Chapter 3: Equations in 2 Variables

<u>Essential Question:</u> Why are graphs helpful?

Lesson 3.1 Constant Rate of Change
Lesson 3.2 Slope
Lesson 3.3 Equations in y=mx Form

* 3.1-3.3 Formative Assessment *

Lesson 3.4 Slope-Intercept Form
Lesson 3.5 Graph a Line Using Intercepts
Lesson 3.6 Write Linear Equations

* 3.4-3.6 Formative Assessment *

Lesson 3.7 Solve Systems of Equations by Graphing
Lesson 3.8 Solve Systems of Equations Algebraically

Lesson 7.3 Elimination by Addition and Subtraction
Lesson 7.4 Elimination by Multiplication

* Possible 3.7-7.4 Formative Assessment *

*** CHAPTER 3 Summative Assessment ***

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Lesson 1 Reteach

Constant Rate of Change

Relationships that have straight-line graphs are called **linear relationships**. The rate of change between any two points in a linear relationship is the same, or constant. A linear relationship has a **constant rate of change.**

Example

The height of a hot air balloon after a few seconds is shown. Determine whether the relationship between the two quantities is linear. If so, find the constant rate of change. If not, explain your reasoning.

	Time (s)	Height of Hot Air Balloon (ft)	
+1	1	9	+9
+1	2	18	+9
+1	3	27	+9
	4	36	

As the number of seconds increase by 1, the height of the balloon increases by 9 feet.

Since the rate of change is constant, this is a linear relationship. The constant rate of change is $\frac{9}{1}$ or 9 feet per second. This means that the balloon is rising 9 feet per second.

Exercises

Determine whether the relationship between the two quantities described in each table is linear. If so, find the constant rate of change. If not, explain your reasoning.

1.

Greeting Cards	
Number of Cards	Total Cost($)
1	1.50
2	3.00
3	4.50
4	6.00

2.

Party Table Rental	
Number of Tables	Cost($)
1	10
2	18
3	24
4	28

3.

Donuts	
Dozens Bought	Cost ($)
2	3.25
4	6.50
6	9.75
8	13.00

4.

Running	
Time (min)	Distance(mi)
15	2
30	4
45	5
60	6

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Lesson 2 Reteach

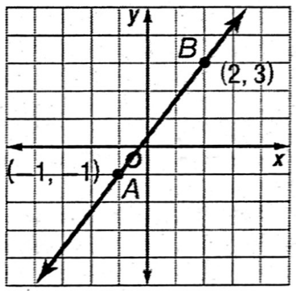
Slope

The slope m of a line passing through points (x_1, y_1) and (x_2, y_2) is the ratio of the difference in the y-coordinates to the corresponding difference in the x-coordinates. As an equation, the slope is given by

$m = \frac{y_2-y_1}{x_2-x_1}$, where $x_1 \neq y_1$

Example 1

Find the slope of the line that passes through *A*(–1, –1) and *B*(2, 3).



$m = \frac{y_2-y_1}{x_2-x_1}$ Slope formula

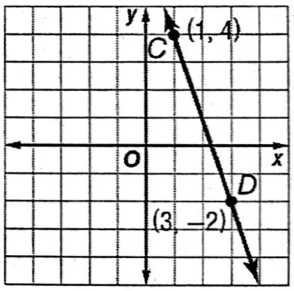
$m = \frac{3-(-1)}{2-(-1)}$ $(x_1, y_1) = (-1, -1)$, $(x_2, y_2) = (2, 3)$

$m = \frac{4}{3}$ Simplify.

Check When going from left to right, the graph of the line slants upward. This is correct for a positive slope.

Example 2

Find the slope of the line that passes through *C*(1, 4) and *D*(3, –2).



$m = \frac{y_2-y_1}{x_2-x_1}$ Slope formula

$m = \frac{-2-4}{3-1}$ $(x_1, y_1) = (-1, 4)$, $(x_2, y_2) = (3, -2)$

$m = \frac{-6}{2}$ or -3 Simplify.

Check When going from left to right, the graph of the line slants downward. This is correct for a negative slope.

Exercises

Find the slope of the line that passes through each pair of points.

