9.5 *t* - test: one μ , σ unknown

GOALS:

1. Recognize the assumptions for a 1 mean t-test (srs, nd or large sample size, population stdev. NOT known).
2. Understand that the actual p-value (area in the tail past the test statistic) is not found on the t-table.
3. Use a calculator to find the p-value (part of t-test)
4. Test hypotheses for population means when population standard deviations are not known by applying the t-test.

Study Ch. 9.5, # 101-113(89 - 101), 117(105), 119 (107)

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Statistics Home Page Class Notes Homework

9.5 *t* - test: one μ , σ unknown

Assumptions for z Test, a Hypothesis Test for One μ:

1. Simple Random Sample (SRS)
2. Normal population or Large Sample
3. σ Known

What if σ is NOT Known?

From knowledge of CI's what would you expect?

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Statistics Home Page Class Notes Homework

9.5 *t* - test: one μ , σ unknown

What if σ is NOT Known?

From knowledge of CI's what would you expect?

Assumptions for *t* Test, the Hypothesis Test for One μ:

1. Simple Random Sample (SRS)
2. Normal population or Large Sample
3. σ NOT Known

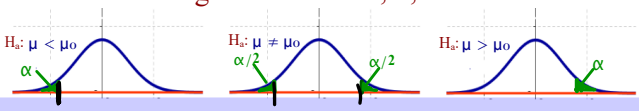
z Test	t Test
$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$	$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$
z distribution	t distribution **df = n - 1**

$\bar{x}$ ← estimates → $\bar{x}$

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Statistics Home Page Class Notes Homework

9.5 *t* - test: one μ , σ unknown

Test	One μ, σ not known **SRS, n.d. or large sample, σ not known**
1.	State the Null and Alternative Hypotheses: H_0, H_a
2.	Decide the significance level, α, and sketch 
3.	Compute the test statistic: t **for df =n -1**
4.	Find the P-value
5.	Decision: Rej. H_0 if $P \le \alpha$
6.	Interpret results

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Statistics Home Page Class Notes Homework

9.5 *t* - test: one μ , σ unknown

G: srs, nd, σ not known.
Right-tailed test, n=11, t= 1.246

skip to word problems

F: a) P - value b) significance level for rej, not rej

*The actual p-value cannot be found on a t-table.

t-table shows only 0.10, 0.05, 0.025, 0.01, 0.005

*The p-value can only be estimated from the t-table.

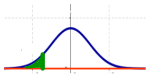
*Use a calculator to compute the p-value:

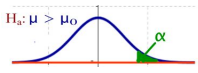
DISTR / 6:tcdf / df = n-1 = _____

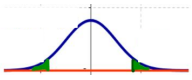
tcdf(lower bound, upper bound, df)

tcdf(_____, 9,_____) p = _____

(9 easier to type, same 4 digit result as 1EE99.
Note that t-distribution has larger tails than z-curve.)







From z get area in 1 tail.
Then multiply by 2 (2-tailed)

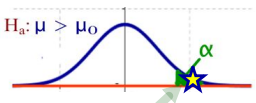
Can also use STAT/TESTS/t-test if assumptions are met.

skip to word problems

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Statistics Home Page Class Notes Homework

9.5 *t* - test: one μ , σ unknown



G: srs, nd, σ not known.
G: Right-tailed test, n=11, t= 1.246
F: a) P - value b) significance level for rej, not rej

DISTR / 6:tcdf / df = n-1 = 11-1 = 10

tcdf(lower bound, upper bound, df)

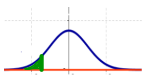
tcdf(1.246,9,10)

p = .1206

a) P > 0.10
b) α to reject?
NOT REJECT at any α

(9 easier to type, same 4 digit result as 1EE99.
Note that t-distribution has larger tails than z-curve.)

or:
STAT/TESTS/T-TEST
Use $u_0 = 0$, $\bar{x} = t/\sqrt{n}$, and s = 1
Use $u_0 = 0$, $\bar{x} = 1.246/\sqrt{11}$, and s = 1
Select the alternative test and calculate
p = .1206



