Essentials to Mathematics

Arithmetic and Algebra Worksheets

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Contents

Acknowledgements

I want to acknowledge that this booklet does not contain all the worksheets needed to cover the entire algebra curriculum. This book began ten years ago when I assisted a colleague, Dr. Keith Calkins, remediate high school students entering a rigorous advanced mathematics program. The worksheets I developed then focused on common weak areas my students needed to strengthen. Since that time I worked a couple of years with Dr. Lynelle Weldon who directed the task to remediate university students before placing them into university mathematics courses. A few of her study guides became a blue print for a few of mine and those got inserted into this book as well. Then I spent three years developing a two-year pre-algebra course for a combined seventh and eighth grade class. Since there was always an influx of new students each year, the curriculum was the same each year with the difference only in the activities and worksheets. The worksheets I developed were for certain days when I could find no resources on hand for what I wanted the students to master. These worksheets found their way into this book as well. So you can conclude that this booklet you are perusing is a compilation of ten years of supplemental writing. Hopefully you will find it useful.

I want to thank Dr Calkins and Dr Weldon for their inspiration and their examples! Pun intended.

Chapter 1 Number System

Prior Skills:
- Convert fractions to decimal for sheet 1c
- Time measurements for sheet 1c
- Basic understanding of decimal and fractions for sheet 1d

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1a- Translating Mathematical Symbols

For each question, translate the equation and then solve by mental math. No calculator!

Example: $3x = 21$.   Translation: Three times a number is 21.   Answer: $x = 7$

1. $x - 4 = 13$

2. $x + 5 = 8$

3. $8 - x = 5$

4. $4x = 12$

5. $2x = 6$

6. $T + 7 = 10$

7. $14 - t = 5$

8. $21 - x = 13$

9. $Y \div 3 = 6$

10. $9 \div P = 1$

11. $8 \times P = 32$

12. $6 \times R = 54$

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1b – Number Systems

Complex Numbers - All numbers are complex. Their form is $a + bi$. These numbers will be taught later!

Real Numbers – numbers found on the "number line". If written as a complex number, they would look like $a+0i$.

Imaginary numbers - points not on the standard number line. If written as complex, they would have form $0+bi$.

Zero - It is both real and imaginary.

Rational Numbers – Real numbers that can be expressed as a ratio of two integers. If written as a decimal, they would be terminating or repeating.

Irrational Numbers - reals that CANNOT be expressed as a ratio of integers. If written as a decimal, they would be nonterminating and nonrepeating decimals.

Transcendental Numbers - irrational numbers that can NOT be solved by algebraic methods

Integers - whole numbers and their opposites

Non-integers - another name for a reduced fraction where 1 is NOT in the denominator.

Whole numbers - 0, 1, 2, 3…

Natural Numbers (counting numbers) - 1, 2, 3…

Digits - whole numbers from 0 to 9, those numbers which make up our numerals

Even - integers divisible by 2

Odd - integers that are NOT divisible by 2

Positive - reals greater than 0

Negative - reals less than 0

Answer the following about numbers:
1. On a separate piece of paper, create a hierarchy for the number systems above.
   For each branch, list three examples of the number system.

2. Which of the following is not a rational number?
        3.1   3.01   3.111...   3.1234322344523...   3½

3. Which of the following is not a rational number?
        3.4   -3.4   3.444...   -3.444   3.040040004...

4. Which is not an integer?   2   -2   0   ½ $\dfrac{4}{2}$

5. What type of number is this: (rational, irrational, integer, real...)
        A. -3.4   B. $\sqrt{5}$ C. $\sqrt{12}$ D. 0

6. Explain which decimals are rational numbers? How can you tell them from an irrational number?

Name: _____   Date: _____

1c - Number Systems

Mathematicians use short-hand notation when referring to number systems:

N - natural, *Z* - integer, *Q* - rational, *R* - real, *C* - complex.

1. Check off which number systems the following numbers are:

	N	Z	Q	R	C
π					
3.4					
$\sqrt{0.81}$					
$\sqrt{27}$					
0					

2. How many minutes are there in two and one half days?

3. How many seconds are there in a day?

4. Some rational numbers can be expressed in decimal form. Express the following in decimal, showing all work:

   a. ¼   b. $\dfrac{1}{6}$ c. $\dfrac{1}{9}$ d. $\dfrac{5}{12}$ e. $\dfrac{7}{100}$

5. Explain which decimals are rational and which are irrational.
   For example: $\pi \approx 3.141592...$ is irrational.

6. Write the following decimals as a ratio of two integers:

   a. 0.315   b. 3.151515...

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1d - Which Number is Bigger?

You may use your calculator to convert into decimal if necessary to answer the questions.

1. Plot the numbers on a real number line: -1⅓, 0, 2 ⅔, 1¾, -⅝, -⅔, $1\frac{3}{5}$, $2\frac{5}{7}$, -3, 2 ½

2. Plot the numbers on a real number line: -2.3, -3.3, 4.3, 4, 2, -2, -3.8

3. Plot on a real number line: -1 ½, 3⅛, -2⅞, 1.5, 1.05, -1¾, 1.7

6. Which is bigger? Fill in the blank with <, >, or =.

     a. -3 ____ 3

     b. - 4 ____ -5

     c. -10.4 ____ -10.3

     d. 4.5 ____ 4.55

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1e-Adding & Subtracting Integers

| Perform the operations without a calculator. Show work by plotting the operations on a number line. |

1. There are several ways to add or subtract integers. Some think of money debts, others think of protons versus electrons. The following example is showing how addition is about gaining and subtraction about losing in terms of the real number line.

$$-4 + 9 \qquad\qquad 7 - 5 \qquad\qquad -3 - 3 \qquad\qquad 1 - 7 \qquad\qquad 1 + 3$$
$$= 5 \qquad\qquad\quad = 2 \qquad\qquad\quad = -6 \qquad\qquad = -6 \qquad\qquad = 4$$
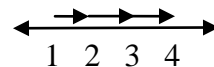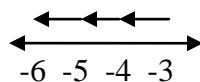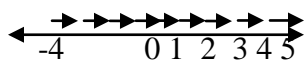
Simplify the following by doing the indicated operation:

2. $- 4 - 9$ 3. $-7 - 5$ 4. $-3 + 3$

5. $-1 - 7$ 6. $1 - 3$ 7. $-5 - 9$

8. $8 - 3$ 9. $53 - 42$ 10. $31 - 82$

11. $- 44 + 53$ 12. $- 35 + 35$ 13. $23 - 17$

14. $2 - 4 - 6$ 15. $2 + (- 4) - 6$ 16. $0 - 2 + 6$

Name: _____   Date: _____

1f- more Adding & Subtracting Integers

You may use your calculator only to check your answers. Simplify the expressions.

Find the result.

1. $7 + 3$

2. $-6 - 3$

3. $-8 - 6$

4. $6 + (-3)$

5. $(+6) + (-3)$

6. $-7 + (-8)$

7. $-4 + (+2)$

8. $4 + (-2)$

9. $5 - 8$

10. $-78 - 21$

11. $-32 - 21$

12. $-55 - 44$

13. $34 - 43$

14. $-34 + 68$

15. $54 - 59$

16. $-90 + 90$

17. $3 - 6$

18. $-4 + 5$

19. $4 - 5$

20. $6 - 5$

21. $7 - 17$

22. $10 - 15$

23. $0 - 1$

24. $-3 + 4$

25. $-14 + 25$

26. $-5 + 10$

27. $-1 + 8$

28. $-7 + 23$

29. $8 - 3$

30. $3 - 6$

31. $10 - 6$

32. $4 - 7$

33. $-1 + 3$

34. $-10 + 6$

35. $-7 + 4$

36. $-7 + 8$

Translate the following expression and find the integer that represents the overall change.

37. The temperature starts at -15°C, drops 10°C, rises 5°C and rises 8°C.

38. A person starts with $50, earns $12, spends $15, earns $18, and spends $22.

39. A submarine starts at sea level, dives down 125 m, dives another 72 m, and rises 42 m.

40. An elevator starts on the seventh floor, descends 5 floors and ascends 9 floors.

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1g-Multiplying & Dividing Integers

You may use your calculator only to check your answers.

1. (-8)(-3)   2. -6(- 4)   3. 5(-9)   4. 10(-3)

5. -6(-3)   6. -2(5)   7. 15(- 4)   8. 16(-3)

9. 17×(-5)   10. (-8)(-9)   11. (-7)(31)   12. 90(100)

13. 8 × (-3)   14. -3 × (-2)   15. -5 × -14   16. 12×12

17. -9 ÷ -3   18. -18 ÷ (-9)   19. 20 ÷ (-5)   20. -72 ÷ (-8)

21. -100 ÷ (-10)   22. -35 ÷ 7   23. 36 ÷ 4   24. 81 ÷ (-3)

25. 95 ÷ (-19)   26. -32 ÷ 4   27. 64 ÷ 8   28. 42 ÷ (-6)

29. -(-9)   30. -0   31. (-1)(-1)(-1)(-1)(-1)   32. 2(-3)(4)(-5)(6)

Name: _____   Date: _____

1h-Expanding Numbers

You may use your calculator only to check your answers.

State the place value of the '5' in each number below:

    1. 78,513   2. 960,500   3. 5,000,732

    4. 85,723   5. 23,985   6. 234,951

Write the following in expanded form:

7. 34

8. 5345

9. 4,000,001

10. 203,432

11. 432

12. 865,342,422

Write each of the following in standard form:

13. $(4\times100,000) + (5\times10,000) + (3\times1000) + (8\times100)$

14. $(9\times1,000,000) + (7\times1)$

15. $(6\times1) + (7\times10) + (8\times100) + (6\times1000) + (7\times10,000)$

16. $(3\times100) + (4\times1000) + (7\times1) + (9\times10) + (4\times1,000,000)$

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Test REVIEW: Integers

SHOW WORK. A calculator is NOT allowed on this test. You must work alone.
Questions regarding interpreting the directions are allowed. Simplify your answers!

1. Solve the following for x:

    a. $x - 12 = 28$ b. $x + 17 = 0$ c. $6 - x = 2$

2. Evaluate the following:

    a. $13 - 12$ b. $-13 + 12$ c. $-13 - 12$

3. Evaluate the following:

    a. $-2(-4)$ b. $4(-8)$ c. $-(-7)$ d. -5×4

4. Evaluate the following:

    a. $25 \div (-5)$ b. $-32 \div 8$ c. $-18 \div (-2)$

5. Write the following number in standard form:

    $4(100,000) + 6(1,000) + 5(100) + 3(10)$

6. Write in expanded form: 43,507.

Chapter 1 Test

Name: _____   Date: _____

SHOW WORK. A calculator is allowed on this test, but work still must be shown for full credit. Only questions regarding interpreting the directions are allowed – no talking to anyone but the teacher until all have finished and have submitted their test.

$\frac{}{5}$

1. Put the following numbers in order from smallest to biggest:

    -9, 9, -1, 1, 7, -7, 0, 2, -3, -5, 6

$\frac{}{5}$

2. Write each number in standard form:
    a. 500,000 + 1,000 + 70 + 2

    b. 20,000 + 300 + 4

    c. 4,000,000 + 800,000 + 2,000 + 900

$\frac{}{5}$

3. Rewrite using algebraic symbols:
    a. three less than a number   B. Five more than twice a number is thirteen.

$\frac{}{5}$

4. Fill in the blanks with either standard form or expanded form.
    a. $62,723 = (6 \times 10^4) + \_\_\_\_\_ + (7 \times 10^2) + _____ + (3 \times 10^0)$

    b. _____ $= (3 \times 10^2) + (9 \times 10^5)$

$\frac{}{5}$

5. Solve: a. x - 2 = 5   b. x -(-3) = 7

$\frac{}{5+0}$

6. Circle each of the following types of numbers that best describes 4.5.
    a. Real   b. Rational   c. Complex   d. Transcendental   e. Integer

Bonus: Fill in the next three numbers that continues the sequence: 1, 3, 6, 10,__, __, __

Chapter 2 Fractions

Prior skills:
- For sheet 2a, know perimeter
- For sheet 2n, know area

An asterisk (*) next to a question, such as question 17 & 18 on sheet 2i implies that the student may find the question challenging. The questions may have come from an activity we did in class prior to the worksheet. If you using the worksheets without other resources, just beware that the students may have difficulty with asterisk questions.

Name: _____   Date: _____

2a-Finding Fractions

For each question, translate the equation and then solve by mental math.

1. Darcy decides to eat only ⅓ of a candy bar. Draw a candy bar and shade in what was eaten.

2. Students at SLA walked 20 laps to help the Terry Fox Foundation. Some walked only ¾ the laps. Make 20 squares to represent the laps and shade in the amount some only walked.

3. The perimeter of the park is about 4 miles. Someone walked only ¼ of it. Draw a circle and color the fraction of the circle walked.

4. Nicole bought four apples. One was eaten this morning. What fraction of apples are left?

5. There are 150 days of school. If students have been in school for 15 days, what fraction of the school year is left?

6. The perimeter of the building is 400 feet and is getting a new coat of paint, what fraction of the building is left to paint if only 100 feet got painted? Draw a picture of the outline of the building and where it's painted. Does your drawing look like others?

Name: _____   Date: _____

2b-Proper and Improper Fractions

Write as a mixed number:

1. $\frac{11}{3}$ 2. $\frac{24}{5}$ 3. $\frac{43}{12}$ 4. $\frac{31}{8}$

5. $\frac{33}{5}$ 6. $\frac{56}{15}$ 7. $\frac{23}{16}$ 8. $\frac{7}{3}$

Write as an improper fraction:

9. $2\frac{7}{8}$ 10. $5\frac{6}{11}$ 11. $10\frac{2}{7}$ 12. $6\frac{9}{11}$

13. $2\frac{5}{7}$ 14. $3\frac{11}{12}$ 15. $5\frac{3}{10}$ 16. $13\frac{1}{5}$

Name: _____   Date: _____

2c-Adding & Subtracting Fractions of Different Denominators

Simplify the following without the use of a calculator. Leave as a proper fraction.

1. $\frac{2}{3} + \frac{5}{12}$

2. $3\frac{1}{5} - 1\frac{3}{8}$

3. $\frac{5}{6} + \frac{5}{12}$

4. $2\frac{1}{3} + 4\frac{4}{5}$

5. $\frac{8}{9} + \frac{3}{4}$

6. $\frac{1}{7} + \frac{3}{4}$

7. $2\frac{1}{2} - 1\frac{3}{4}$

8. $10\frac{5}{6} - 3\frac{1}{9}$

9. $4\frac{5}{9} - 2\frac{13}{15}$

10. $1\frac{1}{3} - \frac{14}{15}$

11. ⅞ + ⅔

12. ⅓ - ⅜

Name: _____   Date: _____

2d-Multiplying & Dividing Fractions

Use your calculator only to check your answer. Leave answer as simplified proper fractions.

1. $\frac{3}{4} \cdot \frac{8}{9}$

2. $\frac{4}{5} \times \frac{10}{17}$

3. $\frac{5}{6} \times \frac{3}{10}$

4. $\frac{6}{7} \times \frac{14}{15}$

5. $\frac{8}{9} \times \frac{27}{28}$

6. $\frac{3}{7}\left(\frac{21}{10}\right)$

7. $2\frac{3}{4} \times 4\frac{2}{5}$

8. $-\frac{4}{5}\left(\frac{15}{24}\right)$

9. $\frac{3}{4} \times 3$

10. $\frac{4}{5}(-15)$

11. $3\left(-\frac{2}{3}\right)$

12. $2\frac{1}{4} \times 3\frac{1}{3}$

13. $3\frac{2}{3} \times 4\frac{1}{11}$

14. $2\frac{1}{17} \times 3\frac{2}{5}$

15. $5\frac{1}{3} \times 7\frac{1}{2}$

16. $2\frac{1}{2} \div \frac{5}{6}$

17. $4\frac{3}{5} \div 11\frac{1}{2}$

18. $\frac{5}{7} \div \frac{10}{21}$

19. $\frac{8}{9} \div 3\frac{1}{9}$

20. $4\,\frac{1}{2} \cdot 8\,\frac{2}{3}$

21. $(8\frac{2}{3})\,/\,(4\frac{1}{2})$

Question 21 is an example why there are many types of parentheses and division symbols. Some symbols make the question cluttered and hard to read.

Name: _____   Date: _____

2e-Adding Fractions of Like Denominator

For each question, translate the equation and then solve by showing your algebraic steps.

1. The perimeter of the room is 248 inches. If two walls of the rectangular room are 80 ½ long, how long is each of the other walls?

2. A rectangular room has perimeter of 320 ½ inches. One wall is 70¼. Find the dimensions of the other three walls.

3. A triangle has sides 1⅜, 2⅝, and 2⅞. Find its perimeter.

4. A rectangle has dimensions 6⅔" by 7⅓". Find its perimeter.

5. A rectangle has perimeter 8⅛ and one known side of 2⅞. What are the dimensions of the rectangle?

6. A hexagon (6-sided shape) has sides 2⅞, 1⅜, 4, 5⅛, 4½ and a perimeter of 20. How long is the missing side?

7. Find the perimeter of an octagon with equal sides of 4⅝. An octagon has 8 sides.

8. A pentagon has perimeter of 30⅔. If four sides are known to be 4⅓, 5⅔, 7⅔, and 6⅔, how long is the remaining side?

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2f - Like Terms

Constant: A number that doesn't change. In the expression $2x + 3$, the constant is 3. The 2 is a **coefficient**.

Variable: A number that may change, usually is represented by a letter. In the expression $2x + 3$, the variable is x.

Term: Any constant or variable that is being added or subtracted. In the expression $2x + 3y$, the terms are $2x$ and $3y$.

Expression: A collection of terms that together represents a number.

Equation: When two expressions are equal. Usually the goal is to find the value of the variable that makes the equation true (equal).

Like Terms: Two or more terms having the same variables with the same exponents. The variables do not need to be in the same order; $2wz$ and $3zw$ are like terms. Coefficients are ignored since they refer to the amount of this term you have.

Combining like terms: When the terms are alike, you add or subtract (depends on signs) the coefficients. Adding $2wz$ and $3zw$ would give you $5wz$. Subtracting $2wz$ from $3zw$ would result in $1wz$ (better written as wz).

1. Write a like term for each of the following:
    a. $-16y$ b. -5 c. $4xy^2z$ d. $5h$

2. Determine which of the following sets are made up of like terms:
    a. $\{4x, 3x^2, 3x^3\}$ b. $\{xyz, -3xyz, 5yz\}$ c. $\{2xy, -3xy, -8xy\}$

For the following exercises, combine like terms.

3. $5x^2 - 6x^2 + 4xy + 3y^2 - 2y^2$

4. $(8y^2 + 6y - \frac{1}{2}) + (-y^2 - 2y + \frac{3}{4})$

5. $(3x^2 + 4x + 4) + (x^2 - 2x - 2)$

6. Add $x^2 - \frac{1}{4}$ and $-x^2 - 5x + \frac{7}{8}$.

7. $(5x^3 - 10x^2 + 3x) - (-3x^3 + 5x^2 - 8x^0)$

8. $(5x-3) - (- 4x^2 + 3x - 1)$

9. Subtract $-3x^3 - 2x^2 - 6x^0$ from $-2x^3 - 3x^2 - 6x$.

10. $(2 - x) - (4 + 5x)$

11. Subtract $2x^3 - 3x^2 + 4x$ from $-2x^3 - 3x^2 + 5x + 4x^0$.

12. $(3 + 3x) + (1 - 2x)$

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2g-Solving Basic Algebraic Equations (one step)

Use your calculator to check your answer. Leave answer as simplified proper fractions.

1. $3x = -21$

2. $-7y = 28$

3. $-196 = -28x$

4. $-15a = -45$

5. $-x = 17$

6. $-22 = 2x$

7. $-12b = -288$

8. $12x = -60$

9. $A \div 5 = -6$

10. $Y \div (-5) = -14$

11. $X \div 4 = -24$

12. $X \div (-3) = 4$

13. $A \div 14 = 7$

14. $3a = -15$

15. $A \div 4 = -81$

16. $Y \div (-1) = 0$

17. $X \div (-3) = 6$

18. $-12 = 4n$

19. $-4w = -12$

20. $Y \div 6 = 2$

21. $4x = 0$

22. $x + 6 = 9$

23. $x - 9 = 1$

24. $7 = x + 3$

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2h-Solving Basic Algebraic Equations (two steps)

Use your calculator to check your answer. Leave answer as simplified proper fractions.

Solve the following equations:

1. $2x + 1 = 5$

2. $-3x + 1 = 10$

3. $4x - 2 = 10$

4. $5x + 3 = -12$

5. $8 - x = 13$

6. $4x - 3 = 5$

7. $-3x + 17 = 14$

8. $-x + 6 = 6$

9. $-2x + 3 = -5$

10. $-4 + 2x = 6$

11. $-7 + 5x = 13$

12. $6 - 5x = 1$

13. $-4 - 6x = -4$

14. $10x - 45 = 45$

15. $-9 + 7x = 12$

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2i-More Solving Basic Algebraic Equations

Use your calculator to check your answer. Leave answer as simplified proper fractions.

1. $3x + 2 = 14$

2. $\frac{x}{-3} + 4 = 10$

3. $x + 3 = 3\,¾$

4. $½ x - 8 = 16$

5. $5x - 10 = 5$

6. $\frac{x}{2} + 3 = 7$

7. $\frac{x}{-7} - 5 = 15$

8. $¼ x = 32$

9. $\frac{x}{6} - 8 = 10$

10. $12 = \frac{x}{8} + 5$

11. $5x - 3 = 17$

12. $4x + 7 = -1$

13. $\frac{2x}{3} - 8 = 6$

14. $\frac{-3x}{4} + 4 = -8$

15. $\frac{-x}{3} + 1 = 4$

16. $5x + 3 = -12$

*17. $2x + 8 = 4x + 2$

*18. $3x - 1 = x + 5$

Name: _____   Date: _____

2j-Solving for a Variable (multiple steps)

When solving for a variable, reverse the order of operations. The objective is to isolate that variable by getting it to one side of the equation and all the constants to the other side. If faced with an equation with a nested variable (see example C), eliminate the outer portion before messing with the interior of an expression. To eliminate terms or coefficients, you will need to apply the inverse operations.

Examples:

A.
$$3x = 24$$
$$(3x) \div 3 = 24 \div 3$$
$$x = 8$$

B. $3x + 1 = 25$
$$3x + 1 - 1 = 25 - 1$$
$$3x = 24$$
$$\frac{3x}{3} = \frac{24}{3}$$
$$x = 8$$

C. $(3x-1) \div 5 = 3$
$$(3x-1) \div 5 \times 5 = 3 \times 5$$
$$3x - 1 = 15$$
$$3x - 1 + 1 = 15 + 1$$
$$3x = 16$$
$$\frac{3x}{3} = \frac{16}{3}$$
$$x = 5\tfrac{1}{3}$$

Solve for the variable in the following equations:

1. $3x - 4 = 11$

2. $5x + 7 = -3$

3. $-3x + 2 = 17$

4. $\dfrac{2x + 3}{3} = 5$

5. $\dfrac{2x + 5}{2} = 3$

6. $2(2x - 1) = 8$

*7. $\dfrac{2x + 5}{2} - 3 = 3$

*8. $\dfrac{3x + 2}{3} + 7 = 6$

*9. $3(x+5) - 2 = 7$

Name: _____   Date: _____

2k-Solving for a Variable (Multiple Like Terms)

When faced with an equation that has multiple terms of the same variable you need to solve, get those terms together on one side of the equation. It doesn't matter what side of the equation you move the terms to, as shown below. You may have to simplify before you can move terms around. There are many alternate methods of solving for a variable; valid methods require the use of the Field Axioms and PEMDAS.

Example: $2(3w + 2) - 12 = 3w - 11$

$$6w + 4 - 12 = 3w - 11$$
$$6w - 8 = 3w - 11$$

$6w - 3w - 8 = 3w - 3w - 11$ or $6w - 6w - 8 = 3w - 6w - 11$

$$3w - 8 = -11 \qquad\qquad -8 = -3w - 11$$
$$3w - 8 + 8 = 8 - 11 \qquad 11 - 8 = -3w -11 + 11$$
$$3w = -3 \qquad\qquad 3 = -3w$$
$$3w \div 3 = -3 \div 3 \qquad 3 \div (-3) = -3w \div (-3)$$
$$w = -1 \qquad\qquad -1 = w$$

Solve for the variable:

1. $8x - 2(x - 8) + 4x = -4$

2. $4x - 2(x+3) - 4x = -3$

3. $-3(x + 5) = 10 - 2x$

4. $12z - 3(z - 7) = -(5z + 7)$

5. $-6(w - 3) = 3w - 9$

6. $16x - 4(x - 8) + 8x = -8$

*7. $5y - [7 - (2y - 1)] = 3(y - 5) + 4(y + 3)$

8. $\frac{x}{4} + \frac{x}{2} + \frac{x}{3} = \frac{13}{6}$

Name: _____   Date: _____

2L-More Practice Solving Equations

You may use your calculator only to check your answers. Leave fractions proper.

