**COMMON CORE**

Learning Standard
HSA-CED.A.1

Essential Question How can you use an inequality to describe a real-life statement?

EXPLORATION 1 Writing and Graphing Inequalities

Work with a partner. Write an inequality for each statement. Then sketch the graph of the numbers that make each inequality true.
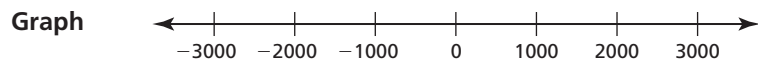
a. Statement The temperature t in Sweden is at least $-10°C$.

Inequality

Graph
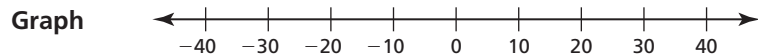
b. Statement The elevation e of Alabama is at most 2407 feet.
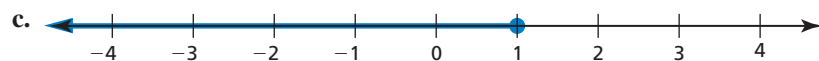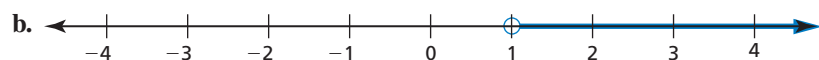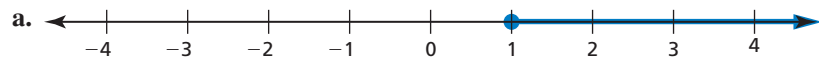
Inequality

Graph

ATTENDING TO PRECISION

To be proficient in math, you need to communicate precisely. You also need to state the meanings of the symbols you use.

EXPLORATION 2 Writing Inequalities

Work with a partner. Write an inequality for each graph. Then, in words, describe all the values of x that make each inequality true.

a.

b.

c.

d.

Communicate Your Answer

3. How can you use an inequality to describe a real-life statement?

4. Write a real-life statement that involves each inequality.

   a. $x < 3.5$

   b. $x \leq 6$

   c. $x > -2$

   d. $x \geq 10$

What You Will Learn

▶ Write linear inequalities.
▶ Sketch the graphs of linear inequalities.
▶ Write linear inequalities from graphs.

Writing Linear Inequalities

An **inequality** is a mathematical sentence that compares expressions. An inequality contains the symbol $<$, $>$, $\leq$, or $\geq$. To write an inequality, look for the following phrases to determine what inequality symbol to use.

Inequality Symbols				
Symbol	$<$	$>$	$\leq$	$\geq$
Key Phrases	• is less than • is fewer than	• is greater than • is more than	• is less than or equal to • is at most • is no more than	• is greater than or equal to • is at least • is no less than

EXAMPLE 1 Writing Inequalities

Write each sentence as an inequality.

a. A number w minus 3.5 is less than or equal to -2.

b. Three is less than a number n plus 5.

c. Zero is greater than or equal to twice a number x plus 1.

SOLUTION

a. A number w minus 3.5 is less than or equal to -2.

$$w - 3.5 \qquad \leq \qquad -2$$

▶ An inequality is $w - 3.5 \leq -2$.

b. Three is less than a number n plus 5.

$$3 \qquad < \qquad n + 5$$

▶ An inequality is $3 < n + 5$.

c. Zero is greater than or equal to twice a number x plus 1.

$$0 \qquad \geq \qquad 2x + 1$$

▶ An inequality is $0 \geq 2x + 1$.

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Write the sentence as an inequality.

1. A number b is fewer than 30.4.

2. $-\frac{7}{10}$ is at least twice a number k minus 4.

Sketching the Graphs of Linear Inequalities

A **solution of an inequality** is a value that makes the inequality true. An inequality can have more than one solution. The set of all solutions of an inequality is called the **solution set**.

Value of x	$x + 5 \geq -2$	Is the inequality true?
-6	$-6 + 5 \overset{?}{\geq} -2$ $-1 \geq -2$ ✔	yes
-7	$-7 + 5 \overset{?}{\geq} -2$ $-2 \geq -2$ ✔	yes
-8	$-8 + 5 \overset{?}{\geq} -2$ $-3 \ngeq -2$ ✘	no

Recall that a diagonal line through an inequality symbol means the inequality is *not* true. For instance, the symbol $\ngeq$ means "is not greater than or equal to."

EXAMPLE 2   Checking Solutions

Tell whether -4 is a solution of each inequality.

a. $x + 8 < -3$

b. $-4.5x > -21$

SOLUTION

a. $x + 8 < -3$ Write the inequality.

$-4 + 8 \overset{?}{<} -3$ Substitute -4 for x.

$4 \nless -3$ ✘   Simplify.

4 is *not* less than -3.