From z get area in 1 tail.
Then multiply by 2 (2-tailed)

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Statistics Home Page Class Notes Homework

9.5 *t* - test: one μ , σ unknown

G: srs, nd, σ not known.
G: Two-tailed test, n=8, t= 3.725
F: a) P - value b) significance level for rej, not rej

df = n-1 = _____

DISTR / 6:tcdf /

tcdf(lower bound, upper bound, df)

tcdf(_____, 9,___) = _____

p = 2(_____) = _____

a) _____ < P = _____ < _____

b) α to reject?
Reject at α = _____, α = _____, α = _____
Do NOT Reject at α = _____

or:
STAT/TESTS/T-TEST
Use $u_0 = 0$, $\bar{x}$ = t /√n , and s = 1
Use $u_0 = 0$, $\bar{x}$ = 3.725 /√8 , and s =
Select the alternative test and calcula
p = .0074

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Statistics Home Page Class Notes Homework

9.5 *t* - test: one μ , σ unknown

G: srs, nd, σ not known.
G: Two-tailed test, n=8, t= 3.725
F: a) P - value b) significance level for rej, not rej

for 2-tailed p<0.010

df = n-1 = 8-1 = 7

calculator
DISTR / 6:tcdf /
tcdf(lower bound, upper bound, df)
tcdf(3.725,9,7) = .0037
p = 2(.0037) = 0.0074

table: compare given test t to t's from table

df	$t_{.10}$	$t_{.05}$	$t_{.025}$	$t_{.01}$	$t_{.005}$
7	1.415	1.895	2.365	2.998	3.499

a) 0.005 < P = 0.0074 < 0.01

b) α to reject?
Reject at α = .10, α = .05, α = .01
Do NOT Reject at α = .005

Calculator t-test
automatically accounts for 2 tails

Class Notes: Prof. G. Battaly, Westchester Community College, NY

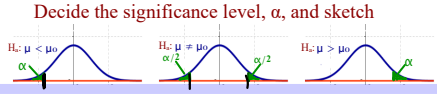
Statistics Home Page Class Notes Homework

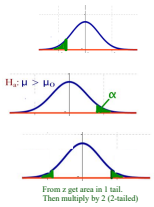
9.5 *t* - test: one μ , σ unknown

G: srs, σ not known.

G: $\bar{x} = 21$, n=32, S = 4 $H_0: \mu = 22$, $H_a: \mu < 22$

F: One mean *t*-test at $\alpha = 0.05$

Test	One μ, σ not known
1.	State the Null and Alternative Hypotheses: H_0, H_a
2.	Decide the significance level, α, and sketch 
3.	Compute the test statistic: $t = \frac{\bar{x}-\mu}{s/\sqrt{n}}$
4.	Find the P-value
5.	Decision: Rej. H_0 if $P \le \alpha$
6.	Interpret results



Class Notes: Prof. G. Battaly, Westchester Community College, NY

Statistics Home Page Class Notes Homework

⑤ $P > \alpha$ ∴ fail to reject

⑥ Conclude that evidence is NOT sufficient to claim $\mu < 22$.

9.4 Hypothesis Tests: one μ, σ Known

G: srs, σ not known.

G: $\bar{x} = 21$, n=32, S = 4 $H_0: \mu = 22$, $H_a: \mu < 22$

F: One mean *t*-test at $\alpha = 0.05$

Solving Word Problems

1. Read the problem. Try to identify the general type of problem. eg: CI, Hyp Test, specific value, etc.
2. Read the problem again, identifying what is given and what you need to find.
3. Use the Procedure Index to select a procedure.
4. Before beginning a procedure, determine if all assumptions are met.
5. If assumptions are not met, look for a different procedure. (Exception: if srs not met, then write "Assuming srs... ")
6. If assumptions are met, follow the procedure including:
   - Draw a sketch to show α as left-tailed, 2-tailed, or right-tailed.
   - For Hypothesis Tests, include null and alternative hypotheses.
   - Include all equations, substitutions, and answers for the equations. (Indicate calculator or tables.)
   - Decide to reject null hypothesis or not. Explain why.
   - Write a verbal interpretation of your decision.
7. Check: Have you satisfied the to find above?

