Chapter 2

Present Value

Road Map

Part A Introduction to finance.

- Financial decisions and financial markets.
- Present value.

Part B Valuation of assets, given discount rates.

Part C Determination of risk-adjusted discount rates.

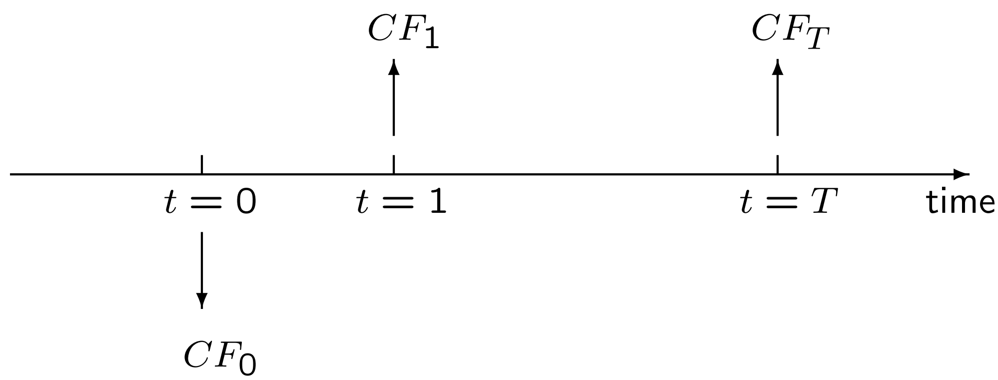
Part D Introduction to derivatives.

Main Issues

- Present Value
- Compound Interest Rates
- Nominal versus Real Cash Flows and Discount Rates
- Shortcuts to Special Cash Flows

1 Valuing Cash Flows

"Visualizing" cash flows.



Example. Drug company develops a flu vaccine.

- Strategy A: To bring to market in 1 year, invest $1 B (billion) now and returns $500 M (million), $400 M and $300 M in years 1, 2 and 3 respectively.
- Strategy B: To bring to market in 2 years, invest $200 M in years 0 and 1. Returns $300 M in years 2 and 3.

Which strategy creates more value?

Problem. How to value/compare CF *streams*.

1.1 Future Value (FV)

How much will \$1 today be worth in one year?

Current interest rate is r, say, 4%.

- \$1 investable at a rate of return $r = 4\%$.
- FV in 1 year is

$$FV = 1 + r = \$1.04.$$

- FV in t years is

$$\begin{aligned} FV &= \$1 \times (1+r) \times \cdots \times (1+r) \\ &= (1+r)^t. \end{aligned}$$

Example. Bank pays an annual interest of 4% on 2-year CDs and you deposit \$10,000. What is your balance two years later?

$$FV = 10,000 \times (1 + 0.04)^2 = \$10,816.$$

1.2 Present Value (PV)

We can ask the question in reverse (interest rate $r = 4\%$).

What is the PV of $1 received a year from now?

- Consider putting away $1/1.04$ today. A year later receive:
$$\frac{1}{1.04} \times (1+0.04) = 1.$$
- The PV of $1 received a year from now is:
$$\frac{1}{1+r} = \frac{1}{1+0.04}.$$
- The present value of $1 received t years from now is:
$$PV = \frac{1}{(1+r)^t}.$$

Example. (A) $10 M in 5 years or (B) $15 M in 15 years. Which is better if $r = 5\%$?

$$PV_A = \frac{10}{1.05^5} = 7.84.$$
$$PV_B = \frac{15}{1.05^{15}} = 7.22.$$

Solution to Example. Flu Vaccine.

Assume that $r = 5\%$.

Strategy A:

Time	0	1	2	3
Cash Flow	-1,000	500.0	400.0	300.0
Present Value	-1,000	476.2	362.8	259.2
			Total PV	98.2

Strategy B:

Time	0	1	2	3
Cash Flow	-200	-200.0	300.0	300.0
Present Value	-200	-190.5	272.1	259.2
			Total PV	140.8

Firm should choose strategy B, and its value would increase by $140.8 M.

