



**Mental Math
Yearly Plan
Grade 7**

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Introduction

Definitions

It is important to clarify the definitions used around mental math. Mental math in Nova Scotia refers to the entire program of mental math and estimation across all strands. It is important to incorporate some aspect of mental math into your mathematics planning everyday, although the time spent each day may vary. While the *Time to Learn* document requires 5 minutes per day, there will be days, especially when introducing strategies, when more time will be needed. Other times, such as when reinforcing a strategy, it may not take 5 minutes to do the practice exercises and discuss the strategies and answers.

For the purpose of this booklet, fact learning will refer to the acquisition of the 100 number facts relating the single digits 0 to 9 for each of the four operations. When students know these facts, they can quickly retrieve them from memory (usually in 3 seconds or less). Ideally, through practice over time, students will achieve automaticity; that is, they will abandon the use of strategies and give instant recall. Computational estimation refers to using strategies to get approximate answers by doing calculations in one's head, while mental calculations refer to using strategies to get exact answers by doing all the calculations in one's head.

While we have defined each term separately, this does not suggest that the three terms are totally separable. Initially, students develop and use strategies to get quick recall of the facts. These strategies and the facts themselves are the foundations for the development of other mental calculation strategies. When the facts are automatic, students are no longer employing strategies to retrieve them from memory. In turn, the facts and mental calculation strategies are the foundations for estimation. Attempts at computational estimation are often thwarted by the lack of knowledge of the related facts and mental calculation strategies.

Rationale

In modern society, the development of mental computation skills needs to be a major goal of any mathematical program for two major reasons. First of all, in their day-to-day activities, most people's calculation needs can be met by having well developed mental computational processes. Secondly, while technology has replaced paper-and-pencil as the major tool for complex computations, people need to have well developed mental strategies to be alert to the reasonableness of answers generated by technology.

Besides being the foundation of the development of number and operation sense, fact learning itself is critical to the overall development of mathematics. Mathematics is about patterns and relationships and many of these patterns and relationships are numerical. Without a command of the basic relationships among numbers (facts), it is very difficult to detect these patterns and relationships. As well, nothing empowers students with confidence and flexibility of thinking more than a command of the number facts.

It is important to establish a rationale for mental math. While it is true that many computations that require exact answers are now done on calculators, it is important that students have the necessary skills to judge the reasonableness of those answers. This is also true for computations students will do using pencil-and-paper strategies. Furthermore, many computations in their daily lives will not require exact answers. (e.g., If three pens each cost \$1.90, can I buy them if I have \$5.00?) Students will also encounter computations in their daily lives for which they can get exact answers quickly in their heads. (e.g., What is the cost of three pens that each cost \$3.00?)

The Implementation of Mental Computational Strategies

General Approach

In general, a strategy should be introduced in isolation from other strategies, a variety of different reinforcement activities should be provided until it is mastered, the strategy should be assessed in a variety of ways, and then it should be combined with other previously learned strategies.

Introducing a Strategy

The approach to highlighting a mental computational strategy is to give the students an example of a computation for which the strategy would be useful to see if any of the students already can apply the strategy. If so, the student(s) can explain the strategy to the class with your help. If not, you could share the strategy yourself. The explanation of a strategy should include anything that will help students see the pattern and logic of the strategy, be that concrete materials, visuals, and/or contexts. The introduction should also include explicit modeling of the mental processes used to carry out the strategy, and explicit discussion of the situations for which the strategy is most appropriate and efficient. The logic of the strategy should be well understood before it is reinforced. (Often it would also be appropriate to show when the strategy would not be appropriate as well as when it would be appropriate.)

Reinforcement

Each strategy for building mental computational skills should be practised in isolation until students can give correct solutions in a reasonable time frame. Students must understand the logic of the strategy, recognize when it is appropriate, and explain the strategy. The amount of time spent on each strategy should be determined by the students' abilities and previous experiences.

The reinforcement activities for a strategy should be varied in type and should focus as much on the discussion of how students obtained their answers as on the answers themselves. The reinforcement activities should be structured to insure maximum participation. Time frames should be generous at first and be narrowed as students internalize the strategy. Student participation should be monitored and their progress assessed in a variety of ways to help determine how long should be spent on a strategy.