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Statistics Home Page Class Notes

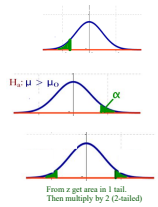
9.5 *t* - test: one μ , σ unknown

G: srs, σ not known.

G: $\bar{x}$ = 21, n=32, S = 4 H_0: μ = 22, H_a: μ < 22

F: One mean *t*-test at α = 0.05

Test	One μ, σ not known
1.	State the Null and Alternative Hypotheses: H_0 H_a
2.	Decide the significance level, α, and sketch
3.	Compute the test statistic: $t = \frac{\bar{x}-\mu}{s/\sqrt{n}}$
4.	Find the P-value
5.	Decision: Rej. H_0 if $P \le \alpha$
6.	Interpret results



P=0.0836

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Statistics Home Page Class Notes Homework

⑤ $p \ge \alpha$ ∴ $p = 0.0836 > 0.05 = \alpha$ Do NOT rej. H_0

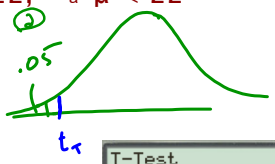
⑥ Concl. that evidence is NOT sufficient to claim μ < 22.

9.5 *t* - test: one μ , σ unknown

Assumptions: srs, large size > ~n.d., s

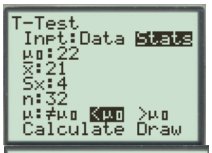
G: $\bar{x}$ = 21, n=32, S = 4 ① H_0: μ = 22, H_a: μ < 22

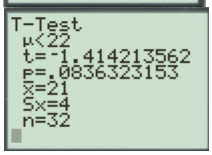
F: One mean *t*-test at α = 0.05

② 

③ $t = \frac{\bar{x}-\mu}{s/\sqrt{n}}$ df = 32 - 1 = 31

$= \frac{21-22}{4/\sqrt{32}} = -1.4142$





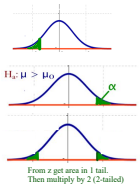
④ p = 0.0836

⑤ $p \ge \alpha$ $p = 0.0836 > 0.05 = \alpha$ ∴ Do NOT rej. H_0.

⑥ Concl. that evidence is NOT sufficient to claim μ < 22.

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Statistics Home Page Class Notes Homework



9.5 *t* - test: one μ , σ unknown

Solving Word Problems

1. Read the problem. Try to identify the general type of problem. eg: CI, Hyp Test, specific value, etc.
2. Read the problem again, identifying what is given and what you need to find.
3. Use the Procedure Index to select a procedure.
4. Before beginning a procedure, determine if all assumptions are met.
5. If assumptions are not met, look for a different procedure. (Exception: if srs not met, then write "Assuming srs... ")
6. If assumptions are met, follow the procedure including:
   - Draw a sketch to show α as left-tailed, 2-tailed, or right-tailed.
   - For Hypothesis Tests, include null and alternative hypotheses.
   - Include all equations, substitutions, and answers for the equations. (Indicate calculator or tables.)
   - Decide to reject null hypothesis or not. Explain why.
   - Write a verbal interpretation of your decision.
7. Check: Have you satisfied the to find above?

A professor wants to know if her introductory statistics class has a good grasp of basic math. Six students are chosen at random from the class and given a math proficiency test. The professor wants the class to be able to score above 70 on the test. The six students get scores of 62, 92, 75, 68, 83, and 95. Can the professor have 90 percent confidence that the mean score for the class on the test would be above 70?

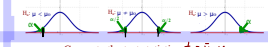
Class Notes: Prof. G. Battaly, Westchester Community College, NY

Statistics Home Page Class Notes Homework

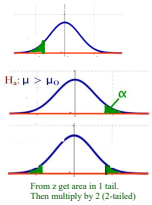
9.5 *t* - test: one μ , σ unknown

A professor wants to know if her introductory statistics class has a good grasp of basic math. Six students are chosen at random from the class and given a math proficiency test. The professor wants the class to be able to score above 70 on the test. The six students get scores of 62, 92, 75, 68, 83, and 95. Can the professor have 90 percent confidence that the mean score for the class on the test would be above 70?