2 Compound Interest Rates

2.1 APR and EAR

Sometimes, interest rate is quoted as an *annual percentage rate* (APR) with an associated compounding interval.

Example. Bank of America's one-year CD offers 5% APR, with semi-annual compounding. If you invest $10,000, how much money do you have at the end of one year? What is the actual annual rate of interest you earn?

- Quoted APR of $r_{\text{APR}} = 5\%$ is not the actual annual rate.
- It is only used to compute the 6-month interest rate as follows:

  $$(5\%)(1/2) = 2.5\%.$$

- Investing $10,000, at the end of one year you have:

  $$10,000(1 + 0.025)(1 + 0.025) = 10,506.25.$$

  In the second 6-month period, you earn interest on interest.
- The actual annual rate, the *effective annual rate* (EAR), is

  $$r_{\text{EAR}} = (1 + 0.025)^2 - 1 = 5.0625\%.$$

Annual rates typically refer to EARs.

2.2 Compounding

Let r_{APR} be the annual percentage rate and k be the number of compounding intervals per year. One dollar invested today yields:

$$\left(1+\frac{r_{\text{APR}}}{k}\right)^k$$

dollars in one year.

Effective annual rate, r_{EAR} is given by:

$$(1+r_{\text{EAR}}) = \left(1+\frac{r_{\text{APR}}}{k}\right)^k$$

or

$$r_{\text{EAR}} = \left(1+\frac{r_{\text{APR}}}{k}\right)^k - 1.$$

Example. Suppose $r_{\text{APR}} = 5\%$:

k	Value of \$1 in a year	r_{EAR}
1	1.050000	5.0000%
2	1.050625	5.0625%
12	1.051162	5.1162%
365	1.051268	5.1267%
8,760	1.051271	5.1271%
$\vdots$	$\vdots$	$\vdots$
∞	$e^{0.05} = 1.051271$	5.1271%

Here, $e \approx 2.71828$.

3 Real vs. Nominal CFs and Rates

Nominal vs. Real CFs

Inflation is 4% per year. You expect to receive \$1.04 in one year, what is this CF really worth next year?

The *real* or *inflation adjusted* value of \$1.04 in a year is

$$\text{Real CF} = \frac{\text{Nominal CF}}{1 + \text{inflation}} = \frac{1.04}{1 + 0.04} = \$1.00.$$

In general, at annual inflation rate of i we have

$$(\text{Real CF})_t = \frac{(\text{Nominal CF})_t}{(1+i)^t}.$$

Nominal vs. Real Rates

- Nominal interest rates - typical market rates.
- Real interest rates - interest rates adjusted for inflation.

Example. \$1.00 invested at a 6% interest rate grows to \$1.06 next year. If inflation is 4% per year, then the real value is \$1.06/1.04 = 1.019. The real return is 1.9%.

$$1 + r_{\text{real}} = \frac{1 + r_{\text{nominal}}}{1 + i}.$$

Example. Sales is \$1M this year and is expected to have real growth of 2% next year. Inflation is expected to be 4%. The appropriate *nominal* discount rate is 5%. What is the present value of next year's sales revenue?

- Next year's *nominal* sales forecast:

$$1 \times 1.02 \times 1.04 = 1.0608.$$

$$PV = \frac{1.0608}{1.05} = 1.0103.$$

- Next year's *real* sales forecast:

$$1 \times 1.02 = 1.02.$$

  Real discount rate:

$$r_{\rm real} = \frac{1+r_{\rm n}}{1+i} - 1 = \frac{1.05}{1.04} - 1 = 0.9615\%.$$

