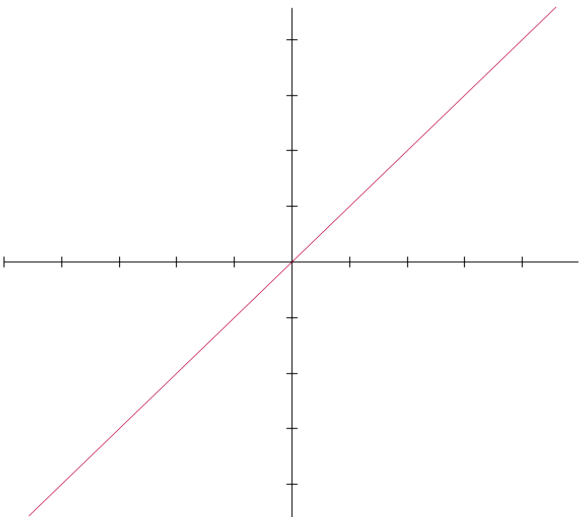
Absolute Value Functions

In order to graph an absolute value function, you will be using many of the same methods you did for quadratics. The standard form of an absolute value function is nearly identical to that of a quadratic function.

$$f_{(x)} = a|x-h|+k$$

The standard graph by which we translate absolute value functions comes from the equation of the diagonal line $y = x$.

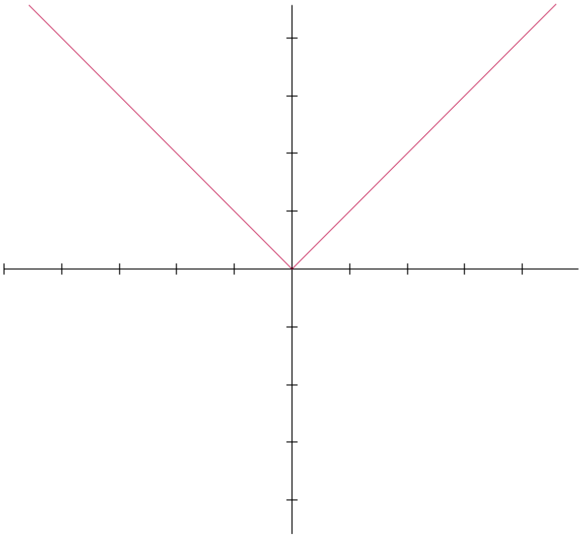
$y = x$



This is the graph of the function $y = x$. In this case, the x and y values of coordinates are identical. For example, (-3,-3). You can see the x and y values are the same.

Now, lets take a look at what happens when I want the absolute value of x.

$y = |x|$

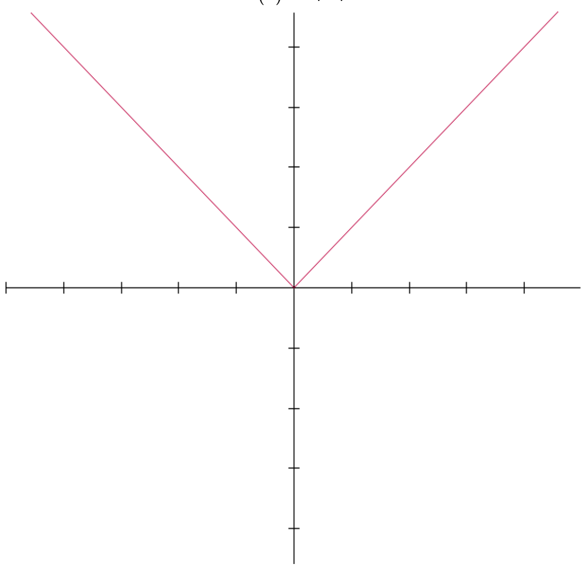


If the graph of $y = x$ above is $f_{(x)}$, the function to the left is $|f_{(x)}|$. We know that the absolute value of a number cannot be negative. If we take the absolute value of $f_{(x)}$, it would cause the left portion of the graph above to reflect above the x axis. Now as you can see, all y values of the function are positive. This is where the graph of the absolute value of x comes from.