1. *A*(0, 1), *B*(3, 4)

2. *C*(1, –2), *D*(3, 2)

3. *E*(4, –4), *F*(2, 2)

4. *G*(3, 1), *H*(6, 3)

5. *I*(4, 3), *J*(2, 4)

6. *K*(–4, 4), *L*(5, 4)

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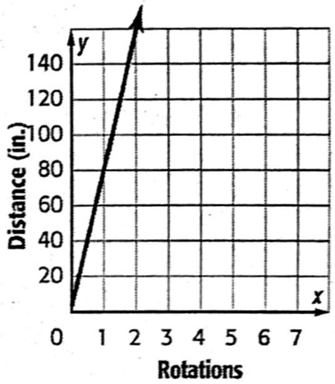
Lesson 3 Reteach

Equations in $y = mx$ Form

When the ratio of two variable quantities is constant, their relationship is called a **direct variation.**

Example 1

The distance that a bicycle travels varies directly with the number of rotations that its tires make. Determine the distance that the bicycle travels for each rotation.



Since the graph of the data forms a line, the rate of change is constant. Use the graph to find the constant ratio.

$$\frac{\text{distance traveled}}{\text{\# of rotations}} \longrightarrow \frac{80}{1} \quad \frac{160}{2} \text{ or } \frac{80}{1} \quad \frac{240}{3} \text{ or } \frac{80}{1} \quad \frac{320}{4} \text{ or } \frac{80}{1}$$

The bicycle travels 80 inches for each rotation of the tires.

Example 2

The number of trading cards varies directly as the number of packages. If there are 84 cards in 7 packages, how many cards are in 12 packages?

Let x = the number of packages and y = the total number of cards.

$y = mx$	Direct variation equation
$84 = m(7)$	$y = 84$, $x = 7$
$12 = m$	Simplify.
$y = 12x$	Substitute for $m = 12$.

Use the equation to find y when $x = 12$.

$y = 12x$	
$y = 12(12)$	$x = 12$
$y = 144$	Multiply.

There are 144 cards in 12 packages.

Exercises

Write an equation and solve the given situation.

1. **TICKETS** Four friends bought movie tickets for \$41. The next day seven friends bought movie tickets for \$71.75. What is the price of one ticket?

2. **JOBS** Barney earns \$24.75 in three hours. If the amount that he earns varies directly with the number of hours, how much would he earn in 20 hours?

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Lesson 4 Reteach

Slope-Intercept Form

Linear equations are often written in the form $y = mx + b$. This is called the **slope-intercept form**. When an equation is written in this form, m is the slope and b is the y-intercept.

Example 1

State the slope and the y-intercept of the graph of $y = x - 3$.

$y = x - 3$ Write the original equation.

$y = \overset{\uparrow}{1}x + \overset{\uparrow}{(-3)}$ Write the equation in the form $y = mx + b$.

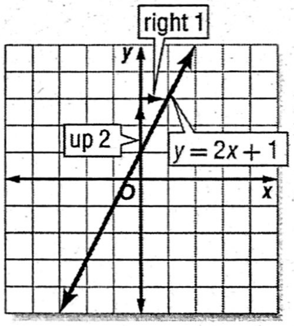
$y = mx + b$ $m = 1$, $b = -3$

The slope of the graph is 1, and the y-intercept is –3.

You can use the slope intercept form of an equation to graph the equation.

Example 2

Graph $y = 2x + 1$ using the slope and y-intercept.



Step 1 Find the slope and y-intercept.
$y = 2x + 1$ slope = 2, y-intercept = 1

Step 2 Graph the y-intercept 1.

Step 3 Write the slope 2 as $\frac{2}{1}$. Use it to locate a second point on the line.

$m = \frac{2}{1}$ ← change in y: up 2 units
← change in x: right 1 unit

Step 4 Draw a line through the two points.

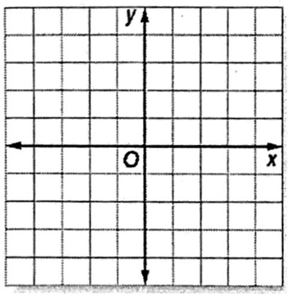
Exercises

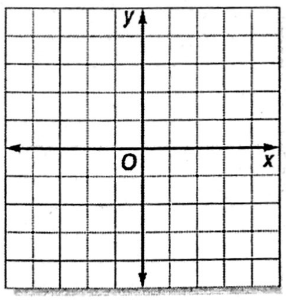
State the slope and the y-intercept for the graph of each equation.