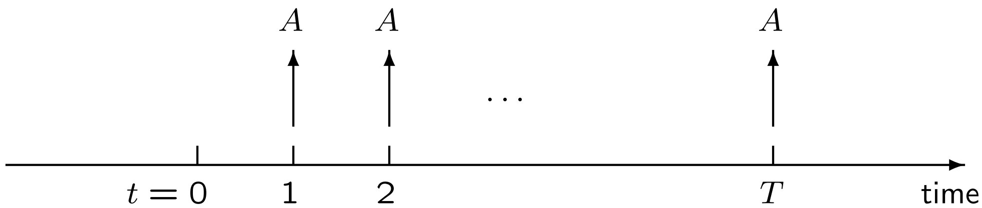
$$PV = \frac{1.02}{1.009615} = 1.0103.$$

Important Rules:

- Discount nominal CFs by nominal discount rates.
- Discount real CFs by real discount rates.

4 Shortcuts to Special Cash Flows

4.1 Annuity



Today is $t = 0$ and cash flow starts at $t = 1$.

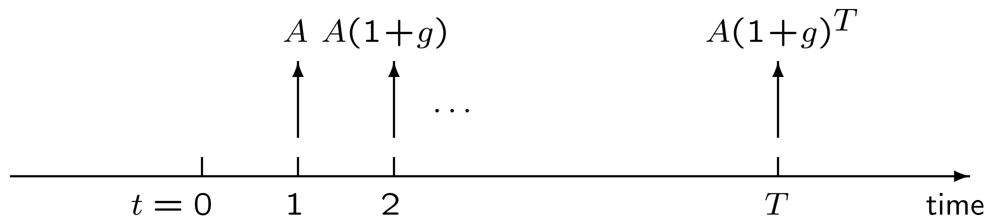
$$\text{PV (Annuity)} = \frac{A}{1+r} + \frac{A}{(1+r)^2} + \cdots + \frac{A}{1+r)^T}$$
$$= A \times \frac{1}{r} \left[1 - \frac{1}{(1+r)^T} \right].$$

Example. An insurance company sells an annuity of \$10,000 per year for 20 years. Suppose $r = 5\%$. What should the company sell it for?

$$PV = 10,000 \times \frac{1}{0.05} \times \left(1 - \frac{1}{1.05^{20}} \right) = 10,000 \times 12.46$$
$$= 124,622.1.$$

$$\text{FV (Annuity)} = \text{PV (Annuity)} \times (1+r)^T.$$

4.2 Annuity with Growth



PV (growing annuity)

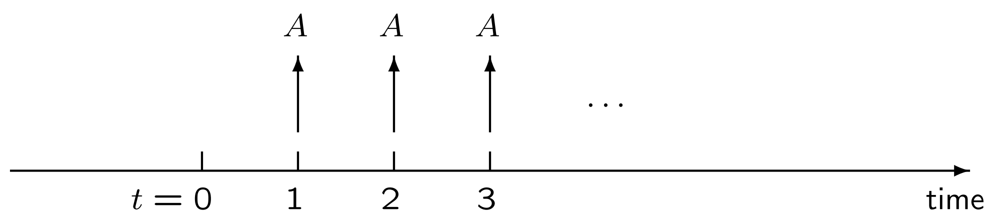
$$= A \times \left[\frac{1}{1+r} + \frac{1+g}{(1+r)^2} + \cdots + \frac{(1+g)^{T-1}}{(1+r)^T} \right]$$

$$= A \times \begin{cases} \frac{1}{r-g} \left[1 - \left(\frac{1+g}{1+r} \right)^T \right] & \text{if } r \neq g \\ \frac{T}{1+r} & \text{if } r = g. \end{cases}$$

Example. Saving for retirement - Suppose that you are now 30 and would like \$2 million at age 65 for your retirement. You would like to save each year an amount that grows by 5% each year. How much should you start saving now, assuming that r=8%?

$$A = \frac{2,000,000}{308.977} = 6,472.97.$$

4.3 Perpetuity



A perpetuity is an annuity with infinite maturity.