After you are confident that most of the students have internalized the strategy, you need to help them integrate it with other strategies they have developed. You can do this by providing activities that includes a mix of number expressions, for which this strategy and others would apply. You should have the students complete the activities and discuss the strategy/strategies that could be used; or you should have students match the number expressions included in the activity to a list of strategies, and discuss the attributes of the number expressions that prompted them to make the matches.

Assessment

Your assessments of mental math and estimation strategies should take a variety of forms. In addition to the traditional quizzes that involve students recording answers to questions that you give one-at-a-time in a certain time frame, you should also record any observations you make during the reinforcements, ask the students for oral responses and explanations, and have them explain strategies in writing. Individual interviews can provide you with many insights into a student's thinking, especially in situations where pencil-and-paper responses are weak.

Assessments, regardless of their form, should shed light on students' abilities to compute efficiently and accurately, to select appropriate strategies, and to explain their thinking.

Response Time

Response time is an effective way for teachers to see if students can use the mental math and estimation strategies efficiently and to determine if students have automaticity of their facts.

For the facts, your goal is to get a response in 3-seconds or less. You would give students more time than this in the initial strategy reinforcement activities, and reduce the time as the students become more proficient applying the strategy until the 3-second goal is reached. In subsequent grades when the facts are extended to 10s, 100s and 1000s, a 3-second response should also be the expectation.

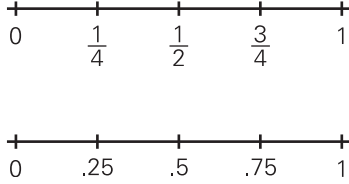
In early grades, the 3-second response goal is a guideline for the teacher and does not need to be shared with the students if it will cause undue anxiety.

With other mental computational strategies, you should allow 5 to 10 seconds, depending upon the complexity of the mental activity required. Again, in the initial application of the strategies, you would allow as much time as needed to insure success, and gradually decrease the wait time until students attain solutions in a reasonable time frame.


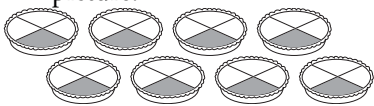
Mental Math: Grade 7 Yearly Plan

In this yearly plan for mental math in grade 7, an attempt has been made to align specific activities with the topic in grade 7. In some areas, the mental math content is too broad to be covered in the time frame allotted for a single chapter. While it is desirable to match this content to the unit being taught, it is quite acceptable to complete some mental math topics when doing subsequent chapters that do not have obvious mental math connections. For example practice with integer operations could continue into the data management and geometry chapters. Integers are so important in grade 7 that they should be interjected into the mental math component over the entire year once they have been taught.

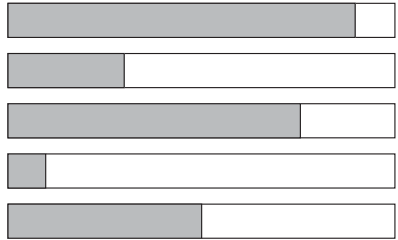
	Skill	Example
Number Sense	<p>Review multiplication and division facts through</p> <p>a) rearrangement, or commutative property/decomposition</p> <p>b) multiplying by multiples of 10</p> <p>c) multiplication strategies such as doubles, double/double, double plus one, halve/double etc.</p> <p><i>(Intent is to practice facts through previously learned strategies)</i></p>	<p>a) $8 \times 7 \times 5 = 8 \times 5 \times 7$ (rearrangement or commutative property) $16 \times 25 = 4 \times 4 \times 25 = 4 \times 100$</p> <p>b) $70 \times 80 = 7 \times 8 \times 10 \times 10$ $4200 \div 6 = (7 \times 600 \div 6)$</p> <p>c) $12.5 \times 4 = 12.5 \times 2 \times 2$ (Double 12.5 and then double again)</p> <p>16×25 8×50 (half 16 then double 25)</p> <p>$3 \times 15 = (2 \times 15) + (1 \times 15) = 30 + 15$</p>
	<p>Link exponents to fact strategies and properties for whole numbers. Use previously learned strategies such as</p> <ul style="list-style-type: none"> - distributive strategy <p>- associative property</p>	<p>a) $7^2 = 7 \times 7 = 49$ Use the above fact along with the distributive property to calculate:</p> <p>b) $7^3 = 7 \times 7 \times 7 = 49 \times 7 = 50 \times 7 - 7 = 350 - 7$</p> <p>c) for 6^3, use distributive property $6 \times 6 \times 6 = 36 \times 6 = (30 \times 6) + (6 \times 6)$</p>