▶   So, -4 is *not* a solution of the inequality.

b. $-4.5x > -21$ Write the inequality.

$-4.5(-4) \overset{?}{>} -21$ Substitute -4 for x.

$18 > -21$ ✔   Simplify.

18 is greater than -21.

▶   So, -4 is a solution of the inequality.

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Tell whether -6 is a solution of the inequality.

3. $c + 4 < -1$ **4.** $10 \leq 3 - m$

5. $21 \div x \geq -3.5$ **6.** $4x - 25 > -2$

The **graph of an inequality** shows the solution set of the inequality on a number line. An open circle, ○, is used when a number is *not* a solution. A closed circle, ●, is used when a number is a solution. An arrow to the left or right shows that the graph continues in that direction.

EXAMPLE 3 **Graphing Inequalities**
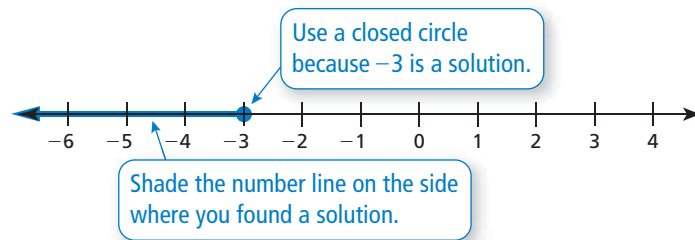
Graph each inequality.

a. $y \le -3$ **b.** $2 < x$ **c.** $x > 0$

SOLUTION

a. Test a number to the left of -3. $y = -4$ is a solution.

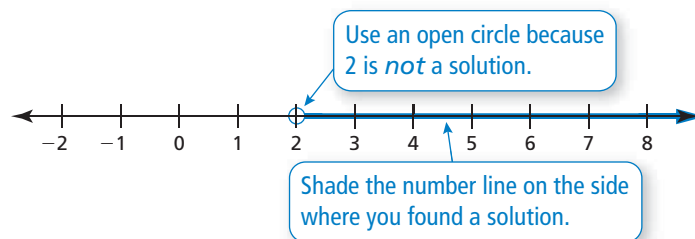Test a number to the right of -3. $y = 0$ is *not* a solution.

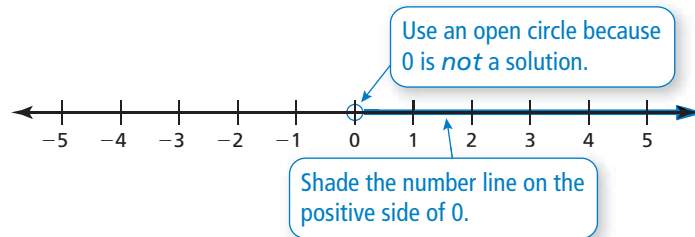Use a closed circle because -3 is a solution.

Shade the number line on the side where you found a solution.

b. Test a number to the left of 2. $x = 0$ is *not* a solution.

Test a number to the right of 2. $x = 4$ is a solution.

Use an open circle because 2 is *not* a solution.

Shade the number line on the side where you found a solution.

c. Just by looking at the inequality, you can see that it represents the set of all positive numbers.

Use an open circle because 0 is *not* a solution.

Shade the number line on the positive side of 0.

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Graph the inequality.

7. $b > -8$ **8.** $1.4 \ge g$
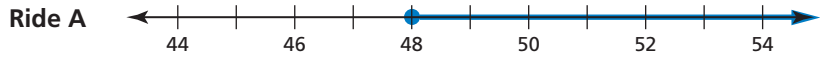
9. $r < \frac{1}{2}$ **10.** $v \ge \sqrt{36}$

Writing Linear Inequalities from Graphs

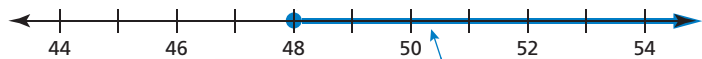**EXAMPLE 4**   **Writing Inequalities from Graphs**

The graphs show the height restrictions h (in inches) for two rides at an amusement park. Write an inequality that represents the height restriction of each ride.

YOU MUST BE THIS TALL TO RIDE

Ride A

Ride B

SOLUTION

Ride A

The closed circle means that 48 is a solution.

Because the arrow points to the right, all numbers greater than 48 are solutions.

Ride B

The open circle means that 52 is *not* a solution.

Because the arrow points to the left, all numbers less than 52 are solutions.

▶ So, $h \geq 48$ represents the height restriction for Ride A, and $h < 52$ represents the height restriction for Ride B.

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11. Write an inequality that represents the graph.