Solve for the variable:

1. $3x - 2 = 5x$ 2. $4x + 6 = 3x$ 3. $5x - 14 = -2x$

4. $6x - 2x - 3 = 9$ 5. $-7x + 4x + 5 = 20$ 6. $-x + 7 = 3x - 1$

7. $2x + 6 = 7x - 14$ 8. $7x - 12x + 4 = 19$ 9. $8 - x = 9 - 2x$

10. $7 - 2x = -3 + 3x$ 11. $54 - (\tfrac{2}{3})x = 38$ 12. $-7 - x = 8 + 4x$

13. $4x = 3x - 2(5-x)$ 14. $2x - 3 = 4(x+3) - 5x$ 15. $\dfrac{x-3}{4} + \dfrac{2x+5}{2} = 7$

Name: _____   Date: _____

2m-Basic Algebraic Equations with Perimeter

For each question, translate the equation and then solve by showing your algebraic steps.

1. The perimeter of the room is 248 inches. If two walls of the rectangular room are 80 long, how long is each of the other walls?

2. A rectangular room has perimeter of 30 inches. One wall is 10. Find the dimensions of the other three walls.

3. A triangle has sides 1, 2, and 2. Find its perimeter.

4. A rectangle has dimensions 6" by 7". Find its perimeter.

5. A rectangle has perimeter 6 and one known side of 2. What are the dimensions of the rectangle?

6. A hexagon (6-sided shape) has sides 2, 1, 4, 5, 4 and a perimeter of 20. How long is the missing side?

7. Find the perimeter of an octagon with equal sides of 4. An octagon has 8 sides.

8. A pentagon has perimeter of 26. If four sides are known to be 4, 5, 7, and 6, how long is the remaining side?

9. A rectangle has dimensions x - 3 and 2x + 6 with a known perimeter of 24, what is x?

10. A triangle has sides 2x, 3x +1, and x - 5 with a known perimeter of 26, what is x?

Name: _____   Date: _____

2n-Basic Algebraic Equations with Area

For each question, translate the equation and then solve by showing your algebraic steps.
Leave answers as proper fractions.

1. A rectangular room is 8 feet by 10 feet. Draw the layout and label. Find the area of the room.

2. A rectangular kitchen is 12 feet by 9 feet. Draw its layout and find the area of the room.

3. A rectangle is 9 ½ by 10 ¼. Find its area.

4. A triangle has a base of 4 ½ and a height of 5¾. Find its area.

5. A rectangle is 3⅝ by 4⅛. Find its area.

6. A square has a side of 3⅓. Find its area.

7. A bedroom is to be carpeted. Its dimensions are 9⅓' by 11⅞'. How much square feet does it have? How much square yardage does it have? (3 ft = 1 yd, 9 ft² = 1 yd²)

Name: _____   Date: _____

2o-Basic Algebraic Equations with Fractions

For each question, translate the equation and then solve by showing your algebraic steps. Leave answers as proper fractions.

1. $2x - 3 = 5$

2. $3x - 1 = 11$

3. $4x - 1 = 15$

4. $3x - \frac{1}{2} = 5\frac{1}{2}$

5. $4x - 1\frac{1}{4} = 3\frac{3}{4}$

6. $6x - 1\frac{1}{8} = 4\frac{3}{8}$

7. $3x - 2 = 5$

8. $4x + 5 = 16$

9. $5x + 30 = 3$

10. $-2x = 9$

11. $-3x + 1 = 6$

12. $-6x + 4 = -9$

13. $\frac{3}{2}x + 4 = -5$

14. $\frac{5}{3}x - 4 = 6$

15. $\frac{4}{5}x - 1\frac{3}{4} = 10\frac{1}{8}$

Name: _____   Date: _____

2p-Basic Algebraic Equations with Fractions

Solve for x by showing your algebraic steps. Leave answers as proper fractions.

1. The area of a rectangle is length times width. If the area of a rectangle is 9⅜ cm² and its width is 4¼ cm, what is its length?

2. If a rectangle's area is 100¾ cm² and its length is 9½ cm, what is its width?

3. Area of a triangle is half its height times base. If the area of a triangle is 10 cm² and has base of 3 ½ cm, what is its height?

4. If the area of a triangle is 4 ½ and its height is 3¼, what is its base?

5. $\frac{3}{2}x = \frac{27}{44}$

6. $\frac{4}{5}x = \frac{6}{25}$

7. $\frac{4}{9}x = \frac{30}{21}$

8. $\frac{3}{4}x - \frac{3}{4} = \frac{11}{12}$

9. $\frac{8}{9}x + 3\frac{1}{2} = 10\frac{7}{8}$

10. $4\frac{1}{3}x + 1\frac{3}{4} = 11\frac{1}{8}$

Name: _____   Date: _____

2q-Basic Algebraic Equations with Fractions

Use your calculator to check your answer. Leave answer as simplified proper fractions.

1. $2x - 4 = 9$

2. $3x + 5 = 7$

3. $7x - 1 = 8$

4. $\frac{2}{3}x - 4 = 10$

5. $\frac{3}{4}x + 5 = 14$

6. $\frac{x}{2} - 3\frac{3}{4} = 5\frac{1}{8}$

7. $\frac{3x}{4} - 6 = 10$

8. $\frac{5x}{3} = 15$

9. $\frac{3}{4}x = \frac{15}{16}$

10. If a square has a side of ½ inch, what is its area?

11. If a rectangle has dimensions of ¾" and ⅝", what is its area?

12. If a rectangle has area of 32 ⅔" and a length of 14", what is its width?

13. If a rectangle has area of 38" and a length of 4¾", what is its width?

14. If a square has area of $\frac{25}{64}$ squared feet, what is its length of side?

Name: _____ Date: _____

2r- More Fractions with Perimeter and Area

| Use your calculator to check your answer. Leave answer as simplified proper fractions. |

1. Find the perimeter of the following triangles with sides of:
   a. 3¾, 4½, 5⅝   b. 7⅔, 8⅓, 13⅓   c. 4⅓, 3½, 5⅛

2. Find the perimeter of the following rectangles with dimensions of:
   a. 4⅜" by $5\frac{3}{5}$ "   b. 4" by 4⅜"   c. 3½" by 3½"

3. Find the area of the following rectangles, with dimensions of:
   a. 4⅜" by $5\frac{3}{5}$ "   b. 4" by 4⅜"   c. 3½" by 3½"

4. Find the area of the following triangles, with dimensions of:
   a. Height 3½", base 8"   b. Height 2⅛", base $4\frac{4}{17}$ "   c. Height 4, base 5 ½

5. If the perimeter of a square is 16, what is its area?

6. If a triangle has area of 35 and length of 16, what is its base?

7. If a triangle has area of 32 and height of 5, what is its base?

*8. If the perimeter of a rectangle is 14 and the length is 5 more than the width, what is its area?

*9. If perimeter of a rectangle is 14 and the length is 4 more than the width, what is its area?

Name: _____   Date: _____

Chapter 2 Test

SHOW WORK. A calculator is NOT allowed on this test. You must work alone.
Questions regarding interpreting the directions are allowed. Simplify your fractions!

5

1. Put in order from smallest to greatest.

    a. $\frac{1}{2},\frac{1}{3},\frac{1}{5},\frac{1}{4},\frac{1}{1}$ B. - ½, ¼, - ⅝, ⅔

5

2. A. Write as an improper fraction: $6\frac{3}{4}$.   B. Write as a mixed number: $\frac{23}{8}$.

5

3. Add the fractions, leaving answer as a proper fraction:

    a. $\frac{3}{8}+\frac{7}{8}$ B. $\frac{4}{5}+3\frac{2}{3}$

5

4. Subtract the fractions, leaving answer as a proper fraction:

    a. $17\frac{8}{9}-5\frac{4}{9}$ B. $6\frac{1}{2}-3\frac{7}{8}$

5

5. Multiply and simplify to proper fractions:

    a. $\frac{5}{6}\times\frac{9}{10}$ B. $5\frac{2}{3}\times\frac{9}{17}$

5

6. Divide and simplify to proper fractions:

    a. $\frac{3}{4}\div\frac{7}{8}$ B. $8\frac{1}{2}\div4$

Chapter 2 Test, continued

7. Match the terms

| 10 |

_____ a. Vinculum   A. Top part of a fraction

_____ b. Denominator   B. Bar separating parts of fraction

_____ c. Numerator   C. Bottom part of a fraction

_____ d. Decimal   D. Dot separating whole from parts

_____ e. Improper fraction   E. $\frac{41}{5}$

_____ f. Proper fraction   F. $3\frac{2}{5}$

_____ g. 0.06   G. 6 thousandths

_____ h. 600   H. 6 hundredths

_____ I. 0.006   I. 6 hundreds

_____ j. 0.6   J. 6 tens

                                K. 6 tenths

| 5 |

8. Find the perimeter of the rectangle whose dimensions are 1¾ cm by 3 ⅓ cm.

| 5 |

9. Find the area of the rectangle whose dimensions are 5 ½" by 10".

| 0 |

Bonus: (2 pts) Let N = 20×30×50×70×110×130. What is the smallest positive prime number which is NOT a factor of N?

(3 pts) What is the value of x which satisfies the equation $\frac{5}{20} + \frac{7}{28} + \frac{9}{36} + \frac{11}{44} + \frac{x}{80} = 1$?

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Chapter 3 Decimals

Prior Skills:
- Fractions
- Area
- Perimeter
- For sheet 3k, volume and surface area
- For sheet 3m and 3o, Pythagorean Theorem
- For chapter 3 test, order of operations and definition of addend

Name: _____   Date: _____

3a-Decimal Notation

| Use your calculator only to check your answer. |

Write the following number in standard form:
1. $(3{\times}100) + (5{\times}10) + (1{\times}1) + (8{\times}0.001)$

2. $(5{\times}1000) + (3{\times}10) + (4{\times}0.1) + (5{\times}0.01)$

3. $(4{\times}1000) + (8{\times}100) + (4{\times}0.1) + (3{\times}0.001) + (3{\times}0.0001) + (5{\times}0.00001)$

4. $(8{\times}1) + (7{\times}0.1) + (4{\times}0.001) + (2{\times}0.000001)$

Write the following in expanded form, use the example as a guide:
$27.6581 = (2{\times}10) + (7{\times}1) + (6{\times}0.1) + (5{\times}0.01) + (8{\times}0.001) + (1{\times}0.0001)$
$$= (2 \times 10) + (7 \times 1) + (6 \times \tfrac{1}{10}) + (5 \times \tfrac{1}{100}) + (8 \times \tfrac{1}{1000}) + (1 \times \tfrac{1}{10000})$$

5. 4.8712

6. 3.140092

7. 62.34

8. 144.987

Write the following lengths of a rectangle as fractions:
9. 0.47 m   10. 0.3609 m   11. 5.63 ft

12. Which area is largest? 3.404, 3.44, 3.40004, 3.4004

13. Which perimeter is smallest? 2.3, 2.31, 2.311, 2.3111, 2.31111

Name: _____   Date: _____

3b-Switching Between Fractions and Decimals

Use your calculator only to check your answer.

The perimeter of the room is given in fraction form. Convert the fractions into decimal

1. 40½   2. 31¾   3. 61⅝   4. 56⅔

5. $\frac{4}{5}$ 6. $\frac{7}{4}$ 7. $\frac{19}{9}$ 8. $\frac{212}{100}$

9. $\frac{82}{5}$ 10. $6\frac{12}{15}$ 11. $\frac{5}{8}$ 12. $3\frac{14}{20}$

13. Which questions had repeating decimals? When does the fraction cause repeating decimals?

14. Rational numbers are those numbers that can be written as fractions. Thus the numbers in problems #1- #12 are all rational numbers. Which of the following decimals are not rational numbers?

    A. 0.121212...   F. 2.718281828459045...
    B. 3.444554445544455...   G. 1.4142643...
    C. 0.5   H. 1.4141414...
    D. 0.567   I. 6.55789789789...
    E. 3.141592...

Name: _____   Date: _____

3c- Operations with Decimals

Order the decimals from least to greatest.

1. 9.33, 9.4, 9.44, 9.45, 9.446

2. 2.11, 2.111, 2.121, 2.112

3. 3.4, 3.43, 3.424, 3.4509, 3.43509

4. 1.2, 1.302, 1.3002, 1.30002

Simplify the expressions.

5. $8.275 - 5.857$

6. $18.93 + 149.42$

7. $87.944 - 6.58$

8. $14.923 + 1.8$

9. $13.245 + 1.4467$

10. $12.3 + 1.43 + 1.5607$

11. $2.3 - 3.12$

12. $0.45 + 45$

13. $0.45 - 45$

14. $4.01 + 0.034$

15. $-0.475 - 3.78$

16. $-7.2 + 10.56$

17. $1.8 + 2.401 + 1.05$

18. $3.45 + 2.356$

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3d-Adding and Subtracting Decimals

Use your calculator only to check your answer.

1. Find the perimeter of the room with dimensions: 8.35' by 11.24'.

2. Find the perimeter of a triangle whose sides are 4.3", 6.71", and 5.901".

3. Find the perimeter of the room with dimensions 9.12' by 10.8'.

4. Find the perimeter of the room with dimensions 7.02' by 11.76'.

5. If the perimeter of a room is 48' and the length is known to be 10.8', what is the other dimension?

6. If the perimeter of a room is 65' and the length is known to be 12.5', what is the other dimension?

7. The perimeter of an equilateral (all sides equal) triangle is 34.8". What is the side length?

8. The perimeter of a square is 38". What is the side length?

9. The perimeter of a regular hexagon (6-sided shape with equal sides) is 39'. What is the side length?

Name: _____   Date: _____

3e-Multiplying & Dividing Decimals

Only use your calculator to check your answer.

1. $(-14.3)\times(-2.1)$

2. $(-0.05)\times(-0.05)\times(-0.05)$

3. -0.003×16.1

4. 15.2×5.2

5. $-18.34(-1.02)$

6. $0.0003(-1.2)$

7. $-18.46 - (-18.79) - 18.46$

8. $-15.37 - 14.35 + 6.2$

9. $-83.26 - (-15.6) +(-18.2)$

10. $9.3 \cdot 1.23$

11. $0.0023 \cdot 1.57$

12. $3.14 \cdot 2.718$

Solve the following problems with the aid of a calculator.

13. Your bank account started off with $40. Over the course of the week you either put money or took money out of your account. Here are the pile of ATM transactions: 15.50, - 4.75, -20, 55.75. How much did you have at the end of the week?

14. Your room has dimensions of 11.23 feet by 9.56 feet. What is its perimeter and area?

15. The monkey cage at a laboratory is 5.3 feet by 5.3 feet. What is the perimeter of the cage and it's area?

16. A zoologist recommended that captive monkeys should have AT LEAST a cage 10 feet by 20 feet. What is its perimeter and area?

17. What is the difference in areas between the two cages in problems #15 and #16?

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Name: _____   Date: _____

3f-Decimals with Area and Perimeter

Use your calculator to check your answer. You must show ALL your work!!!

1. Find the area of the rectangle with dimensions 4.1' by 8.24'.

2. Find the length of the rectangle which has width 3.2' and area 64 feet squared.

3. Find the area of the rectangle with dimensions 3.201' by 8.9'.

4. Find the width of a 32 feet squared rectangle with length 6.4'.

5. Find the area of the triangle with height 5.4" and base 8.2".

6. Find the area of the triangle with height 4.35" and base 2".

7. Find the base of the triangle with area of 35 squared inches and height of 1.5".

8. Find the height of the triangle with area of 36 squared inches and base of 1.8".

Name: _____   Date: _____

3g-Multiplying & Dividing Decimals

Simplify without the use of a calculator.

1. $13.5 \div 1000$

2. 1.004×100

3. $0.36 \div 0.009$

4. 2.3×3.5

5. 4.56×3.2

6. 3.4×5.6

7. $2.3 \div 100$

8. 5.34×1000

9. $1002.1 \div 10$

10. $3.2 \div 0.8$

11. $0.42 \div 0.07$

12. $0.0036 \div 0.06$

Solve without the use of a calculator.

13. If the available cars hold only 4 people, and 34 are going to the beach, how many cars will be needed?

14. If you spent $48.75 on three pair of jeans, how much is each?

15. If you drive 450 miles in a car that uses 27 gallons, what is your average mile per gallon?

16. If a sack of potatoes weigh 1.2 kilograms, then how much is it in grams? How much in milligrams?

17. A teacher wants to show a video in her 45 minute class. If the video lasts 153 minutes, how many classes must she reserve for its use?

18. A lawn of 150 ft by 250 ft will have a house built on it. If the house will be 75 ft by 45 ft, how much lawn will remain? Put answer in units of feet and yard.

Name: _____   Date: _____

3h-More Multiplying & Dividing Decimals

Simplify without the use of a calculator.

1. 2.9×3.1 2. 4.06×1.2 3. $27 \div 100$

4. $34.45 \div 1000$ 5. 0.45×100 6. $53.4 \times 10,000$

7. $3.2 \div 0.4$ 8. $0.45 \div 0.9$

Solve without the use of a calculator.

9. If 42 people are going to the beach, and cars seat at most 5, how many cars are needed? (We are omitting vans and other modes of transportation.)

10. If you spent $39.75 on three pair of jeans, what is the average price of one?

11. If you drive 500 miles in a car that uses 22 gallons, what is your average mile per gallon?

12. If a can weighs 10 grams, then how much is it in kilograms? How much in milligrams?

13. A teacher wants to show a video in her 35 minute class. If the video lasts 112 minutes, how many classes must she reserve for its use?

14. A lawn of 162 ft by 210 ft will have a house built on it. If the house will be 51 ft by 81 ft, how much lawn will remain? Put answer in units of feet and yard.

Name: _____   Date: _____

3i-Circumference with Decimals

No calculator is allowed. Use 3.14 when approximating π. Round to the nearest hundredth.

1. Calculate the circumference of the circle with radius 9".

2. Calculate the circumference of the circle with diameter 2".

3. Calculate the circumference of the circle with radius 3".

4. Find the radius of a circle with circumference of 31.41 inches.

5. Find the diameter of a circle with circumference of 6.28 inches.

6. Find the radius of a circle with circumference of 9.42 inches.

7. Find the diameter of a circle with circumference of 4.71 inches.

Name: _____   Date: _____

3j-Area with Decimals

No calculator is allowed. Use 3.14 when approximating π. Round to the nearest tenth.

1. Calculate the area of the circle with radius 9".

2. Calculate the area of the circle with diameter 2".

3. Calculate the area of the circle with radius 3".

4. Calculate the area of the circle with diameter 3".

5. Calculate the radius of a circle with area of 28.26 squared inches.

6. Calculate the radius of the circle with area of 50.24 squared inches.

7. Calculate the diameter of the circle with area of 314 squared inches.

Name: _____   Date: _____

3k-Surface Area & Volume with Decimals

A calculator is allowed. If your calculator does not have a π button, use 3.14 to approximate π.

1. Find the surface area and volume of the box:

8

20

10
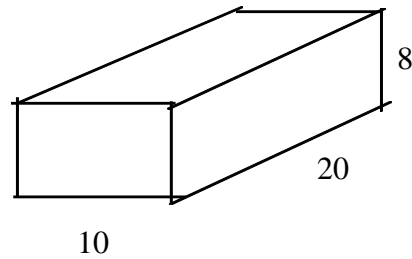
2. Find the surface area and volume of the box with dimensions 6" by 8" by 10".

6   10

8
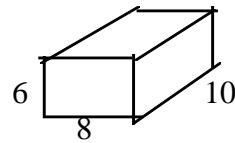
3. Find the surface area and volume of the cylinder with a radius of 5" and height of 15".

5

15
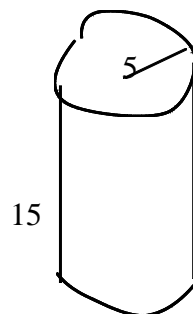
Name: _____   Date: _____

3L- Similar Shapes and their Surface Area & Volume

Show your work in spite of having a calculator. If your calculator does not have a π button, use 3.14 to approximate π.

1. Find the volume of a box with dimensions of 3 by 4 by 5.

2. Find the volume of a box with dimensions 6 by 8 by 10.

3. Find the volume of a box with dimensions of 12 by 16 by 20.

4. By how much, are the dimensions of the second box bigger than the first box? How does that compare to their volumes?

5. By how much, are the dimensions of the third box bigger than the first box? How does that compare to their volumes?

6. Find the volume of a cylinder that has a diameter of 2 and height of 4.

7. Find the volume of a cylinder that has a diameter of 4 and a height of 8.

8. Find a volume of a cylinder that has a diameter of 6 and a height of 12.

9. How do the dimensions in problems #6 and #7 compare? How do their volumes compare?

10. How do the dimensions in problems #6 and #8 compare? How do their volumes compare?

11. What rule can you create to make it easier to compare the volumes of similar shapes?

Name: _____   Date: _____

3m-Surface Area & Volume of Pyramids & Spheres

A calculator is allowed. If your calculator does not have a π button, use 3.14 to approximate π.

1. A pyramid is twice as big as a similar pyramid with volume of 25 in^3. Find the volume of the other pyramid.

2. A pyramid has a square base of sides 5 inches and a height of 3 inches. What's the volume of the pyramid?

3. A triangular pyramid is made up of equilateral triangles with a side of 2 feet. What is the pyramid's surface area? (Use Pythagorean Theorem to find the height of a triangle.)

4. A pyramid has a triangular base with area of 15 squared inches and a height of 6 inches. What's the volume of the pyramid.

5. A man decides to make a playhouse that's a triangular pyramid like the one in question #4, but three times as large. What would be the new volume?

6. Find the volume of a sphere that has a radius of 9 inches.

7. Find the volume of a sphere that has a radius of 4 inches.

8. Find the surface area of a sphere that has a radius of 9 inches.

9. Find the surface area of a sphere that has 4 inches.

10. A. Finding the amount of leather to cover a basketball is an example of (area, volume).
      B. Finding the amount of air contained in a basketball is an example of (area, volume).

Name: _____   Date: _____

3n-Mixed Review of Shapes and Objects

A calculator is allowed. If your calculator does not have a π button, use 3.14 to approximate π.

1. The area of a square is 625 cm^2. Find the length of the sides.

2. The area of a rectangle is 600 cm^2. Find the width of the rectangle if the length is 40 cm.

3. The area of a triangle is 45 cm^2. Find the base of the triangle if the height is 9 cm.

4. The area of the circle is 8π cm^2. Find the circumference of the circle.

5. The area of a square is 400 cm^2. Find the perimeter of the square.

6. The perimeter of a rectangle is 30 cm with the length being 8 cm. Find the area of the rectangle.

7. If the surface area of a box is known to be 300 cm^2 and have a base of 5 cm by 4 cm, what is the height of the box?

8. If the surface area of a box is known to be 320 cm^2 and have a height of 36 cm and a width of 2 cm, what is the depth of the box?

9. If the surface area of a sphere is 400π cm^2, what is its volume?

10. If the surface area of a sphere is 100π cm^2, what is its volume?

11. If the surface area of a cube is 600 cm^2, what is its volume?

12. If the surface area of a cube is 96 cm^2, what is its volume?

13. If the volume of a cube is 27 cm^3, then what is its surface area?

14. If a square pyramid has height of 10 cm and a volume of 60 cm^3, what is the dimension of its base?

15. If a square pyramid has a height of 5 cm and a base side of 6 cm, what is its volume?

Name: _____   Date: _____

3o-Mixed Review of Volume & Surface Area

A calculator is allowed. Leave answers in terms of π.

1. A cube has surface area of 72 in².
   What is the area of each face?
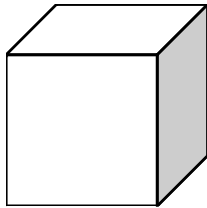
2. A tetrahedron has surface area of
   of 27.7128 in². What is the length
   of each
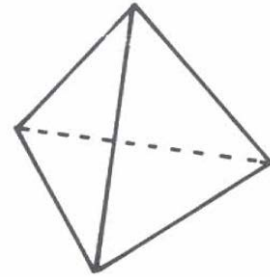
3. A prism has a base of 4" by 8" and surface area
   of 112 in². What is its height?
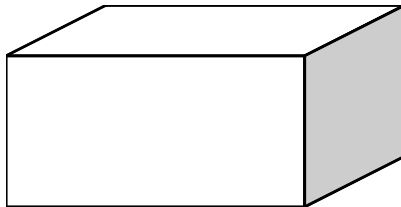
4. A prism has dimensions
   of 3" by 4" by 5".
   What's its surface area?
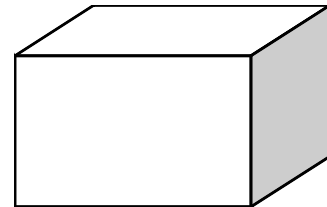
5. A cylinder has radius of 2" and
   height of 6". What's the surface area?

6. A cylinder has diameter of 2"
   and height of 3". What's the
   surface area?
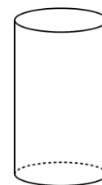
Name: _____   Date: _____

3p-Mixed Review of Volume & Surface Area

A calculator is allowed. Leave your answer in terms of π.