	Skill	Example
	Apply the divisibility rules to help create multiples of numbers	<p>multiples of 26 (<i>in c and d use pencils to record answers</i>)</p> <p>Fill in the missing digit(s) so that the number is</p> <p>a) divisible by 9: 3419__ b) divisible by 6: 7__158__ c) divisible by 6 and 9: 5601__ d) divisible by 5 and 6: 70__81__</p>
Fractions and Decimals	The mental math material connected to this topic is extensive and consideration needs to be given as to what should be addressed during the fraction unit and what can be done at a later time.	
	<i>Review</i> mentally converting between improper fractions and mixed numbers	
	<p>Teach benchmarks for fractions ($0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1$) and decimals ($0, 0.25, 0.50, 0.75, 1$) -treat fractions and decimals together</p> <p>a) where they are located on a number line</p> <p>b) compare and order numbers with the benchmarks</p>	 <p>a) Place these fractions in the appropriate place on the number line</p> <p>i. 63, $\frac{1}{17}, \frac{44}{51}, \frac{7}{16}$ ii. $\frac{1}{8}, \frac{1}{20}, \frac{1}{99}, 0.001$ iii. $\frac{33}{20}, \frac{6}{20}, \frac{17}{20}, 2/20$ iv. $\frac{7}{8}, 0.42, 1 \frac{1}{3}, \frac{13}{12}, 0.15$</p> <p>b) Is $\frac{2}{5} < \frac{3}{4}$?</p>

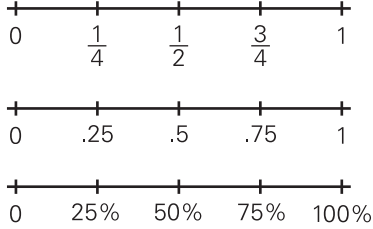
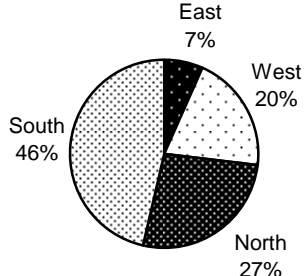
	Skill	Example
	<p>c) Create fractions close to the bench marks</p>	<p> $0.51 > \frac{7}{8}$? $\frac{4}{3} < 1\frac{1}{2}$? </p> <p>Which benchmark is each of the following numbers closest to? 0.26, 0.81, 0.95, 0.00099</p> <p>Which benchmark is each of the following numbers closest to? 0.51, 0.501, $\frac{1}{5}$, $\frac{4}{5}$, 0.9</p> <p>c) complete the fraction so that it is close to the benchmark given:</p> <p>i. close to $\frac{1}{2}$: $\frac{\square}{11}$</p> <p>ii. close to 1: $\frac{13}{\square}$</p> <p>iii. close to 0.5: $\frac{25}{\square}$</p> <p>iv. close to 0: $\frac{\square}{9}$</p>
	<p>Practice until students have automaticity of equivalence between certain fractions and decimals (halves, fourths, eights, tenths, fifths, thirds, ninths,) This can be revisited during the probability unit so the equivalencies are remembered and practiced.</p> <p><i>Review mentally converting between improper fractions and mixed numbers</i></p>	<p>Can use flashcards with fractions on one side and decimals on the other.</p> <p>If you have a class that works well together you can put fractions and decimals on separate cards and hand a ‘class set’ out. Go down the rows and as one student calls out their fraction or decimal the others have to listen and call out the equivalent if they have the card.</p>
	<p>- practice estimation using</p>	<p>Estimate: $\frac{2}{3} + \frac{1}{10}$</p>