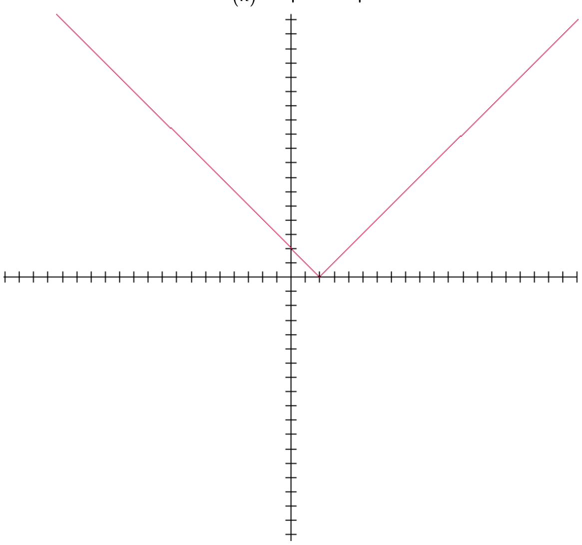
As we look at the following absolute value functions, you will notice how similar they are to quadratic functions. The vertex of an absolute value function is also given by (h,k). Horizontal and vertical shifts are identical, as well as the effect the value of a has on the graph. The rules for finding the range and domain of an absolute value function are also the same as a quadratic. Sometimes, an axis of symmetry must be used to graph your function. Intercepts are found by substituting zero for either x or y, and solving for the remaining variable.

$$f_{(x)} = a|x-h|+k$$

$$f_{(x)} = |x|$$

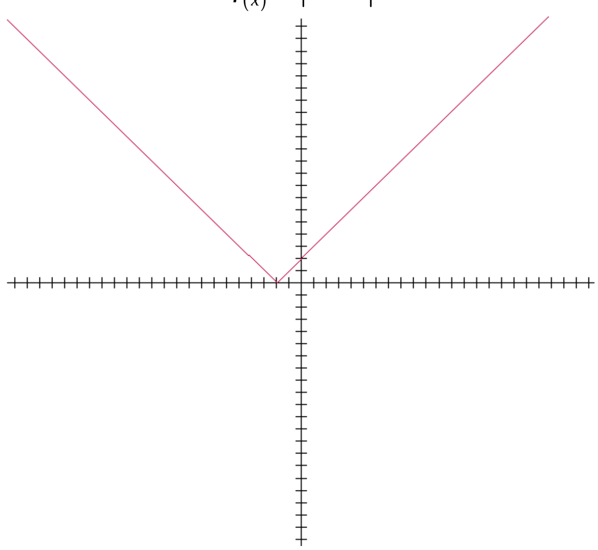


$$f_{(x)} = |x-2|$$



The graph of this function shifts to the right 2.

$$f_{(x)} = |x+2|$$

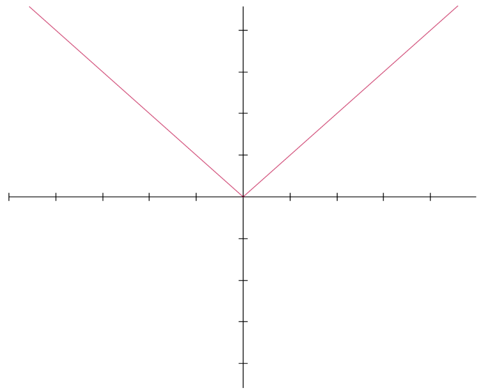


The graph of this function shifts to the left 2.