1. What the volume of a cylinder whose diameter is 10 and height is 3?
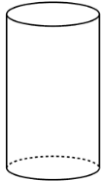
2. A cylinder has radius of 4" and surface area of 64π. What's the height?
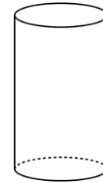
3. What's the volume of a rectangular prism of dimensions 3 by 4 by 5?
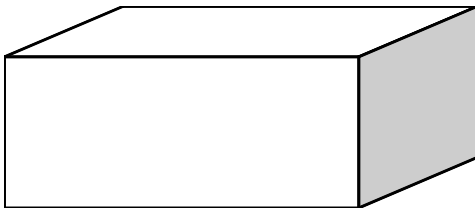
4. What's the volume of a right triangular prism whose base has sides 3, 4, and 5 and height of 10?
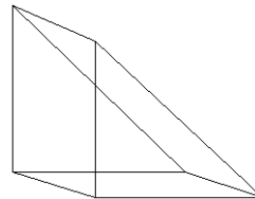
5. What the volume of a cylinder whose radius is 6 and height is 10?

*6. A cylinder has height of 3" and surface area of 36π. What is the radius?

Chapter 3 Test

Name: _____   Date: _____

| SHOW WORK. A calculator is NOT allowed on this test. You must work alone. |

[5]

1. Put in order from smallest to greatest.
   a. 6.1, 6.01, 6.11, 6.001   B. -3.2, -3.22, -3.3

[5]

2. A. Write as decimal: $6\frac{3}{4}$.   B. Write as a decimal: $\frac{23}{8}$.

[5]

3. Add:
   a. $4.3 + 7$ b. $4.3 + 13.7$ c. $4.3 + 8.007$

[5]

4. Subtract:
   a. 12.3 - 3.4   b. 12.3 - 5.444   c. 1.23 - 5.444

[5]

5. Multiply:
   a. 1.3×6 b. 1.3×6.44 c. $-2.3 \times (-8.55)$

[5]

6. Divide:
   a. $17.34 \div 1.7$ b. $42.9 \div 0.03$ c. $-12.56 \div (-8)$

7. Find the perimeter of the rectangle whose dimensions are 2.45 cm by 3.1 cm.

$$\boxed{\frac{}{5}}$$

8. Find the area of the rectangle whose dimensions are 4.03" by 11".

$$\boxed{\frac{}{5}}$$

9. Simplify

   a. $4(3 - 4)^2 - 5 \div 10$ b. $[3(2-1)\times 5 - 4] \div 10 + 1$

$$\boxed{\frac{}{10}}$$

10. Find the volume and surface area of a cylinder whose diameter is 10 cm and its height is 20 cm.

$$\boxed{\frac{}{10}}$$

Bonus: (3 pts) The sum of ten positive odd numbers is 20. What is the largest number which can be used as an addend in this sum?

$$\boxed{\frac{}{5}}$$

Bonus: (3 pts) Two 5×5 squares overlap to form a 5×7 rectangle, as shown. What is the area of the region in which the two squares overlap?
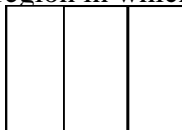
Chapter 4 Percents and Proportions

Prior Skills:
- For sheet 4f, placement values
- For sheet 4g, powers of ten
- For sheet 4h, an introduction to other bases would be good for students to get a historical sense of mathematics.
- For sheet 4i, decimal representations of fractions
- For sheet 4k, practice understanding and solving with percents
- For sheet 4m, practice with understanding and solving proportions

Name: _____   Date: _____

4a- Field Axioms

Simplifying any expression or solving any equation requires the use of the following axioms:
Closure: A set of numbers is closed if the unique sum or product of an operation is also in the same set. For example: $3+8 = 11$, so the set of reals would be closed, but not digits.

Commutativity: $x+ y = y + x$ or $xy = yx$. The order to how the sum or product is obtain is not important, e.g. $2 + 1 = 1 + 2$.

Associativity: How the sum or product is grouped isn't important. Examples can be seen in $\mathbf{x} + (y + z) = (x + y) + z$ or $2(3\times4) = (2\times3)\times4$.

Distribution of **Multiplication** Over **Addition**: $x(y + z) = xy + xz$

Identity: Zero plus any number remains that number. One times any number remains that number. For example: $0+2 = 2$, $1\times2 = 2$. Identity is about leaving the value unchanged!

Inverse: To obtain zero, sum the number and its opposite. To obtain a one, multiply the number and its reciprocal. The Inverse defines subtraction and division! For example: $2 + (-2) = 0$ which is the same as $2 - 2 = 0$. You can see it with $2(½) = 1$ which is $2/2 = 1$.

1. What is the additive inverse of ⅝?
2. Write an example of the distribution property.
3. What is the multiplicative inverse of ⅓?
4. What is the multiplicative inverse of x?
5. Rewrite the following using addition: $5 - 5 = 0$.
6. Rewrite the following using multiplication: $5/3$.
7. Fill in the justifications (axioms) used in the proof of the Multiplicative Property of Zero:
     a. $0 = 0$ Reflexive Property
     b. $0 + 0 = 0$ _____
     c. $x(0 + 0) = x(0)$ Multiplication Property of Equality
     d. $x(0+0) = 0 + x(0)$ _____
     e. $x(0) + x(0) = 0 + x(0)$ _____
     f. $x(0) = 0$ Add/Subtraction Property of Equality
8. Fill in the justifications to the following problem:
     a. $4x - 3 = 5$ Given
     b. $4x - 3 + 3 = 5 + 3$ _____, Add Property of Equality
     c. $4x + 0 = 8$ _____
     d. $4x = 8$ Add Property of Zero
     e. $(¼)(4x) = (¼)(8)$ _____
     f. $(¼*4)x = 2$ _____
     g. $1*x = 2$ _____
     h. $x = 2$ Identity

Name: _____   Date: _____

4b – Exponents

An exponent is a shorthand notation for multiplication. A power is an expression that uses exponents. In the example, $5^3 = 5 \cdot 5 \cdot 5 = 125$, the three is the exponent and five is the base. The exponent of three says to multiply five three times. $4^2 = 16$ because $4 \cdot 4$ is 16.

Write the following as a product and then simplify:

1. 3^2 2. 2^4 3. 5^3 4. 10^6

5. 4^2 6. 3^3 7. 8^2 8. 4^5

9. 10^3 10. 1^{10} 11. 0^2 12. $(-4)^2$

13. $(-3)^4$ 14. $(-2)^3$ 15. $(-3)^5$ 16. 4^0

Please note that anything with a zero in the exponent is reduced to equal 1.

Negative exponents are first simplified by making them positive. By definition they mean to reciprocate the value.

17. $10^{-2} = 1/10^2 = 1/100$ 18. $2^{-4} = 1/2^4 = 1/16$ 19. 3^{-3} 20. 4^{-2}

21. 3^{-3} 22. 2^{-5} 23. 6^{-3} 24. 10^{-5}

4c- Order of Operations

In English, there is a standard way of writing; there is an order of words that belie the meaning. The subject is first, followed by verb, and then the direct object. The sentence, "the dog chased the cat" has a different meaning from "the cat chased the dog". The same can be said about mathematical operations. There is a prescribed order of simplifying expressions. This order can be remembered by the mnemonic "Please excuse my dear aunt Sally" or "Please eat Ms. Daisy's apple sauce." The first letter of each word represents the mathematical operations: **Parentheses, Exponents, Multiplication, Division, Addition**, and **Subtraction**.

Actually division and subtraction are a special type of multiplication and addition. So whenever you have multiplication and division (addition and subtraction) together, they are treated equally. That means you do the operations as they present themselves from **left to right**.

Example A: $2 \times 3 \div 2 + 1 - 2$
$\qquad 6 \div 2 + 1 - 2$
$\qquad 3 + 1 - 2$
$\qquad 4 - 2$
$\qquad 2$

Example B: $-(-3) - [-(-4) + 2] + 7$
$\qquad 3 - [4+2] + 7$
$\qquad 3 - 6 + 7$
$\qquad -3 + 7$
$\qquad 4$

Simplify the following expressions:

1. $(2 \times 3 \times 4 - 1 \times 3)^2$

2. $3(2 - 3)^2 \times 3 - 1$

3. $(2 + 2^2) \div 3 - 6 \div 3$

4. $1 + 4^2 \div (5-3)$

5. $2 - 2 \times 3^2 + 1$

6. $(2 \div 2) \times [3^2 - 1]$

7. $2x - 3x(5x) + 3x$

8. $2x - 2x(5x + 3)$

9. $(2-3)x + 4x + 3$

Name: _____   Date: _____

4d- Using Absolute Values within the Order of Operations

The American way to remember the order is to say "Please Excuse My Dear Aunt Sally" where the beginning of each word represents the operation: Parentheses, Exponents, Multiplication, Division, Addition, and Subtraction. Since division is definition a multiplication of fractions, whenever you see them together, you go left to right. This can explain why the Canadian expression "Bed Mas" is still valid in spite of them listing division first. Again when multiplication and division are together, the order is the left operator goes first. The same can be said of addition and subtraction.

1. $2 - 3(4+5) + 3 \cdot 5$

2. $2 \cdot 3 + 3(2-1)$

3. $4(3-2^2) - 3 \cdot 7 + 8 \cdot 4$

4. $3 \div 9 \cdot 6 - 4^2 + 5(3-3^2)$

5. $5 - 2^2 + 3^4$

6. $-4*2*6 - 12/3 + 3^3$

7. $2 + 3 \times 4(3 - 2^2) \div 6 + 8$

8. $(3 - 3^2)(4 - 3 \times 5)$

9. $3 \times 4 \div 6 + 5 \times 2$

There are various ways to write a parenthesis: { }, ( ), [ ], ‾, | |. The absolute value is a special type of parenthesis because it makes the expression contained within a positive value. The first two questions have been done for you.

1. $| -3 | = 3$

2. $| 1-4 | = | -3| = 3$

3. $-2 | 3+8-10 |$

4. $| 2-6 | \cdot 3$

5. $| -\frac{1}{2} |$

6. $(-3+4^2)(-6)$

7. $-3(2^3 + 4) \cdot 5$

8. $1-5 | 5 - (9+1)|$

9. $-2|3-(7-3)|$

10. $[3 - (2 - 4)][3 + | 2 - 4 |]$

11. $4 - | 1 - 7 |$

12. $8 - 3| 5 - 4^2 + 1|$

Name: _____

4e - Evaluating

Most times you need to determine the value of an expression (the number an expression represents).   In order to find the value, you need to know what the variables represent. By substituting the value of the variables and simplifying, the value of the expression will be found. Note the vertical alignment of the work; try to show your work in similar manner.

Example A: Evaluate the expression when $y = 2$:

Step I - substitute value of y.

Step II - simplify.

$3y + 2y^2 - 6(y - 4) + 8y^3$.

$3(2) + 2(2)^2 - 6(2 - 4) + 8(2)^3$.

$6 + 2(4) - 6(-2) + 8(8)$

$6 + 8 + 12 + 64$

90

Example B: Evaluate $\dfrac{3(2x+1) - 2(x-3)}{x+6}$ when $x = -3$

$$\dfrac{3(2(-3)+1) - 2(-3-3)}{-3+6}$$

$$\dfrac{3(-6+1) - 2(-6)}{3}$$

$$\dfrac{3(-5)+12}{3}$$

$$\dfrac{-3}{3} = -1$$

Evaluate when $a = -1$.

1. $(4a + 3)a - (2 + 2a)$

2. $12a^2 - 3a + 2$

3. $3a^4 + 5a^3 - 6a^2 - 2a$

Evaluate when $x = 1$, $y = 2$, and $w = 2$

4. $|2x + 3| + |5 - 3x|$

5. $-(-x)$

6. $x - (x - \{x - y\})$

7. $4w + 7 + 3w - 2 + 2w$

8. $\dfrac{\frac{1}{w} + \frac{w}{3} \times \frac{3}{w} + \frac{1}{w}}{w \times w - w}$

9. $\dfrac{3(x+y) - 2(x-y)}{5x+y}$

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4f- Expanded Form

Fill in the missing parts by writing out the numeric value.
1. 8,375 = eight thousand, three hundred _____
2. 312, 435, 600 = three hundred twelve million, _____, six hundred
3. 412 = _____ twelve
4. 8,300,567 = eight million, _____, five hundred sixty-seven

Identify the numeric value of the number indicated:
Example 567,890   5 represents the _____hundred thousands_____
5. 320,045   2 represents the _____
6. 1030   3 represents the _____
7. 321   3 represents the _____

Write out in expanded form:
Example: 431.24 $4(100) + 3(10) + 1(1) + 2(1/10) + 4(1/100)$
8. 3, 243

9. 345,645

10. -23,120

11. 304.125

12. 300,561, 100

Write out in standard form:
Example: $3(1000) + 4(10) + 8(1) + 9(1/10) = 3,048.9$

13. $4(1,000,000)+8(10,000)+3(1,000)+7(100) =$

14. $2(10,000) + 8(100)+3(10)+4(1/10) =$

Name: _____   Date: _____

4g-Expanding Numbers Using Powers

Use calculator only when checking answers!

Write the following as a power of 10:
1. 10,000   2. 100   3. 10,000,000   4. 1

Write each of the following in standard form:
5. $(4 \times 10^4) + (9 \times 10^3) + (5 \times 10^2) + (6 \times 1)$

6. $(6 \times 10^5) + (6 \times 10^2)$

7. $(7 \times 10^6) + (8 \times 10^5) + (6 \times 10^4)$

8. $(8 \times 10^4) + (2 \times 10^3) + (3 \times 10^2)$

9. $(4 \times 10^2) + (8 \times 10^4) + (4 \times 1) + (7 \times 10^3)$

Write the following numbers in expanded form, using powers:
10. 34,500

11. 403

12. 1,234

13. 50,500,000

14. 271,828,459,045

Name: _____   Date: _____

4h- Expanded Form using Powers

Do without the aid of a calculator. Check your answers with a calculator at the end.

Use the powers of 10 to expand this number:
example: $8,321 = 8(10^3) + 3(10^2) + 2(10^1) + 1(10^0)$

1. 340, 073   2. 53,100   3. 830,401

4. 234,876,912   5. 32   6. 144

7. 300,045   8. 16, 230, 030

Write the following numbers in standard form:
9. $7(10^3) + 4(10^2) + 2(10^1) + 6(10^0)$
10. $8(10^5) + 8(10^3) + 4(10^1) + 7(10^0)$
11. $8(10^6) + 1(10^5) + 2(10^4) + 1(10^3) + 5(10^2) + 3(10^1)$
12. $5(10^8) + 8(10^6) + 7(10^4) + 9(10^2) + 2(10^0)$

We work with base 10, probably because we have 10 fingers. But other societies used different bases. France used to have base 20 (they used fingers and toes). It still exists in some names for numbers, like *quatre-vingt-neuf*. The Babylonians used base 60 which remains in our time: 60 minutes in an hour and 60 seconds to a minute. Computers made wide use of counting by 2s or 16s. Follow the example of how to convert other bases into base 10 (standard form).

Example: $11101_2 = 1(2)^4 + 1(2)^3 + 1(2)^2 + 0(2)^1 + 1(2)^0 = 16 + 8 + 4 + 0 + 1 = 29$

10. $10111_2 = 1(2)^4 + 0(2)^3 + 1(2)^2 + 1(2)^1 + 1(2)^0 =$ _____

11. $101_2 = 1(2)^2 + 0(2)^1 + 1(2)^0 =$ _____

12. $110101_2 = 1(2)^5 + 1(2)^4 + 0(2)^3 + 1(2)^2 + 0(2)^1 + 1(2)^0 =$ _____

13. $FACE_{16} = F(16)^3 + A(16)^2 + C(16)^1 + E(16)^0 = 15(16)^3 + 10(16)^2 + 12(16)^1 + 14(16)^0$
       = _____ (calculator allowed)

Name: _____   Date: _____

4i-More Practice with Standard and Non-Standard Forms

Exercise A. Write the following numbers in expanded form:
Example: $835 = 8(10^2) + 3(10^1) + 5(10^0)$

1. 435,800
2. 12,300,506
3. 1000
4. 15,000
5. 4,000,000,000,000,000
6. 5,000,000,000,000

Exercise B. Write the following numbers in expanded form, using the example as a guide:
Example: $1.2345 = 1 + 2(\frac{1}{10}) + 3(\frac{1}{100}) + 4(\frac{1}{1000}) + 5(\frac{1}{10,000})$

7. 12.005
8. 0.0004
9. 0.100304
10. 4.500005

Exercise C. Write the following numbers in expanded form, using the example as a guide:
Example: $1.2345 = 1 + 2(10^{-1}) + 3(10^{-2}) + 4(10^{-3}) + 5(10^{-4})$

11. 12.005
12. 0.0004
13. 0.100304
14. 4.500005

Exercise D. Write the following numbers in standard form.
15. $8(10^6) + 3(10^5) + 5(10^4) + 1(10^0) =$

16. $4(10^{13}) + 8(10^{10}) + 5(10^5) =$

17. $3(10^3) + 5(10^0) + 4(10^{-1}) + 5(10^{-2}) =$

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4j - Scientific Notation

Since mathematicians and scientists have tried to find ways of simplifying written expressions, scientific notation is common for writing VERY large or small numbers. This is very useful when talking about the distance between stars, light years, the number of molecules in an object, the people on the planet, etc.

Note how this large number is rewritten: $64000000 = 6.4 \cdot 10^7$ The 6.4 is the mantissa and is always a number between 1 and 9, inclusively. The exponent 7 is the characteristic and says what power of 10 you are multiplying by. Some people just remember that is how many spaces to move the decimal point. A negative moves it left and a positive moves the decimal right. A small decimal like 0.000007 is rewritten as $0.000007 = 7 \cdot 10^{-6}$.

1. 0.000123   2. 2,340,000,000   3. 0.0000000345

4. 435000000000000   5. 76,000,000,000   6. 0.0000308

Express each number into scientific notation. For some you may have to round to three significant digits in the mantissa.

7. 23,000,000,000,000   7. _____
8. 134,000,000,000   8. _____
9. 456,000,340,000,000   9. _____
10. 788,999,213,543,345,532,345   10. _____

Express each number in standard form.

11. 2.34×10^4 11. _____

12. 3.4×10^{14} 12. _____

13. 6.7×10^7 13. _____

14. 2.6×10^9 14. _____

15. 3.4×10^{-5} 15. _____

16. 1×10^{-10} 16. _____

17. 3.25×10^{-6} 17. _____

Fill in the missing spots of the equation.

18. $0.0000234 = 2.34 \times 10$ 19. $3.1 \times 10^{-5} = 00000031$

20. $0.0045 = 45 \cdot 10^{-3}$ 21. $6.1 \cdot 10 = 610,000$

Name: _____   Date: _____

4k - Percents

Write the equivalent percent for each of the following:
1. 0.04   2. 0.12   3. 0.45   4. 1.23   5. 1

Convert the percent into standard decimal form:
6. 1%   7. 23%   8. 123%   9. 0.3%   10. 56%

Solve using percents:
11. What is 10% of $32?   12. What is 5% of $32?   13. What is 15% of $32?

14. What is 5% of $42?   15. What is 10% of $42?   16. What is 20% of $42?

17. What is 15% of $42?   18. What is 8% of $40?   19. What is 110% of 80?

20. A suit at JcPenny's has a $185 tag. But it's on sale for 30% off. How much will it cost without tax?
    If the Nova Scotia government has a provincial tax of 15%, how much will the suit cost?

21. If you spent $42.75 on 3 CDs, how much does each one cost?
    What percent of the total cost is one CD?

22. A dress has a price tag of $90. The dress is on sale for 20% off and there is a 6% sales tax.
    What is the total cost of the dress? How much money do you save?

23. A dress has a price tag of $86.
    a. If it is on sale for 20% off, what will be the cost? (Ignore tax)
    b. What is the price of the dress on sale with a 6% tax?
    c. How much would the dress cost when not on sale?

Name: _____   Date: _____

4L – Using Proportions with Conversions

Show your work! Calculator is allowed.

Going between Canada and the United States, you will find that the two countries use different measuring systems. If you want to know how fast to go or how far away some city is, you might have to do a bit of converting. Here are some basic units:

1 mile = 1.6 kilometer
10 mm = 1 cm   10 cm = 1 dm   10 dm = 100 cm = 1 m   1000 m = 1 km
2.5 cm = 1 in
12 in = 1 ft   3 ft = 1 yd   5280 ft = 1 mi (mile)

Metric Distance Conversion

1. 5 m = _____ cm = _____ mm = _____ km

2. 60 km = _____ m = _____ cm = _____ dm

3. 45 mm = _____ m = _____ km

4. 0.23 km = _____ m = _____ mm

5. 0.003 km = _____ m = _____ cm

Area Unit Conversion

1.   10 feet squared = _____ inches squared   2. 196 in^2 = _____ ft^2

3. 105 in^2 = _____ yd^2 4. 144 mm^2 = _____ cm^2

5. 10000 cm^2 = _____ m^2 6. 100 m^2 = _____ km^2

7. 145 m^2 = _____ km^2 8. 5 ½ m^2 = _____ cm^2

9. 10 ¾ yd^2 = _____ ft^2 10. 12 ¼ ft^2 = _____ in^2

11. 400 ½ m^2 = _____ km^2 12. 2 ¼ ft^2 = _____ in^2

13. 3 1/3 yd^2 = _____ in^2 14. 2 ¾ km^2 = _____ cm^2

Name: _____   Date: _____

4m - Proportions

Show your work! Use a proportion to solve each problem. Calculator is allowed.

1. $\frac{3}{5} = \frac{x}{15}$

2. $\frac{x+1}{5} = \frac{x-1}{2}$

3. $\frac{x}{x+8} = \frac{2}{3}$

4. The ratio of seniors to juniors is 2:3. If there are 21 juniors, how many seniors are there?

5. A 15-foot building casts a 9-foot shadow. How tall is a building that casts a 30-foot shadow at the same time?

6. A photo that is 3 inches wide by 5 inches high was enlarged so that it is 12 inches wide. How high is the enlargement?

7. Philip has been eating 2 hamburgers every 5 days. At that rate, how many hamburgers will he eat in 30 days?

8. An architect wants to build a model of the structure he is making. The structure is 80 feet tall and 35 feet wide. His model will be 50 cm wide. How tall will it be?

9. Draw a rectangle whose dimensions are 3 cm by 4 cm. Find its area and perimeter. Then draw an enlarged rectangle whose dimensions are 9 cm by 12 cm. Find its area and perimeter. How do those numbers relate to the scale factor between the two images?

The following is a review of shapes. You may use a calculator.

10. A right triangle has two sides of 5 and 12. What is the distance of the hypotenuse?
11. What is a 4-sided polygon called?
12. What is a 6-sided polygon called?
13. If two angles in a triangle are 30 and 45, what is the third angle?
14. What is a scalene triangle, isosceles triangle, and equilateral triangle?
        What relationship is there between the angles and sides being equal?

15. Draw an example of a reflection, rotation, and translation. Label each drawing.

Name: _____   Date: _____

4n – Mixed Review

Solve the following. Show your work! Calculator is allowed.

1. If you drive 650 miles in a car uses 30 gallons, what is your average mpg?

2. If a house is 2.3 km away, how far is it in meters? Millimeters?

3. A teacher wants to show a movie in her 50 minute class. If the movie is 210 minutes long, how many classes must she need in order to watch the entire movie?

4. A lawn of 100 ft by 150 ft has a house built on it. The house is 50 ft by 30 ft. What percent of the lawn is left to landscape?

5. Amy used to work 60 hours each week. Now she works 45 hours a week. What is the percent decrease?

Chapter 4 Test

Name: _____   Date: _____

SHOW WORK for full credit!!! You must work alone. Only a calculator is allowed.

| 5 |
1. Determine which ratio is larger. Use the symbols $<$, $>$, or $=$ to fill the blank:

  a. $\frac{5}{6}$ ——— $\frac{8}{9}$ b. $\frac{11}{15}$ ——— $\frac{23}{27}$ c. $\frac{15}{18}$ ——— $\frac{5}{6}$

| 5 |
2. Write the ratios:

  a. There are fourteen P-3 students, fourteen 4-6th grade students, six 7-8th graders, fourteen10th graders and nine 11-12th grade students at SLA. Write the ratio of the 4-6th to the 7-8th graders.

  b. Ethan and Ben played checkers. Ethan won 8 games and Ben won 12. Write the ratio, in simplest form that compares Ethan's score to Ben's.

| 5 |
3. Write each rate in simplest form, and give its units:
  a. Kelsey skied downhill twice in 30 minutes.
  b. The Morash family visits Martock 20 times in five months.
  C. My granny bought 4 tickets to the Tattoo for $160.

| 5 |
4. Solve: a. $\frac{4}{x} = \frac{24}{54}$ b. $\frac{40}{n} = \frac{8}{5}$ C. $\frac{12}{13} = \frac{5}{x}$

| 5 |
5. Solve these word problems:
  a. The ratio of the boy's shadow to the flag pole's shadow is 1:10. If the boy is 160 cm tall, how high is the pole?

  b. On a map 2.5 cm represents 300 km. What distance would a 3.5 cm line represent?

| 5 |
6. A. Which is cheaper: $3.20 for 8 L or $2.40 for 4 L?

  b. Paul's family travelled 190 km in 3 hours. At this speed, how far do they go in 5 hours?