	Skill	Example
	<p>benchmarks to add and subtract fractions and mixed numbers</p> <p>- is the sum or difference greater than, less than or equal to the closest benchmark</p>	$\frac{5}{8} \cdot \frac{10}{39}$ $4\frac{5}{6} - \frac{4}{5}$ <p>Is $\frac{1}{6} + \frac{1}{8} > \frac{1}{2}$?</p> <p>Is $\frac{2}{3} + \frac{3}{4} > 1.5$?</p> <p>Which sum or difference is larger? Estimate only?</p> <p>a) $\frac{3}{4} + \frac{4}{7}$ or $\frac{3}{8} + \frac{1}{10}$</p> <p>b) $4\frac{5}{6} - \frac{2}{5}$ or $4\frac{5}{6} - \frac{3}{4}$</p>
	<p>a) Link multiplying a whole number by a fraction to division.</p> <p>b) Link multiplying a fraction by a whole number to visually accumulating sets</p> <p>c) When the 2 separate visual pictures are firmly established,</p>	<p>a) i. For $\frac{3}{4} \times 20$, think $\frac{1}{4}$ of 20 is the same as $20 \div 4$ which is 5 so</p> <p>$\frac{3}{4} \times 20$ is 3 sets of 5 which is 15</p> <p>ii. Write a fraction sentence for this picture:</p>  <p>b) i. Write a fraction sentence for this picture:</p>  <p>ii. $6 \times \frac{1}{3} = 6 \text{ thirds} = 2 \text{ wholes}$</p> <p>c) i. $16 \times \frac{1}{8}$</p>

	Skill	Example
	<p>practice should consist of problems using both types</p> <p><i>(The intent here is that students keep a firm connection between number sentences and visuals at this time.)</i></p>	<p>ii. $\frac{4}{5} \times 10$</p> <p>iii. $14 \times \frac{3}{7}$</p> <p>iv. $\frac{2}{3} \times 9$</p>
	<p>Revisit the 4 properties associative, commutative, distributive, and identity</p> <p>a) mentally we want students to (1) recognize when a problem can be done mentally and (2) do the mental calculation</p> <p>b) create problems using whole numbers, decimals</p> <p>c) create problems that involve combinations of properties using rearrangement etc.</p> <p>Since students need much practice in this area, it is advisable to revisit this topic several times during the year, where appropriate.</p>	<p>Calculate:</p> <p>a) $1.33 + 8.25 + 6.75$</p> <p>b) $6 \times 98 = 6 \times (100 - 2)$ $= (6 \times 100) - (6 \times 2)$</p> <p>c) 4×2.25</p> <p>c) $7 \times 2.50 \times 6$</p> <p>d) $25 \times 2.08 \times 4$</p> <p>e) $46 \times 23 \times 0 \times 55$</p> <p>Judgment questions are found in the resource <u>Number Sense: Grades 6-8</u> (Dale Seymour Publications) pages 18 – 24</p>
	<p>Incorporate the “Make 1, Make 10, etc” strategy for decimals as well as properties stated above</p> <ul style="list-style-type: none"> do the 4 operations and incorporate other strategies 	<p>Practice can start with simple whole numbers, order of operations, and extend to decimals and use multiple strategies:</p> <p>a) $38 + 14$ could be $38 + 2 + 12$</p> <p>b) $4 \times 7 - 3 \times 7$ could be $(4-3) \times 7$</p> <p>c) $6 + 42 \div 7$</p> <p>d) $17 - 4^2$</p> <p>e) $1.25 + 3.81 = 1.25 + 3.75 + 0.06$</p> <p>f) $4 \times 0.26 = 4 \times 0.25 + 4 \times 0.01$</p> <p>g) $4 - 1.98$ could be $4.02 - 2.00$ or $4 - 2 + 0.02$</p> <p>h) $42 \div 0.07 = 4200 \div 7$</p>
Decimals and Percent	<p>a) This is an extension of the percents work done earlier.</p> <p>b) Establish benchmarks for percents: 0%, 25%, 50%, 75 %, and 100 %.</p>	<p>a) Use flashcards with fractions on one side and percents on the other. A suggested progression is to work with halves, fourths, tenths, and fifths on the first day and eighths, thirds and ninths on the next day. Mixed practice continues until automaticity is achieved.</p> <p>b) State the % benchmark closest to each of these:</p>

