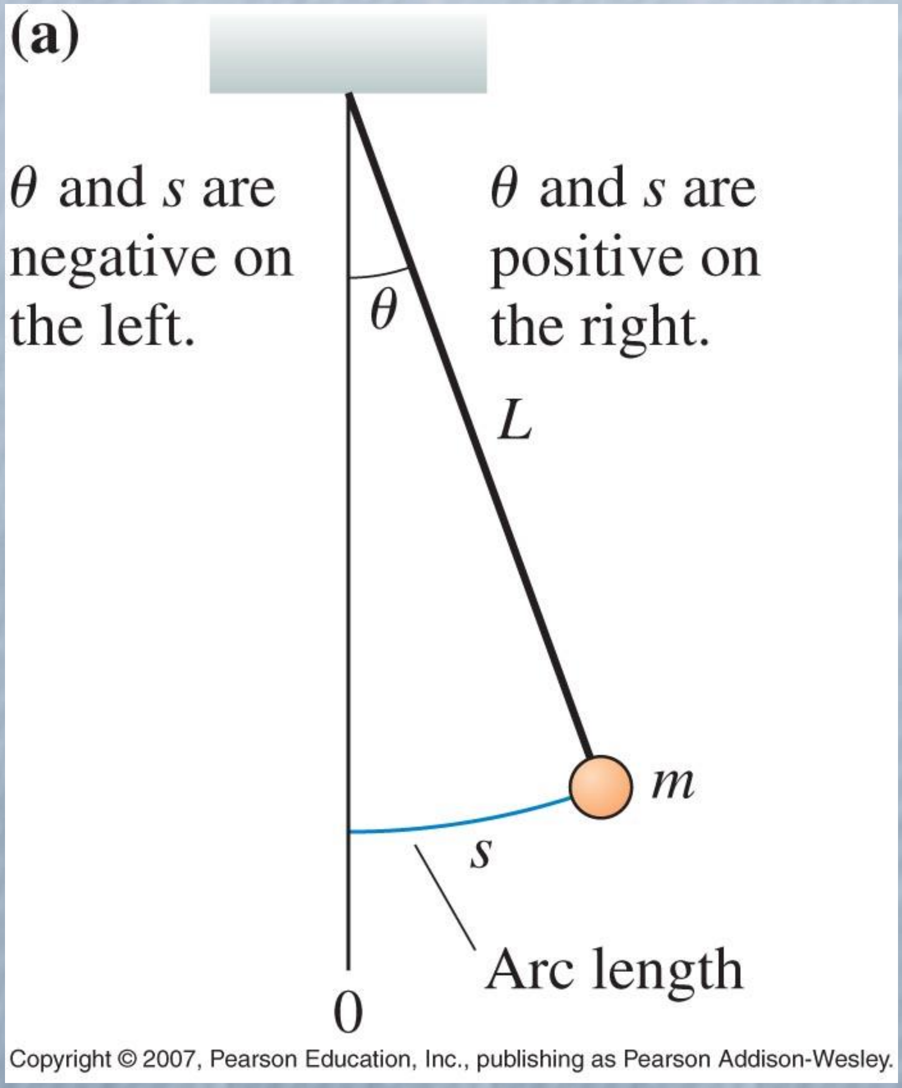


Pendulum

For a small angle, the return force is proportional to the distance from the equilibrium point:

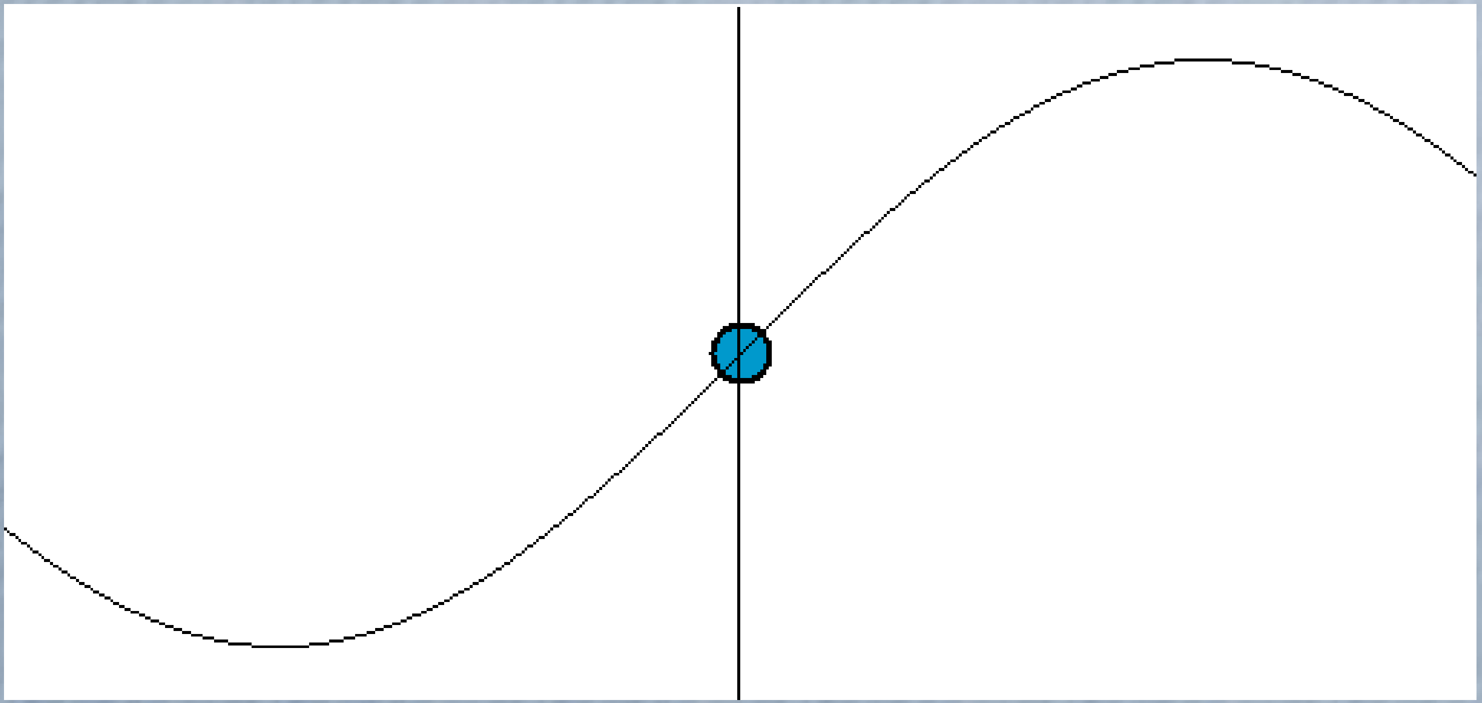
$$\theta \approx \sin\theta = \frac{s}{L}$$

$$F_{return} = -mg\theta = -\left(\frac{mg}{L}\right)s$$



Kinematics of SHM

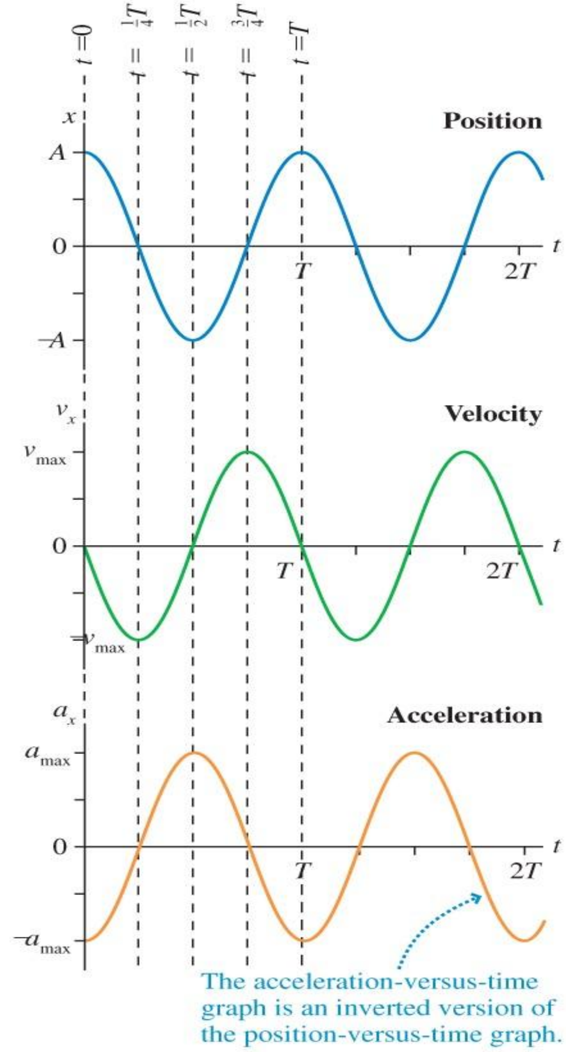
Simple Harmonic motion can be described by a sinusoidal wave for displacement, velocity and acceleration:



Kinematics of SHM

- The angle for the sinusoidal wave changes with time.
- It goes full circle 0 to 2π radians in one period of revolution, T.

$$x(t) = A\cos\left(\frac{2\pi t}{T}\right)$$



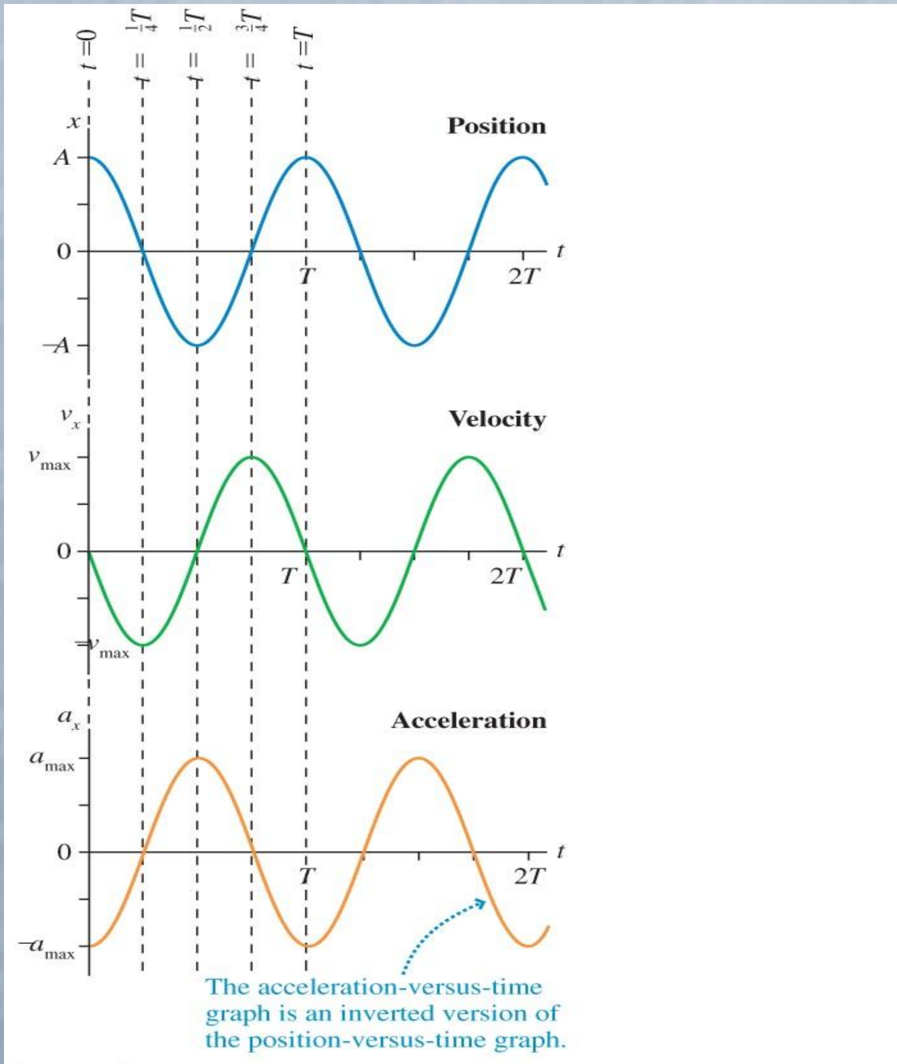
Kinematics of SHM

•We define the frequency of revolution as

$$f = \frac{1}{T}$$

$$x(t) = A\cos\left(2\pi f t\right)$$

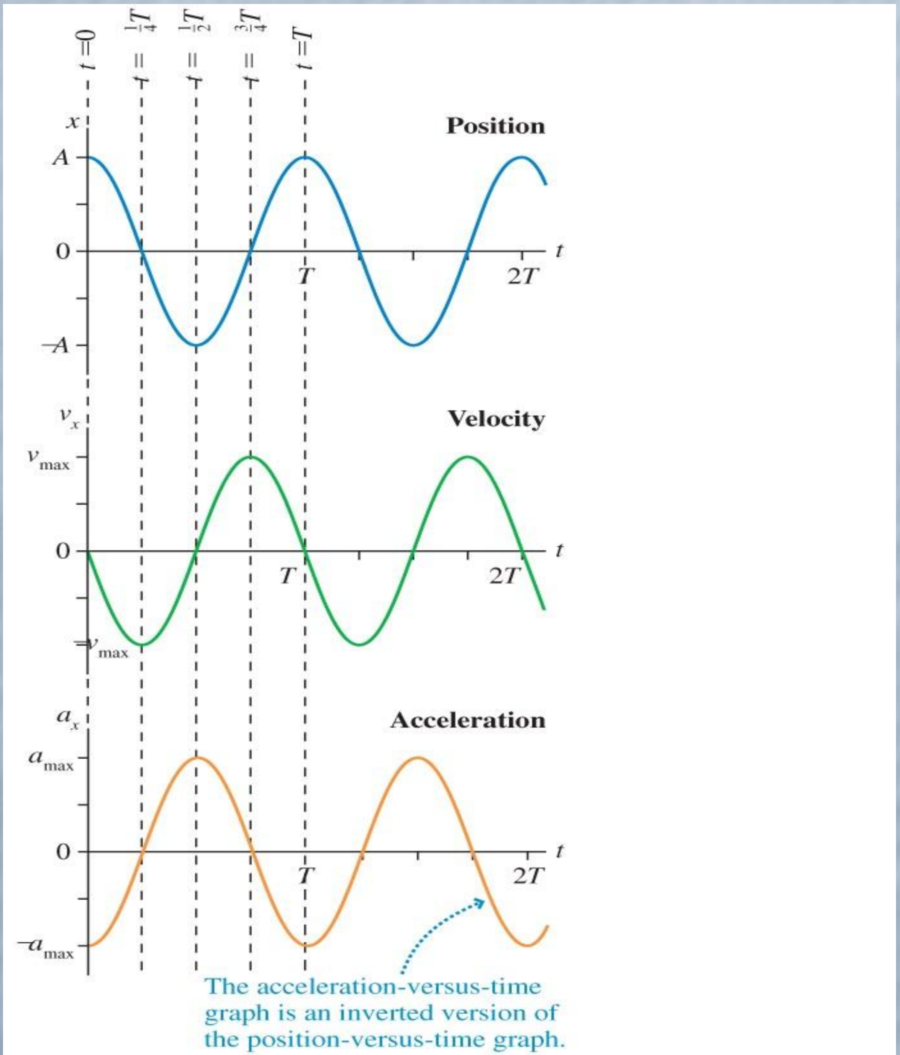
Frequency, f, has units s^{-1} or Hertz, Hz



Kinematics of SHM

- Velocity is 90º or π/2 radians out of phase:

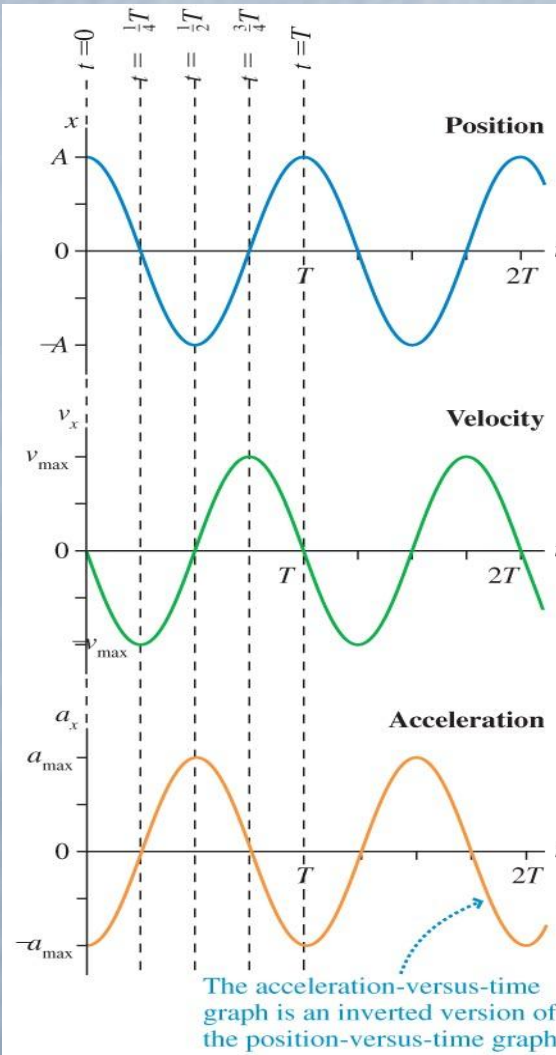
$$v(t) = -v_{\max} \sin(2\pi f t)$$



Kinematics of SHM

- Acceleration is 180º or π radians out of phase

$$a(t) = -a_{\max} \cos(2\pi f t)$$



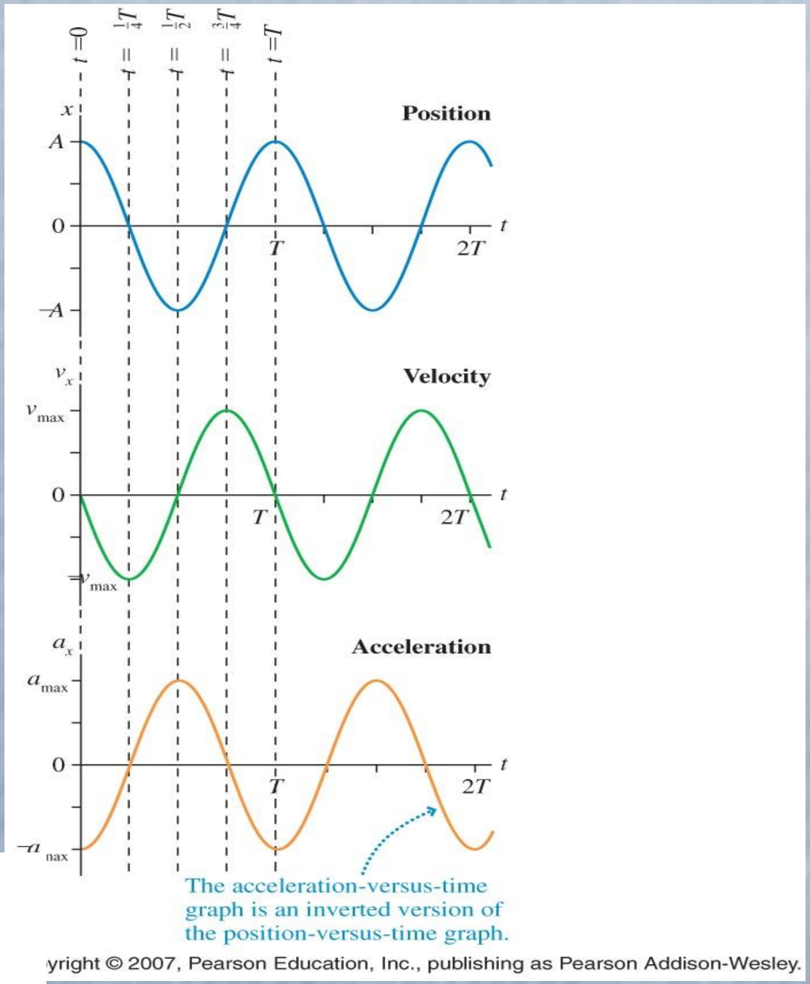
Kinematics of SHM

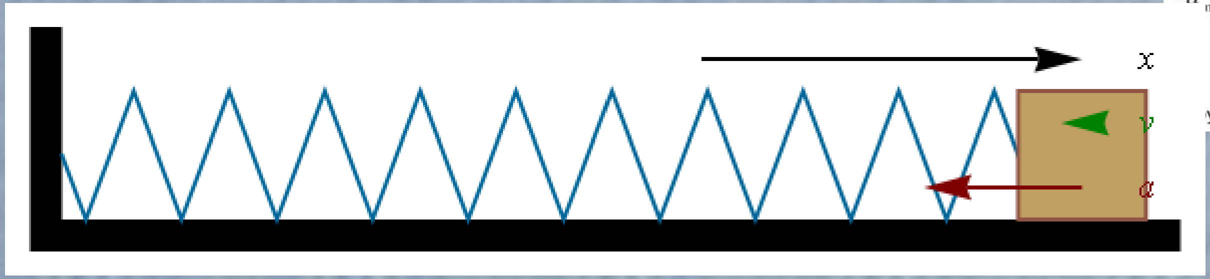
SHM equations of motion

$$x(t) = A\cos(2\pi f t)$$

$$v(t) = -v_{\max}\sin(2\pi f t)$$

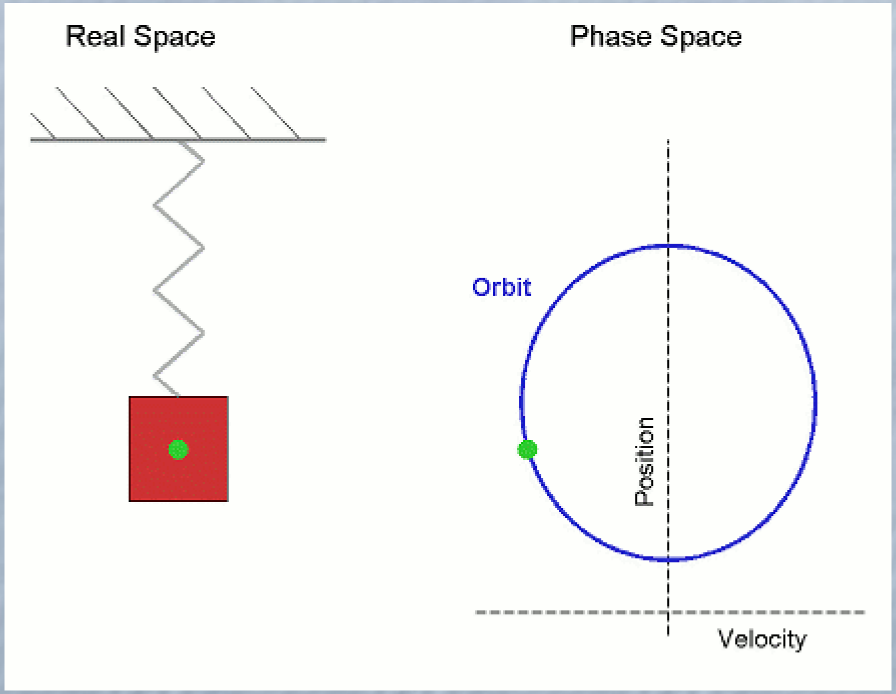
$$a(t) = -a_{\max}\cos(2\pi f t)$$





Calculating v_{max}

A circular motion when looked end-on gives us a velocity like:



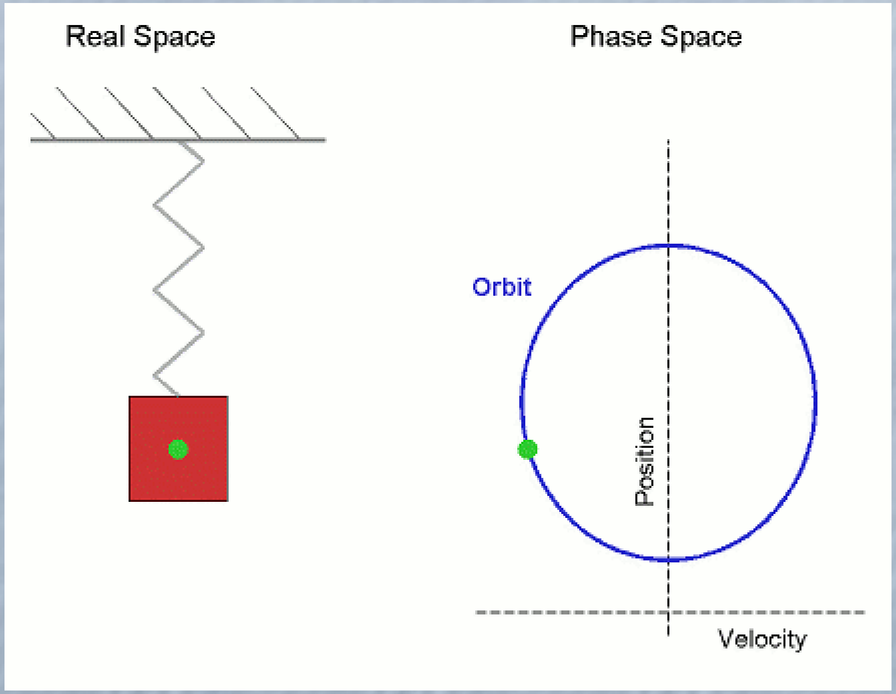
$$v = -v_{max} \sin(2\pi f t)$$

Calculating v_{max}

The velocity around the circle will be

$$v_{max} = \frac{D}{T} = \frac{2\pi A}{T}$$

$$v_{max} = 2\pi f A$$

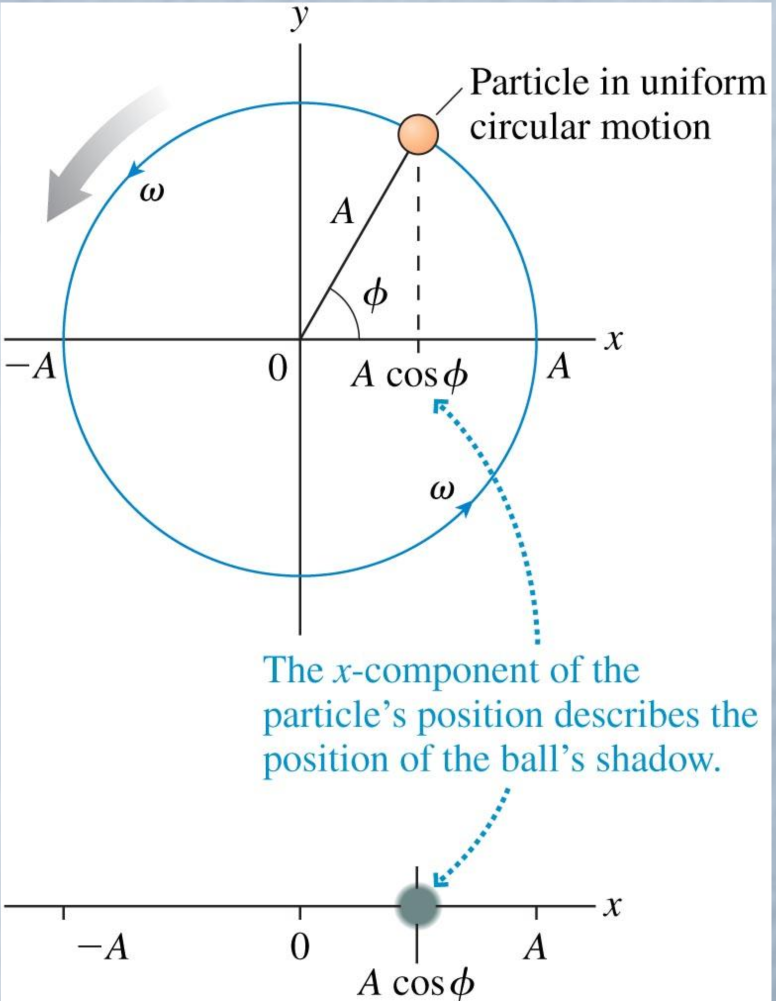


Calculating a_{max}

For circular motion, we know about acceleration and forces

$$F = ma, F = \frac{mv^2}{r}$$

$$a_{max} = \frac{v_{max}^2}{A}$$



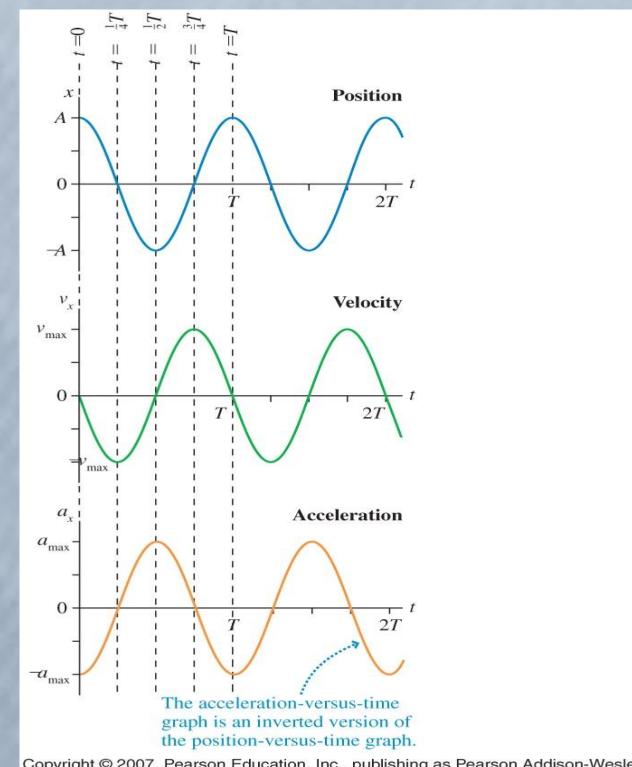
Kinematics of SHM

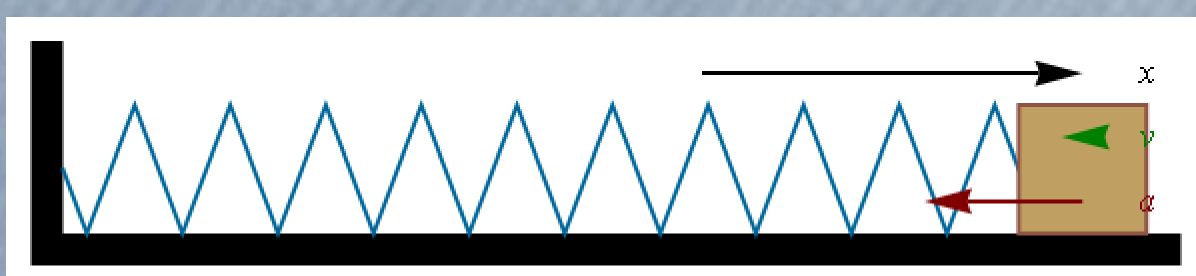
SHM equations of motion

$$x(t) = A\cos(2\pi f t)$$

$$v(t) = -2\pi f A\sin(2\pi f t)$$

$$a(t) = -(2\pi f)^2 A\cos(2\pi f t)$$





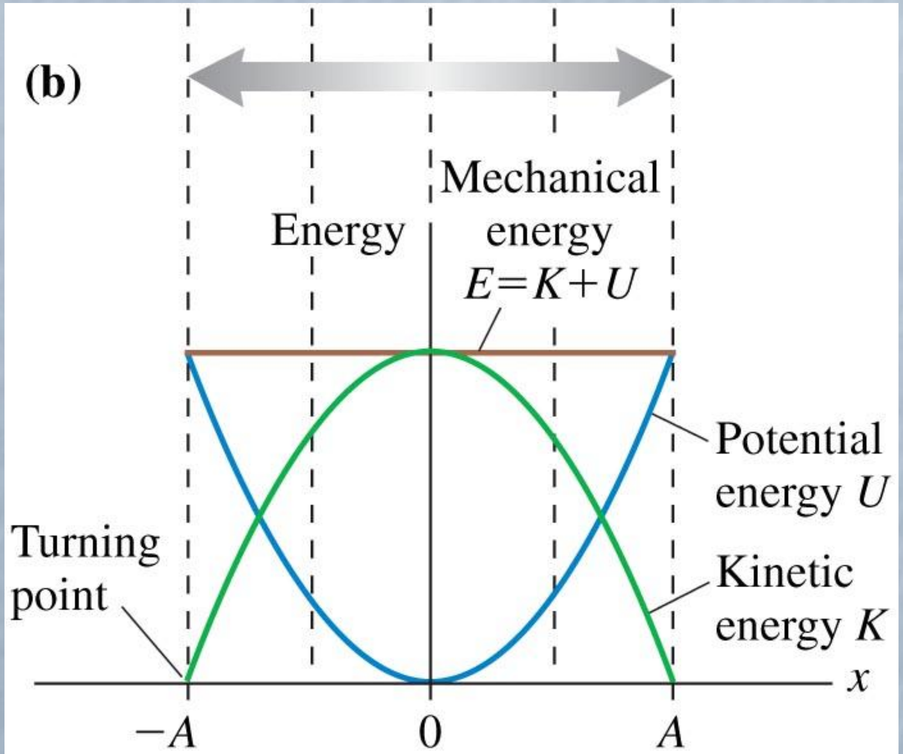
SHM and Energy

- Energy is conserved:
- Bounces between kinetic and potential energy

$$E_{total} = E_{kinetic} + E_{potential}$$