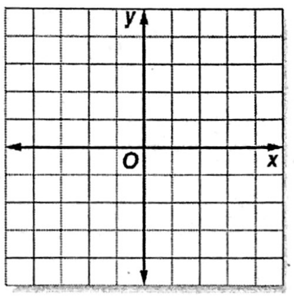
1. $y = x + 1$ **2.** $y = 2x - 4$ **3.** $y = \frac{1}{2}x - 1$

Graph each equation using the slope and the y-intercept.

4. $y = 2x + 2$ **5.** $y = x - 1$ **6.** $y = \frac{1}{2}x + 2$







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Lesson 5 Reteach

Graph a Line Using Intercepts

Standard form is when an equation is written in the form $Ax + By = C$.

Example

State the x- and y-intercepts of $3x + 2y = 6$. Then graph the function.

Step 1 Find the x-intercept.

To find the x-intercept, let $y = 0$.

$3x + 2y = 6$	Write the equation.
$3x + 2(0) = 6$	Replace y with 0.
$3x + 0 = 6$	Multiply.
$3x = 6$	Simplify.
$x = 2$	Divide each side by 3.

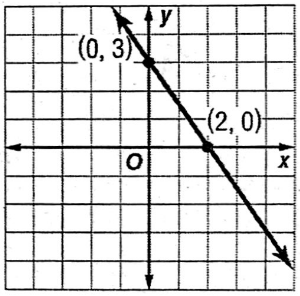
The x-intercept is 2.

Step 2 Find the y-intercept.

To find the y-intercept, let $x = 0$.

$3x + 2y = 6$	Write the equation.
$3(0) + 2y = 6$	Replace x with 0.
$0 + 2y = 6$	Multiply.
$2y = 6$	Simplify.
$y = 3$	Divide each side by 2.

The y-intercept is 3.

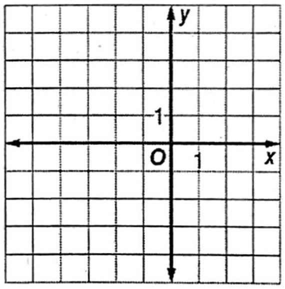


Step 3 Graph the points (2, 0) and (0, 3) on a coordinate plane. Then connect the points.

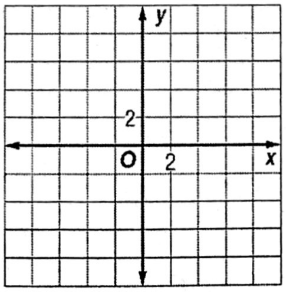
Exercises

State the x- and y-intercepts of each function. Then graph the function.

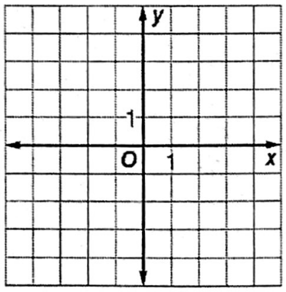
1. $3x + 5y = -15$



2. $-2x + y = 8$



3. $-4x - 3y = -12$



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Lesson 6 Reteach

Write Linear Equations

Point-slope form is when an equation is written in the form $y - y_1 = m(x - x_1)$, where (x_1, y_1) is a given point on a nonvertical line and m is the slope of the line.

Example

Write an equation in point-slope form and slope-intercept form for a line that passes through (2, –5) and has a slope of 4.

Step 1	$y - y_1 = m(x - x_1)$	Point-slope form
	$y - (-5) = 4(x - 2)$	$(x_1, y_1) = (2, -5)$, $m = 4$
	$y + 5 = 4(x - 2)$	Simplify.
Step 2	$y + 5 = 4(x - 2)$	Write the equation.
	$y + 5 = 4x - 8$	Distributive Property
	$\underline{\quad -5 = \quad -5}$	Addition Property of Equality
	$y = 4x - 13$	Simplify.

Check: Substitute the coordinates of the given point in the equation.

$$y = 4x - 13$$
$$-5 \stackrel{?}{=} 4(2) - 13$$
$$-5 = -5 \checkmark$$

Exercises

Write an equation in point-slope form and slope-intercept form for each line.