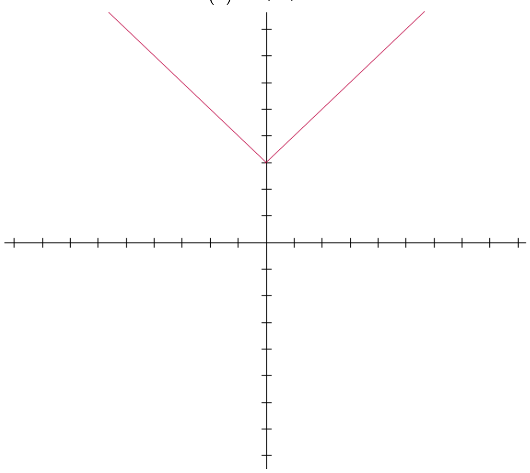
Once again, notice that the value of h determines the horizontal shift of the function. If the function is defined as $f_{(x)}$, the graph on the left is $f_{(x-2)}$, while the graph on the right is $f_{(x+2)}$.

$$f_{(x)} = a|x-h|+k$$

$$f_{(x)} = |x|$$

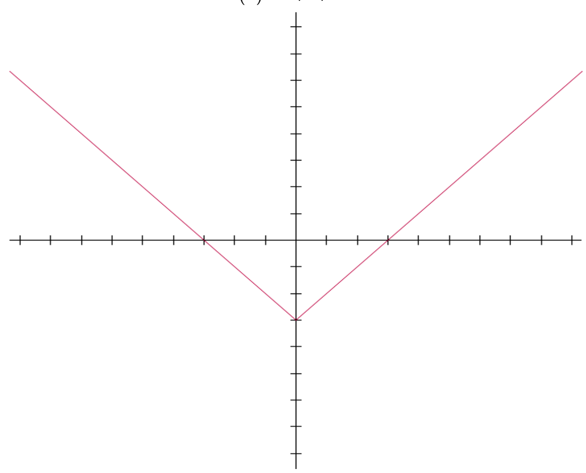


$$f_{(x)} = |x|+3$$



The graph of this function shifts up 3.

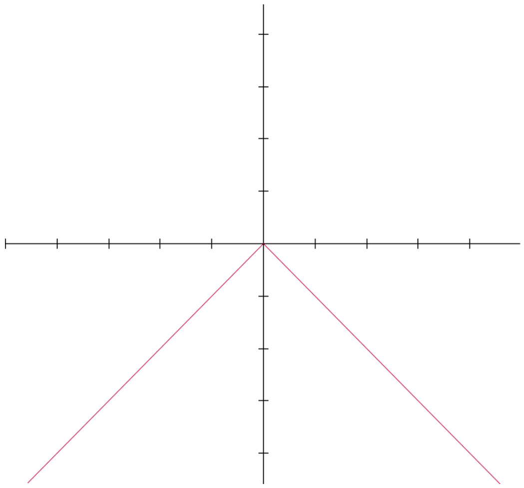
$$f_{(x)} = |x|-3$$



The graph of this function shifts down 3.

The value of k, for an absolute value function in standard form determines the vertical shift of the function. As before, if the function is simply defined as $f_{(x)}$, we are looking at $f_{(x)}+3$ and $f_{(x)}-3$ respectively.

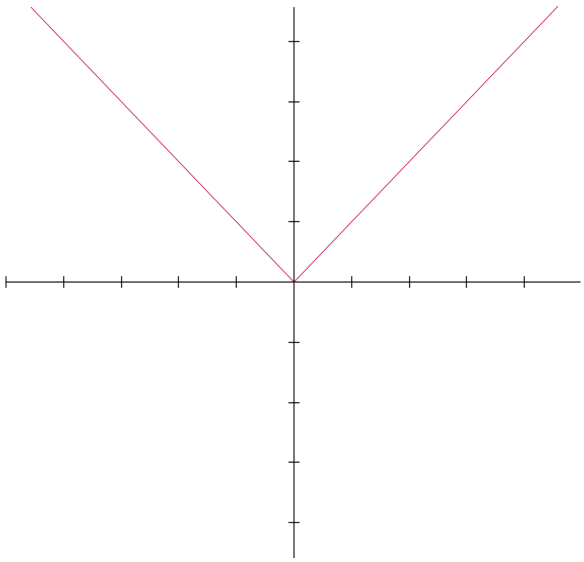
$$f_{(x)} = -|x|$$



On the left we have the opposite of the parent function. In this example, the value of a, in the standard form is -1. A negative reflects the graph of the function about the horizontal axis. This is read as the opposite of the absolute value of x. If the parent function given is referred to as $f_{(x)}$, this function is $-f_{(x)}$.