$$E_{kinetic} = \frac{1}{2} m v^2$$

$$E_{potential} = \frac{1}{2} k x^2$$

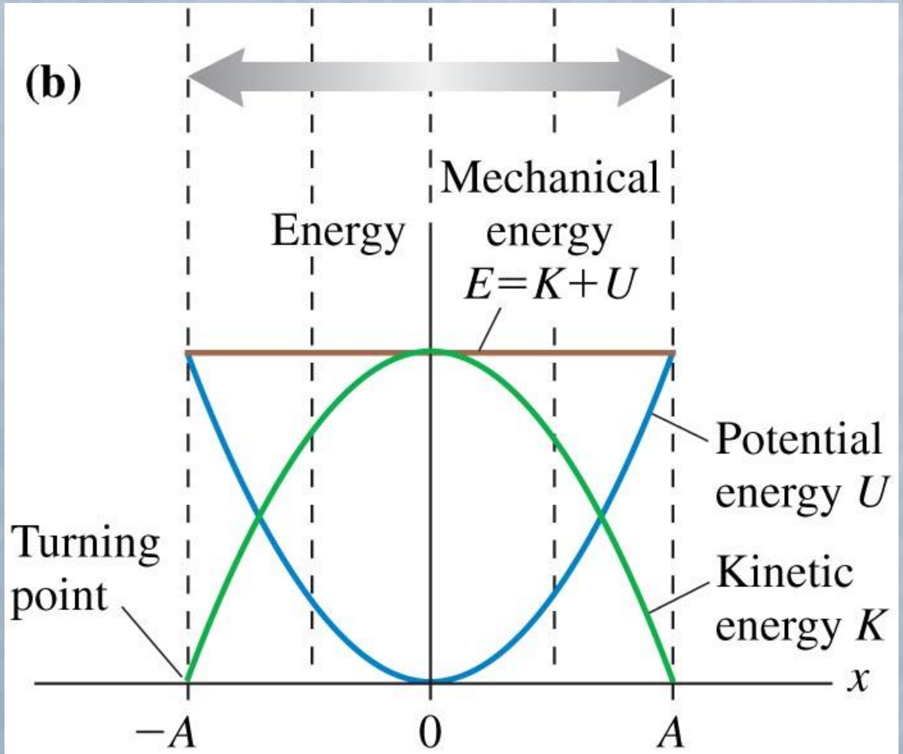


SHM and Energy

- The max KE must equal the max PE:

$$\frac{1}{2}m(v_{\max})^2 = \frac{1}{2}kA^2$$

$$v_{\max} = \sqrt{\frac{k}{m}}A$$



Finding the period of oscillation for a spring

We now have 2 equations for v_{max}:

$$v_{\max} = \sqrt{\frac{k}{m}}A = 2\pi f A$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}, T = 2\pi\sqrt{\frac{m}{k}}$$

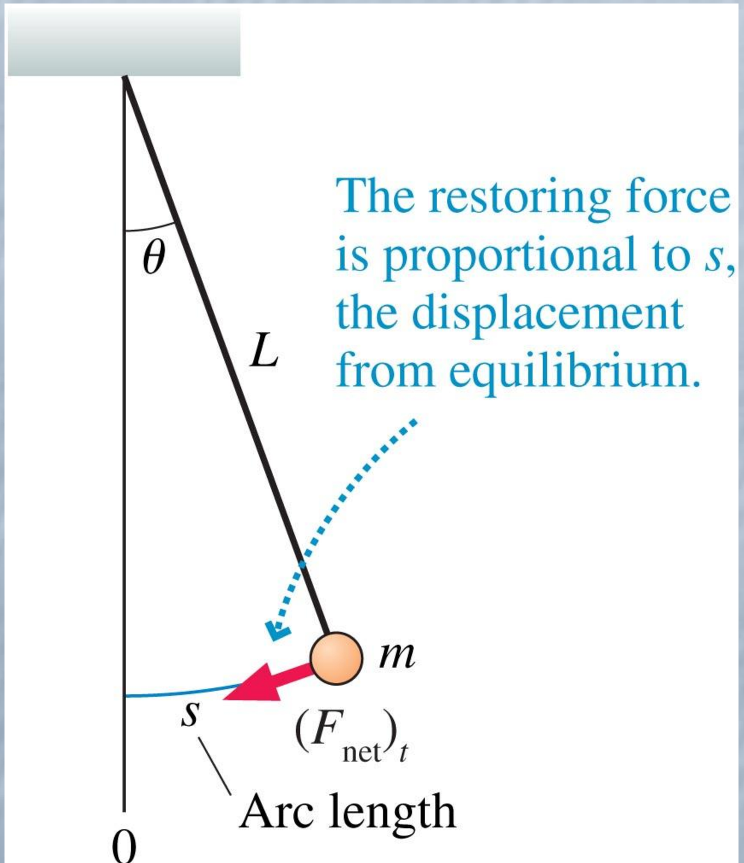
Period of oscillation is independent of the amplitude of the oscillation.

Finding the period of oscillation for a pendulum

Consider the acceleration using the equation for the return force, and the relation between acceleration and displacement:

$$a = \frac{F}{m} = \frac{1}{m}\left(\frac{mg}{L}\right)s$$

$$a_{\max} = (2\pi f)^2 A = \frac{g}{L} A$$

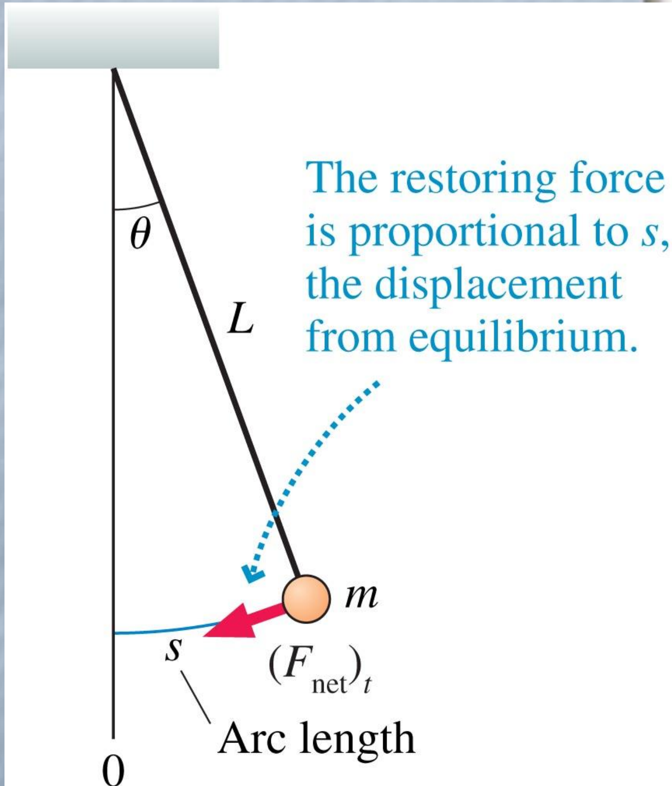


Finding the period of oscillation for a pendulum

We can calculate the period of oscillation

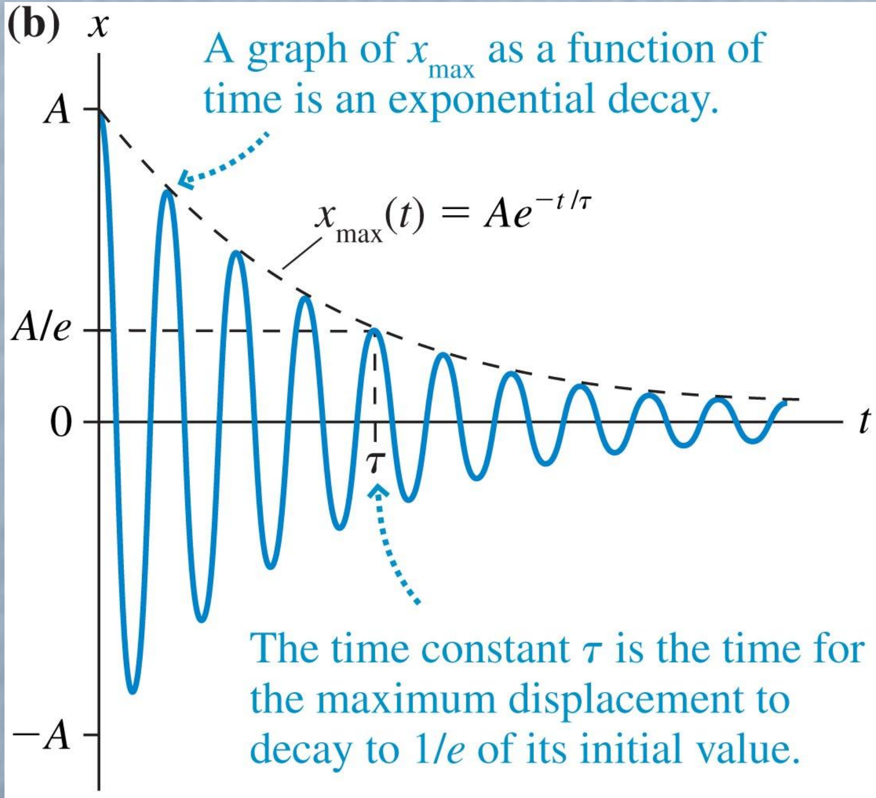
$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}, T = 2\pi\sqrt{\frac{L}{g}}$$

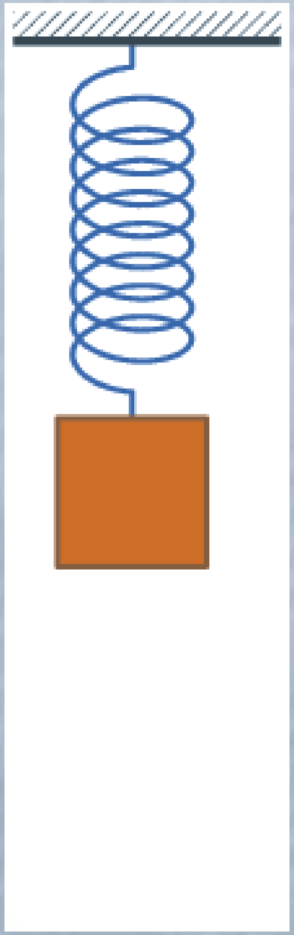
Period is independent of the mass, and depends on the effective length of the pendulum.



Damped Oscillations

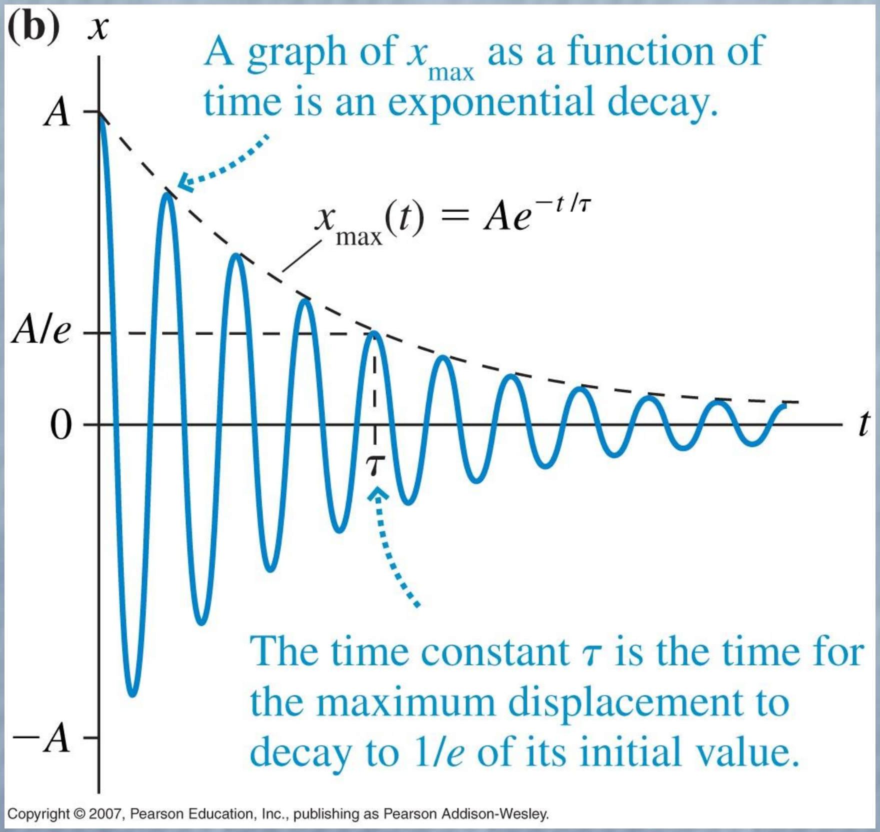
All the oscillating systems have friction, which removes energy, damping the oscillations