Concept Summary

Representing Linear Inequalities

Words	Algebra	Graph
x is less than 2	$x < 2$	
x is greater than 2	$x > 2$	
x is less than or equal to 2	$x \leq 2$	
x is greater than or equal to 2	$x \geq 2$	

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** A mathematical sentence using the symbols $<$, $>$, $\leq$, or $\geq$ is called a(n)_____.

2. **VOCABULARY** Is 5 in the solution set of $x + 3 > 8$? Explain.

3. **ATTENDING TO PRECISION** Describe how to graph an inequality.

4. **DIFFERENT WORDS, SAME QUESTION** Which is different? Write "both" inequalities.

w is greater than or equal to -7.	w is no less than -7.
w is no more than -7.	w is at least -7.

Monitoring Progress and Modeling with Mathematics

In Exercises 5–12, write the sentence as an inequality. *(See Example 1.)*

5. A number x is greater than 3.

6. A number n plus 7 is less than or equal to 9.

7. Fifteen is no more than a number t divided by 5.

8. Three times a number w is less than 18.

9. One-half of a number y is more than 22.

10. Three is less than the sum of a number s and 4.

11. Thirteen is at least the difference of a number v and 1.

12. Four is no less than the quotient of a number x and 2.1.

13. **MODELING WITH MATHEMATICS** On a fishing trip, you catch two fish. The weight of the first fish is shown. The second fish weighs at least 0.5 pound more than the first fish. Write an inequality that represents the possible weights of the second fish.

1.2 LB

14. **MODELING WITH MATHEMATICS** There are 430 people in a wave pool. Write an inequality that represents how many more people can enter the pool.

HOURS
Monday – Friday: 10 A.M. – 6 P.M.
Saturday – Sunday: 10 A.M. – 7 P.M.
Maximum Capacity: 600

In Exercises 15–24, tell whether the value is a solution of the inequality. *(See Example 2.)*

15. $r + 4 > 8$; $r = 2$

16. $5 - x < 8$; $x = -3$

17. $3s \leq 19$; $s = -6$

18. $17 \geq 2y$; $y = 7$

19. $-1 > -\dfrac{x}{2}$; $x = 3$

20. $\dfrac{4}{z} \geq 3$; $z = 2$

21. $14 \geq -2n + 4$; $n = -5$

22. $-5 \div (2s) < -1$; $s = 10$

23. $20 \leq \dfrac{10}{2z} + 20$; $z = 5$

24. $\dfrac{3m}{6} - 2 > 3$; $m = 8$

25. **MODELING WITH MATHEMATICS** The tallest person who ever lived was approximately 8 feet 11 inches tall.

    a. Write an inequality that represents the heights of every other person who has ever lived.

    b. Is 9 feet a solution of the inequality? Explain.

26. DRAWING CONCLUSIONS The winner of a weight-lifting competition bench-pressed 400 pounds. The other competitors all bench-pressed at least 23 pounds less.

    a. Write an inequality that represents the weights that the other competitors bench-pressed.

    b. Was one of the other competitors able to bench-press 379 pounds? Explain.

ERROR ANALYSIS In Exercises 27 and 28, describe and correct the error in determining whether 8 is in the solution set of the inequality.

27.

✗
$$-y + 7 < -4$$
$$-8 + 7 \overset{?}{<} -4$$
$$-1 < -4$$
8 is in the solution set.

28.

✗
$$\tfrac{1}{2}x + 2 \le 6$$
$$\tfrac{1}{2}(8) + 2 \overset{?}{\le} 6$$
$$4 + 2 \overset{?}{\le} 6$$
$$6 \le 6$$
8 is *not* in the solution set.

In Exercises 29–36, graph the inequality. *(See Example 3.)*

29. $x \ge 2$ **30.** $z \le 5$

31. $-1 > t$ **32.** $-2 < w$

33. $v \le -4$ **34.** $s < 1$

35. $\tfrac{1}{4} < p$ **36.** $r \ge -|5|$

In Exercises 37–40, write and graph an inequality for the given solution set.

37. $\{x \,|\, x < 7\}$ **38.** $\{n \,|\, n \ge -2\}$

39. $\{z \,|\, 1.3 \le z\}$ **40.** $\{w \,|\, 5.2 > w\}$

In Exercises 41–44, write an inequality that represents the graph. *(See Example 4.)*

41.

42.

43.

44.
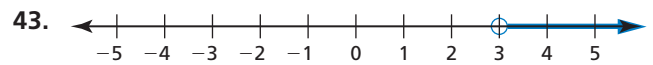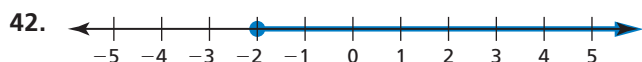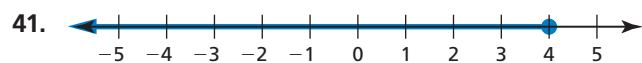
45. ANALYZING RELATIONSHIPS The water temperature of a swimming pool must be no less than 76°F. The temperature is currently 74°F. Which graph correctly shows how much the temperature needs to increase? Explain your reasoning.