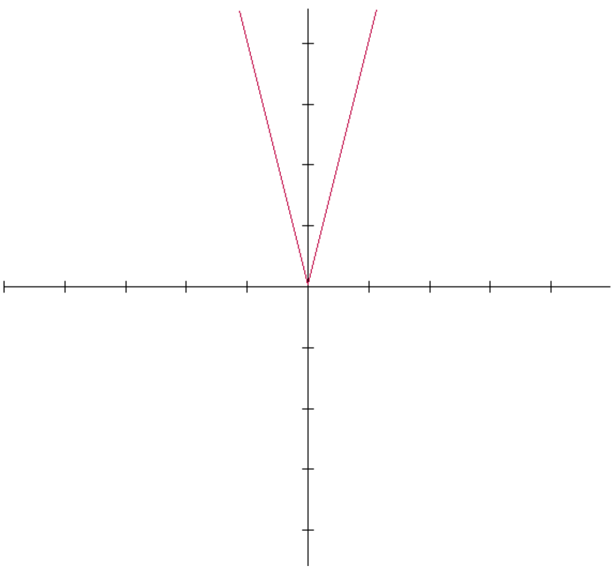
$$f_{(x)} = a|x-h|+k$$

$$f_{(x)} = |x|$$

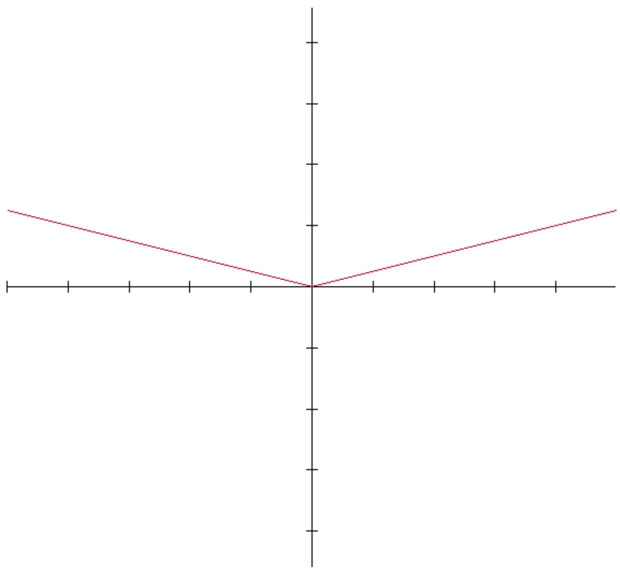


Here we will see how the value of a in an absolute value function in standard form affects the graph of the function. To illustrate this, we will look at the following graphs that have their vertices on the origin.

$$f_{(x)} = 4|x|$$



$$f_{(x)} = \frac{1}{4}|x|$$



This graph seems very narrow, but what is actually happening, is the value of the function is increasing very rapidly. The y values are increasing at 4 times their normal rate. The rapid increase causes the graph to appear narrow.

This graph is wider than the parent function. In this case, the y values of the function are increasing at ¼ their normal rate, causing a more gradual increase.

If the value of the leading coefficient is a whole number, the y values of the graph will increase rapidly causing a narrow graph and more extreme slope. If the leading coefficient is a fraction, the y values of the function will increase mildly, yielding a more gradual slope.

Describe the movement of each of the following absolute value functions. Describe how the graph of the function opens and if there is any horizontal or vertical movement. Be sure to tell identify how many spaces it moves, for example: ***This graph opens up, and shifts left 6, up 3.***

A) $f_{(x)} = 3|x-4|+1$ **B)** $f_{(x)} = -|x+1|+6$ **C)** $f_{(x)} = |x|+4$

D) $f_{(x)} = |x+5|-2$ **E)** $f_{(x)} = |x-3|$ **F)** $f_{(x)} = -3|x+1|+3$

G) $f_{(x)} = \frac{1}{2}|x-3|+2$ **H)** $f_{(x)} = -|x+6|$ **I)** $f_{(x)} = \frac{2}{3}|x|-4$

State the range and domain for each of the following.