5

7. Answer each part:
    a. What is 40% of 70?
    b. What is 25% of 40?
    c. Fifteen is 10% of what number?
    d. Sixteen is 25% of what number?
    e. Write as a percentage: 7 girls in a class of 18 students.

5

8. A city recycles 78% of the newspapers sold there. The Chronicle Herald has a readership of 316,700. How many of the Chronicle Herald gets recycled?

5

9. It costs you $25 to make a sweater and want to sell it for profit at $45. What percent markup do you have (percent increase)?

5

10. SLA has 73 students and is praying for 100 next year. What is the percent increase?

10

11. Answer the following short questions:
    a. Write as standard form: 2.34×10^5.
    b. Write as a sum of powers: 300.04.
    c. Write as standard form: $6(10^3) + 3(10^2) + 4(10^0)$
    d. Simplify: $3\text{-}3 \times 2^3/4 - 3$
    e. Evaluate $3x^2 - 4x + 1$ when x = -1.

Bonus: (5pts) Two identical jars are filled with equal number of marbles. The marbles are colored red or white. The ratio of red to white in jar I is 7:1 and 9:1 in jar II. If there are 90 white marbles all together, how many red marbles are in Jar II?

Chapter 5 Number Line and Cartesian Plane

This chapter contains worksheets on the topics of solving linear equations and inequalities, absolute value equations and inequalities, graphing on number lines, and plotting on the Cartesian coordinate system.

Prior Skills
- For sheet 5h, students need to understand what constitutes a polynomial.
- For sheet 5i, domain and range need to be introduced.

5a – Solving for a Variable (Multiple Variables present)

When solving for a variable, use the inverse functions. To remove a division, multiply. To remove an addition, subtract. Here is a list of some common inverse pairs:

add	multiply	square, cubes...	logarithms f(x)s	trigonometr ic f(x)s	derivatives
subtract	divide	square root, cube root...	exponential f(x)s	inverse trig	integrals

When faced with multiple variables, just focus on the variable you need to solve for. Remove all the terms and coefficients that surround the needed variable. Here is an example:

Solve for b_1: $A = \frac{1}{2}(b_1 + b_2)h$

$2A = (b_1 + b_2)h$

outside

$\frac{2A}{h} = b_1 + b_2$

$\frac{2A}{h} - b_2 = b_1$

Since b_1 is interior, remove constants of the parentheses first. Undo division of 2 by multiplying by 2. Undo multiplication of h by dividing. And so forth.

1. Solve for h: $V = \frac{1}{3}\pi r^2 h$

2. Solve for m: $y = mx + b$

3. Solve for b: $V = \frac{1}{3}b^2 h$

4. Solve for r: $V = \pi r^2 h$

5. Solve for y_1: $m = \dfrac{y_1 - y_2}{x_1 - x_2}$

6. Solve for x_1: $m = \dfrac{y_1 - y_2}{x_1 - x_2}$

7. Solve for h: $A = \frac{1}{2}bh$

8. Solve for y: $x^2 + y^2 = r^2$

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5b – More Practice Translating

Algebra was founded by people who needed to find answers to problems. For centuries, people would try to solve problems without the use of variables. It wasn't until Fibonacci in the 1400s made using symbols (mathematical operators and variables) popular. Note the difference between the two equations below:

> Tom's age plus 4 equals Sally's age $T + 4 = S$

Students typically have problems translating sentences into expressions or equations. Once they get the equation written, they no longer have as much difficulty.

Translate the following into expressions or equations. Define your variables:

1. Sally is five years less than twice Tom's age. 2. One less than Sally's hourly wage.

3. Three times as many rocks   4. Thrice as many hours

5. Sally's and Tom's wages sum to be $50,000.   6. Four less pounds

7. The sum of two numbers   8. A number and 3

9. My height is 4 inches more than yours.   10. Five less than a number

11. The height of the room is half the length.   12. Twice the age of Sally.

13. Tom earns 1.25 times as much as Sally.   14. Four more pounds

Solve:

15. I ate ½ my daily peanut butter sandwiches for lunch. Had I eaten one more sandwich, I would have eaten ⅝ my daily sandwiches. How many sandwiches do I eat daily?

16. Tom wanted Sally's telephone number. Knowing she lived in Berrien Springs, with an exchange number of 473, Tom just needed the last four digits. Sally slyly stated that 45 added to his age of 30 equaled to 1000 less than half her telephone number (ignoring the exchange).

5c – Number Line

Real numbers can be plotted on a line from left to right with the numbers in ascending order. That means negatives are placed on the left end and positives on the right. Remember -100 is smaller than -1, so -100 would be further left than -1. For equalities, a solid dark dot on the number line indicates the value of the variable that makes the equation true. For example, an equation is found that $x = 4$. The graph of $x = 4$ is shown below:

-2 -1 0 1 2 3 4 5 6
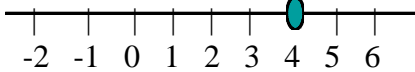
Below are examples of inequalities and their solutions graphed. Remember that the inequality symbols switch when multiplying (dividing) by a negative. Also an open circle or) shows a strict inequality, as in the constant is not included in the solution set. On the other hand, a closed circle or] shows that the constant is included.

Example: $4x - 1 < 7$

$4x < 8$

$x < 2$

-2 -1 0 1 2 3 4 5 6

$-x - 2 \geq 2$

$-x \geq 4$

$x \leq -4$

-6 -5 -4 -3 -2 -1 0 1
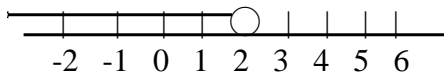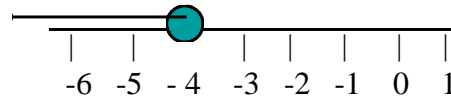
Solve and graph the solution on a number line:

1. $3x - 2 = 13$

2. $2x + 2 = 12$

3. $(4 - 2)x = 12$

4. $-4 \leq x - 1 \leq 1$

5. $-3w < 12$

6. $6(1 - x) - 3x \leq 12$

7. $-2z > 12$

8. $2y \geq -11$

9. $3x + 4 > 5$

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5d – Absolute Values

On a real number line, what is the distance between: 5 and 12, -3 and 5, or 4 and 7? We find the distance by subtracting the smaller value from the larger: 12-5, 5 - (-3), 7 - 4. So what is the distance between 0 and x? It would be written as x - 0. But what if we knew x was 5 units from zero? The number could be five less or five more than zero. Then symbolically it would be written as $|x - 0| = 5$, then x is either -5 or 5. The absolute value, **magnitude**, of x - 0 (or x) gives the distance, without specifying direction.

The equation $|x - 1| = 5$, can be thought as the distance x from 1 is 5 units. So starting on the real number line at 1, you would count to the right or left 5 units, getting two answers: 6 and - 4. Expressions with absolute values can get more complicated, so you may want to remember a certain rule: $|xy| = |x| \cdot |y|$. For example: $|-x| = 3$ can be written as $|-1| \cdot |x| = 3$. Then $|-1|$ really is 1 since the absolute value is asking for the magnitude of -1. So the equation really is $1 \cdot |x| = 3$, which gives the answer of -3 and 3.

Simplify each expression and graph the solution.

1. The distance between 4 and 6 is 2.

2. The distance between 5 and 11 is 6.

3. The distance between 7 and -3 is 10.

4. The distance between x and 3 is 5.

5. The distance between x and -2 is 4.

6. The distance between x and 3 is more than 6.

7. The distance between x and -2 is less than 4.

Solve each equation by translating its symbolic meaning first. Graph the solution.

8. $| x - 4 | = 2$ 9. $| x - 8| = 3$ 10. $| x + 3| = 5$ 11. $|x + 2| = 6$

5e – Graphing Absolute Values

Sometimes it is quite impossible to simplify absolute equations unless you get rid of them altogether. That is done only by finding two equivalent equations and solving each. The reason for two equations is because the expression inside the absolute value could very well be a positive or negative value. Now if the expression were positive, the absolute values are redundant and can simply be dropped. But if the expression were negative, the only way to make it positive like an absolute value would be to negate the expression.

Example: $|-x + 2| \leq 5$

   $+(-x + 2) \leq 5$ and $-(-x + 2) \leq 5$

   $-x \leq 3$ and $x - 2 \leq 5$

   $x \geq -3$ and $x \leq 7$
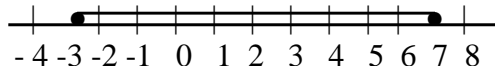
$|x - 3| > 2$

$x - 3 > 2$ or $-(x - 3) > 2$

$x > 5$ or $x - 3 < -2$

$x > 5$ or $x < 1$

- 4 -3 -2 -1 0 1 2 3 4 5 6 7 8

-2 -1 0 1 2 3 4 5 6
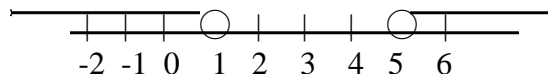
Explain why *and* is used in the first example and *or* in the second. _____
_____

Simplify each expression and graph the solution.

1. $x = |-2|$

2. $x = |-5|$

3. $|x| = 4$

4. $|x| = 2$

5. $x < |-3|$

6. $x \geq |-1|$

7. $|x| \leq 3$

8. $|x| \geq 7$

Solve each equation by converting into two equivalent equations first. Graph the solution.

9. $|x - 3| = 5$

10. $|x - 1| = 4$

11. $|x - 5| = 6$

12. $|x - 2| = 3$

Name: _____   Date: _____

5f – Solving Absolute Values

| Show the two equivalent equations. Solve and graph the solution set. |

1. $|x - 2| < 3$

2. $|x-4| > 2$

3. $|x+7| < 1$

4. $|x+2| > 3$

5. $|x - 1| \geq 5$

6. $|x + 2| \leq 7$

7. $|x - 2| \leq 4$

8. $|3x + 1| < 5$

9. $|4x - 1| > 3$

10. $|2 - y| < 2$

11. $|2x - 4| \leq 5$

12. $|3x - 6| \geq 1$

13. $|2x - 5| \leq 4$

14. $|x - 12| \leq 3$

15. $|x + 3| \leq 2$

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5g – Cartesian Coordinate Plane

Rene Descartes came up with a way to systematize giving directions. He took two real number lines and had them intersecting at zero to form perpendicular angles. At each integer, you can draw a vertical or horizontal line. After a while you will have formed a grid, with each line intersecting at integer coordinates (lattice points). The horizontal real number line is commonly referred to as the *x*-axis and the vertical number line is called the *y*-axis. The point (x,y) can be found by moving left/right along the *x*-axis and then from that new point, moving up/down y-spaces. For example the ordered pair (2,-1) would be found by starting at the origin (0,0) and moving right 2 spaces and down 1 space.

There are other coordinates, such as polar coordinates, but those are for a later course.

1. Draw a coordinate system, label the integers from -10 to 10 on both axes.
2. Draw a dot and label the points R(-2, 4), E(1, 5), S(4, -3) and T(-7,-2).

3. Draw a coordinate system, label the integers from -10 to 10 on both axes.
4. Label the points A(-2,4), B(-1, 2), and C(3, -6). Connect the dots. What does figure does it look like? How could you tell that it really is that shape?

5h – Polynomials and Variation

Polynomials are algebraic expressions that involve only the operations of addition, subtraction, and multiplication of variables. Examples of polynomials are 3, $3x$, $3x + 1$, $3x^2 + 6x+1$, and $x^3 + x$. Polynomials are described by the number of terms in the expression. Special names are used for one-term through three-termed polynomials: monomial, binomial, trinomial. Polynomials are also described by the highest-degree term in the expression: constant (zero-degree), linear (first-degree), quadratic, cubic, quartic, quintic, hextic (sextic), heptic (septic), any higher degrees are labeled by the ordinal value.

Match the following polynomials. If it isn't a polynomial, explain wy:

___ A. $3x^2 + 6x+1$ 1. Quadratic trinomial

___ B. $3x^2 + 8x^4$ 2. Quintic monomial

___ C. $\frac{3}{x} + 4$ 3. Non-polynomial; since _____

___ D. $3x^5$ 4. Non-polynomial, since division by variable

___ E. 5   5. Hextic binomial

___ F. $2^3x^5 + x^4 + 3$ 6. Quartic binomial

___ G. $x^6 + x^4$ 7. Constant

___ H. $8\sqrt{x} + 4$ 8. Quintic trinomial

Ever had such a conversation?
>   "This recipe calls for eggs and flour."
>   "How much flour?"
>   "One egg is needed for every cup of flour."

The first statement tells what is needed, a simple list of ingredients. The third statement gives the proportion of each amount. Expressing how variables relate in an equation is very similar. In the following exercises, you'll get a chance to symbolically express that relationship of variables. In the first couple of examples, you see the use of k. This letter represent the proportion; its value is of secondary importance to the actually relationship of the variables. As noted in the dialogue, the most important was getting the ingredients and then the proportion.

9. y varies directly with x.   (answer: $y = kx$)   10. y varies indirectly with x. ($y = k/x$)

11. y varies with the reciprocal of x.   12. y varies with the cube of x.

13. y varies with the square root of x.   14. y is a constant.

15. y varies linearly with x.   16. y varies inversely with x.

17. Solve: The number of pounds you weigh is directly proportional to the number of kilograms you are. Sally steps onto a scale calibrated in kilograms and finds that she is 60 kilograms. Her bathroom scale says 140 pounds. Write the equation expressing pounds in terms of kilograms. How much would she weigh if she was 25 kilograms?

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5i – Identifying Functional Relations

Identify which of the following are functions. Determine the domain and range of each relation.

1.
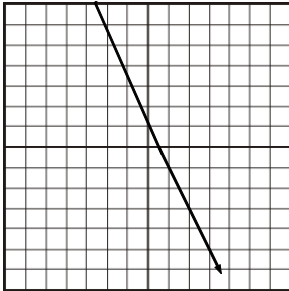
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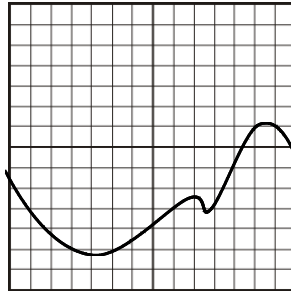
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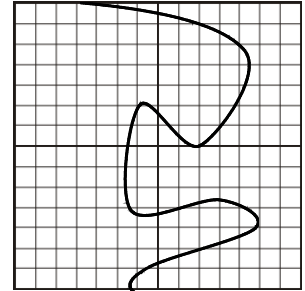
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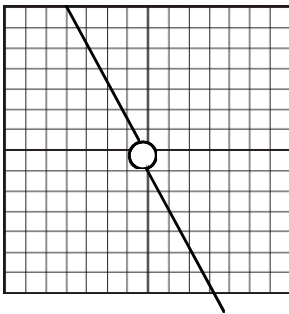
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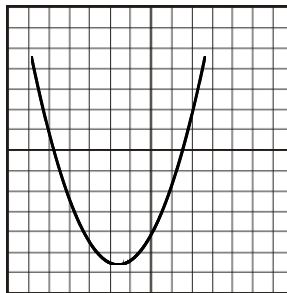
6.
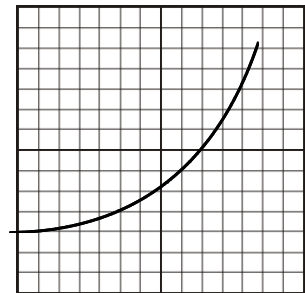
7.
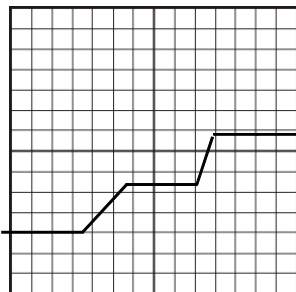
8.
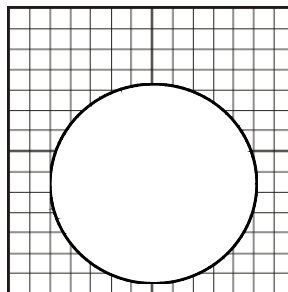
9.
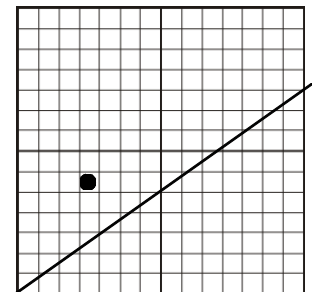
Chapter 5 Test

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| 10 |

1. Solve for x: A. 2x - 6 = 9   B. -3(2 - x) = 18   C. $\frac{1}{8}x = \frac{3}{4}$

| 5 |

2. A truck contains crude oil. The mass of the empty truck is 14,000 kg. The mass of one barrel of oil is 180 kg. Let T kilograms represent the total mass of the truck and the oil. Let b represent the number of barrels of oil. Write an equation relating T and b. How much would the total mass be, if the truck has 13 barrels?

| 10 |

3. Solve for *h*:   A. $A = \frac{1}{2}bh$ B. $V = \pi r^2 h$

| 5 |

4. Graph the solutions:
   a. $2y < -6$ b. (5, -3)   c. $10 > y \geq 5$

| 10 |

5. Graph the solutions:
   a. $|x - 3| < 5$ b. $|x + 1| \leq 6$ c. $|3x - 2| = 4$

Chapter 6 Linear Equations

6a – Linear Equations

Polynomials with x and y to the first power and are expressed either as $Ax + By = C$ or $y = mx + b$ are linear equations. They are called linear equations because when graphically represented, the solutions form the shape of a line. There is a pattern between the solutions that makes it easy to distinguish linear equations. In the spaces below, fill in the missing pieces to the pattern:

Equations: A.   B.   C.

$y = 2x - 1$	$y = -3x + 2$	$2x + 3y = 5$
(-3, -7)	(0, 2)	(-5, 5)
(0, -1)	(1, -1)	(1, 1)
(3, 5)	(2, - 4)	(7, -3)
(6, 11)	(3, -7)	(13, -7)

In equation A, as x increased by___, y values increased by___. In equation B, x increased by___ each time y decreased by___. In equation C, increasing x by___ made y decrease by___.

It is necessary to find the solutions to a linear equation. Some people make a t-chart, where they choose values of x (or y) and find the corresponding value of y (or x).

1. Fill in the t-chart for $x - 2y = 6$.

x	-2			1	2
y		-3.5	-3		-2

2. Fill in the t-chart for the following data: St. Joseph's ice arena has a $3 admittance per person. Let x represent the number of people in a group and y be the admittance price.

x	1	3	5		
y				21	42

3. Let x represent a bag of apples that are being sold at $1.99 a bag. If y represents cost, fill in the chart below:

x	4			12
y		11.94	19.90	

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6b – Graphs of Linear Equations

Some of the solutions to $2x - y = 1$ are (-2,-5), (0,-1), and (1, 1). Recognizing $2x$-y $= 1$ as a linear equation, the conclusion can be made that there is a pattern between the ordered pairs. Between the first two pairs, x increases by 2. But between the last two pairs, it increases by 1. With such inconsistency, it will give some difficulty for writing an equation when only given coordinates. So to circumvent this problem, slope was defined. Slope is the ratio of the change between y-values to the change between x-values. As any ratio can be reduced to simplest terms, the ratio 4/2 between the first two ordered pairs reduces to 2, which is the ratio between the last two pairs.

Algebraically, slope (m) is defined as $m = \dfrac{y_1 - y_2}{x_1 - x_2}$. Other expressions are $\dfrac{rise}{run} = \dfrac{\Delta y}{\Delta x}$.

Slope can be found by taking the ratio between two ordered pairs, or by looking at the equation when it is in the form $y = mx + b$ (slope-intercept form). Solving for y in the example gives $y = 2x - 1$. If $x = 0$, the equation gives y $= -1$, just like the -1 in the equation. The y-intercept (b) is represented by the -1. Go ahead and plot the point on the graph. By evaluating more values of x, more ordered pairs are obtained: (1,1) and (2, 3). The same can be done by starting at the y-intercept and moving right 1 and up 2 and make a dot. Notice the slope was 2 and there is a x-coefficient of 2 in the equation. So graphing can be done by knowing your m's and b's.

Graph the following:

1. $2x + y = 3$ 2. $6x + 2y = 4$ 3. $-6x + 3y = 9$

4. $y = x - 3$ 5. $y = 5x - 2$ 6. $y = (½)x - 3$

7. $y = -2x + 1$ 8. $-x - y = 3$ 9. $y = 3x$

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6c –Writing Linear Equations

A needed skill is being able to write a model to represent values on a graph. A **model** is an equation that represents the data. It may be an approximation, especially when the values do not form a true shape of a line, parabola, etc. There are four equations to represent a line, and each are useful when depending on a set of questions or given data. These equations are: slope-intercept ($y = mx + b$), point-slope ($y - y_1 = m(x - x_1)$), standard form ($Ax + By = C$, where A, B, and C are integers), and double-intercept form ($x/a + y/b = 1$).

To write the equation of a line, you need either two points or a point and the slope. Let's follow two methods below with the points (1,-1) and (3,5). First find slope: $\dfrac{5 - (-1)}{3 - 1} = \dfrac{6}{2} = 3$.

Method 1:
$y = mx + b$
$y = 3x + b$
$5 = 3(3) + b$
$5 = 9 + b$
$- 4 = b$
$y = 3x - 4$

Method 2:
$y - y_1 = m(x - x_1)$
$y - 5 = 3(x - 3)$
$y - 5 = 3x - 9$
$y = 3x - 4$

1. Write the equation that has slope of ⅔ and y-intercept of -7.

2. Write the equation that has m = 3 and passes through (0,-2).

3. Write the equation that passes through (0,2) and (3,1).

4. Write the equation that passes through (0,3) and (2,1).

5. Write the equation that passes through (2,1) and (4,-3).

6. Write the equation that passes through (1,2) and (10,2).

7. Write the equation that passes through (-6, 3) and (3,7).

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6d – More on Writing & Graphing Linear Equations

Write the equation of the line that has the following characteristics.

1. Slope = -3, point: (0,2)

2. Slope = ⅔, point: (4,5)

3. Points: (6, 8) and (1, 7)

4. Points: (-2, 3) and (3, 8)

5. Slope = ¾ and passes through (4, 3)

6. Slope = -2 and passes through (3,-1)

7. Passes through (2,-3) and (4,5)

8. Passes through (-1,-3) and (3,-3)

Graph the equations after simplifying it.

9. $6x = 36$

10. $7x = 42$

11. $5x = -15$

12. $3y = 12$

13. $4y = -8$

14. $y = 0$

15. $y = 3x - 3$

16. $4x - 3y = 6$

17. $3x - 5y = 10$

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6e –Parallel Lines

A set of lines may be either parallel or not. No fuzzy logic here! Parallel lines never intersect. Looking at a graph isn't a good indicator because the lines could be getting closer little by little. So determine if the lines are parallel by comparing the slopes. **If the slopes are equal, then the lines are parallel.** If the slopes and intercepts are equal, then the lines are the same!

Example A: Is $2x - y = 5$ parallel to $4x - 2y = 1$?
    Solution: Writing the equations into slope-intercept form: $y = 2x - 5$ and $y = 2x - ½$, it is
               obvious both have slopes of 2. *Y*es, they are parallel.

Example B: Find the linear equation parallel to $3x - y = 5$ that is passes through (0,2).
    Solution: First find the slope, which is 3. Then using point-slope form, fill in the point and
             the slope: $y - 2 = 3(x\text{-}0)$. Simplify the equation to slope-intercept form: $y = 3x + 2$.
             This equation can also be written as $-3x + y = 2$ or $3x - y = -2$.

Example C: Find the linear equation parallel to $2x - 3y = 1$ that passes through (1,2).
    Solution: Slope of the line is ⅔, plug into point-slope form: $y - 2 = ⅔(x\text{-}1)$. Simplify into
             standard form by getting rid of fraction - multiply by denominator of slope:
             $3y - 6 = 2(x\text{-}1)$. Bring x and y together on one side of equation: $-2x + 3y = 4$.
             This equation can also be written as $2x - 3y = - 4$.

There is a pattern in the examples that make finding parallel lines easier. Be on the look-out!

1. Determine which of the following lines are parallel. Show your work.
      A. $3x - 2y = 5$ B. $6x + 9y = 1$ C. $6x - 4y = 4$ D. $9x + 6y = 1$

2. Write the linear equation that is parallel to $x - 3y = 1$ and passes through the point (4, 2).

3. Write the linear equation that is parallel to $5x - 3y = 1$ and passes through the point (- 4, 2).

4. Write the linear equation in standard form that is parallel to $4x + 9y = 3$ and intersects (4,-2).

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6f – Perpendicular Lines

Lines that form at right angles are said to be perpendicular or orthogonal. If one line has a slope of m, then the other has the slope of -1/m. The product between the slopes of perpendicular lines is always -1. Another way of expressing the slopes is to say the slopes are negative reciprocals of each other.

Example 1: Determine if the line $2x - y = 5$ is perpendicular to $x + 2y = 3$.
   Solution: The equations can be written as $y = 2x - 5$ and $y = (-\frac{1}{2})x + 1.5$.
   Since $2(-\frac{1}{2}) = -1$, the lines are perpendicular.