	Skill	Example
		$\frac{11}{20}, \frac{1}{6}, 0.98, \frac{5}{8}, \frac{7}{15}, \frac{1}{11}, 0.2$
	<p>a) Estimate % from a visual</p> <p>b) Convert easy fractions and decimals to a percent.</p> <p>c) Introduce “friendly fractions” in estimation of percents.</p>	<p>a) Estimate what % is shaded.</p>  <p>b) $\frac{4}{25} = _\% , \frac{7}{20} = _\% , \frac{1}{50} = _\% ,$ $0.16 = _\% , 0.165 = _\% ,$ $\frac{25}{75} = _\% , \frac{20}{40} = _\% , \frac{6}{8} = _\%$</p> <p>c) Estimate the percentage:</p> <p>i. $\frac{23}{61} \rightarrow \frac{20}{60} = \frac{1}{3} = 33 \frac{1}{3}\%$</p> <p>ii. $\frac{141}{352} \rightarrow \frac{140}{350} = \frac{14}{35} = \frac{2}{5} = 40\%$</p> <p>iii. $\frac{18234}{909000} \rightarrow \frac{18000}{900000} = \frac{18}{900}$ $= \frac{2}{100} = 2\%$</p>
	<p>a) Practice making judgments when given a problem as to whether to use the 1% method, convert the % to a fraction or convert the % to a decimal to solve the problem.</p> <p>b) Part to Whole problems using 1% method</p> <p>c) Estimation</p>	<p>a) Find</p> <p>i. 25 % of 80 (think $\frac{1}{4} \times 80$)</p> <p>ii. 23 % of 40 (think 0.23×40)</p> <p>iii. 6 % of 400 \rightarrow 1 % of 400 = 4 , so 6 % of 400 = $6 \times 4 = 24$</p> <p>b) Find \square if 20% of \square is 8.</p> <p>c) 39% of 78 \rightarrow 40 % of 80</p>
	<p>Mental math activities on sales tax will now become estimation activities instead of calculation activities. Students can</p>	

	Skill	Example
	<p>use the idea that 14 % is close to 15 % (10 + 5) to help them estimate</p> <p>a) Estimate sales tax mentally. 14% is close to 15% (10 %+ 5%)</p> <p>b) Create problems that can be done mentally using strategies learned previously.</p> <p>c) Estimate for discounts: -Applications to percents should be used when ever possible.</p>	<p>a) 14% of \$60 = think 14% is close to 15% so 10% x \$60 + 5% x 60</p> <p>b) Create a problem where you would convert the percent to i. a fraction. ii. a decimal iii. Use the 1% method</p> <p>c) i. Estimate the sale price for: 33 % discount of \$39.99 →</p> <p>ii. Bring in a sales flyer and give the original price and the sale. Ask students to estimate the sale price of items from the flyer. As a motivator have the students pick an item from the flyer. Most flyers give the sale price so this will be easy to correct!</p>
	<p>Mixed practice using three kinds of percent problems: Work on judgment as well as solutions. This is an opportunity to review the Work by Parts, Halve / Double strategies.</p> <p>a) Find percentage</p> <p>b) Find the percent of a number</p> <p>c) Find the number when given the part (Use proportional reasoning) Look for the <i>Proportional Reasoning</i> resource in your school. There are lots of exercises at the front of the book to use to practice proportional reasoning with percents.</p>	<p>a) 6 is what % of 20?</p> <p>b) i. Work by Parts: 35% discount on \$90.00 = (30 % of 90) + (5 % of 90) ii. Halve/double: $13 \frac{1}{2} \% \text{ of } 200 = 26\% \text{ of } 100$</p> <p>c) If 15% of $\square = 12$, find \square Think 5 % of $\square = 12 \div 3 = 4$ so 10 % of $\square = 8$ and 100 % of \square is 80</p>

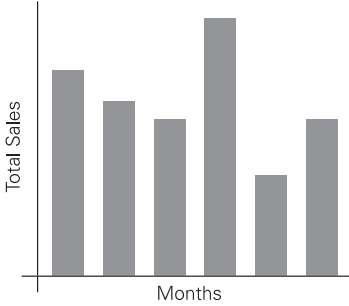
<p>Probability</p>	<p>There are opportunities to build on mental math content from previous work.</p>	
	<p>a) Reinforce equivalency among common fractions, decimals, and percent.</p> <p>b) Develop automaticity for conversions as well as associations with Never, Seldom, About half of the time, Often, and Always.</p>	 <p>a) Place on a number line: 25%, 0.49, $\frac{3}{4}$, 33% 90%, 0.10, 0.55, $\frac{2}{3}$, $\frac{4}{9}$</p> <p>b) Use the descriptors “Never”, “Seldom”, “About Half the Time”, “Often” and “Always” to describe the following probabilities: $\frac{3}{4}$ 0.10 60% 88% 5% 0.45 $\frac{5}{9}$</p>
	<p>Finding a fraction of a whole number</p> <p>Estimation: a) Friendly fractions b) Interpreting graphs</p>	<p>In a survey of 30 members of a class, 12 answered Yes, and 16 No. What is the probability that the next person surveyed will say Yes?</p> <p>a) How many in a school of 700 would you expect to say No ?</p> <p>b) If the circle graph below describes a city population of 24 000, estimate how many people live in the northern part?</p> 