1. passes through (–4,0), slope = 2

2. passes through (–2, –1), slope = $\frac{1}{2}$

3. passes through (3, –6), slope = 2 – 3

4. passes through (–4, –3), slope = –2

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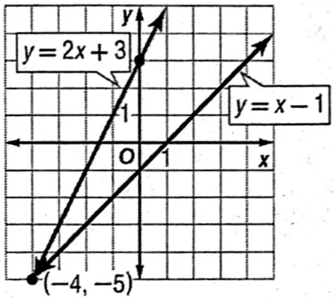
Lesson 7 Reteach

Solve Systems of Equations by Graphing

Example

Solve the system $y = 2x + 3$ and $y = x - 1$ by graphing.

Graph each equation on the same coordinate plane.



The graphs appear to intersect at (–4, –5).

Check this estimate by replacing x with –4 and y with –5.

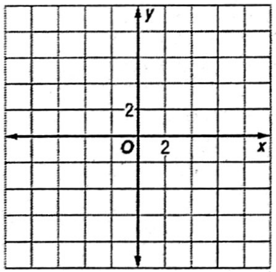
Check	$y = 2x + 3$	$y = x - 1$
	$-5 \stackrel{?}{=} 2(-4) + 3$	$-5 \stackrel{?}{=} -4 - 1$
	$-5 = -5$ ✓	$-5 = -5$ ✓

The solution of the system is (–4, –5).

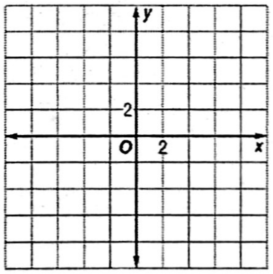
Exercises

Solve each system of equations by graphing.

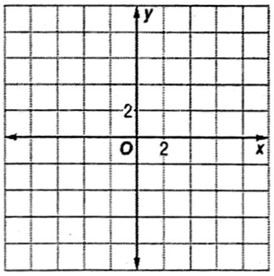
1. $y = 2x + 5$
$y = -x + 8$



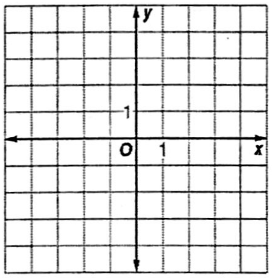
2. $y = -x - 3y$
$y = x + 1$



3. $y = -3x + 9$
$y = -3x + 3$



4. $y = -2x + 4$
$y = -x + 3$



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Lesson 8 Reteach

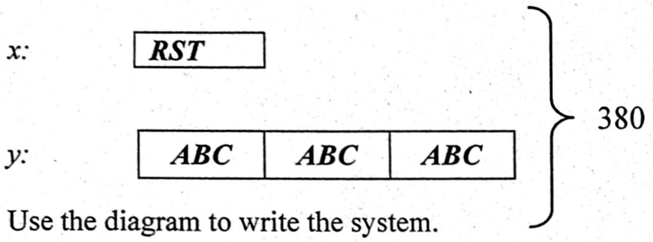
Solve Systems of Equations Algebraically

Example

You own three times as many shares of *ABC* stock as you do of *RST* stock. Altogether you have 380 shares of stock.

a. Write a system of equations to represent this situation.

Draw a bar diagram.



Use the diagram to write the system.

$y = 3x$ There are 3 times as many shares *ABC* stocks as *RST* stocks.

$x + y = 380$ The total number of stocks owned is 380.

b. Solve the system algebraically. Interpret the solution.

Since y is equal to $3x$, you can replace y with $3x$ in the second equation.

$x + y = 380$ Write the equation.

$x + 3x = 380$ Replace y with $3x$.

$4x = 380$ Simplify.

$\frac{4x}{4} = \frac{380}{4}$ Division Property of Equality

$x = 95$ Simplify.

Since $x = 95$ and $y = 3x$, then $y = 285$ when $x = 95$. The solution of this system of equations is (95, 285). This means that you own 95 shares of *RST* stock and 285 shares of *ABC* stock.

Exercises

Solve each system of equations algebraically.

1. $y = x + 3$
$y = 4x$

2. $y = -x - 2$
$y = -2x$

3. $y = x + 14$
$y = 8x$

4. $y = x - 6$
$y = 2x$

5. $y = -x + 8$
$y = 3x$

6. $y = -x$
$y = -2x$