91261

# Level 2 Mathematics and Statistics, 2016 <br> 91261 Apply algebraic methods in solving problems 

9.30 a.m. Thursday 24 November 2016<br>Credits: Four

| Achievement | Achievement with Merit | Achievement with Excellence |
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| Apply algebraic methods in solving <br> problems. | Apply algebraic methods, using <br> relational thinking, in solving problems. | Apply algebraic methods, using <br> extended abstract thinking, in solving <br> problems. |

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.
Make sure that you have Formulae Sheet L2-MATHF.
Show ALL working.
If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

You are required to show algebraic working in this paper. Guess-and-check methods and correct answer(s) only will generally limit grades to Achievement.

Check that this booklet has pages $2-11$ in the correct order and that none of these pages is blank.
YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

## QUESTION ONE

(a) Simplify $\left(\frac{3 b}{c^{2}}\right)^{-4}$ leaving your answer with positive indices.
(b) Write $x^{2}-8 x+10$ in the form $(x-p)^{2}+q$.
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(c) (i) Show that the solutions of the equation $x^{2}+x-56=0$ are four times the solutions of the equation $4 x^{2}+x-14=0$.
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(ii) Find the relationship between the solutions of the equation $d x^{2}+e x+f=0$ and the solutions of the equation $x^{2}+e x+d f=0$, where $d, e$, and $f$ are real numbers.
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(d) A quadratic equation of the form $a x^{2}+b x+c=0$ has solutions $-\frac{1}{2}$ and $\frac{2}{3}$.

Find a possible set of values for $a, b$, and $c$.
(e) Find positive integer value(s) for $k$ so that the quadratic equation $2 x^{2}+4 k x+\left(2 k^{2}+3 k-11\right)=0$ has real rational solutions.

Justify your answer.

## QUESTION TWO

(a) Find the discriminant of the quadratic equation $x^{2}=10 x+3$.
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(b) Simplify $\frac{4 \log \left(u^{3}\right)}{\log u}$.
(c) Marie buys a new car for $\$ 24990$.

The car's value decreases continuously by $12 \%$ each year.
The value of the car, $\$ P, t$ years after she first bought it, can be modelled by a function of the form $P=A(r)^{t}$.

How long will it take for the value of the car to halve?
(d) (i) Solve the equation $\log _{8} x=\frac{2}{3}$.
(ii) Solve the equation $6\left(\log _{8} x\right)^{2}+2 \log _{8} x-4=0$.
(e) The diagram below shows a triangular garden with a path around it.


The triangular garden has sides with lengths in the ratio 3:4:5.
The path is 1 m wide.
At each corner of the garden, the path is a sector (part) of a circle with a radius of 1 m .
The difference between twice the total area of the path and the area of the garden is $2 \pi \mathrm{~m}^{2}$.
Find the length of the longest side of the garden.
(Area of circle $=\pi r^{2}$ )
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## QUESTION THREE

(a) Where would the graph of $y=12 x^{2}-x-6$ cut the $x$-axis?
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(b) For what value(s) of $x$ does $\log _{x}(216)=3$ ?
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(c) Rearrange the following formula to make $x$ the subject: $\frac{4 x}{5}=\frac{y(x+3)}{2}$.

Question Three continues on the following page.
(d) Solve the equation $9^{8 n+6}=27^{n^{2}-1} \times 3^{1-3 n}$.
(e) A symmetrical bridge has its central cable in the shape of a parabola, as shown in the diagram below.

The towers supporting the cable are each 15 m high and 40 m apart.
At the point midway between the towers, the height of the cable above the road is 3 m .
A vertical post (shown dotted in the diagram) is placed 10 m from the centre of the bridge and just touches the cable.

(i) Use algebra to show that the post is 6 m high.
(ii) The length of the bridge AB is 60 m .

The outside cables are also parabolic and symmetrical in shape, and touch the road at their vertices $A$ and $B$.


Find the distance, CD, between the two parabolas at a height of 6 m above the road (the distance CD is shown in the diagram).
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Extra paper if required.

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