

SIMULTANEOUS EQUATIONS

Solving by substitution

When we solve a pair of simultaneous equations what we are actually looking for is the intersection of two straight lines because it is this point that satisfies both equations at the same time.

So how can we handle the two equations algebraically so that we do not have to draw graphs? Let us use two equations.

Example 1: $2x - y = 3$ (equation 1)
 $3x + 2y = 8$ (equation 2)

By rearranging this equation we get: $y = 2x - 3$ (equation 3)

Now when substitute this expression for y we get: $3x + 2y = 8$ (equation 4)

$$\begin{aligned} 3x + 2(2x - 3) &= 8 \\ 3x + 4x - 6 &= 8 \\ 7x - 6 &= 8 \\ 7x &= 14 \\ x &= 2 \end{aligned}$$

This can be checked by substituting back into both original equations to ensure that the left-hand and right-hand (LHS) and (RHS) sides are equal for these values of x and y .

Finally, using equation 3 we substitute our x from equation 2 to get: $y = 2x(2) - 3 = 1$

So $x = 2, y = 1$ is the solution to the pair of simultaneous equations.

So, with $x = 2, y = 1$, the LHS of equation 1 is $2(2) - 1 = 3$, which is the same as the RHS.

With $x = 2, y = 1$, the LHS of equation 2 is $3(2) + 2(1) = 8$, which is the same as RHS.

Example 2: $7x + 2y = 47$ (equation 1)
 $5x - 4y = 1$ (equation 2)

In this example we need to choose one of these two equations and rearrange it to obtain an expression for y (or x).

The choice is entirely ours and we have to make the choice based upon what we feel will be the simplest. Looking at a pair of equations like this, it is often difficult to know which is the simplest.

Let's choose equation 2 and rearrange it to find the expression for x .

$$\begin{aligned} 5x - 4y &= 1 \\ 5x &= 1 + 4y && \text{by adding } 4y \text{ to each side} \\ x &= \frac{1 + 4y}{5} && \text{by dividing both sides by } 5 \end{aligned}$$

We now use this expression for x and substitute it in Equation (1). $7\left(\frac{1 + 4y}{5}\right) + 2y = 47$

Now multiply throughout by 5.

Why? Because we want to get rid of the fraction and the way to do that is to multiply everything by 5:
 $7(1 + 4y) + 10y = 235$

Now we need to multiply out the brackets: $7 + 28y + 10y = 235$

Gather the y 's and subtract 7 from each side to get: $38y = 228$

$$\begin{aligned} \text{So } y &= \frac{228}{38} \\ &= 6 \end{aligned}$$

So we have established that $y = 6$. Having done this we can substitute it back into the equation that we first had for x .

$$x = \frac{1 + 4y}{5} = \frac{1 + 24}{5}$$

and so $x = 5$

So again, we have our pair of values - our solution to the pair of simultaneous equations. In order to check that our solution is correct these values should be substituted into both equations to ensure they balance.

So, with $x = 5$, $y = 6$, the left-hand side of Equation (1) is $7(5) + 2(6) = 47$,

which is the same as the right-hand side. With $x = 5$, $y = 6$, the left-hand side of Equation (2) is $5(5) - 4(6) = 1$, which is the same as the right-hand side.

Exercises:

1. Solve the following pairs of simultaneous equations:

- a) $y = 2x + 3$
 $y = 5x - 3$
- b) $y = 3x - 1$
 $2x + 4y = 10$
- c) $6x + y = 4$
 $5x + 2y = 1$
- d) $x - 3y = 1$
 $2x + 5y = 35$

2. Solve the following pairs of simultaneous equations:

- a) $5x + 3y = 9$
 $2x - 3y = 12$
- b) $2x - 3y = 9$
 $2x + y = 13$
- c) $x + 7y = 10$
 $3x - 2y = 7$
- d) $5x + y = 10$
 $7x - 3y = 14$
- e) $\frac{1}{3}x + y = \frac{10}{3}$
 $2x + \frac{1}{4}y = \frac{11}{4}$
- f) $3x - 2y = \frac{5}{2}$
 $\frac{1}{3}x + 3y = -\frac{4}{3}$

ANSWERS 1:
a) $x = 3, y = -2$ b) $x = 6, y = 1$ c) $x = 3, y = 1$
d) $x = 2, y = 0$ e) $x = 1, y = 3$ f) $x = 1/2, y = -1/2$

ANSWERS 2:
a) $x = 2, y = 7$
b) $x = 1, y = 2$
c) $x = 1, y = -2$
d) $x = 10, y = 3$

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