MUST DO all steps of procedure

Test	One μ, σ not known
1.	State the Null and Alternative Hypotheses: $H_0 H_a$
2.	Decide the significance level, α, and sketch 
3.	Compute the test statistic: $t = \frac{\bar{x}-\mu_0}{s/\sqrt{n}}$
4.	Find the P-value
5.	Decision: Rej. H_0 if $P \le \alpha$
6.	Interpret results

Can use calculator
STAT / TESTS
2: T-Test
Inpt: Data
μ_0: _____
List: _____
μ: ≠ μ_0, < μ_0, > μ_0
Calculate Draw



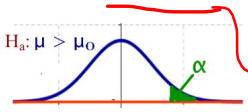
Class Notes: Prof. G. Battaly, Westchester Community College, NY

Statistics Home Page Class Notes Homework

9.5 *t* - test: one μ , σ unknown

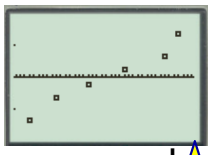
SYS

A professor wants to know if her introductory statistics class has a good grasp of basic math. Six students are chosen at random from the class and given a math proficiency test. The professor wants the class to be able to score above 70 on the test. The six students get scores of 62, 92, 75, 68, 83, and 95. Can the professor have 90 percent confidence that the mean score for the class on the test would be above 70?



$H_0: \mu = 70$

$H_a: \mu > 70$



~nd

σ not known

∴ t-test

CL = 0.90

$\alpha = 1 - 0.90 = 0.10$

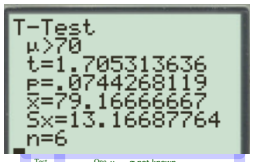
$t_{\bar{x}} = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{79.17 - 70}{13.167/\sqrt{6}} = 1.705$

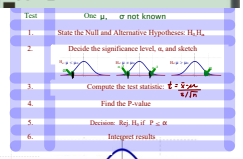
$p = 0.0744$

$p = 0.0744 < 0.10 = \alpha$

∴ reject H_0

Can use calculator
STAT / TESTS
2: T-Test
Inpt: Data
μ_0: 70
List: L1
μ: $\neq \mu_0$, $< \mu_0$, $> \mu_0$
Calculate Draw

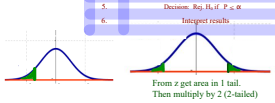




conclude: that prof can have ~92.6% confidence that the mean >70

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Statistics Home Page Class Notes Homework



From z get area in 1 tail.
Then multiply by 2 (2-tailed)

9.5 *t* - test: one μ , σ unknown

students do

A health researcher read that a 200-pound male can burn an average of 524 calories per hour playing tennis. 37 males were randomly selected and the mean number of calories burned per hour playing squash was 534.8 with a standard deviation of 45.9 calories. At the 1% significance level, do squash players burn more calories per hour than tennis players?

Problem type: _____.

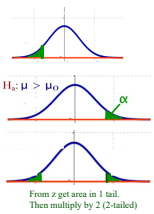
G: _____.

F: _____

MUST DO all steps of procedure

Test	One μ, σ not known
1.	State the Null and Alternative Hypotheses: H_0, H_a
2.	Decide the significance level, α, and sketch
3.	Compute the test statistic: $t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$
4.	Find the P-value
5.	Decision: Rej. H_0 if $P \le \alpha$
6.	Interpret results

Can use calculator
STAT / TESTS
2: T-Test
Inpt: Data
μ_0: _____
List: _____
μ: $\neq \mu_0$, $< \mu_0$, $> \mu_0$
Calculate Draw



$\bar{x}$
S
n

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Statistics Home Page Class Notes Homework

9.5 *t* - test: one μ , σ unknown

A health researcher read that a 200-pound male can burn an average of 524 calories per hour playing tennis. 37 males were randomly selected and the mean number of calories burned per hour playing squash was 534.8 with a standard deviation of 45.9 calories. At the 1% significance level, do squash players burn more calories per hour than tennis players?