Example 2: Determine if the lines $2x - 3y = 5$ and $6x + 4y = 3$ are perpendicular.
   Solution: The equations in slope-intercept form are $y = (\frac{2}{3})x - 5/3$ and $y = (-6/4)x + \frac{3}{4}$.
   Since $-6/4 \times \frac{2}{3} = -1$, the lines are perpendicular. Note: $-6/4 = -3/2$ which is the negative reciprocal of $\frac{2}{3}$.

Example 3: Write the linear equation perpendicular to $2x - y = 5$ which passes through $(-4, 2)$.
   Solution: The given line has slope of 2, so the perpendicular line must have $m = -\frac{1}{2}$.
   Substitute the point and slope into the point-slope form to get $y - 2 = -\frac{1}{2}(x + 4)$.
   Simplify into slope-intercept form, $y = (-\frac{1}{2})x$, or standard form, $x + 2y = 0$.

There is a pattern in the examples that make finding perpendicular lines easier. Look out!
1. Determine which of the following lines are perpendicular. Show your work.
   A. $3x - 2y = 5$ B. $6x + 9y = 1$ C. $6x - 4y = 4$ D. $9x + 6y = 1$

2. Which of the following is perpendicular to $-3x - y = 4$? Show work.
   A. $x + 3y = 2$ B. $9x + 3y = 3$ C. $3x - y = 3$ D. $x - 3y = 5$

3. Write the linear equation perpendicular to $5x - 3y = 1$ that passes through $(4, 2)$.

4. Write the linear equation in standard form that is perpendicular to $4x - 3y = 7$ at $(1, -1)$.

6g – Linear Inequalities

The solution to a linear inequality is a set of points that make the inequality true. In the case of the inequality, the solutions include points that lie on one side of the line. The points on the line, the boundary to half-planar solutions, are solutions only if the inequality includes an =. Use the following example as a guide.

Example: $6x + 3y > 12$
$\qquad 3y > 12 - 6x$
$\qquad y > 4 - 2x$ 　　　　Plot some points on the line $y = -2x + 4$.
Connect the dots with a dashed line since the line itself contain no solution to the inequality. Now pick a point that does NOT lie on the line. (0,0) is an easy choice. Since $0 + 0 > 12$ is false, then the other side of the dashed line must contain the solutions. Shade the solution side.

Graph the solutions.

1. $-2x - y > 4$ 　　　　　　2. $3x + y > 5$ 　　　　　　3. $x - y > 4$

4. $-2x + y \leq 4$ 　　　　　　5. $x + 2y \geq 5$ 　　　　　　6. $4x - 3y < 9$

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6h – Systems of Equations- Substitution

System is to equation as a paragraph is to a sentence. Since all the sentences in the paragraph relate to the same idea, all the equations in a system relate to the same variables. The solution to a system is the ordered pair(s) that makes all the equations true.

One way of finding the solution is to graph each equation and see where the graphs intersect. Unfortunately if the graphs intersect anywhere but a lattice point, it is hard to determine the exact values of the ordered pair(s). Instead most mathematicians either do a substitution or an elimination process.

Example: solve the system of $2x + y = 3$ and $3x - 5y = -2$.

Step I: choose one variable in one equation to solve	$y = 3 - 2x$
Step II: Substitute the expression into other equation	$3x - 5(3-2x) = -2$
Step III: Simplify and solve for remaining variable	$3x - 15 + 10x = -2$
	$13x - 15 = -2$
	$13x = 13$
	$x = 1$
Step IV: Substitute known variable into either equation	$2(1) + y = 3$
	$2 + y = 3$
	$y = 1$
Step V: write answer	Ans: $(1,1)$

Solve. Sketch a graph to confirm results.

1. $x + y = 3$
   $x - y = 2$

2. $5x - y = 6$
   $3x - 2y = -2$

3. $y = -x + 1$
   $y = -3x + 5$

4. $y = 3x + 5$
   $y = 2x + 2$

5. $x - 4y = 5$
   $3x - 4y = -1$

6. $4x - 2y = 2$
   $-3x + 2y = 1$

Name: _____   Date: _____

6i – Systems of Equations - Elimination

Solving a system with Substitution either means you are good with fractions or you use it mostly when a variable has a coefficient of one. Most times elimination will prove easier.

Example: Solve the system: $5x + 3y = 18$
                                $3x - 2y = 7$

Step I: choose variable you want to eliminate. We choose to eliminate y for this example.
Step II: Multiply each row by a number so that the coefficients for y are opposites. The first equation was multiplied by 2 and the second by 3. The equations are still balanced!

$$10x + 6y = 36$$
$$9x - 6y = 21$$

Step III: Add the equations together. One variable must disappear or else we made a mistake! By adding equal items to both sides, the result is a balanced equation.

$$19x + 0 = 57$$

Step IV: Solve for remaining variable.

$$19x = 57$$
$$x = 3$$

Step V: Repeat process with other variable or do a substitution.

$$3(3) - 2y = 7$$
$$9 - 2y = 7$$
$$-2y = -2$$
$$y = 1$$

Solve. Sketch a graph to confirm results.

1. $x + y = 5$ 2. $4x + 3y = 7$ 3. $y = -2x + 1$
    $x - y = 1$ $3x - 2y = 1$ $y = -3x + 3$

4. $5x - y = 6$ 5. $5x - 3y = 12$ 6. $4x + 5y = 2$
    $3x - 2y = -2$ $3x + 2y = 11$ $3x + 2y = 5$

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6j – Systems of Equations – Mixed Review

Solve the following systems algebraically. Decide whether to use substitution or elimination. But you **cannot** do only one method for the entire worksheet!!!   Graph the system and confirm results.

1. $2x + 3y = 2$
    $4x - 9y = -1$

2. $5x + 2y = 11$
    $x + y = 4$

3. $y = 2x - 1$
    $y = 3x - 5$

4. $x - y = -11$
    $7x + 4y = -22$

5. $6x - 7y = 47$
    $2x + 5y = -21$

Complete the sentence!

6. Two lines intersect zero, _____, or infinite times. The systems that contain these lines are referred to as inconsistent (formed by parallel lines), independent (lines intersecting), or dependent (lines that are the same).

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6k – Linear Word Problems

You can identify a linear equation even if it is written in the jargon of a word problem. The biggest clue is to identify a constant change in the values. If you read the cost for a pencil is $0.50, then two pencils should cost $1.00. Then identify your variables, choosing those not easily mistaken for a number. Convert given values into ordered pairs. Find the slope and then write the equation. Once you have an equation, you can predict almost anything.

Solve:
1. Pencils are sold at the bookstore for 49 cents each. How much would 75 pencils cost?

2. Some reception halls have a flat fee for use of the hall and then a fee of $25 or more for each person they will be serving. For a wedding reception, Black Forest Inn charges about $64 per person plus $5500 for the use of their inn. How much will it cost my brother to host a wedding of 250 people?

3. An auto repair shop charges $25 an hour. They say your muffler needs to be replaced for $65. It takes them 2.5 hours to fix your exhaust system. How much will the bill be?

4. To frame an oil painting, framers charge a rate based on the perimeter of the painting. You choose a polished wooden frame that has a price tag of $8 a foot. Will the price of framing exceed your $125 budget if your painting is 3.5' by 2.5'?

5. A Nissan Sentra sold in 1996 for $17,000. Ten years later its worth is $3,400. How much would it be worth in another five years, if the depreciation followed a linear model? (In real life, cars depreciate exponentially.)

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6L – More Word Problems (Mixed Review)

Solve the word problems:

1. When water freezes, it is $0°C$ (or $32°F$). When water reaches its boiling point, it is $100°C$ (or $212°F$). What is the temperature in Fahrenheit if the water is $72°C$?

2. Machinery is purchased for $450, but five years later is worth $0. What would be the worth of the machinery after two years? (Assume linear depreciation.)

3. A room is twice as long as the width. If the area of the room is 72 square feet, what is the perimeter?

4. A lawn is 1000 feet around. If the length is three times the width, how much square feet is the lawn?

5. A recipe wants 4 cups of flour and 1 cup of oats. How much flour is needed for ⅔ cups of oats?

6. A recipe wants 3 cups of flour and ½ cup of butter. How much butter would you use for 8 cups of flour?

7. Keith is 16 years older than Shirleen and three times as old as Rachel. If the sum of their ages is 96, how old are they?

Chapter 6 Test

Name: _____   Date: _____

> SHOW WORK. A calculator is allowed on this test. Attach any scratch paper that's used.

$\dfrac{}{5}$

1. Solve for the variable:

   A. $-3(n + 2) = 21$ B. $\frac{3}{4}p = \frac{3}{8}$ C. $1 - \frac{x}{5} = 3$

$\dfrac{}{5}$

2. An airplane travels eight times as fast as a car. The difference in their speeds is 420 km/h. How fast is each vehicle travelling?

$\dfrac{}{10}$

3. Graph the following lines. Label intercepts.

   a. $y = -3$ B. $x = 4$ C. $y = 2x - 3$

$\dfrac{}{10}$

4. Graph the solutions:

   a. $2y < -6$ b. $y < 2x - 3$ c. $2x - y \geq 5$

$\dfrac{}{5}$

5. Solve the system:

   $3x - 2y = 6$
   $x + 2y = 10$

5

6. Rewrite each equation into standard form (Ax + By = C) using integer coefficients.

      A. $y = \frac{2}{3}x - \frac{8}{9}$ B. y - 3 = ½ (x + 4)

5

7. Write the linear equation that passes through (4, 3) and (3, 9). Show work for slope.

5

8. Plot the ordered pairs and label: (2, -3), (-4, 5) and (0, 6).

5

9. Write the linear equation that passes through (3, 5) and (7, 8).

5

10. Write the equation of the line parallel to 2x – 3y = 5 and passes through the point (4,-3). For three bonus points, find the equation of the line perpendicular to 2x – 3y = 5 that passes through (4,-3).

0

Bonus: (3 pts) If x + y = 5 and x - y = 1, what is the value of $2^{x^2 - y^2}$?

         (3 pts) If 2010x + 2010y = 2011(x + y), what is the value of x/y?

Chapter 7 Exponents

Prior Skills:
- Sheet 7a: division by zero

7 - Rules of Exponents

The operation of raising a number to a power is exponentiation. In the expression x^3, 3 x's are being multiplied together so that $x^3 = x \cdot x \cdot x$. For the expression x^n, x is called the base, n is the exponent and the whole expression is a power.

Remember that exponents occur second in the order of operations. So that $4x^3$ does not mean $4x \times 4x \times 4x$, rather $4 \times x \times x \times x$. One of the common mistakes when simplifying -2^4 is to wrongly use the negative. The expression -2^4 means the negative of 2 to the fourth or the negative of 16 which is -16. If it is -2 that is raised to the fourth power, then it needs to be written as $(-2)^4$.

The properties of Exponents:

1. Product of two powers with equal bases: $x^a \cdot x^b = x^{a+b}$

2. Quotient of two powers with equal bases: $\dfrac{x^a}{x^b} = x^{a-b}$

3. Power of a power: $(x^a)^b = x^{ab}$

4. Power of a product: $(xy)^a = x^a y^a$

5. Power of a quotient: $\left(\frac{x}{y}\right)^a = \dfrac{x^a}{y^a}$

6. $x0 = 1$, as long as $x \neq 0$.

7. $x^{-a} = \dfrac{1}{x^a}$

8. $x^{\frac{a}{b}} = \sqrt[b]{x^a}$

Reminders:
1. Never confuse distribution of exponents - ONLY distribute over multiplication and division, never over addition or subtraction. e.g. $(x^2+y^3)^2 \neq x^4 +y^6$ but $(x^2y^3)^2 = x^4y^6$.
2. The product of two fractions is made from the product of the numerators over the product of the denominators. e.g. $(2/3)(5/6) = 10/18$

Example a: $x^2 y^3 \bullet y^5 = x^2 y^8$

Example b: $\dfrac{x^2}{y^3} \bullet \dfrac{(xy)^2}{xy^3} = \dfrac{x^2 x^2 y^2}{y^3 xy^3} = \dfrac{x^4 y^2}{xy^6} = x^{4-1} y^{2-6} = x^3 y^{-4} = \dfrac{x^3}{y^4}$

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7a – Simplifying Exponents

Simplify each of the following expressions:

1. $3(-2)^2$

2. $(3 \cdot 2)^2$

3. $\frac{2}{7}\left(\frac{3}{2}\right)^2$

4. $\dfrac{(3x^2)^3}{6x^5}$

5. $\dfrac{8(2006)^{1003}}{1003(2006)^{1002}}$

6. $\dfrac{8(2-3)^3 + 3^2}{2(5-\{2+3\})}$

7. $\left(\frac{100xy}{x^3y}\right)^0 \left(\frac{5x}{100x^3}\right)$

8. $8y + (-7y)^2$

9. $(4x)^2 + 10(4x)^2$

10. $zy^4 \cdot (x^3)^2 y \div z^2$

11. $\dfrac{(2x^2)^4}{4x^4}$

12. $\dfrac{6 \cdot 2^{148}}{3 \cdot 2^{150}}$

13. $\dfrac{(-5x)^2}{30x^3}$

14. $zz(y^2x^4)^3 \div x^3 \cdot z$

15. $\dfrac{(5-3)^3 + 7}{6^2 - 5^2 + (-2)^2}$

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7b – More Simplifying Exponents

Simplify each of the following expressions:

1. $y^2 y^4 y^5$

2. $-(x^4 y^3)^2$

3. $\dfrac{z^{12}}{z^6}$

4. $(a^5 b^3)^4 \div (a^2 b)$

5. $(x^2)^4$

6. $\dfrac{13}{a} \cdot a \cdot \dfrac{a}{26}$

7. $\left(\dfrac{2}{3} t\right)^2$

8. $\dfrac{1}{x^2 y} \cdot 2xy \cdot \dfrac{y}{x}$

9. $(-x^3 y^5)^2$

10. $4a \div (2b)^4$

11. $\left(\dfrac{2}{3} xy^3\right)\left(\dfrac{3}{4} x^2 y^5\right)$

12. $(2x^{-2} y^{-1} z^0)^{-3}$

13. $\dfrac{3a^2 c^2}{n^4} \div \dfrac{12ac}{n^3}$

14. $\dfrac{3x^3}{4y^4} \div \dfrac{15x}{12y^5} \cdot \dfrac{3y}{2x^2}$

15. $(3x^{-4} y)^{-2}$

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7c – Prime Numbers

Prime numbers are any number that cannot be divided evenly by another number except for one and itself.

1. Shade any number that is NOT a prime number in the list below:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Here are a few tricks by which you can tell whether a number is divisible:
- Divided by 2: all even numbers.
- Divided by 3: if the sum of the digits is divisible by 3.
- Divided by 5: number ends with 0 or 5
- Divided by 6: even and rule for 3 works.
- Divided by 9: if sum of digits is divisible by 9.
- Divided by 11: if a 3-digit number has the first and last digit sum to be the middle digit.

Using the number tricks above, fill in the blanks.

2. 22,245 is divisible by ____ and ____.

3. 473 is divisible by _____.

4. 6561 is divisible by _____ and _____.

5. 792 is divisible by 2, 3, ____, _____, and ____.

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7d – Prime Factorization

Often times it is necessary to break down an item into smaller pieces, whether it be a digestive system, rearranging a postal package contents, troubleshooting a computer problem, or any other instance. Working with numbers, the **factor** is a number that divides into another evenly. For example, the factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24. A factor tree is a usual algorithm for finding the factors of a number.

(American) Factor Tree:

```
       18
      /  \
     2    9
         / \
        3   3
```
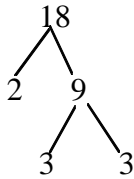
(European) Factor Chart
- looks like long division

```
  |18
2 |9
3 |3
3 |1
```
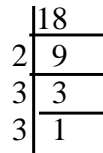
The usefulness of a factor tree is that you have divided up the number (18) into its smallest factors. Those factors of 2 and 3 are only divisible by 1 and itself. That makes those factors **prime**. One is not considered prime; it is **unique**. While the goal of the factor tree is to get prime factors, the other factors can be found by combining the different primes. The **prime factorization** is a list of all the prime factors in ascending order. 18 would have a prime factorization of $2 \times 3 \times 3$ or 2×3^2. The prime factorization of 24 is $2^3 \times 3$ because $24 = 2 \times 12 = 2 \times 2 \times 6 = 2 \times 2 \times 2 \times 3$. By combining different prime numbers of 24, you can get the entire factors of 24: 2, 3, 4, 6, 8, 12, and 24. The factors of 18 are 2, 3, 6, and 9. Factoring can also take a polynomial and split it into smaller polynomials. This idea that polynomials like x^2 - 2x - 3 can be factored into x - 3 and x + 1 will be developed later.

When adding fractions together, you need to get a common denominator. Find the Least Common Multiple (LCM) in order to reduce the work. For 18 and 24 the least common multiple would be 72. There are two methods for finding the LCM. A common elementary method is to list the multiples of each number until you find a common number between the two.

18	36	54	**72**	90	108	126	144	162	180	198	216
24	48	**72**	96	120	144	168	192	216	240	264	288

Another method is to use the greatest power of each prime in the prime factorization. The prime factorization of 18 is 2×3^2 and 24 is $2^3 \times 3$, so the LCM would be $2^3 \times 3^2$ which is $8 \times 9 = 72$. A good way to visualize the result is to use a Venn diagram of the prime factors. Place the prime factors of 18 in the left circle and the prime factors of 24 in the right circle. The factors in common should be placed in the shared region. Now take each part of the circles and multiply the factors together and you get $3 \cdot 2 \cdot 3 \cdot 2 \cdot 2 = 72$.

Note: GCF is the shared region. This will be explained later.

```
  (  3  ( 2*3 )  2*2 )
```
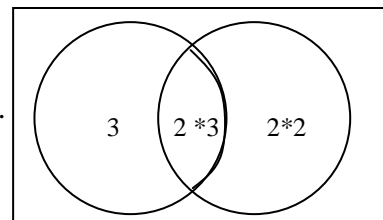
Name: _____   Date: _____

7d – Prime Factorization continued

Whereas the LCM, least common multiple, is the **union** of the two circles in a Venn diagram, the GCF would be the **intersection**. The GCF, short for Greatest Common Factor (Divisor), is the largest factor that divides into each number evenly. In the Venn diagram, it is the region overlapped by both circles. So the GCF(18, 24) is 6.

Another way of finding the GCF is to write the prime factorization of each number and take only the primes that are in common, and with the least exponent. The prime factorization of 18 is 2×3^2 and 24 is $2^3 \times 3$, so the GCF would be $2^1 \times 3^1$ which is 6. Of course the way most elementary and middle schools teach is to list the factors of each number and take the greatest common number: 18 is 1, 2, 3, 6, 9, 18 and 24 is 1, 2, 4, 6, 8, 12, 24.

The GCF is useful in simplifying equations. Say you had an equation such as $4x^2 + 12x - 20 = 0$. The greatest common factor of 4, 12, and 20 is 4. So you could divide both sides by 4 to get $x^2 + 3x - 5 = 0$, making it easier to solve.

The word factoring can be used in other manners. *Factoring out* implies dividing the GCF from an expression. The example above would be written as $4x^2 + 12x - 20 = 4(x^2 + 3x - 5)$. The formal name is Converse to Distribution. Another example of *factoring out* is $2x - 4y = 2(x - 2y)$.

Find the prime factorization of each:
1. 116   2. 175   3. 216   4. 40

Find the Lowest Common Multiple and Greatest Common Factor of each. Label.
5. 35 and 21   6. 18 and 42   7. 21 and 54   8. 3, 12, and 20

9. 24, 42   10. 15, 36   11. 35, 25, 50   12. 12, 28, 32

13. 64, 32, 56   14. 10, 42, 72   15. 36, 72, 84   16. 8, 15, 20

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7e –Simplifying Radicals

One of the properties of exponents that was not discussed before was that of fractional exponents. As you can tell in the example below, fractional exponents are another way of writing radicals.

$$x^{\frac{a}{b}} = \sqrt[b]{x^a} \qquad\qquad 81^{\frac{3}{4}} = \sqrt[4]{81^3} = 3^3 = 27$$

Before manipulating radicals, you'll need to understand the pieces to the radical. In the expression $\sqrt[b]{x^a}$ the b is the *root index*. It says how many of the same number is being multiplied together to get x^a. The $\sqrt{}$ is the radical sign; it implies what operation needs to be performed. The line over the x^a is the *vinculum*; it is a fancy name for parentheses. So $\sqrt{4}$ can be reduced to 2 because the same two numbers that multiply to get 4 is 2. Note that for square roots, the root index is usually dropped. $\sqrt{144}$ reduces to 12. But what happens when the number is not a perfect square? You simplify the *radicand* (expression inside the radical) so it contains no factors that are perfect squares. For n[th] roots, you want no factors that are n[th] powers.

Example A: $\sqrt{24} = \sqrt{4 \cdot 6} = \sqrt{4} \cdot \sqrt{6} = \sqrt[2]{2^2}\sqrt{6} = 2\sqrt{6}$

Example B: $\sqrt{146} = \sqrt{2 \cdot 73} = \sqrt{146}$

Example C: $\sqrt{108} = \sqrt{4 \cdot 27} = \sqrt{4 \cdot 9 \cdot 3} = 2 \cdot 3\sqrt{3} = 6\sqrt{3}$

Example D: $\sqrt{56} = \sqrt{2 \cdot 28} = \sqrt{(2 \cdot 2) \cdot 2 \cdot 7} = 2\sqrt{14}$

Another way to simplify a radicand is to make a factor tree and look for pairs. Better yet is to use prime factorization.

Simplify exactly the following:

1. $\sqrt{24}$ 2. $\sqrt{75}$ 3. $\sqrt{96}$ 4. $\sqrt{102}$

5. $\sqrt{144}$ 6. $\sqrt{225}$ 7. $\sqrt{625}$ 8. $\sqrt{525}$

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7f –More Rational Exponents

Given: $x^{\frac{a}{b}} = \sqrt[b]{x^a}$

Examples of rewriting fractional exponents:

$$16^{\frac{1}{2}} = \sqrt{16} = \sqrt[2]{16} = \sqrt{4 \cdot 4} = 4$$
$$27^{\frac{1}{3}} = \sqrt[3]{27} = \sqrt[3]{3 \cdot 3 \cdot 3} = 3$$
$$81^{\frac{1}{4}} = \sqrt[4]{81} = \sqrt[4]{3 \cdot 3 \cdot 3 \cdot 3} = 3$$

For rational exponents, most times it's easier to simplify with the denominator of the exponent first. See the examples below:

$$4^{\frac{3}{2}} = 2^3 = 8 \qquad 16^{\frac{3}{4}} = 2^3 = 8 \qquad 625^{\frac{3}{4}} = 5^3 = 125$$

Simplify the following without a calculator into an integer or a simplified radical:

1. $64^{\frac{2}{3}}$

2. $64^{-\frac{7}{6}}$

3. $64^{\frac{5}{6}}$

4. $(-64)^{\frac{2}{3}}$

5. $-32^{-\frac{6}{5}}$

6. $343^{\frac{2}{3}}$

7. $-100^{-\frac{3}{2}}$

8. $\left(\frac{256}{625}\right)^{-\frac{3}{4}}$

9. $\left(\frac{125}{27}\right)$

10. $\sqrt{6} \div \sqrt{2}$

11. $\sqrt[3]{81} \cdot \sqrt[3]{9}$

12. $\sqrt[3]{128} \cdot \sqrt{32}$

13. $\sqrt{\sqrt[3]{64}}$

14. $\sqrt{\sqrt[3]{36}}$

15. $\sqrt[3]{\sqrt[5]{\sqrt[4]{64}}}$

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7g – Combining Radicals

Adding radicals is similar to adding like terms. Like terms are combined by adding the coefficients. When radicals have the same radicands and root indices, then add the coefficients.

Example A: $2\sqrt{3} + 4\sqrt{3} = 6\sqrt{3}$ Example B: $2\sqrt{3} + 4\sqrt[3]{3}$

Example B cannot be simplified because the root indices are different.

Simplify exactly:

1. $-3\sqrt{24} + 2\sqrt{54}$ 2. $-3\sqrt{3} + 2\sqrt{27}$ 3. $5\sqrt{3} + \sqrt{48}$

4. $3\sqrt{1} - \sqrt{4}$ 5. $2x\sqrt{0} - 3x\sqrt{4}$ 6. $\sqrt{8} + 2\sqrt{32}$

Multiplying radicals can only be done easily with those of the same root index. If the root indices are the same, multiply the radicands. Those with differing indices will be dealt with later.

Example C: $\sqrt{2}\sqrt{24} = \sqrt{48} = 4\sqrt{3}$

Example D: $(2\sqrt{5})^2 = (2\sqrt{5})(2\sqrt{5}) = 4 \cdot 5 = 20$

See a trick? Share it!_____

Simplify each expression:

7. $\sqrt{6}\sqrt{12}$ 8. $2\sqrt{8}\sqrt{4}$ 9. $(3\sqrt{6})^2$

10. $(3\sqrt{8})(2\sqrt{12})$ 11. $\sqrt{3}(\sqrt{8} - \sqrt{2})$ 12. $(5\sqrt{3})^2$

7h – Rationalizing Denominators

Dividing by radicals could require a calculator. But before those were invented, mathematicians used a trick to get the radical out of a denominator. What's the purpose? Dividing by a never-ending number is quite impossible to do! But to divide by an integer is not bad at all! Even with the invention of the calculator, most people prefer the answer to be written with the radical on in the numerator. Mathematicians don't consider a fraction with radicals simplified until it has been rationalized.