A) $f_{(x)} = 3|x-4|+1$ **B)** $f_{(x)} = -|x+1|+6$ **C)** $f_{(x)} = |x|+4$

D) $f_{(x)} = |x+5|-2$ **E)** $f_{(x)} = |x-3|$ **F)** $f_{(x)} = -3|x+1|+3$

G) $f_{(x)} = \frac{1}{2}|x-3|+2$ **H)** $f_{(x)} = -|x+6|$ **I)** $f_{(x)} = \frac{2}{3}|x|-4$

Find the vertex of each of the following absolute value functions.

A) $f_{(x)} = -|x+3|$ **B)** $f_{(x)} = 3|x-2|-4$ **C)** $f_{(x)} = \frac{1}{2}|x|-2$

D) $f_{(x)} = |x-2|+3$ **E)** $f_{(x)} = 2|x+6|+9$ **F)** $f_{(x)} = -|x-4|+7$

Solve each of the following absolute value equations. ***This is what you will need to do to find the x intercepts of absolute value functions. Remember, first isolate the absolute value, then set up two separate equations to find your solutions.***

$$|x+3|-6=0$$
$$|x+3| \quad = 6$$

Set equation equal to 6 and solve by subtracting 3 to both sides.

$$x+3=6$$
$$x=3$$

and

$$x+3=-6$$
$$x=-9$$

Set equation equal to -6 and solve by subtracting 3 to both sides.

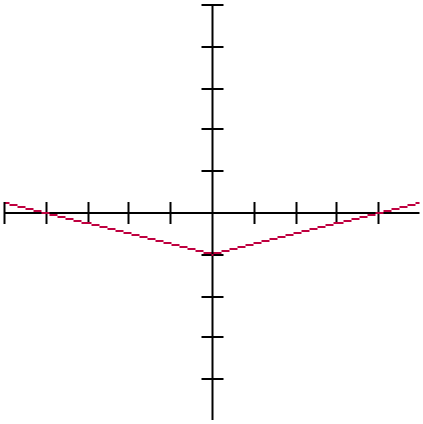
So the two solutions are 3 and -9. These would be the x intercepts of the graph of the function $y = |x+3|-6$ **.**
Watch for abnormalities. Iif the absolute value equals a negative number, you cannot create two problems. If this happens, there will be no solutions to the problem, which in terms of the graph of the function, tells you that there are no x intercepts.

A) $|x-4|-2=0$ **B)** $2|x+4|-12=0$ **C)** $-\frac{1}{2}|x|+1=0$

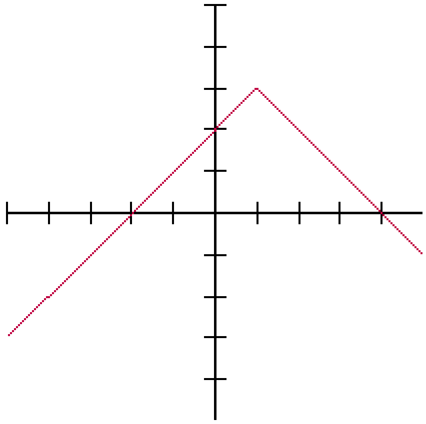
D) $-2|x-3|=0$ **E)** $|x-6|-5=0$ **F)** $-3|x-1|+2=0$

Match the appropriate graph with its equation below. Explain why each of your solutions is true.