Example. You just won the lottery and it pays $100,000 a year for 20 years. Are you a millionaire? Suppose that r = 10%.

$$PV = 100,000 \times \frac{1}{0.10}\left(1 - \frac{1}{1.10^{20}}\right) = 100,000 \times 8.514$$
$$= 851,356.$$

- What if the payments last for 50 years?

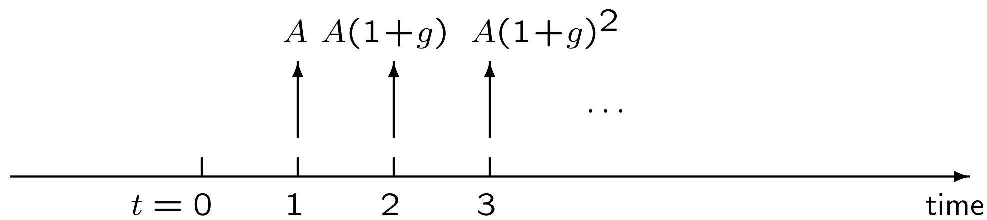
$$\text{PV} = 100,000 \times \frac{1}{0.10}\left(1 - \frac{1}{1.10^{50}}\right) = 100,000 \times 9.915$$
$$= 991,481.$$

- How about forever - a perpetuity?

$$PV = 100,000/0.10 = 1,000,000.$$

$$\text{PV (Perpetuity)} = \frac{A}{r}.$$

4.4 Perpetuity with Growth



$$\text{PV (Perpetuity with growth)} = \frac{A}{r-g}$$

where A is the first CF one period from now.

Example. Super Growth Inc. will pay an annual dividend next year of \$3. After analyzing the company, you expect the dividend after that to grow at the rate of 5% per year forever. Also for companies of this risk class, the expected return is 10%. What should be Super Growth's price per share?

$$\begin{aligned} \text{PV} &= \frac{3}{1.10} + \frac{3(1+0.05)}{1.10^2} + \frac{3(1+0.05)^2}{1.10^3} + \cdots \\ &= \frac{3}{0.10-0.05} \\ &= 60. \end{aligned}$$

Example. Mortgage calculation in the U.S.

- Pay 20% down payment, and borrow the rest from the bank using the property as collateral.
- Pay a fixed monthly payment for the life of the mortgage.
- Have the option to prepay the mortgage anytime before the maturity date of the mortgage.

Suppose that you bought a house for \$500,000 with \$100,000 down payment and financed the rest with a thirty-year fixed rate mortgage at 8.5% APR compounded monthly.

- The monthly payment M is determined by

$$\begin{aligned} 400,000 &= \sum_{t=1}^{360} \frac{M}{[1+(0.085/12)]^t} \\ &= \frac{M}{(0.085/12)} \left\{ 1 - \frac{1}{[1+(0.085/12)]^{360}} \right\} \\ &= M \times \frac{(0.9212)}{(0.085/12)}. \end{aligned}$$

$M = \$3,075.65.$

- Your effective annual interest rate (EAR):

$[1 + (0.085/12)]^{12} - 1 = 1.08839 - 1 = 8.839\%.$

The monthly payments are as follows:

t (month)	Principal	Interest	Sum	Remaining P.
1	242.37	2833.33	3075.7	399,757.63
2	244.08	2831.62	3075.7	399,513.55
3	245.81	2829.89	3075.7	399,267.74
⋮	⋮	⋮	⋮	⋮
120	561.29	2514.42	3075.7	354,415.49
121	565.26	2510.44	3075.7	353,850.23
⋮	⋮	⋮	⋮	⋮
240	1309.27	1766.43	3075.7	248,068.95
241	1318.54	1757.16	3075.7	246,750.41
⋮	⋮	⋮	⋮	⋮
359	3032.60	43.10	3075.7	3,054.07
360	3054.07	21.63	3075.7	0.00

- Total monthly payment is the same for each month.
- The percentage of principal payment increases over time.
- The percentage of interest payment decreases over time.

5 Homework

Readings:

- BMA Chapter 3.

Assignment:

- Problem Set 1.