The whole process of rationalizing the denominator is to multiply by one. That way the value doesn't change, but its appearance does. Write in *your* own words what occurs in each step.

Example: $\sqrt{\dfrac{2}{3}} = \dfrac{\sqrt{2}}{\sqrt{3}}$ Definition of division

$\dfrac{\sqrt{2}}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}}$ _____

$\dfrac{\sqrt{6}}{\sqrt{9}}$ _____

$\dfrac{\sqrt{6}}{3}$ _____

Simplify:

1. $\dfrac{\sqrt{6}}{\sqrt{2}}$

2. $\dfrac{3}{\sqrt{5}}$

3. $\sqrt{\dfrac{5}{6}}$

4. $\dfrac{2}{\sqrt{6}}$

5. $\dfrac{2}{\sqrt{8}}$

6. $\dfrac{4}{2\sqrt{3}}$

7. $\dfrac{3}{4\sqrt[2]{8}}$

8. $\sqrt{\dfrac{2}{56}}$

Name: _____   Date: _____

7i – Solving Radical Equations

Solve the following equations:

1. $2x^2 - 32 = 0$ 2. $3x^2 = 75$ 3. $4x^3 = 32$

4. $\sqrt{x} - 8 = 4$ 5. $\sqrt{x} + 5 = 20$ 6. $\sqrt{x+6} = 6$

7. $\sqrt{x-1} = 7$ 8. $\sqrt{x+3} = 5$ 9. $5x^4 = 625$

Chapter 7 Test

Name: _____   Date: _____

> **SHOW WORK.** A calculator is allowed on this test. Attach any scratch paper that's used.

$\dfrac{\quad}{5}$

1. With the numbers 16 and 24, find the
    A. Greatest Common Factor.   B. Least Common Multiple.

$\dfrac{\quad}{5}$

2. With the numbers 18, 24, and 28, find the
    A. Greatest Common Factor.   B. Least Common Multiple.

$\dfrac{\quad}{5}$

3. Write as a product and then evaluate:

    A. $\left(\dfrac{2}{3}\right)$ B. $-\left(\dfrac{1}{4}\right)$

$\dfrac{\quad}{5}$

4. Write each product as a single power and then evaluate:

    A. $(-3)^3 \cdot \left(\dfrac{1}{3}\right)$ B. $7^3 \cdot 3^3$

$\dfrac{\quad}{5}$

5. Simplify each quotient:
    A. $4^3 \times 5^2 \div (4^2 \times 5)$ B. $x^3 y^7 \div (x^4 y^3)$

$\dfrac{\quad}{5}$

6. Simplify each expression:
    A. $8^{\,5} \times 8^{\,-11} \div 8^{\,-3}$ B. $x^3 (x^3)^4 \div (x^2)^0$

5

7. Simplify into scientific notation

A. 0.000 000 000 034

B. $3\times10^8 + 4.7\times10^9$

10

8. Solve for x:

a. $4^x = 64$

B. $(125)^{\frac{2}{3}} = x^2$

C. $\sqrt{x+8} = 6$

5

9. Short answer:

A. How can you tell if a radical (square root, cube root...) is a rational number?

B. Simplify: $4\sqrt{3} + (3 - \sqrt{3})$

5

10. Simplify:

A. $\dfrac{\sqrt{18}}{\sqrt{6}}$

B. $3\sqrt{27}\sqrt{6}$

0

Bonus: (3pt) Let $10^{101}-1$ be written as an integer in standard form. What is the sum of the digits?
(2pt) Suppose $N^{1982} = 1982^{1982}$. If $N \neq 1982$, what is the real value of N?

Chapter 8 Expanding & Solving Polynomials

Name: _____   Date: _____

8a – Multiplying Polynomials

If the teacher were to ask the class to multiply $2x + 5$ to $x - 3$, many students will give the incorrect answer of $2x^2 - 15$. These students didn't distribute correctly, causing them to lose the middle terms. There are four methods to ensure a complete answer: traditional distribution, box method, FOIL, or multiplying like you did with real numbers. The letters of FOIL stand for the product of first terms, outer terms, inner terms, and last terms of each binomial. The box method uses a multiplication table or a Punnett Square (see biology class) with the binomials. The answer for both methods is the sum of these products.

Traditional distribution: $(2x^2 + 4)(x - 1) = (2x^2 + 4)x + (2x^2 + 4)(-1) = 2x^3 - 2x^2 + 4x - 4$

Box Method:

	x	-1
$2x^2$	$2x^3$	$-2x^2$
4	$4x$	$- 4$

$$\begin{array}{cccc} F & O & I & L \end{array}$$
$(2x + 5)(x - 3) = (2x)(x) + (-3)(2x) + (5)(x) + (5)(-3)$
$ = (2x^2) + (-6x) + (5x) + (-15)$
$ = 2x^2 - 6x + 5x - 15$
$ = 2x^2 - x - 15$

Multiply like Reals:
$$\begin{array}{r} 2x + 5 \\ \times \quad x - 3 \\ \hline -6x - 15 \\ 2x^2 + 5x \\ \hline 2x^2 - x - 15 \end{array}$$

Expand.

1. $(x - 3y)(x + 2y)$ 2. $(x+5)^2$ 3. $(x-2)(3-x)$ 4. $(4x-3)(4x+3)$

5. $(2a+3b)(a-5b)$ 6. $(x + 2)(10 - 4x)$ 7. $(W + 2)(W - 2)$ 8. $(a - 4)(a + 4)$

9. $(11-x)(x+3)$ 10. $(3x^2 - 4)(x + 2)$ 11. $(x^2 + 2)(2x^2 + 1)$ 12. $(2x^2 - 6)(x^2 + 4)$

Name: _____   Date: _____

8b – Factoring

Factoring when $a = 1, c > 0$:
A quadratic expression of $ax^2 \pm bx + c$ where $a = 1$ looks like $x^2 \pm bx + c$. This expression can be factored into $(x+d)(x+e)$ or $(x-d)(x-e)$ whenever the c term divisors sum up to the middle term b. Note how both factors have the same sign as b whenever $c>0$.

Example A: $x^2 + 5x + 6$ You probably won't have more than one line of
          ( )( )   work on your paper. But these are the steps to fill
          $(x\ )(x\ )$ in the parentheses. Note $(x\ 2)(x\ 3)$ was chosen
  Choices: $(x\ 1)(x\ 6)$ Or $(x\ 2)(x\ 3)$ because $2 + 3 = 5$. The signs are usually the last.
  Answer: $(x + 2)(x + 3)$

Example B: $x^2 - 12x + 36$
        ( )( )   Fill in the missing numbers!
        $(x\ )(x\ )$
Choices: $(x \pm)(x\ 3)$ or $(x\ 1)(x\ 36)$ or $(x\ 2)(x\ 18)$ or $(x\ 4)(x\ \pm)$ or $(x\ 6)(x\ \pm)$
Answer: $(x - 6)(x - 6)$

Factor. Check work.
1. $x^2 + 4x + 3$ 2. $x^2 - 2x + 1$ 3. $x^2 + 6x + 8$

4. $x^2 + 7x + 12$ 5. $x^2 - 14x + 24$ 6. $x^2 - 11x + 24$

7. $x^2 + 9x + 18$ 8. $x^2 - 10x + 24$ 9. $x^2 + 11x + 18$

10. $x^2 + 9x + 20$ 11. $x^2 + 13x + 40$ 12. $x^2 - 5x + 4$

Name: _____   Date: _____

8c – Factoring

Factoring when $a = 1$, $c < 0$:
A quadratic in the form of $x^2 \pm bx - c$ can be factored into $(x-d)(x+e)$ whenever the c term divisors differ by the middle term b. The larger factor will have the same sign as b; the other factor will have the opposite sign.

Example A: $x^2 + 5x - 6$
        (   )(   )
        $(x$ $)(x$ $)$
  Choices: $(x$ 1$)(x$ 6$)$ Or $(x\,2)(x\,3)$
        $(x + 6)(x - 1)$

You probably won't have more than one line of work on your paper. But these are the steps to fill in the parentheses. Note $(x\,6)(x\,1)$ was chosen because $6 - 1 = 5$. The signs are last to be done.

Example B: $x^2 - x - 30$
        (   )(   )   Fill in the missing numbers!
        $(x$ $)(x$ $)$
Choices: $(x \pm)(x$ 3$)$ or $(x$ 1$)(x$ 30$)$ or $(x$ 2$)(x$ 18$)$ or $(x$ 5$)(x$ $\pm)$
        $(x - 6)(x + 5)$

Factor. Check work.

1. $x^2 + x - 6$

2. $x^2 + 3x - 10$

3. $x^2 - 8x - 20$

4. $x^2 + 4x - 12$

5. $x^2 - 6x - 16$

6. $x^2 - x - 12$

7. $x^2 - 16$

8. $x^2 + 3x - 18$

9. $x^2 - 7x - 18$

10. $x^2 - 36$

11. $x^2 + 8x - 9$

12. $x^2 - 9x - 36$

Name: _____   Date: _____

8d – Factoring

Factoring when $a \neq 1$:

A quadratic expression of $ax^2 \pm bx \pm c$ can be factored in the same way as lessons 28a and 28b. There is one extra step, and that is factoring the a term as well. This can create many combinations to try. But after lots of practice, you will start to recognize patterns. Ask for one.

Example A: $4x^2$ - $4x$ - 3   With an $a \neq 1$ term, it just means
       ( )( )   there are more combinations to try.
       (x)(x)
  Choices: ($2x$ 1)($2x$ 3)   Or   ($4x$ 1)(x 3) Or ($4x$ 3)(x 1)
  Answer: ($2x$ + 1)($2x$ - 3)

Example B: $2x^2$ - $11x$ - 30
        ( )( )   Fill in the missing numbers!
        (x)(x)
  Choices: ($2x$ ±)(x 3) or ($2x$ 3)(x 10) or ($2x$ 2)(x 15) or ($2x$ 15)(x ±)
           ($2x$ 1)(x ±) or ($2x$ 30)(x 1) or ($2x$ ±)(x 6)   or ($2x$ 5)(x 6)
  Answer: ($2x$ - 15)(x + 2)

Factor. Check work.

1. $2x^2 + 5x$ - 7

2. $3x^2$ - $12x$ - 15

3. $3x^2$ - $8x$ + 4

4. $4x^2$ - x - 5

5. $3x^2$ - $11x$ + 6

6. $4x^2$ - $8x$ + 3

7. $3x^2$ - x - 4

8. $3x^2$ - $4x$ - 4

9. $8x^2$ - $6x$ - 9

10. $6x^2$ - $5x$ - 6

11. $6x^2$ + $15x$ + 6

12. $6x^2$ - $37x$ + 6

Name: _____   Date: _____

8e – Solve by Factoring

Once you know how to factor, then you are able to solve many quadratic equations for the variable. Most equations will follow the steps as in the example below.

Example: $2x^2 = x^2 + 5x - 6$
$2x^2 - x^2 - 5x + 6 = 0$ Bring terms to one side of the equation.
$x^2 - 5x + 6 = 0$ Simplify
$(x - 2)(x - 3) = 0$ Factor
$x - 2 = 0$ Or $x - 3 = 0$ If $ab = 0$, then either $a = 0$ or $b = 0$.
$x \in \{2, 3\}$ Solve each equation for the variable.

Example: $0 = 2x^2 + 6x - 20$ Fill in the blanks!
$0 = 2(x^2 + 3x - \pm)$
$0 = 2(x + \pm)(x - \pm)$
$x + \pm = 0$ Or $x - 2 = 0$
$x \in \{2, -5\}$

Solve by factoring:

1. $x^2 - 7x + 6 = 0$ 2. $x^2 - 3x + 2 = 0$ 3. $x^2 - 7x + 12 = 0$

4. $x^2 - 3x = 4$ 5. $x^2 + 5x = -4$ 6. $x^2 + 13x + 12 = 0$

7. $x^2 + 5x = 14$ 8. $x^2 - 8x + 12 = 0$ 9. $x^2 + 9x = -14$

10. $2x^2 + x = 6$ 11. $4x^2 + 9x = -5$ 12. $12x^2 - x = 20$

Name: _____   Date: _____

8f – Difference of Squares

When a quadratic expression of $ax^2 \pm bx \pm c$ is missing the bx, it probably still can be factored. As you noted before in a previous lesson, $x^2 - c$ can factored into $(x - d)(x + d)$. Then it can be concluded c is a perfect square. The factors of c are equal, so when foiling $dx - dx = bx = 0$.

Example A: $x^2 - 49 = (x - 7)(x + 7)$.
      Check work: $x^2 - 7x + 7x - 49$ can be simplified to $x^2 - 49$.

A leading coefficient must be factored into equal quantities as well.
Example B: $9x^2 - 100 = (3x - 10)(3x + 10)$

Maybe you can simplify before factoring
Example C: $25x^2 - 100 = 25(x^2 - 4) = 25(x - 2)(x + 2)$

Summarize the following factor rules:

Perfect Trinomial	$a^2x^2 \pm 2abx + b^2 =$
Difference of Squares	$a^2x^2 - b^2 =$

Solve by factoring:
1. $x^2 - 9 = 0$ 2. $x^2 = 16$ 3. $x^2 - 25 = 0$

4. $x^2 - 36 = 0$ 5. $25x^3 - 100x = 0$ 6. $2x^3 - 32x = 0$

7. $3x^2 - 27 = 0$ 8. $4x^2 - 36 = 0$ 9. $25x^2 - 9 = 0$

Name: _____   Date: _____

8g – More Practice Factoring

Use the following formulas to help factor the questions below.
$a^2 - b^2 = (a - b)(a + b)$
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Factor:

1. $25x^2 - 9$ 2. $4 - 81x^2$ 3. $16c^2 - 64$ 4. $27 - 3h^2$

5. $3c^4 - 81c$ 6. $x^4 - x$ 7. $5x^5 - 5000x^2$ 8. $y^3 + 64$

9. $a^4b - ab^4$ 10. $4x^2 - 16y^2$ bonus. $x^{12} - y^{12}$

Name: _____   Date: _____

8h –Quadratic Formula

All quadratic equations are of the form $y = ax^2 + bx + c$, where a, b, and c are real numbers. When $y = 0$, you can solve for x by factoring or using the quadratic formula. Factoring doesn't always work, but the quadratic formula will!

Quadratic Formula:
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example A: $x^2 + 2x - 3 = 0$
   Identify parts: a = 1, b = 2, c = -3

   Substitute into formula: $x = \dfrac{-2 \pm \sqrt{2^2 - 4(1)(-3)}}{2(1)} = \dfrac{-2 \pm \sqrt{16}}{2} = \dfrac{-2 \pm 4}{2} = 1 \text{ or } -3$

Find the solutions (roots, zeros, x-intercepts):

1. $5x^2 + 2x - 3 = 0$

2. $4x^2 - 3x - 7 = 0$

3. $4x^2 + 5x - 6 = 4$

4. $3x^2 + 7x - 10 = 0$

5. $x^2 - 5x + 6 = 0$

6. $3x^2 + 11x - 4 = 0$

7. $x^2 - 5x - 6 = 0$

8. $x^2 + 10x + 21 = 0$

9. $x^2 - 2x + 2 = 2x$

Name: _____   Date: _____

8i – more Quadratic Formula

Solve for the given variable, leaving answer in simplest form.

1. $3x^2 - 5x - 8 = 0$ 2. $w^2 + 5w - 6 = 0$ 3. $z^2 + 7z - 8 = 0$

4. $g^2 + 5g - 5 = 0$ 5. $y^2 - 2y - 3 = 0$ 6. $4x^2 + 10x - 14 = 0$

Solve for x:

7. $3a^2 + 10a + 5 = 0$ 8. $-2x^2 + 5x - 3 = 0$ bonus: $3x^2 + ax - 4a^2 = 0$

Name: _____   Date: _____

8j – Graphing Quadratics

Polynomials with a general form of $y = ax^2 + bx + c$ are called quadratic equations. When a quadratic is graphed, the shape of the curve is referred to as a parabola. One of the easiest, but time-consuming methods of graphing a parabola is to complete a t-chart. Then plot the points.

Example: Graph $y = 2x^2 - 3x + 1$.
Start by choosing common values for x or y: In the end it should look like:

x	-1	0		1	2
y			0		

x	-1	0	½	1	2
y	6	1	0	0	3

Substitute the values you picked:
        When $x = -1$, then $y = 2 + 3 + 1 = 6$.
        When $x = 0$, then $y = 0 - 0 + 1 = 1$.
        When $x = 1$, then $y = 2 - 3 + 1 = 0$.
         When $y = 0$, then $0 = 2x^2 - 3x + 1$.
Then by factoring, we can solve $0 = (2x-1)(x-1)$ for $x = ½$ or 1.

Place the corresponding values in the t-chart. Then plot on the axis provided. Connect the dots. The shape of the curve should look like a *u*.
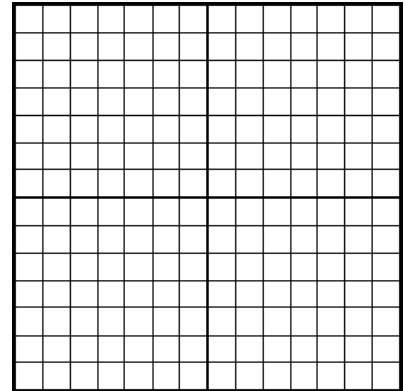
Fill in the t-charts. Then graph to the right. Please label!
1. $y = x^2 + 4x - 5$

x	-5	-3	-1	0	1	3
y						

2. $y = 2x^2 + 3x - 5$

x	-3	-2	-1	0	1	2
y						

3. $y = x^2 + 5x + 6$

x	- 4	-3	-2	-1	0	1
y						

4. $y = -x^2 + 3x + 4$

x	-1	0	1	2	3	4
y						

Name: _____   Date: _____

8k – Graphing Quadratics

Parabolas have certain defining characteristics. If we know those characteristics, then we can use them to make graphing easier. One of the first characteristics to see is the Line of symmetry. If you 'cut' down that line, you would cut the parabola in halves. And each half would be a reflection of the other. If your parabola was already graphed (see #4 from lesson 8j), then take two ordered pairs with the same y-value. Find the midpoint between those ordered pairs. The line of symmetry will pass through this point so that it cuts the parabola in reflected halves. The line of symmetry will also pass through the vertex, which is lowest point or highest point on the graph. If the parabola were a string necklace, the vertex would be where the single charm would hang. Without the graph, some refer to the equation $y = ax^2 + bx + c$ and determine the vertex x-coordinate with the formula: $x = -\frac{b}{2a}$. Then they can evaluate for the y-coordinate. Another quick calculation is done to find the y-intercept, where the parabola crosses the y-axis. Evaluate for y when $x = 0$. You'll notice that y-value is the same as c in $y = ax^2 + bx + c$. Reflect the y-intercept over the line of symmetry to get another point. With those three points, you can graph.

You can always check your work by knowing the direction of the parabola. From lesson 32a, you'll notice that when a is positive the parabola opens upward. When a is negative, it opens downward.

Graph the following equations:

1. $y = x^2 + 4x + 4$ 2. $y = x^2 - 6x + 8$ 3. $y = x^2 - 6x + 4$

direction: up
y-intercept: $y = 0+0+4$
        (0,4)
vertex: $x = -\frac{4}{2(1)} = -2$
        $y = 4 +4(-2)+4 = 0$
        (-2,0)
symmetry point: (- 4,4)

4. $y = x^2 + 8x + 15$ 5. $y = -x^2 + 3x - 2$ 6. $y = -x^2 + x + 12$

Name: _____   Date: _____

8L – Quadratic Word Problems

A quadratic equation can be recognized by the shape of the data when plotted. Or maybe an equation was given and it was in the form $y = ax^2 + bx + c$ where a $\neq 0$. If you need to extrapolate or interpolate (predict) some values, it is best to find the equation first. Then using the equation, the prediction of the needed value should be accurate.

Solve:

1. A ball is kicked so that it lands 50 feet away 3 seconds later. The height of the ball at any given moment is found by $h = -12x^2 + 36x$.   According to the equation, what is the maximum height the ball travelled?

2. Wind chill temperature is given by the formula $C = -\frac{1}{6}w^2 + F$, where C is the wind chill temperature, w is the wind speed in miles per hour, and F is the Fahrenheit temperature in still air. On July 12, the wind speed was 15 mph and the still air was 88°F. What was the wind chill temperature for that day? Which variable has greater effect on C? Explain your conclusion.

3. Carbon Dioxide emissions were found to be increasing since 1975. The following equation has been simplified to make calculations easier, x represents the number of years since 1975 and y represents CO_2 measured in parts per million: $y = 0.005x^2 + x + 300$. What would you expect for the year 2006?

4. The horsepower required to overcome wind drag on an automobile is approximated by
    $H = 0.01s^2 + 0.01s - 0.1$ where s is the speed of the car in miles per hour. What is the speed of the car when the horse power is 2?

Name: _____ Date: _____

8m –Complex Numbers

Imaginary Numbers are complex numbers that are square roots of negative numbers. They are called imaginary because at the time of their discovery, no one could imagine such numbers existing. But they do! You'll encounter them in engineering.

The unit for an imaginary number is i which is equal to $\sqrt{-1}$. Any imaginary number can be written as a multiple of i. Take $\sqrt{-4}$ as an example. It can be written as a product, $\sqrt{-1}\sqrt{4}$ which can be written as $2i$. Of course the example could be irreducible like $\sqrt{-13} = i\sqrt{13}$.

Imaginary numbers can be plotted on the "imaginary" plane. It looks very similar to the real number line! Now if you intersect both the real and imaginary lines at zero. You should have the complex plane. The complex plane contains all numbers. It's only a matter of finding them on the plane. A complex number has the form $a + bi$, where a is the real number and bi is the imaginary number. You plot the number as if you would in the cartesian coordinate system. You go left/right from the origin as many units as your real number (a) and up/down the number of b units.

The graphs of 2 + i, 3 - 2i, and 3i are plotted to the right. (in their approximate spot)
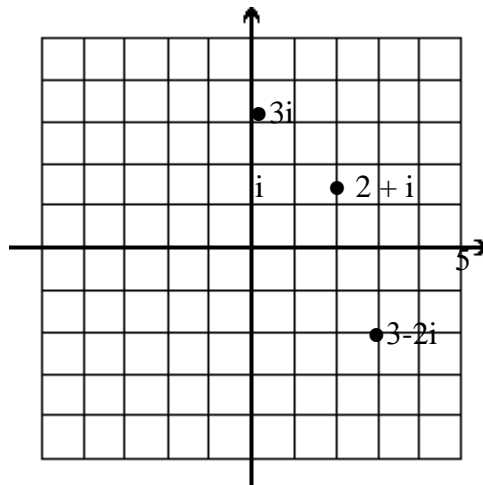
You can combine complex numbers like any real number, except you can only add reals together and imaginaries together. (Like Terms:)

Simplify the following. Graph the results.
1. (-3 - 2i) + (-3 + 2i)   2. 3 - 2i -(3 - 2i)   3. 5 - i+ 3 - 3i   4. -3-2i-(10-12i)

Solve the following and simplify in terms of its complex solution:
5. $4x^2+8x +9 = 0$ 6. $x^2 +x + 1 = 0$

Name: _____   Date: _____

8n –Complex Numbers continued

Multiplying complex numbers is similar to multiplying polynomials. There is just one extra step of simplification and that is to remember $i^2 = (\sqrt{-1})^2 = -1$.

For example: $(3 - 2i)(2 - i) = 6 - 3i - 4i + 2i^2 = 6 - 7i - 2 = 4 - 7i$.

Multiply and simplify:
1. $(2 + 3i)(3 - 2i)$ 2. $(3 + 2i)(3 - 2i)$ 3. $(1 - 2i)(1 - 2i)$ 4. $(1- i)(2 + 2i)$

Sometimes it is necessary to know how far a point is from the origin, otherwise called modulus or magnitude. The magnitude is indicated by vertical bars around the complex number. So $| 4 + 3i |$ would be 5. How do you contrive that? By using the Pythagorean Theorem, use the origin and given point as vertices of a right triangle.

Evaluate. Graph and show work with triangles.
5. $| 3 - 2i|$ 6. $|-2 - 2i|$ 7. $|1 + 2i|$ 8. $|-5 + 12i|$

What can you do really fast to determine (without actually solving) if the equation has real roots? If the discriminant ($b^2 - 4ac$) is negative there are no real solutions. Remember, negative numbers under a square root are not possible with real numbers.

Complex numbers are commonly found when solving quadratic equations. Solve the following for its x-intercepts, simplifying your answers completely.

9. $y = 3x^2 - 6x + 4$ 10. $y = x^2 - 3x + 3$ 11. $y = 2x^2 + 7x + 8$

Chapter 8 Test

Name: _____   Date: _____

SHOW WORK. A calculator is allowed on this test. Attach any scratch paper that's used.