	Find theoretical probability mentally for common situations	<p>What is the probability of</p> <ul style="list-style-type: none"> – getting a 4 on a single roll of a die? – getting a prime number on a single roll of a die? – drawing a queen from a complete deck of cards – drawing a face card from a complete deck of cards?
Integers	Integers is a new topic for grade 7 and requires substantial teaching before students are able to practice working with the integers mentally. This reinforcement takes time and can be extended through the geometry work utilizing the beginning 5 – 7 minutes of class time.	
	Much time should be spent on the Zero principle. Students should realize that all integers can be expressed as a sum in many ways.	<ul style="list-style-type: none"> a) Using 2 color counters, make zero in three different ways. b) Give a context for each way. c) Can you make zero with ___ counters? d) Can you make ___ with ___ counters etc.? e) Write the integer that is: <ul style="list-style-type: none"> i. 5 larger than -2 ii. 4 less than -6
Addition	<p>Much teaching needs to happen before the Mental Math can start. It should be tied to context.</p> <p>Begin with integers between -20 and +20 in size. Have students visualize the counters or relate to a context to help them work mentally.</p> <p>Come at the operations of addition and subtraction in as many ways as possible to help the students feel comfortable using the integers. This is also solidifying the conceptual knowledge of the operation as well as practicing the thinking behind the procedural knowledge.</p>	<p>Calculate:</p> <ul style="list-style-type: none"> a) <ul style="list-style-type: none"> i. $(+3) + (+4)$ ii. $(+3) + (-4)$ iii. $(-13) + (-4)$ iv. $(+3) + (-3)$ b) Determine if a positive or a negative value has been added to give the sum in each: <ul style="list-style-type: none"> i. $\bullet 2 + \square = \bullet 10$ ii. $5 + \square = 8$ iii. $7 + \square = 0$ iv. $\square \bullet 8 = \bullet 16$ c) Determine if the sum of the following integers would be positive or negative. <u>Do Not</u> give the sum: <ul style="list-style-type: none"> i. $\bullet 8, 10$

		<ul style="list-style-type: none"> ii. •12, •14 iii. 15, 20 iv. •4, •6, 13 v. 24, •25, •12 <p>d) Give two integers, with different signs, that have a sum of : (Keep the integer values between 10 and -10)</p> <ul style="list-style-type: none"> i. -2 (example: 4+-6 or -5+7) ii. -4 iii. 0 iv. 5 v. 7 vi. •1
<p>Subtraction</p>	<p>This is the most difficult operation for students to internalize as their individual schemas for addition and subtraction are being rebuilt.</p> <p>Practice in the beginning should focus on context:</p> <ul style="list-style-type: none"> - give a context and have students give the sentence - give a sentence and have students create a story. <p>As the students become comfortable, longer number sentences with both addition and</p>	<ul style="list-style-type: none"> a) Write an integer sentence to describe the person's net worth. <ul style="list-style-type: none"> i. John had 10 points in a card game, but lost 15 points in the next hand. ii. Peggy owed her mother \$3.00, but received \$8.00 for walking her aunt's dog. iii. Barbara's net worth was \$6.00 until Donald forgave a debt of \$5.00 that she had owed him. b) Create a story to fit each number sentence: <ul style="list-style-type: none"> i. $(-3) - (+4) = (-7)$ ii. $(-8) - (-10) = (+2)$ c) $(+3) + (+4) - (-8) + (+7)$ $(-4) + (-5) - (-4) + (-8)$