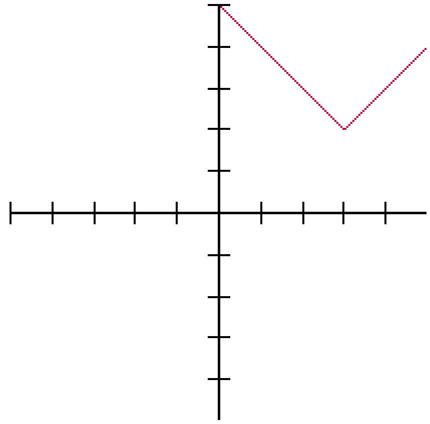
A



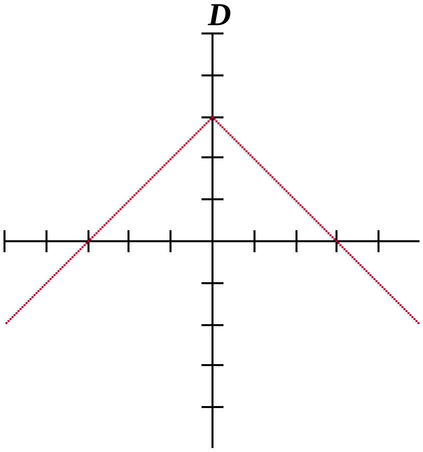
B



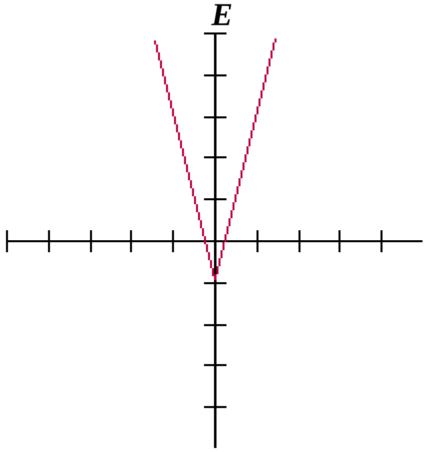
C



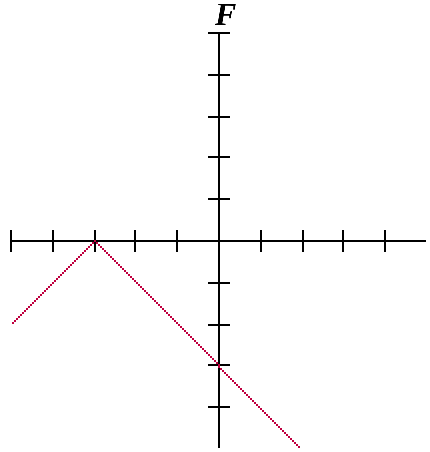
D



E



F



1) $f_{(x)} = 4|x| - 1$

2) $f_{(x)} = \frac{1}{4}|x| - 1$

3) $f_{(x)} = -|x| + 3$

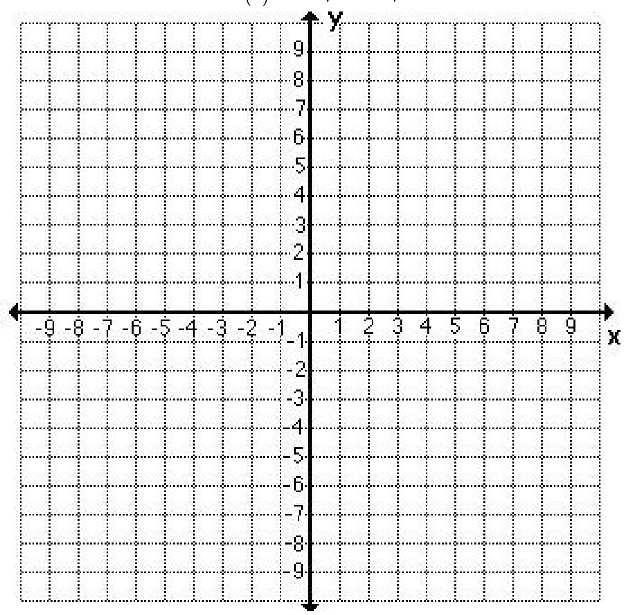
4) $f_{(x)} = |x - 3| + 2$

5) $f_{(x)} = -|x - 1| + 3$

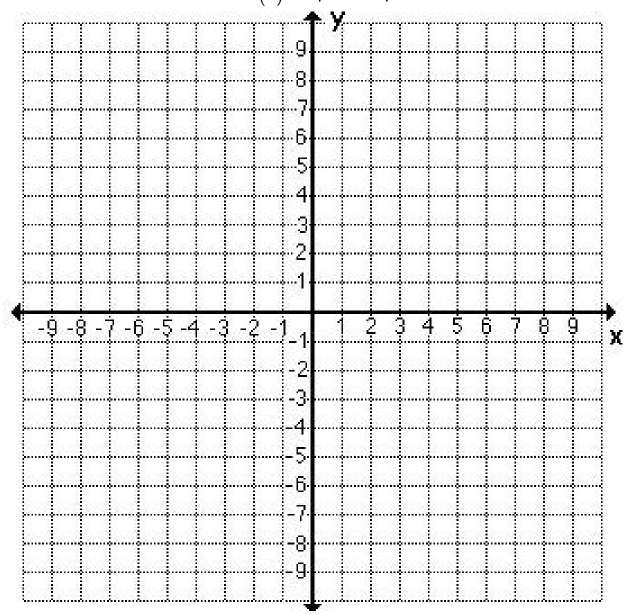
6) $f_{(x)} = -|x + 3|$

Graph each of the following functions. You may need to use an axis of symmetry to graph some of these. Label the vertex, y-intercept, and all x-intercepts. Remember, to find the x intercepts of an absolute value function you will need to set the function equal to zero and solve an absolute value equation.