Ⓐ

Ⓑ

Ⓒ

Ⓓ
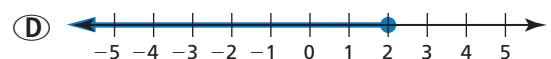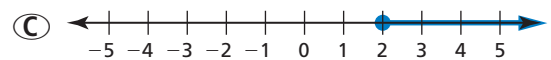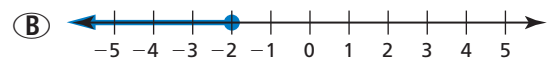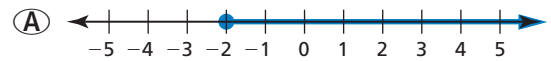
46. MODELING WITH MATHEMATICS According to a state law for vehicles traveling on state roads, the maximum total weight of a vehicle and its contents depends on the number of axles on the vehicle. For each type of vehicle, write and graph an inequality that represents the possible total weights w (in pounds) of the vehicle and its contents.

| Maximum Total Weights		
2 axles, 40,000 lb	3 axles, 60,000 lb	4 axles, 80,000 lb

47. PROBLEM SOLVING The Xianren Bridge is located in Guangxi Province, China. This arch is the world's longest natural arch, with a length of 400 feet. Write and graph an inequality that represents the lengths ℓ (in *inches*) of all other natural arches.

48. THOUGHT PROVOKING A student works no more than 25 hours each week at a part-time job. Write an inequality that represents how many hours the student can work each day.

49. WRITING Describe a real-life situation modeled by the inequality $23 + x \le 31$.

50. HOW DO YOU SEE IT? The graph represents the known melting points of all metallic elements (in degrees Celsius).
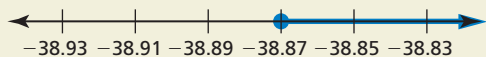
a. Write an inequality represented by the graph.

b. Is it possible for a metallic element to have a melting point of $-38.87°C$? Explain.

51. DRAWING CONCLUSIONS A one-way ride on a subway costs $0.90. A monthly pass costs $24. Write an inequality that represents how many one-way rides you can buy before it is cheaper to buy the monthly pass. Is it cheaper to pay the one-way fare for 25 rides? Explain.

Subway Prices

One-way ride...................$0.90
Monthly pass................$24.00

52. MAKING AN ARGUMENT The inequality $x \le 1324$ represents the weights (in pounds) of all mako sharks ever caught using a rod and reel. Your friend says this means no one using a rod and reel has ever caught a mako shark that weighs 1324 pounds. Your cousin says this means someone using a rod and reel *has* caught a mako shark that weighs 1324 pounds. Who is correct? Explain your reasoning.

53. CRITICAL THINKING Describe a real-life situation that can be modeled by more than one inequality.

54. MODELING WITH MATHEMATICS In 1997, Superman's cape from the 1978 movie *Superman* was sold at an auction. The winning bid was $17,000. Write and graph an inequality that represents the amounts all the losing bids.

MATHEMATICAL CONNECTIONS In Exercises 55–58, write an inequality that represents the missing dimension x.

55. The area is less than 42 square meters.

56. The area is greater than or equal to 8 square feet.
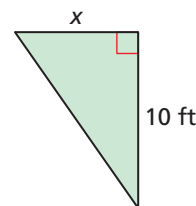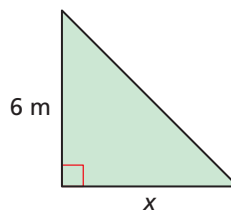
57. The area is less than 18 square centimeters.

58. The area is greater than 12 square inches.
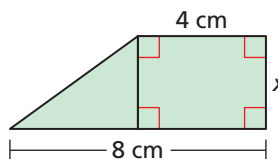
59. WRITING A runner finishes a 200-meter dash in 35 seconds. Let r represent any speed (in meters per second) faster than the runner's speed.

a. Write an inequality that represents r. Then graph the inequality.

b. Every point on the graph represents a speed faster than the runner's speed. Do you think every point could represent the speed of a runner? Explain.

Maintaining Mathematical Proficiency
Reviewing what you learned in previous grades and lessons

Solve the equation. Check your solution. *(Section 1.1)*

60. $x + 2 = 3$

61. $y - 9 = 5$

62. $6 = 4 + y$

63. $-12 = y - 11$

Solve the literal equation for x. *(Section 1.5)*

64. $v = x \cdot y \cdot z$

65. $s = 2r + 3x$

66. $w = 5 + 3(x - 1)$

67. $n = \dfrac{2x + 1}{2}$