<p>Division</p>	<p>In the elementary grades, students have learned that every multiplication sentence has 2 related division sentences.</p> <p>Students can examine the sign patterns “discovered” in multiplication of integers and through the writing of the related division sentences, extend these patterns to division.</p> <p>As with multiplication, early practice should be with smaller numbers until the sign patterns are automatic.</p> <p>Contexts should be asked for where possible.</p> <p>As students internalize sign patterns, strategies used for dividing whole numbers can be extended to integers See Guide 7-30</p> <p>Balance before dividing (Multiply or divide both dividend and divisor by the same number):</p> <p>Work by parts:</p>	<p>a) Write two related division sentences for:</p> <ol style="list-style-type: none"> i. $(+3) \times (+9)$ ii. $(+3) \times (-9)$ iii. $(-3) \times (+9)$ iv. $(-3) \times (-9)$ <p>b) Write one related division sentence and one related multiplication sentence for:</p> <ol style="list-style-type: none"> i. $(+81) \div (-9)$ ii. $(+32) \div (+4)$ iii. $(-42) \div (-7)$ iv. $(-54) \div (+9)$ v. $0 \div (-4)$ <p>c) i. $(-125) \div (+5) = (-250) \div (+10)$ ii. $(-90) \div (-15) = (-30) \div (-5)$ iii. $(-16) \div (+0.25) = (-64) \div (+1)$</p> <p>d) i. $(-1232) \div (-4)$ $= (-1200) \div (-4) + (-32) \div (-4)$ ii. $(-128) \div (+8) = [(-80) + (+40)] \div 8$</p>
	<p>Order of Operations with Integers:</p> <p>Create problems combining the 4 operations using smaller numbers at first and then moving to larger numbers.</p> <p>Include problems with brackets and exponents as well as some that can be done with strategies such as compatible numbers and front-end.</p>	<p>Calculate</p> <ol style="list-style-type: none"> a) $(+3) + (+5) \times (-2)$ b) $20 - (-30) \div (+5)$ c) $(-2)^3$ d) $(-2)^4$ e) $(-48) \div [(+32) \div (+4)]$ f) $(-152) + (-248) - (-48)$ g) 12.5% of (-80)

<p>Geometry</p>	<p>There are opportunities to sharpen subtraction skills when working with finding the sizes of angles.</p> <p>Subtract from the left</p> <p>Much of the Mental Math time in this unit could be spent on practice with integer skills.</p> <p>Students can practice creating angles close to 45 degrees, 90 degrees, and 180 degrees. This can be alternated throughout the week with integer skills or rational number skills.</p>	<p>a) i. $180 - 46 = 180 - 40 - 6 = 140 - 6$ ii. $360 - 128 = 360 - 100 - 20 - 8$</p> <p>b) Put angles on the overhead, one at a time and have the students estimate the size of the angle. They should try to come within 5 degrees of the angle.</p> <p>c) Using only a straight edge and your pencil create the following angle measurements: i. 50 ii. 100 iii. 170</p>
<p>Data Management</p>	<p>Teachers can still use mental math time to reinforce integer operations.</p>	
	<p>Estimation activities that arise from interpreting data displays</p>	<p>a) Given a circle graph representing a total income of \$50 000, use the sectors to estimate what percentage and what amount is spent on; Housing, Food, Clothing and Other.</p> <div data-bbox="938 1350 1192 1602" data-label="Figure"> </div> <p>b) Given a bar graph showing Mr. Jones's sales for the past six months, estimate his mean and median sales.</p>

		
	<p>Compensation technique for finding mean. This gives a valuable application of adding and dividing integers</p>	<p>a) Find the mean of these grades: 85, 76, 71, 72, 86</p> <ul style="list-style-type: none"> · Choose 80 as a convenient central value, and then mentally compute the total of positive and negative differences of the central value from the mean. Divide this total by 5 (the number of integers given) and add to 80. · $+5 + (-4) + (-9) + (-8) + (+6) = -10$ · Average difference: $10 \div (5) = -2$ · Mean = $80 + (-2) = 78$ <p>b) Find the mean of this set: 46, 57, 49, 60, 48, 46</p>
<p>Patterns</p>	<p>At the beginning of the unit, mental math should consist of practice using positive and negative numbers and order of operations, including brackets and exponents. This prepares the student for later work, such as evaluating algebraic expressions and solving single variable equations.</p> <p>After the concept of the variable is firmly established, students should be given experiences where they can evaluate simple expressions and equations mentally.</p>	
	<p>Have students extend number sequences using the 4 operations, exponents etc, visual patterns, tables</p>	<p>Give the next 3 terms in each sequence</p> <p>a) $-2, -4, -6, -8\dots$ b) $-5, -3, -1, \dots$ c) $-2, 4, -8, 16\dots$ d) $100, 94, 88, \dots$ e) $-81, -27, -9\dots$ f) $1, 4, 9, 16\dots$ g) $1, 8, 27, 81\dots$</p>