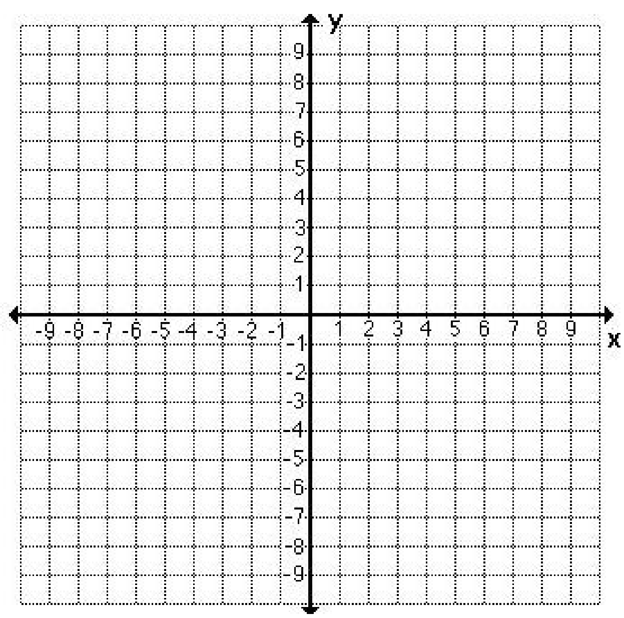
A) $f_{(x)} = -|x-2|+3$



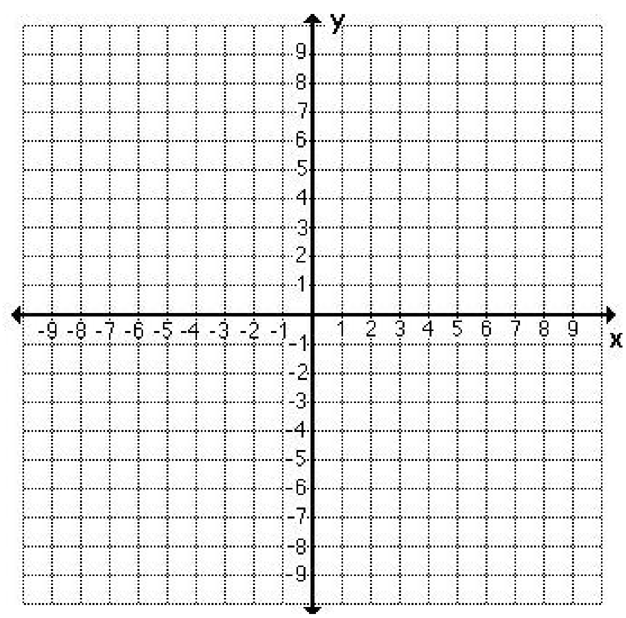
B) $f_{(x)} = |x+4|-4$



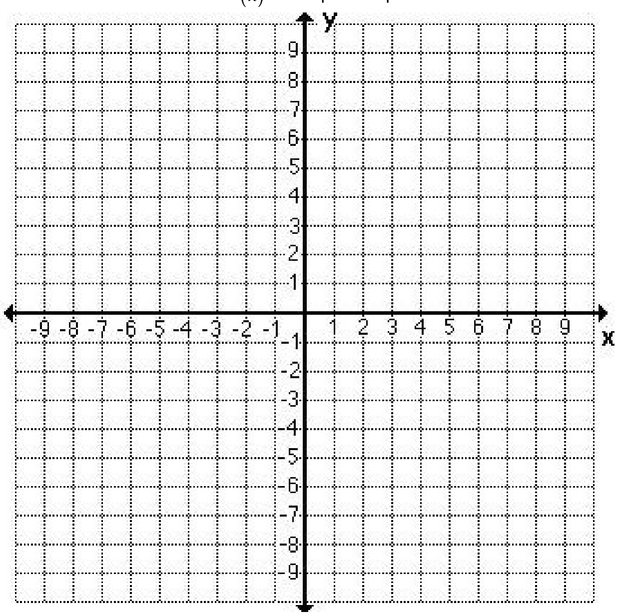
C) $f_{(x)} = -|x-5|$