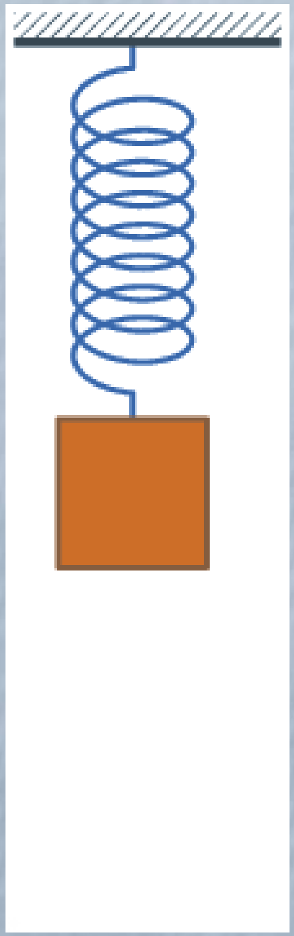


Damped Oscillations

We have an exponential decay of the total amplitude

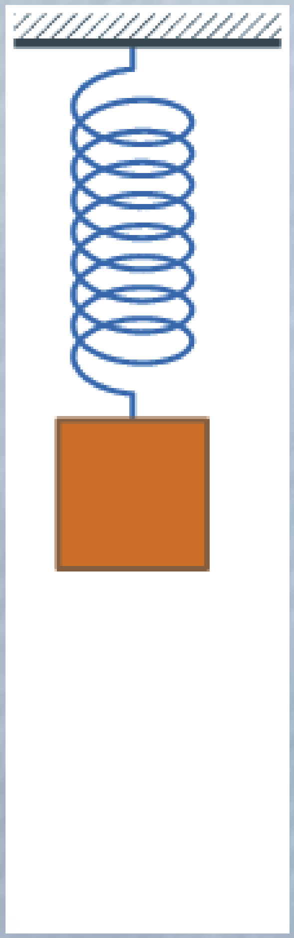


$$x_{max}(t) = Ae^{-t/\tau}$$



Damped Oscillations

The **time constant**, τ, is a property of the system, measured in seconds



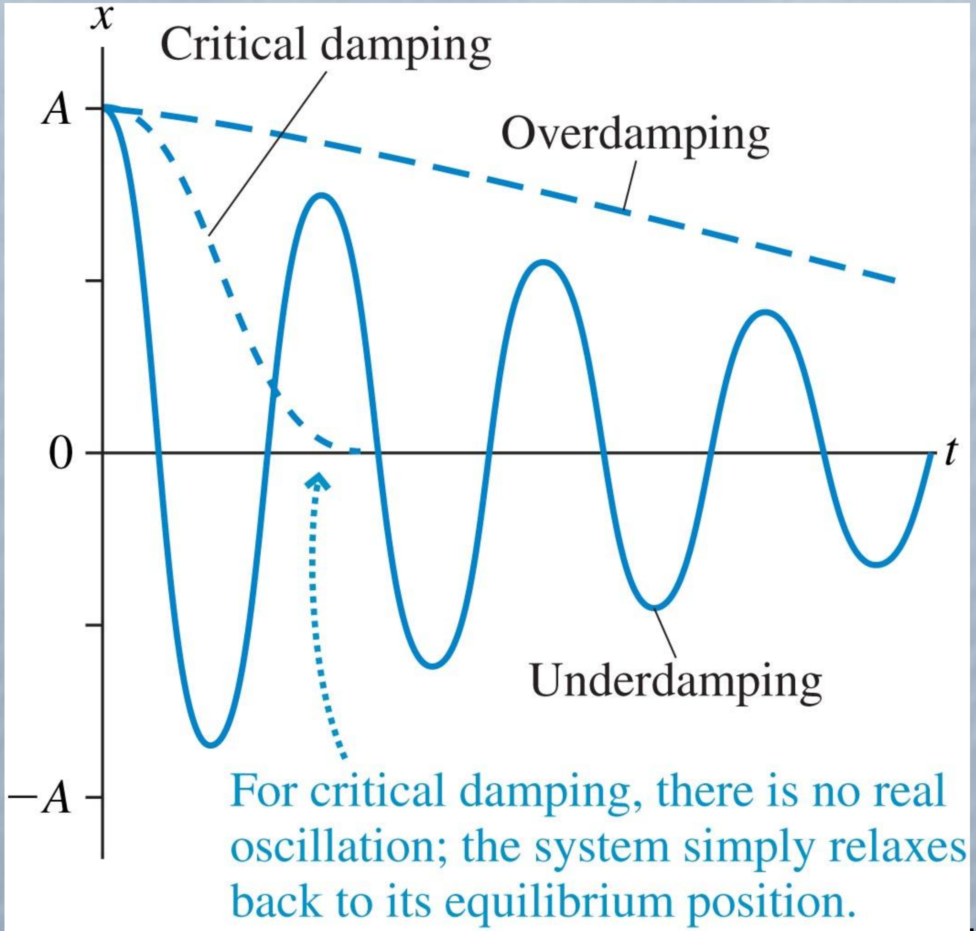
$$x_{\max}(t) = Ae^{-t/\tau}$$

•A smaller value of τ means more damping – the oscillations will die out more quickly.

•A larger value of τ means less damping, the oscillations will carry on longer.

Damped Oscillations

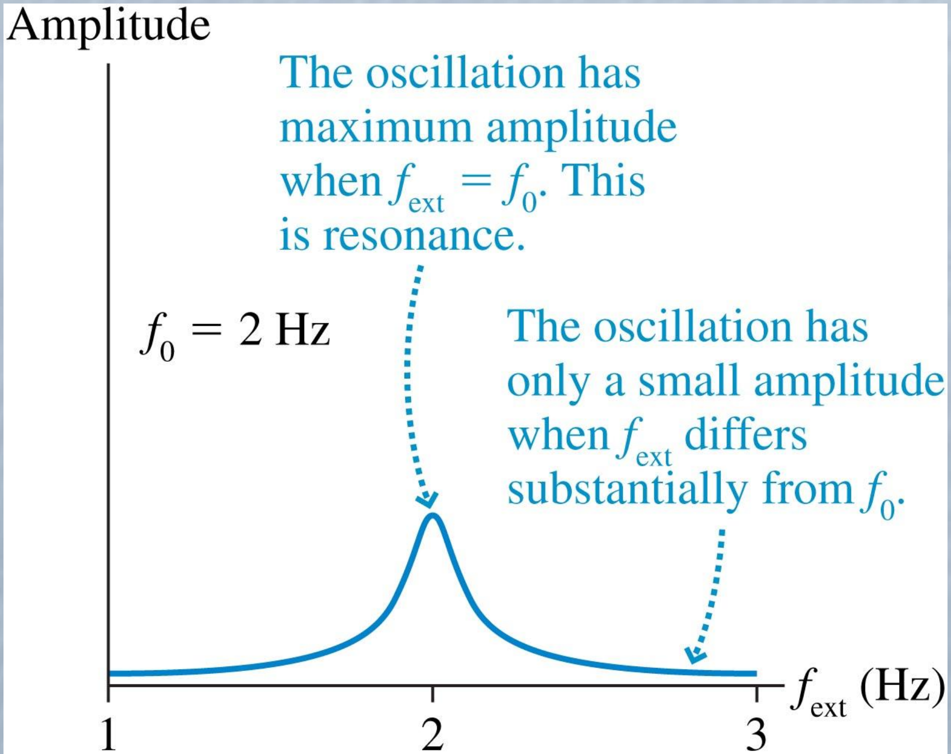
- under-damped τ>>T
- critically-damped τ~T
- over-damped τ<<T



Driven Oscillations and Resonance

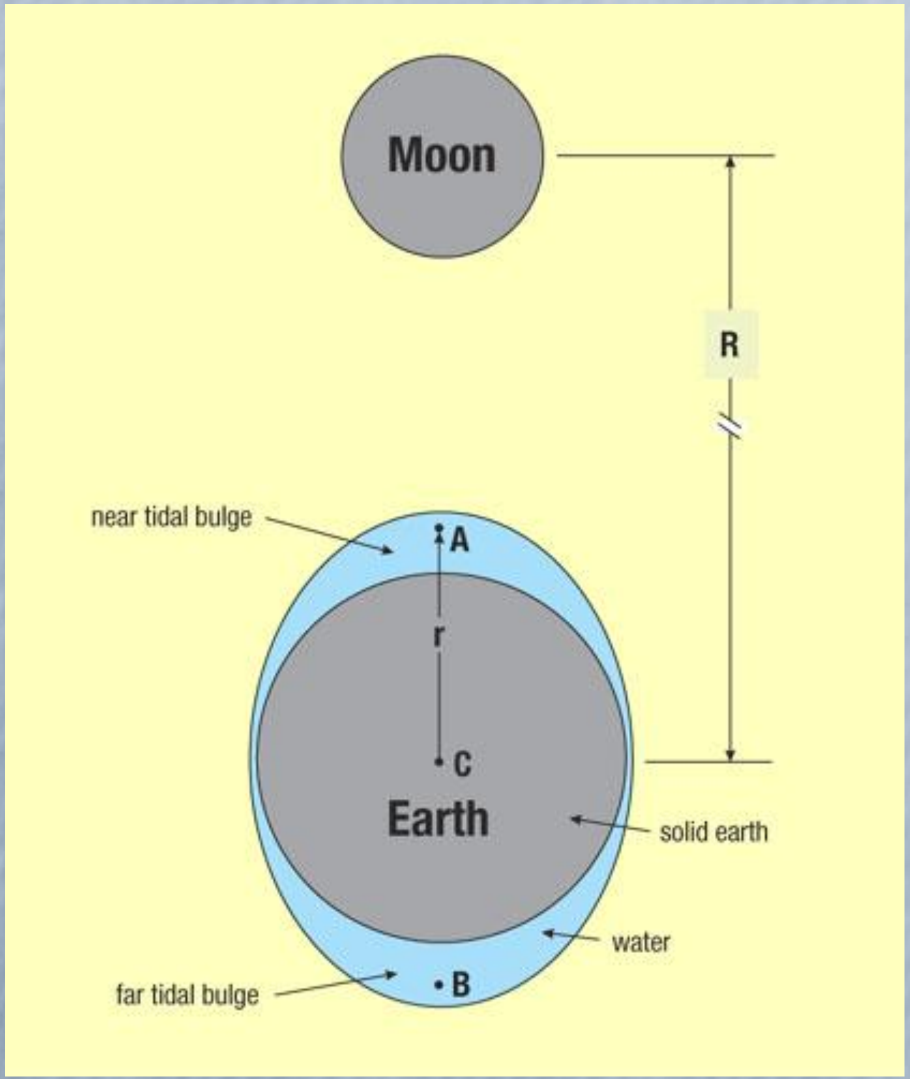
An oscillator can be driven at a different frequency than its resonance or **natural frequency**.

The amplitude can be large if the system is undamped.