Problem type: _____test: z or t?_____.

G: $\mu = 524$; srs; n=37>30 ~n.d.; $\bar{x}$ = 534.8, s = 45.9; α=0.01

σ not given --> Use t-test

F: __Do squash players burn more calories than tennis players?_

MUST DO all steps of procedure

Test	One μ, σ not known
1.	State the Null and Alternative Hypotheses: H_0 H_a
2.	Decide the significance level, α, and sketch
3.	Compute the test statistic: $t = \frac{\bar{x}-\mu}{s/\sqrt{n}}$
4.	Find the P-value
5.	Decision: Rej. H_0 if $P \le \alpha$
6.	Interpret results

Can use calculator

STAT / TESTS

2: T-Test

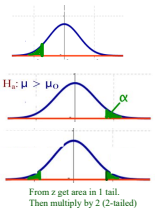
Inpt: Data

μ_0: _____

List: _____

μ: ≠ μ_0, < μ_0, > μ_0

Calculate Draw



S

n

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Statistics Home Page Class Notes Homework

9.5 *t* - test: one μ , σ unknown

A health researcher read that a 200-pound male can burn an average of 524 calories per hour playing tennis. 37 males were randomly selected and the mean number of calories burned per hour playing squash was 534.8 with a standard deviation of 45.9 calories. At the 1% significance level, do squash players burn more calories per hour than tennis players?

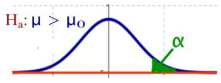
Problem type: _____test: z or t?_____.

G: $\mu = 524$; srs; n=37>30 ~n.d.; $\bar{x}$ = 534.8, s = 45.9; α=0.01

σ not given --> Use t-test

F: __Do squash players burn more calories than tennis players?_

H_0: μ = 524 ; H_a: μ > 524



$t_{\bar{x}} = \frac{\bar{x}-\mu}{s/\sqrt{n}} = \frac{534.8-524}{45.9/\sqrt{37}} = 1.431$

$p = 0.0805$

$p = 0.0805 > 0.01 = \alpha$

$\therefore$ DO NOT reject H_0

Can use calculator

STAT / TESTS

2: T-Test

Inpt: ~~Data~~ Stats

μ_0: 524

$\bar{x}$: 534.8

s_x: 45.9

μ: ≠ μ_0, < μ_0, **> μ_0**

Calculate Draw

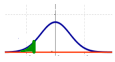
Conclude: At 1% s.l. squash players to not burn more calories than tennis players.

S

n

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Statistics Home Page Class Notes Homework



9.5 *t* - test: one μ , σ unknown

G: srs, nd, σ not known.

G: $\bar{x}$= 182.7yds, n=6, S =2.7yds H_0: μ = 180, H_a: μ > 180

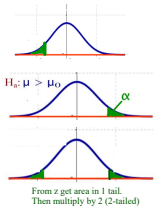
F: a) One mean *t*-test at α = 0.05 b) at α = 0.01

180
187
181
182
185
181

MUST DO all steps of procedure

Test	One μ, σ not known
1.	State the Null and Alternative Hypotheses: H_0,H_a
2.	Decide the significance level, α, and sketch
3.	Compute the test statistic: $t = \frac{\bar{x}-\mu}{s/\sqrt{n}}$
4.	Find the P-value
5.	Decision: Rej. H_0 if P ≤ α
6.	Interpret results

Can use calculator
STAT / TESTS
2: T-Test
Inpt: Data
μ_0: _____
List: _____
μ: ≠ μ_0, < μ_0, > μ_0
Calculate Draw



9.5 *t* - test: one μ , σ unknown

G: 180, 187, 181, 182, 185, 181

① H_0: $\mu = 180$ H_a: $\mu > 180$

② $\alpha = 0.05$

③ $t_T = \frac{\bar{x}-\mu}{s/\sqrt{n}} =$

$p = 0.03 < \alpha = 0.05$

∴ rej. H_0.

Conclude that golf ball is greater than 180