$\boxed{\dfrac{}{5}}$ 1. Write two examples of a like term for each of the following:
       A. 7x   B. $-3x^2y^7$

$\boxed{\dfrac{}{5}}$ 2. Combine the like terms:
       A. $(x^2 + 2x - 1) - (2x^2 + 3x + 3)$ B. $5x^2y^2 - 4x^2y^2 + 3xy^3$

$\boxed{\dfrac{}{5}}$ 3. Determine each product:
       A. 4(3b)   B. $-2p^2(3p^3)$

$\boxed{\dfrac{}{5}}$ 4. Determine each product:
       A. 2x (2x + 3)   B. -12(3 + 2t)

$\boxed{\dfrac{}{5}}$ 5. Factor each binomial:
       A. $25a + 30a^2$ B. $9c^3 - 15c$

$\boxed{\dfrac{}{5}}$ 6. Factor each binomial:
       A. a(a + 6) + 7(a + 6)   B. -16(x - 2) + 48(x - 2)

$\boxed{\dfrac{}{5}}$ 7. Expand (foil):
       A. (a + 1)(a - 3)   B. (2x - 3)(3x + 4)

8. Factor what you can and then reduce each fraction:

| 10 |

A. $(6a - 12) \div 3$ B. $(4a^2 + 12a - 16) \div (a + 4)$ C. $(x^2 + 7x + 12) \div (x + 3)$

9. Solve by factoring:

| 5 |

     A. $a^2 + 5a - 14 = 0$ B. $121 + 22m + m^2 = 0$

10. Solve by the quadratic formula:

| 5 |

     A. $3a^2 + 2a - 5 = 0$ B. $12 + 7m + m^2 = 0$

11. Graph $y = 2x^2 - 5x + 3$.

| 5 |

12. Use the discriminant to determine how many x-intercepts the graph $y = 2x^2 - 5x - 3$.

| 5 |

Bonus of three points: $(x^2 + x - 2) \div (x^2 - 4x - 12) \cdot (x^2 - 5x - 6) \div (x^2 - 1)$.

| 0 |

Name: _____ Date: _____

Algebra Cumulative Review

1. Simplify: $\dfrac{2x^2}{15xy^3} \cdot (3x^2 y^3)^2$.

2. What is the additive inverse to -3?

3. Solve for p: $I = prt$.

4. Graph the equation $y = 2x - 3$.

5. Graph the solution to $|3x + 7| < 8$.

6. Transform the given equation into Standard ($Ax + By = C$) form: $5y = \frac{2}{3}x - 4$.

7. Find the linear equation perpendicular to $5y = \frac{2}{3}x - 4$ through the point (0,1).

8. Compare the point-slope formula of a line and the slope formula.

9. Write a linear equation through the points (3,2) and (5,8).

10. Expand: $(2x-5)^2$.

11. Solve the following by two methods: $0 = 3x^2 - 5x + 2$.

12. Solve the equation for r: $A = \pi r^2$.

13. Solve the following over the complex plane: $0 = 5x^2 - 6x + 5$.

14. Simplify: $\dfrac{4}{x} + \dfrac{5}{xy}$.

Appendix

Algebra II Review Sheets

The appendix contains worksheets that were designed for students taking Algebra II with a textbook by Paul A. Foerster entitled Second Edition of Algebra and Trigonometry: Functions and Applications. These were questions were pulled from his book for students seeking extra questions to practice for a test. You will find these worksheets handy as a test or exam review whether or not you are using the same textbook.

So note that each sheet in the appendix covers the major themes of an Algebra II course. The material to build these topics would have to be found in another document.

Name: _____   Date: _____

1 Sets and Operations of Numbers

Directions: Use your algebra textbook to determine how to do the following questions. Read carefully the text as well as the examples in the textbook. Try a few questions out of the textbook for additional practice.   Help is provided to clarify any concept.

1. Identify the different sets to which each number belongs.

Type number	Integers (Z)	Natural (N)	Rational (Q)	Imaginary	Whole	Complex (C)
0						
2.71828...						
$\sqrt[3]{-27}$						
3.14						
6						

2. Simplify: (2x-3)(x+5)

3. Simplify: 4x - 2[3x - (x-2x)]

Explicitly solve two equivalent forms for absolute equations (inequalities). Then graph the solution:

4. 4 - 3x > -5

5. | 3x + 2 | < 5

6. | x + 1 | < -3

7. 2 < | 2x + 3 | ≤ 5

8. | 5x - 3 | = 12

9. 8(x − 2) < 12

10. Expand: $(x + y)^3$

11. Solve (x + 4)(x - 5)(2x + 3) = 0.

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2 Functions and Relations

1. Solve: $x^3 = 16x$, domain $\in \{x \mid x > 0\}$

2. Show the algebra to expressing
    5.1121212... as a ratio of integers.

3. Determine if the following is a polynomial. If it is, give its name by both degree and term.
    A. $x^2yz^5 - 9x^2y^3z$ B. $4xy^3 - 7xy^7 + 5x^2y^5$ C. $3x^2y^{-3} - (2xy^2)^3$

4. Graph: $y = -x^2 - 4x$.

    What is the range?

5. Graph $f(x) = \begin{cases} 2x, & x \le 4 \\ 3 - x, & x > 4 \end{cases}$

    What is the range? What is f(4)? f(0)?

6. Explain or show the different ways you can determine if $3x^2 - 3y^2 - 6x + 5 = 0$ is a function.
    Is it a function?

7. Name the axiom(s):
         a. $12(4 \times 3) = (12 \times 4)(3)$ _____
         b. $3 + (-3) = 3 - 3 = 0$ _____   _____
         c. $2(\frac{1}{2}) = 1$ _____
         d. $3 \times 4 \times 2 = 2 \times 3 \times 4$ _____

8. Fill in the justifications for each step of the proof:

Multiplication Property of Zero

Step	Justification
$0 = 0$	
$0 + 0 = 0$	
$x(0+0) = x(0)$	
$x(0+0) = 0 + x(0)$	
$x(0) + x(0) = 0 + x(0)$	
$x(0) = 0$	

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3 - Linear Functions

1. Graph $y + 4 = -2(x - 1)$.   2. Graph $4y - x = 8$.

3. Find the y-intercept and slope of   4. Transform $y - 3 = \frac{3}{4}(x + 5)$
    $3x - y + 5 = 0$.   into $Ax + By = C$ form.

5. Graph the plane: $x - 3y + 2z = 6$.   6. Determine the type of system: dependent,
    Label the intercepts and axes.   inconsistent, or independent.
    $$2x - 3y = 1$$
    $$-5x + 7.5y = 4$$

7. Write the linear equation perpendicular   8. Write the equation of the line
    to $2x + 4y = 5$ that passes through $(3,1)$.   parallel to $2x + 4y = 5$ that passes
                                                       through $(3,1)$.

Name: _____   Date: _____

4- Systems of Linear Functions

Solve the following systems by each of the following methods: Substitution, Elimination, Determinants (Cramer's Rule), Augmented Matrices, Graphing, Inverse Matrices. Attach any additional sheets of paper you used to complete the different methods.

1. $2x + y = 3$
   $x - 4y = -3$

2. $7x + 3y = 6$
   $5x - 2y = 25$

3. $x - 5y = -6$
   $3x - 6y = 0$

Solve each of the following systems by a different method: substitution, elimination, determinants, and augmented matrices.

4. $2x - y - z = 6$
   $x + 5y + 3z = -10$
   $3x - y - 5z = 4$

5. $3x + 2y - z = 3$
   $x + 4y - 4z = 7$
   $-2x - 3y + 5z = 0$

Name: _____   Date: _____

5 - Quadratic Functions

Convert the quadratic equations into Vertex Form. Graph.

1. $y = 3x^2 - 12x + 5$ 2. $y = -2x^2 + 8x - 3$ 3. $y > 5x^2 + 10x + 12$

Verify the number of solutions by using the discriminant.

4. $0 = 3x^2 + 4x + 5$ 5. $2x^2 - 3x = 3$ 6. $x^2 + 2x = 5 - x$

7. Write the quadratic equation that passes through (-3, 37), (1,1), and (2,7).

For #8- #10,use $f(x) = 3x^2 + x - 2$ to find x such that the equation is true.

8. $f(x) = 0$ 9. $f(x) = 3$ 10. $f(5) = x$

Name: _____   Date: _____

6a - Exponentials

Simplify. No complex number should remain in the denominator!

1. $(3x - 2)^2$

2. -7^2

3. $(-2x^3y^2)^{-3}(6xy^3)^2$

4. $\dfrac{(9^{35})^{10}}{(9^{32})^{11}}$

5. $(1001x^{-4}) \div (77x^6y^{-7})$

6. $(36x^2)^{\frac{3}{2}}$

7. $(128x^4)^{\frac{2}{7}}$

8. $\dfrac{(3x^3)^3}{6x^5}$

9. $\sqrt[4]{x^{12}y^8}$

10. $\dfrac{5}{2i-1}$

11. $\dfrac{2+i}{1-4i}$

12. $\dfrac{8i^4}{\sqrt{-4}+2i^9}$

Name: _____   Date: _____

6b - Logarithmic Functions

1. What is the definition of a logarithm?

Simplify into one logarithm:

2. log 2 + log 4 - log 3   3. 2log 4 - log 5 + 3log 3

Expand into several logarithms:

4. $\log(\frac{x^2 y^3}{z})$ 5. $\log(\frac{x^2(x+1)}{2})$ 6. $\log(x^2 + x)$

Change into a natural logarithm:

7. log 5   8. $\log_2 6$ 9. $\log_3 7$

Simplify:

10. $\frac{5}{6}\log_{64} 64$ 11. $10^{\log 2005}$ 12. $\log_8 2$

13. log 1   14. log 0   15. log 1000

16. $\log_4 64$ 17. $2\log_{27} 3$ 18. log 2 + log 5

Name: _____   Date: _____

6c - Solving Logarithmic Functions

Solve exactly:

1. $4^x = 64$ 2. $2^{x+1} = 8$ 3. $3^x = 25$ 4. $10^x = 144$

Solve simultaneously these systems in questions 5-6.

5. $\begin{aligned} y &= 3 \cdot 2^x \\ y &= 6 \cdot 2^{3x} \end{aligned}$

6. $\begin{aligned} y &= 3^{2x} \\ y &= 2 \cdot 3^x \end{aligned}$

7. $\log_2 (x - 4) + \log_2 (x - 2) = 3$ 8. $\log_3 (x - 2) + \log_3 x = 1$

9. What are properties of inverses? How do you find an inverse to a function?

10. Graph and determine if $y = -5^x$ and $y = -\log_5 x$ are inverses.

Find the inverse of:

11. $f(x) = 3x^2 + 2$ 12. $y = 3^x$ 13. $f(x) = (x - 1)^2$

Determine if the following pairs of equations are inverses:

14. $\begin{aligned} f(x) &= x^2 + 2 \\ g(x) &= (x - 2)^2 \end{aligned}$

15. $\begin{aligned} f(x) &= 2 \cdot 3^x \\ g(x) &= \log_3 2x \end{aligned}$

Name: _____   Date: _____

7 - Rational Functions

Expand these polynomials:
1. $(6x + 5)^2$ 2. $(x - y)^3$ 3. $(2x + 4)^2$

Factor these polynomials:
4. $x^2 - 6x - 27$ 5. $12x^2 + 25x + 12$ 6. $7a^4 - 28b^2$

7. $9x^2 - 4y^6$ 8. $8x^3 - 27y^{12}$ 9. $4xy - 3y - 8xy^2 + 6y^2$

Simplify these rational expressions:

10. $\dfrac{x^2 + 2xy + y^2}{x^2 - y^2} \div \dfrac{y + x}{y - x}$

11. $\dfrac{1 - \frac{4}{x+1}}{1 - \frac{2}{x-1}}$

12. $\dfrac{x + 2}{x^3 - 1} + \dfrac{x + 1}{x^2 + x + 1}$

Solve these equations:

13. $2x^2 - 5x = 7$

14. $\dfrac{x + 3}{2x - 3} = \dfrac{18x}{4x^2 - 9}$

15. $\dfrac{2}{x + 1} - \dfrac{3}{x - 1} = 5$

16. A rectangular piece of cardboard has area of 200 cm². The length is four times its width. Find the dimensions of the cardboard.

17. A metal worker wants to make sure an open box made from 6" by 8" sheet of metal has maximum volume. The box was made by cutting out equal squares from the corners of the sheet and then bending edges. Once you determine the equation to represent volume, use a calculator to find the maximum volume and corresponding dimensions of the box. Explain how you got your answer.

Reduce the equations. Graph, labelling the intercepts and discontinuities. Differentiate between removable and nonremovable asymptotes. No calculator!

18. $f(x) = \dfrac{x^2 - x - 6}{(x + 2)(x + 3)^2}$

19. $f(x) = \dfrac{3x + 3}{x^3 + 1}$

8 - Irrational Functions

Answer the following questions based on $f(x) = 4 - \sqrt{3-x}$.

1. What is the domain?

2. What is the range?

3. When will $f(x) = 2$?

4. Where does the maximum occur?

Simplify:

5. $3\sqrt{125} - 2\sqrt{80} + \sqrt{405}$

6. $(2\sqrt{3} - \sqrt{2})(2\sqrt{3} + \sqrt{2})$

7. $\dfrac{7}{\sqrt[4]{49}}$

Solve:

8. $\sqrt{x+6} - x = 4$

9. $2\sqrt{x} - \sqrt{4x-3} = \dfrac{1}{\sqrt{4x-3}}$

10. $\dfrac{\sqrt{x}}{\sqrt{x}-1} + 3 = \dfrac{1}{\sqrt{x}-1} - 1$

9 - Polynomials of any Degree

1. Describe the difference in the shape of the curves between even-powered polynomials and odd-powered polynomials.

2. Complex solutions come in _____. They are _____ to each other.

Use synthetic division to rewrite as linear (or non-reducible quadratic) factors:

3. $x^3 - x^2 - 4x + 4$ 4. $5x^3 - 3x^2 - 8x + 20$ 5. $x^4 + x^3 - 5x^2 + x - 6$

6. How many possible positive and negative zeros does $y = x^4 - x^3 - 11x^2 + 9x + 8$ have?

Final Exam

Let me say a word about this final exam. This is an excerpt of an exam where grades seven through twelve had a common portion of questions. The pages were broken down by subject material so that everyone always had to show competence on their basics. This allowed me as a teacher to assess students' progress. This also allowed for reducing my exam writing so I didn't have six completely different exams to write, on top of the other courses I taught. The idea was continue to drill the basics as they learned more concepts each year. This was essential as I noted when arriving at my school a lot of students were weak in their arithmetic and oblivious to their need to shore up their weak areas before graduating.

Thus the exam starts off where everyone has the same pages but slowly different grades will drop out. The bonus page is numbered in base 2 so that it can be attached at any point of the exam. For example the first five pages are for grades seven and eight. So I would attach the bonus page at the end of the fifth page, and not bother printing the rest of the exam for them. I would circle the grade at the top of the page after correlating and stapling the pages. Then I would ask students when passing out the test to verify they had the correct exam by checking the grade level.

You will also note that the first two pages were to be done without a calculator. The students would have to submit it before receiving the rest of the exam and allowing calculators to be used. I would color code the exam sheets so the non-calculator pages were different. Then I would have a visual reminder when scanning the examinees for calculator misuse. I got this idea of having a calculator/non-calculator parts from my colleague Dr Keith Calkins, who would also color code.

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Final Exam

Name: _____   Your grade level: 7, 8, 9, 10
Date: _____

Follow directions! **No calculator** is allowed on this part of the exam. Show work for full credit. One 8.5" by 11" sheet of notes is allowed on the entire exam.

Real Numbers and Their Operations
1. Simplify:

     a. 15 - 7   b. 3 -17   c. -5(-21)   d. -120 ÷ -3

2. Simplify, leaving answer as proper fraction (mixed number):

     a. $\frac{4}{5}+\frac{3}{5}$ b. $\frac{4}{5}-\frac{2}{3}$ c. $\frac{3}{4}\times\frac{2}{15}$ d. $-\frac{14}{15}\div\frac{2}{5}$

3. Simplify:

     a. 3.4 - 5.25   b. - 4.1 - 3.001   c. 0.011(3.5)   d. 5.46 ÷ 0.6

4. Simplify:

     a. 4^2 b. 75% of 88   c. ___ % = 0.241   d. $\sqrt{16}$

Bonus (2 pts): Simplify, exactly: $\sqrt{32}$

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Leave answers as exact or proper fractions (mixed numbers). Demerits given for not leaving answer in correct form.

5. Solve for x:

         a. $4x = -10$ b. $4x - 3 = 7$ c. $3(x + 2) = 9$

6. Simplify:

         a. $(6x^3 + 24x^2) \div (3x)$ b. $-6x^2(2x + 5x^2)$

7. Simplify:

         a. $(x^3 + 3x^2) + (5x - 4x^2)$ b. $(5x^3 - 4x) - (3x^3 - x)$

8. Write a proportion for the similar triangles below. Solve for x.
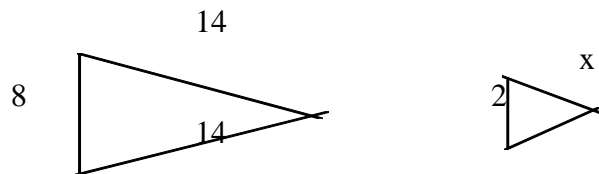
9. Paul eats twice as fast as Nathan, who eats eats three times slower than Ms Luttrell. She finishes eating her lunch in 12 minutes, how quickly does Paul eat?

Bonus (2 pts): An $56 sweater is on sale for $42. What is the percent of discount?

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Basic Geometry

| Calculator is allowed from this point onward.   You must show work for full credit!! |

10. What is the perimeter of the rectangle?

4.2 cm

3.1 cm
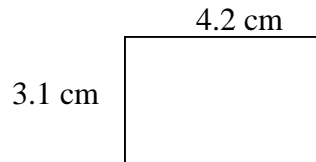
11.   A rectangle with dimensions x and $3x$ - 4 has a perimeter of 24. Find x. Then find its area.

12. If the diameter of a circle is 6 cm, what is its circumference? What is its area?
(Leave the answer in terms of π.)

13.   What is the area of the triangle?

6

9 cm
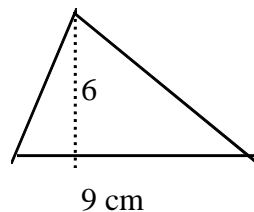
14. If a cube has surface area of 150 cm^2, then what is the volume of the cube?

15

Bonus (2 pts): Find the area of the right triangle: 12
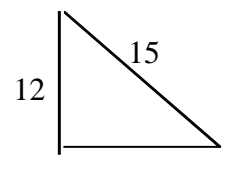
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Use the following test scores to answer the questions #14 to #16:
  61   65   68   71   71 73   77   78   78   82   82   82   87   89   91   91   95 98.

15. Construct a stem-and-leaf plot.

16. What is the mode? The median? The mean?

17. Create a histogram, labeling the axes.

18. The class has 3 boys and 5 girls. What is your chance of randomly picking a boy's name from the list?

19. There are 4 pants (beige, blue, green, black) and 2 matching shirts (white, yellow). How many ways can you match a pants with a shirt? Write out the combinations if you must.

Bonus (2 pts): A class average on an exam is 85 where the boys in the class averaged 90. The class of eight students has only three boys. What is the girls' average?

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20. Factor the following numbers using a factor tree:
    a. 48   B. 144

21. Complete the chart:

Scientific Notation	Standard form
2.1×10^7	
	0.0000456
	2,456,000,000,000

22. Identify the type of angle:

            a.   b.   c.   d.

23. Find the volume: **6**

       12

                      5
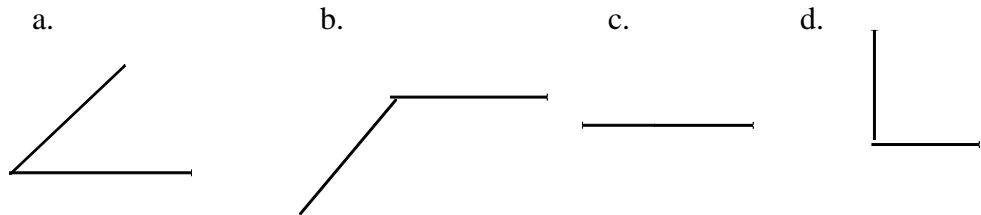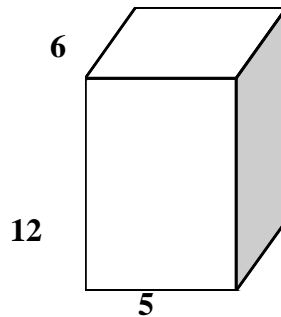
24. Circle whether true or false:
 true   false   If all the sides of one triangle equal to all the sides of another triangle, then the
                 triangles are congruent (equal).
 true   false   If two sides of an isosceles are equal to two sides of another isosceles, then the
                 triangles are congruent (equal).
 true   false   An equilateral triangle has three equal angles as well.
 true   false   The angles in a triangle sum to be 180⁰.
 true   false   The cube with side length of 4 cm has a surface area of 64 cm².

Bonus (3pts):
A regular (all sides and angles are equal) octagon has perimeter of 96 cm. Find its area.
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25. Write the equation of a line that passes through (2,5) and (4, 6).
       Bonus (3 pts): Write the line perpendicular that passes through (3,4).

26. Solve the system: $2x + 3y = 3$ Bonus (3pts each) for doing
                                    $- 3x - 3y = -9$ two other methods.

27. Graph the solutions to the system: $y \leq 3x - 2$ and $y < x + 1$.

28. Answer the questions based on the equation: $y = 5x + 6$.
         a. Slope = _____
         b. y-intercept = _____
         C. x-intercept = _____
         Bonus (1 pt): $f(7)$ = _____

Bonus: Solve by one method (3 pts), by an additional method (2pts): $2x^2 - 7x + 5 = 0$

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29. Simplify each:

    a. $(4x^0y^3)^2(3x^2y^0)^2$ b. $3^3 \times 3^{-3}$

    c. (3 - 2i) + (1 + 3i)   D. (3 - 2i)(1 + 3i)

30.   A. Expand: (x - 3)(x - 5)   B. Solve for x: 3 - 2(1 - x) = -5.

    C. Solve for x: 3 - x < 6   D. Solve for x: | x - 3 | > 3

    E. Expand: (x - 3)(x²-5x + 6)

31. Do the matrix operations:

    a. [2 3] + [-1 2]   b. [2 -2] × [4 3]   c. 2[4   -1]

32. Simplify: $\dfrac{x^2 - 4x - 5}{x^2 - 8x - 9}$

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33. Graph: $y = 3x^2 - 6x - 4$, labeling the intercepts, vertex and symmetry point.

34. Graph $x^2 + y^2 = 9$

35. A. Evaluate $f(x) = 8 - 2x^3 + 4x + 7x^2$ when $f(-2)$.

B. Evaluate $f(x) = 3x^3 + 4x^2$ when $x = -1$.

36. Solve: $\sqrt{x-1} = 7$

37. Solve $\sqrt[3]{x-1} + 4 = 6$

Bonus: (3 pts) Solve $\frac{3}{x-1} + 3 = \frac{5}{x-1}$

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38. Write the next three terms for the sequence: 2, 8, 14, ____, ____, ____ ... and then write a general equation to represent the terms of the sequence.

39. Write in summation notation: 2+8 + 14 +... 236.

40. Evaluate: $\displaystyle\sum_{k=3}^{6}(k^2 - k)$

41. A. Expand this logarithm: $\log_4 3x^2$

   B. Simplify into one logarithm log(x-1) + log(x+1) - log 4

42. Solve: $e^x = 6$

Bonus (2 pts): Find the zeroes of the equation: $y = 3x^4 - 6x^2 + 8x - 16$.

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43. A right triangle has a hypotenuse of 60 and leg of 24. What is the measure of the other leg?
    What are the angle measures?

44. The oldest tree in town casts a shadow of 52 m long on level ground. At the same time, a
    boy 2 m tall casts a shadow 4 m long.
        A. How tall is the tree?
        B. If the boy, standing 60 m away, looks into binoculars at an osprey at the top of the
            tree, what is the angle of elevation?

45. In a right triangle where sin x = $\frac{\sqrt{3}}{2}$, find cos x.

46. A surveyor is measuring the distance across a small lake. He has set up his transit on side of
the lake 150 meters from a piling that is directly across from a pier on the other side of the lake.
From his transit, the angle between the piling and the pier is 72°. What is the distance between
the piling and the pier to the nearest foot?

Bonus (2 pts): For what angle is sinθ = ½ and tanθ = $-\sqrt{3}$?

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Each bonus question is worth 4! ÷ 3!. Show work for full credit!!

0. How can 7 + 8 = 12? One possibility is that the number system being used is to the base 13.
However, what we are looking for here is a different digit to be substituted for each of the letters in the
following example in order to give a correct addition.

```
    S E V E N
  + E I G H T
  T WE L V E
```

01. If the city of *A* is 9000 miles from *B*, and *B* is 9000 miles from the city of *C*, what is the probability
that *C* is closer to *A* than to *B*?