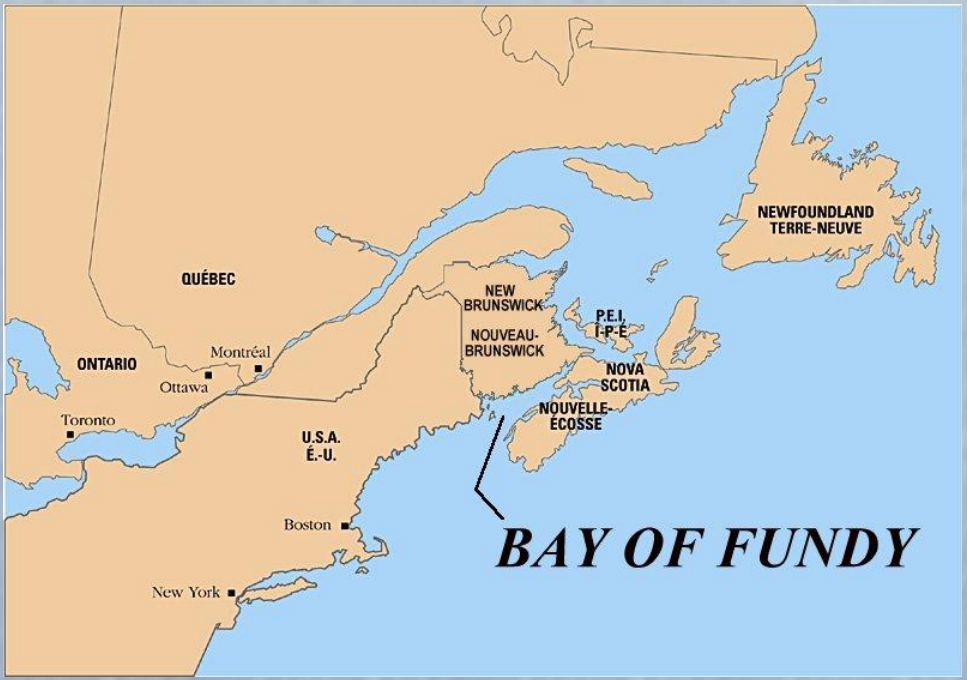
Tidal resonances

- Ocean tides are produced from the Moon (and Sun) gravitational pull on the oceans to make a 20cm wave.
- Moon drives the wave at 12 hours 25 minutes



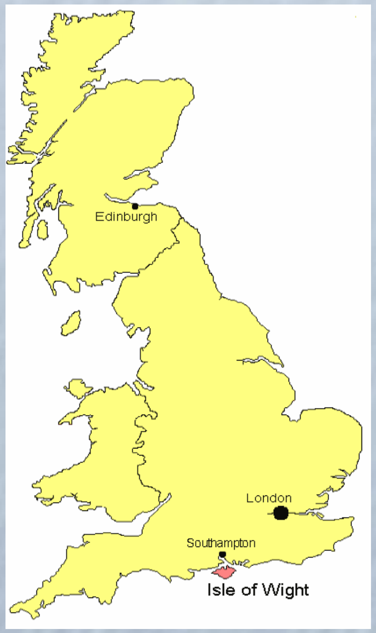
Tidal resonances

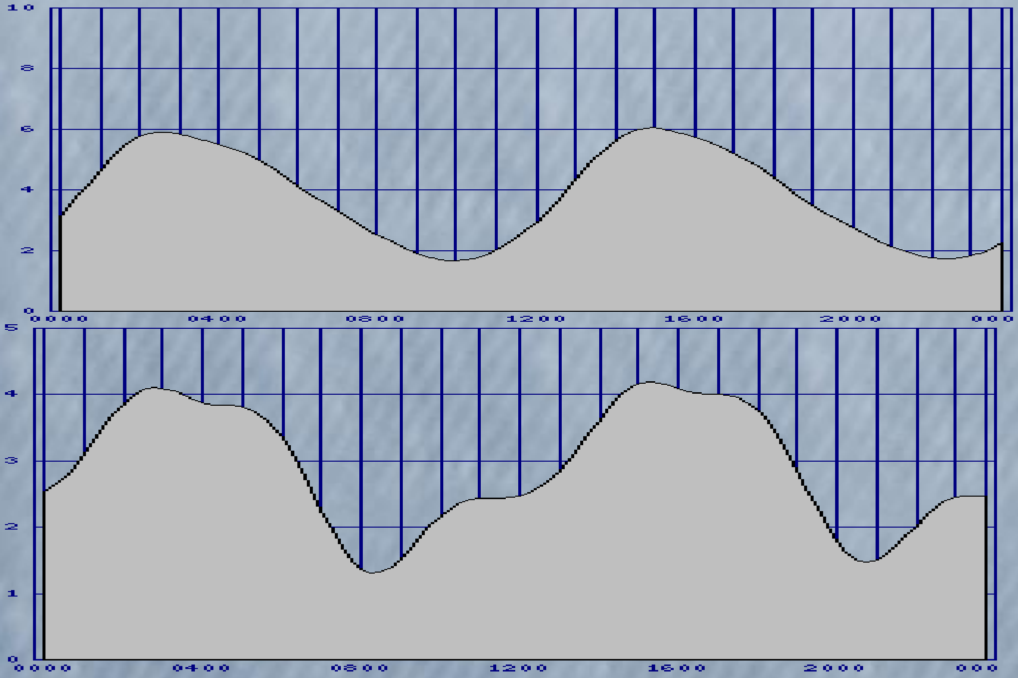
The natural resonance of local geography can affect this: e.g. Bay of Fundy in Canada where the tidal range is amplified from the 20cm wave to 16m.



Tidal resonances

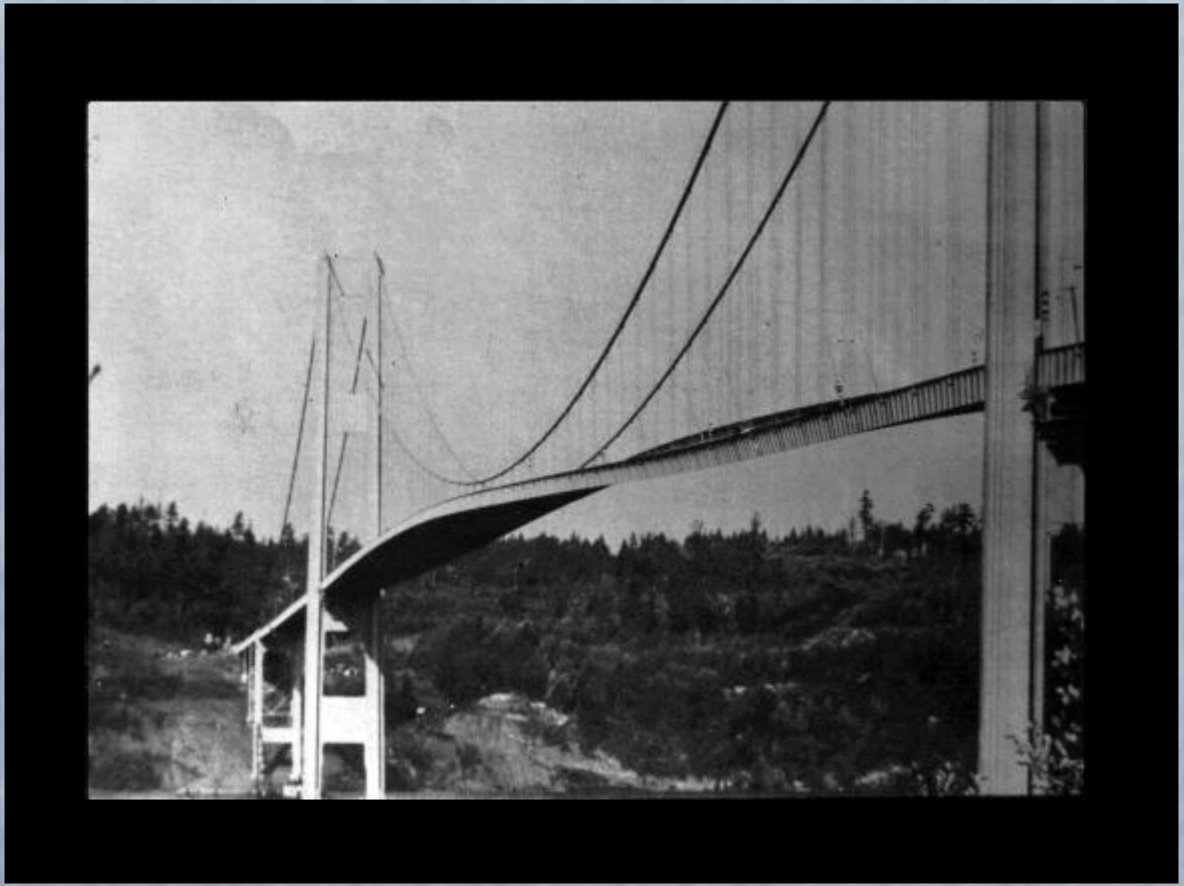
Natural geography can also make double tides:





Undamped driven resonance

Tacoma Narrows Bridge,
Washington State, 1940



Summary

- Simple Harmonic Motion (SHM)
- Position, Velocity, Acceleration
- SHM Forces
- SHM Energy
- Period of oscillation
- Damping and Resonance

Homework problems

Chapter 14 Problems

48, 49, 50, 52, 54, 59, 62, 63