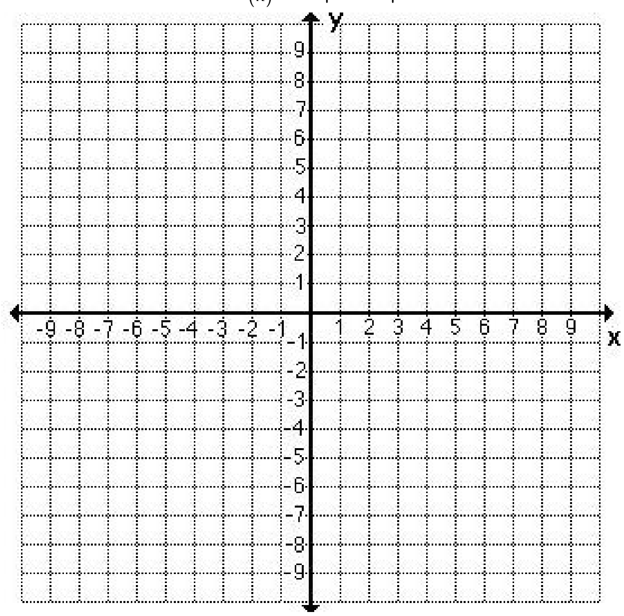
D) $f_{(x)} = \frac{1}{2}|x|-3$



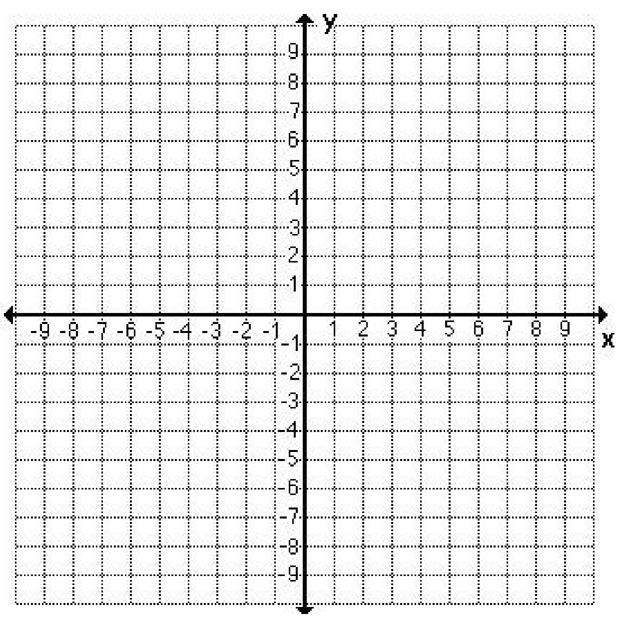
E) $f_{(x)} = -|x-3| - 2$



F) $f_{(x)} = 3|x+5| - 6$



G) $f_{(x)} = |x-3| + 2$



H) $f_{(x)} = -2|x| + 4$