10. What number should be replace the x?

7	8	9
4	6	8
1	x	7

11. With the weights given below, find x so the weights balance the scale.
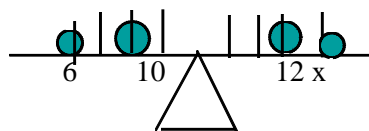
100. What is X?   131   517   192   X

I have been careful not to allow others to see my work and the work on the examination is completely my own. This examination
may be returned for my own personal use and I will not share it except with concurrent classmates. Other uses are not condoned.
I will properly dispose it. I will not talk about the exam until after June 30.

Signature/Date: _____

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Answer Key

A select portion of the answers are given below. Any answer with graphics was not included.

Sheet 1a:
1. 17, Four less than a number is thirteen.
2. 3, Five more than a number is eight.
3. 3, A number less than eight is five.
9. 18, A number divided by three is six.
12. 9, A number times six is fifty-four.

Sheet 1b:
2. 3.1234322...   3. 3.040040004...   4. ½

Sheet 1c:
2. 3600   3. 86400   6. A. 315/100   6. B. 104/33

Sheet 1d:
6. A. < B. >   C. <   D. <

Sheet 1e:
2. -13   3. -12   4. 0   5. -8   6. -2
7. -14   11. 9   12. 0   13. 6

Sheet 1f:
1. 10   2. -9   3. -14   4. 3   5. 3   6. -15
7. -2   8. 2   25. 11   26. 5   27. 7   28. 16
33. 2   34. - 4   35. -3   36. 1   38. $43   40. 11

Sheet 1g:
1. 24   3. -45   5. 18   7. -60   9. -85   11. -217
13. -24   15. 70   17. 3   19. -4   21. 10   23. 9
25. -5   27. 8   29. 9   31. -1

Sheet 1h:
1. Hundred   3. Million   6. Tens   8. $(5\times1000) + (3\times100) + (4\times10) + (5\times1)$
13. 453,800   15. 76,876

Sheet Test review: 1. A. 40   2. A. 1   3. A. 8   C. 7   D. -20
4. C. 9   5. 406,530   6. 4(10,000) + 3(100) + 5(100) +7(1)

Chapter 1 Test: 2. A. 501,072   3. A. x – 3   4B. 900,300   5. A. 3

Sheet 2a:
4. 3⁄4   5. 9⁄10   6. 3⁄4

Sheet 2b:
1. 3 ⅔   2. 3 7⁄12   8. 2⅓   9. 23⁄8   11. 72⁄7
15. 53⁄10   16. 66⁄5

Sheet 2c:
1. 1 1⁄12   3. 1 1⁄4   5. 1 23⁄26   7. 3⁄4   9. 1 21⁄45
11. 1 13⁄24

Sheet 2d:
1. ⅔   3. ¼   5. 6⁄7   7. 12 1⁄10   9. 2¼   11. -2
13. 15   15. 40   17. 2⁄5   19. 2⁄7   21. 1 25⁄27

Sheet 2e:
1. 43 ½   3. 6⅞   7. 37   8. 6⅓

Sheet 2f:
3. $-x^2 + 4xy + y^2$ 5. $4x^2 + 2x + 2$ 7. $8x^3 - 15x^2 + 3x + 8$
11. x + 4   12. x + 4

Sheet 2g:
1. -7   3. 7   5. -17   7. 24   9. -30   11. -96
13. 98   15. -324   17. -18   19. 3   21. 0   23. 10

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Answer Key, chapters 2-3

Sheet 2h: 1. 2   3. 3   5. -5   7. 1   9. 4   11. 4
              13. 0   15. 3

Sheet 2i: 1. 4   3. ¾   5. 3   7. -140   9. 108   11. 4
              13. 21   15. -9   17. 3

Sheet 2j: 1. 5   3. -5   5. ½   7. 3.5   9. -2

Sheet 2k: 1. -2   3. -25   5. 3   7. { }

Sheet 2L: 1. -1   3. 2   5. -5   7. 4   9. 1   11. 24
              13. 10   15. 4 1⁄5

Sheet 2m: 1. 80 by 44   3. 5   5. 1 by 2   7. 32   9. 3

Sheet 2n: 1. 80   3. 97⅜   5. 14 61⁄64   7. 110 5⁄6 ft^2 or 12 17⁄54 yd^2

Sheet 2o: 1. 4   3. 4   5. 1¼   7. 2⅓   9. -5 2⁄5
              11. -1⅔ 13. -6   15. 14 27⁄32

Sheet 2p: 1. 2 7⁄34   3. 5 5⁄7   5. 9⁄22   7. 3 3⁄14   9. 8 19⁄64

Sheet 2q: 1. 6 ½   3. 1 2⁄7   5. 12   7. 21⅓   9. 1¼
              11. 15⁄32 13. 8

Sheet 2r: 1. A. 13⅞   B. 29⅓   C. 12 23⁄24 3. A. 24 ½   B. 17 ½   C. 12 ¼
              5. 16   7. 12 4⁄5   9. W = 1 ½, area = 8¼

Chapter 2 Test: 2. 27⁄4; 2⅞   4. A. 12 4⁄9, B. 2 5⁄8   6. A. 6⁄7, B. 2 1⁄8   8. 10 1⁄6

Sheet 3a: 1. 351.008   3. 4800.40335   9. 47⁄100   11. 5 63⁄100   13. 2.3

Sheet 3b: 1. 40.5 3. 61.0625   5. 0.8   7. 2.1   9. 16.4   13. 4, 7

Sheet 3c: 2. 2.11, 2.111, 2.112, 2.121   5. 2.418   7. 81.364   9. 14.6917
              11. -0.82   13. -44.55   15. -4.255   17. 5.251

Sheet 3d: 1. 39.18'   3. 39.84'   5. 13.2'   7. 11.6"   9. 6.5'

Sheet 3e: 1. 30.03   3. -0.0483   5. 18.7068   7. -18.13   9. -85.86
              11. 0.003611   13. $86.50   15. P=21.2 ft, A=28.09 ft^2 17. 171.91 ft^2

Sheet 3f: 1. 33.784   3. 28.4889   5. 22.14   7. 46.7

Sheet 3g: 1. 0.0135   3. 40   5. 14.592   7. 0.023   9. 100.21
              11. 6   13. 9   15. $16.67   17. 3.4 (4)

Sheet 3h: 1. 8.99   3. 0.27   5. 45   7. 8   9. 9
              11. 22.72   13. 3.2

Sheet 3i: 1. 56.52   2. 6.28   5. 2.00   6. 1.50

Sheet 3j: 1. 254.3   2. 3.1   5. 3   6. 4
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Answer Key, chapters 3-4

Sheet 3k: 1. SA=880, V = 1600   3. SA≈ 628, V ≈ 1,177.5

Sheet 3L: 1. 60   3. 3840   5. 4 times, 64 times   7. 32π
9. double, 8 times   11. K → K^3

Sheet 3m: 1. 200   3. 6.93   5. 810   7. 267.9   9. 198.4

Sheet 3n: 1. 25   3. 10   5. 80   7. 300   9. 4187
11. 1000   13. 54   15. 60

Sheet 3o: 1. 12   3. 2   5. 100.5

Sheet 3p: 1. 235.5   3. 60   5. 1130

Chapter 3 Test: 1. A. 6.001, 6.01, 6.1, 6.11   B. -3.3, -3.22, -3.2   3. A. 11.3   B. 18
3. C. 12.307   7. 11.1   8. 44.33   10. V = 500π cm^2, SA = 250π cm

Sheet 4a: 1. -5/8   2. A(b+c) = Ab+Ac   3. 3   4. 1/x   5. 5+(-5) = 0

Sheet 4b: 1. 3×3 = 9   3. 5×5×5 = 125   5. 4×4 = 16   7. 8×8 = 64
9. 1000   11. 0   13. 81   15. -243
19. 1⁄27   21. 1⁄27   23. 1⁄216

Sheet 4c: 1. 441   3. 0   5. -15   7. $5x - 5x^2$ 9. 3x + 3

Sheet 4d: 3. -2   5. ½   7. -180   9. -2   11. -2

Sheet 4e: 1. 1   3. -6   5. 1   7. 23   9. 1 4/7

Sheet 4f: 1. Seventy-five   3. Four hundred   5. Ten thousands   7. Hundreds
9. 3(100,000) + 4(10,000) + 5(1,000) + 6(100) + 4(10) + 5(1)   13. 4,083,700

Sheet 4g: 1. 10^4 3. 10^7 5. 49,506   7. 7,860,000   9. 87,401
11. $4(10^2) + 3(10^0)$ 13. $5(10^7) + 5(10^5)$

Sheet 4h: 2. $5(10^4) + 3(10^3) + 1(10^2)$ 5. $3(10^1) + 2(10^0)$ 9. 7,426   11. 8,121,530
10. 23   11. 5   12. 53   13. 64,206

Sheet 4i: 3. $1(10^3)$ 5. $4(10^{16})$ 11. $1(10^1) + 2(10^0) + 5(10^{-3})$ 15. 8,450,001

Sheet 4j: 1. $1.23×10^{-4}$ 3. $3.45×10^{-8}$ 5. $7.6×10^{10}$ 7. $2.3×10^{13}$ 9. $4.56×10^{14}$
11. 23,400   13. 67,000,000   15. 0.000 034   17. 0.000 003 25

Sheet 4k: 1. 4%   3. 45%   5. 100%   7. 0.23   9. 0.003
11. $3.20   13. $4.80   15. $4.20   17. $6.30   19. $88
21. $14.25; 33.3%   23. $68.80; $72.93; $91.16

Sheet 4L: 1. 500; 5000; 0.005   3. 0.045; 0.000045   5. 3; 300
1. 1440   3. 0.081   5. 1   7. 0.000145   9. 96.75
11. 0.0004005   13. 480

Answer Key, chapters 4-6

Sheet 4m: 1. 9   3. 16   5. 50   7. 12   9. 12; 14; 108; 42
11. Quadrilateral   12. Hexagon   13. 105

Sheet 4n: 1. 20.8   2. 2300m; 2,300,000 mm   3. 4.2 classes   4. 90%   5. 25%

Chapter 4 Test: 1. <; <; =   2. 7/3; 2/3   3. 15; 4; 40   4. 9; 25; 5.42   5. 1600; 420
6. $3.20 is cheaper; 326.7 km   7. 28; 10; 150; 64; 39%   8. 247,026
9. 80%   10. 37%   11. A. 234000   11. C. 6,304   11. D. -6
11.E. 8   Bonus Jar I: 350 red, 50 white; Jar II: 360 red, 40 white

Sheet 5a: 1. $H = 3V/(\pi r^2)$ 2. $m = (y-b)/x$ 5. $Y_1 = m(x_1-x_2) + y_2$ 7. $h = 2A/b$

Sheet 5b: 1. $S = 2T-5$ 3. $3R$ 5. $S+T = 50,000$ 7. $x + y$
9. $H = 4 + y$ 11. $H = ½ L$ 13. $T = 1.25S$ 15. $D = 8$

Sheet 5c: 1. $x = 5$ 3. $x = 6$ 5. $w > -4$ 7. $z < -6$

Sheet 5d: 1. $|6 - 4| = 2$ 3. $|7 – (-3)| = 10$ 5. $|x – (-2)| = 4$ 7. $|x – (-2)| < 4$
9. 5, 11   11. -8, 4

Sheet 5e: 1. 2   3. -4, 4   5. $x < 3$ 7. $-3 \le x \le 3$ 9. -2, 8
11. -1, 11   12. -1, 5

Sheet 5f: 1. $-1 < x < 5$ 3. $-8 < x < -6$ 5. $x < -4$ or $x > 6$ 7. $-2 \le x \le 6$ 9. $x > 1$ or $x < -1/2$
10. $0 < y < 4$ 14. $9 \le x \le 15$ 15. $-5 \le x \le -1$

Sheet 5g: 4. The figure is a line.

Sheet 5h: 1, 6, 4, 2, 7, 8, 5, 3   9. $y = kx$ 11. $y = k/x$ 12. $y = k$ 15. $y = kx + b$

Sheet 5i: 1. Yes, Domain: $-1 \le x \le 4$; Range: $-5 \le y \le 6$ 2. Yes   3. No   4. Yes
5. Yes   6. Yes   7. Yes   8. No   9. No

Chapter 5 Test: 1. 7.5; 8; 6   2. $T = 180B + 14000$; 16,340   3. $h = 2A/b$; $h = V/(\pi r^2)$
4. $y < -3…$ 5. $-2 < x < 8$; $-7 \le x \le 5$; 2, -⅔

Sheet 6a: Fill in blanks: 3, 6, 1, 3, 6, 4   1. $x = \{-1, 0\}$, $y = \{4, -2.5\}$
2. $x = \{7, 14\}$, $y = \{3, 9, 15\}$ 3. $x = \{6, 10\}$, $y = \{7.96, 23.88\}$

Sheet 6b: Graphs are not included.

Sheet 6c: 1. $y = ⅔x – 7$ 2. $y = 3x – 2$ 3. $y = -⅓x + 2$ 4. $y = -x + 3$
5. $y = -2x + 5$ 6. $y = 2$ 7. $-4x + 9y = 51$

Sheet 6d: 1. $y = -3x + 2$ 2. $y = ⅔x + 2⅓$ 4. $y = x + 5$ 6. $y = -2x + 5$
7. $y = 4x – 11$ 8. $y = -3$ Graphs are not included (#9-#17)
9. $x = 6$ 10. $x = 6$ 11. $x = -3$ 12. $y = 4$

Sheet 6e: 1. A & C   2. $y = ⅓x + ⅔$ 3. $5x – 3y = -26$ 4. $4x + 9y = -2$
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Answer Key, chapters 6-8

Sheet 6f: 1. A&B, B&C 2. D 3. $3x + 5y = 22$ 4. $3x + 4y = -1$

Sheet 6g: Graphs are not included. Simplified equations: 1. $y < -2x -4$ 2. $y > -3x + 5$
3. $y < x – 4$ 4. $y \le 2x + 4$ 5. $y \ge -\frac{1}{2}x + 2.5$ 6. $y > 4/3\, x - 3$

Sheet 6h: 1. (2.5, 0.5) 2. (2, 4) 3. (2,-1) 4. (-3, -4) 5. (-3, -2) 6. (3, 5)
Sheet 6i: 1. (3, 2) 2. (1, 1) 3. (2, -3) 4. (2,4) 5. (3, 1) 6. (3, -2)

Sheet 6j: 1. (½, ⅓) 2. (1, 3) 3. (4, 7) 4. (-6, 5) 5. (2, -5) 6. once

Sheet 6k: 1. $Y = 0.49x$; $36.75 2. $Y = 64x + 5500$; $21,500 3. $Y = 25x + 65$; $127.50
4. $y = 8x$; $96 5. $Y = -1360x + 17000$; $-3400 (cost you to get rid of it!!!)

Sheet 6L: 1. 161.6°F 2. $270 3. 36 ft 4. 46,875 ft^2 5. 2 ⅔ cups
6. 1⅓ cup 7. Keith = 48, Shirleen = 32, Rachel = 16

Chapter 6 Test: 1. -9, ½, -10 2. C = 60, A = 480 5. (4, 3) 6. $6x – 9y = 8$; $-x + 2y = 10$
7. $y = -6x + 27$ 9. $Y = 0.75x + 2.75$ 10. $2x – 3y = 17$; $3x + 2y = 6$ Bonus: 32; -1

Sheet 7a: 1. 12 2. 36 5. 16 6. {} 8. $8y + 49y^2$
9. $176x^2$ 11. $4x^4$ 12. ½ 14. $Z^3y^6x^9$ 15. 1

Sheet 7b: 1. Y^{11} 2. $-x^8y^6$ 3. Z^6 4. $A^{18}b^{11}$ 5. X^8 6. a/2

Sheet 7c: 1. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97
2. 3 and 5 3. 11 4. 3 and 9 5. 6, 9, and 11

Sheet 7d: 1. $2^2 \times 29$ 2. $5^2 \times 7$ 5. GCF = 7; LCM = 105 7. GCF = 3, LCM 378
9. GCF = 6; LCM = 168 11. GCF = 5; LCM = 350 13. GCF = 8; LCM = 448

Sheet 7e: 1. $2\sqrt{6}$ 2. $5\sqrt{3}$ 3. $4\sqrt{6}$ 5. 12 6. 15 7. 25

Sheet 7f: 1. 16 3. 32 4. 16 6. 49 10. $\sqrt{3}$ 11. 9

Sheet 7g: 1. 0 2. $3\sqrt{3}$ 3. $9\sqrt{3}$ 4. 1 5. -6x 6. $10\sqrt{2}$
7. $6\sqrt{2}$ 8. $8\sqrt{2}$ 9. 54 10. $24\sqrt{6}$ 11. $\sqrt{6}$ 12. 75

Sheet 7h: 1. $\sqrt{3}$ 2. $3\sqrt{5}/5$ 3. $\sqrt{30}/6$ 4. $\sqrt{6}/3$ 5. $\sqrt{2}/2$

Sheet 7i: 1. ±4 2. ±5 3. 2 4. 144 5. 225 6. 30
7. 50 8. 22

Chapter 7 Test: 1. 8; 48 2. 2; 504 3. 8⁄27; -256 4. -⅓; 9261 5. 20; $y^4\!/x$
6. 1⁄512; x^{15} 7. 3.4×10^{-11}; 5×10^9 8. 3; 5; 28 bonus: 909; -1982

Sheet 8a: 1. $x^2 – xy – 6y^2$ 3. $-x^2 + 5x – 6$ 5. $2a^2 – 7ab – 15b^2$ 7. $W^2 – 4$ 9. $-x^2 + 8x + 33$

Sheet 8b: 1. (x + 1)(x + 3) 3. (x + 4)(x + 2) 5. (x – 2)(x – 12) 7. (x + 3)(x + 6) 9. (x + 4)(x + 5)
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Answer Key, chapter 8 & appendix

Sheet 8c:　　　　1. $(x - 2)(x + 3)$　3. $(x + 2)(x - 10)$　　　5. $(x + 2)(x - 8)$　7. $(x-4)(x+4)$　　9. $(x - 9)(x + 2)$

Sheet 8d:　　　　1. $(2x + 7)(x - 1)$　　　4. $(x + 1)(4x - 5)$　　　　7. $(3x - 4)(x + 1)$　　10. $(3x + 2)(2x - 3)$

Sheet 8e:　　　　1. $\{1, 6\}$　　　　　　2. $\{2, 1\}$　　　　3. $\{3, 4\}$　　　　4. $\{4, -1\}$　　　5. $\{-1, -4\}$
　　　　　　　　　6. $\{-12, -1\}$　　　　7. $\{-7, 2\}$　　　8. $\{2, 6\}$　　　9. $\{-2, -6\}$　　　10. $\{1.5, -2\}$

Sheet 8f:　　　　$(ax \pm b)^2$; $(ax - b)(ax + b)$　1. $\{\pm 3\}$　　　　2. $\{\pm 4\}$　　　3. $\{\pm 5\}$　　　4. $\{\pm 6\}$
　　　　　　　　　5. $\{0, \pm 2\}$　　　　　　　6. $\{0, \pm 4\}$　　　7. $\{\pm 3\}$　　　8. $\{\pm 3\}$　　　9. $\{\pm 0.6\}$

Sheet 8g:　　　　1. $(5x+3)(5x-3)$　　2. $(2-9x)(2+9x)$　3. $16(c-2)(c+2)$　　4. $3(3-n)(3+n)$　5. $3c(c-3)(c^2 +3c+9)$
　　　　　　　　　6. $x(x-1)(x^2+x+1)$　　　7. $5x^2(x-10)(x^2+10x+100)$　　　9. $ab(a-b)(a^2+ab+b^2)$

Sheet 8h:　　　　1. $\{0.6, -1\}$　　2. $\{1.75, -1\}$　　4. $\{-3\frac{1}{3}, 1\}$　　5. $\{2, 3\}$　　6. $\{\frac{1}{3}, -4\}$　　7. $\{6,-1\}$

Sheet 8i:　　　　1. $\{2\frac{2}{3}, -1\}$　　2. $\{-6, 1\}$　　3. $\{1, -8\}$　　5. $\{3, -1\}$　　6. $\{-3.5, 1\}$　　8. $\{1.5, 1\}$

Sheet 8j:　　　　1. $\{0, -8, -8, -5, 0, 16\}$　　　　2. $\{4, -3, -6, -5, 0, 9\}$　　　3. $\{2, 0, 0, 2, 6, 12\}$

Sheet 8k:　　　　2. Up; y-int $(0,8)$; vertex $(3,-1)$; sym $(6,8)$　　5. Down; y-int $(0,-2)$; vertex $(1.5, 0.25)$; sym $(3,-2)$

Sheet 8L:　　　　1. 27 ft　　　　　2. C = 50.5; Wind speed　　　　3. 365.8 parts per million　　4. 14 mph

Sheet 8m:　　　　1. -6　　　　　2. 0　　　　　3. $8 - 4i$　　　　4. $-13 + 10i$

Sheet 8n:　　　　1. $12 + 5i$　　　2. 13　　　　3. $-3 - 4i$　　　4. 4　　　　　7. $\sqrt{5}$　　　　8. 13

Chapter 8 Test:　2. $-x^2 - x - 4$; $x^2y^2 + 3xy^3$　　　3. $12b$; $-6p^5$　　　　4. $4x^2 + 6x$; $-36 - 24t$
　　　　　　　　　5. $5a(5 + 6a)$; $3c(c^2 - 5)$　　　　6. $(a+7)(a+6)$; $32(x-2)$　7. $a^2 -2a - 3$; $6x^2 - x -12$
　　　　　　　　　8. $2a - 4$; $4(a - 1)$　　　　　9. $\{2, -7\}$; $\{-11\}$　　　10. $\{-1, -1\frac{2}{3}\}$
　　　　　　　　　11. up; vertex $(1.25; -0.125)$, y-int $(0, 3)$; sym $(2.5; -0.125)$　12. 49; two real　　　bonus. 1

Cumulative Review:　　　1. $1.2x^5y^3$　　　2. 3　　3. $P = I/(rt)$　　5. $-5 < x < \frac{1}{3}$　　6. $-2x + 15y = -12$
　　　　　　　　　　　　　7. $y = -7.5x + 1$　　9. $y = 3x - 7$　　10. $4x^2 - 20x + 25$　　11. $\frac{2}{3}$, 1
　　　　　　　　　　　　　13. $0.6 \pm 0.8i$　　　　14. $(4y + 5)/(xy)$

A-1　　　　　　4. $x < 3$　　　5. $x > 1$　　8. $\{3, -1.8\}$　　9. $x^3 + 3x^2y + 3xy^2 + y^3$　10. $\{-4, 5, -1.5\}$

A-2　　　　　　1. $\{4\}$　　　2. 5061/990　　5. Range $\{y|\ y \le 8\}$; 8, 0
　　　　　　　　6. No. There are two values for y for an x.

A-3　　　　　　3. Slope is 3 and y-intercept is 5.　4. $3x - 4y = -27$　　　6. Inconsistent　7. $y = 2x - 5$

A-4　　　　　　1. (1, 1)　　　2. (3, -5)　　　3. (4, 2)　　　4. (2, -3, 1)　　　　5. (-1, 4, 2)

A-5　　　　　　1. $y = 3(x - 2)^2 - 7$　　　3. $y > 5(x + 1)^2 + 7$　　　5. 33; 2 real
　　　　　　　　7. $y = 3x^2 - 3x + 1$　　　8. $\{-1, \frac{2}{3}\}$　　　10. $\{78\}$

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Answer Key, appendix

A-6a   2. -49   6. $216x^3$ 7. $4x^4$ 8. $4.5x^4$ 10. $-1 - 2i$ 12. $-2i$

A-6b   2. log 8/3   3. log 86.4   4. $2\log x + 3\log y - \log z$ 6. $\log x + \log(x+1)$
           7. ln5/ln10   8. ln6/ln2   10. 5/6   11. 2005   12. ⅓   13. 0
           15. 3   16. 3   17. ⅔   18. 1

A-6c   1. 3   2. 2   4. log 144   6. $(\log_3 2, 4)$ 7. 6   8. 3
           10. no   12. $y = \log_3 x$ 14. No   15. no

A-7   1. $36x^2 + 60x + 25$ 2. $x^3 - 3x^2y + 3xy^2 - y^3$ 3. $4x^2 + 16x + 16$ 4. $(x - 9)(x + 3)$
           5. $(3x + 4)(4x + 3)$ 6. $7(a^2 - 2b)(a^2 + 2b0$ 7. $(3x - 2y^3)(3x + 2y^3)$
           9. $y(4x - 3)(1 - 2y)$ 10. -1   13. {-1, 3.5}   14. {3}   15. {0, -0.2}
           18. Removable at x = -2; non-removable at x = -3   19. Removable at x = -1

A-8   1. $x \le 3$ 2. $y \le 4$ 3. -1   4. (3,4)   6. 10   8. -2   9. 1

A-9   2. pairs; conjugates   3. $(x - 2)(x + 2)(x - 1)$ 4. $(x - 1)(5x^2 + 2x - 6)$
           5. $(x^2 + 1)(x + 3)(x - 2)$ 6. Two positive and two negative zeroes

Final Exam: You can always email me if you need my answers. I might use this exam again in the future…

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