	Have students examine tables to mentally determine the next term, a missing term or the nth term.	Determine the missing terms in these tables <table border="1" style="display: inline-table; margin-right: 20px;"> <tr><td>1</td><td>2</td><td>3</td><td>4</td><td>10</td><td>20</td><td></td><td>n</td></tr> <tr><td>2</td><td>5</td><td>8</td><td></td><td></td><td></td><td>74</td><td></td></tr> </table> <table border="1" style="display: inline-table;"> <tr><td>1</td><td>2</td><td>3</td><td>4</td><td>10</td><td>20</td><td></td><td>n</td></tr> <tr><td>-4</td><td>-2</td><td>0</td><td>2</td><td></td><td></td><td>50</td><td></td></tr> </table>	1	2	3	4	10	20		n	2	5	8				74		1	2	3	4	10	20		n	-4	-2	0	2			50	
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-4	-2	0	2			50																												
	Mentally evaluating expressions	Evaluate each expression for $x = -2, \frac{1}{2}, 0.2$ a) $-6x$ b) $2x + 5$ c) x^2 d) $15 - x$																																
	Mentally be able to combine like terms and recognize the parallels to working with integers.	Simplify: a) $3x - 2y + 4x - y$ b) $2a - 5 - 3a + 10$ c) $a - 2a - 3b + 6 - 4a + 5b - 4$																																
Linear Equations and Relations	Work from the unit on Patterns can continue into linear relations and equations																																	
	Once students have extensively practiced solving equations on paper, they can be introduced to some that can be done mentally and asked to orally explain the steps for solution.	a) $x + 5 = 7$ b) $z - 4 = -1$ c) $3w = -9$ d) $m \div 4 = -5$ e) $2q - 1 = 5$																																
(9.5)	Have students mentally calculate rates such as: Better buy, beats/min, km/hr, \$/hr – include conversions	a) Which is the better buy? One dozen peaches for \$ 3.00 or 4 peaches for 90 cents. b) John ran 35 metres in 15 seconds. How far could he run in 1 minute at the same speed? c) If Kay earned \$36.00 in 2.5 hours, how much would she earn in 5 hours? 10 hours? 1 hour?																																



**Mental Math
Yearly Plan
Grade 8**

Draft — September 2006

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Introduction

Definitions

It is important to clarify the definitions used around mental math. Mental math in Nova Scotia refers to the entire program of mental math and estimation across all strands. It is important to incorporate some aspect of mental math into your mathematics planning everyday, although the time spent each day may vary. While the *Time to Learn* document requires 5 minutes per day, there will be days, especially when introducing strategies, when more time will be needed. Other times, such as when reinforcing a strategy, it may not take 5 minutes to do the practice exercises and discuss the strategies and answers.

For the purpose of this booklet, fact learning will refer to the acquisition of the 100 number facts relating the single digits 0 to 9 for each of the four operations. When students know these facts, they can quickly retrieve them from memory (usually in 3 seconds or less). Ideally, through practice over time, students will achieve automaticity; that is, they will abandon the use of strategies and give instant recall. Computational estimation refers to using strategies to get approximate answers by doing calculations in one's head, while mental calculations refer to using strategies to get exact answers by doing all the calculations in one's head.

While we have defined each term separately, this does not suggest that the three terms are totally separable. Initially, students develop and use strategies to get quick recall of the facts. These strategies and the facts themselves are the foundations for the development of other mental calculation strategies. When the facts are automatic, students are no longer employing strategies to retrieve them from memory. In turn, the facts and mental calculation strategies are the foundations for estimation. Attempts at computational estimation are often thwarted by the lack of knowledge of the related facts and mental calculation strategies.

Rationale

In modern society, the development of mental computation skills needs to be a major goal of any mathematical program for two major reasons. First of all, in their day-to-day activities, most people's calculation needs can be met by having well developed mental computational processes. Secondly, while technology has replaced paper-and-pencil as the major tool for complex computations, people need to have well developed mental strategies to be alert to the reasonableness of answers generated by technology.

Besides being the foundation of the development of number and operation sense, fact learning itself is critical to the overall development of mathematics. Mathematics is about patterns and relationships and many of these patterns and relationships are numerical. Without a command of the basic relationships among numbers (facts), it is very difficult to detect these patterns and relationships. As well, nothing empowers students with confidence and flexibility of thinking more than a command of the number facts.

It is important to establish a rationale for mental math. While it is true that many computations that require exact answers are now done on calculators, it is important that students have the necessary skills to judge the reasonableness of those answers. This is also true for computations students will do using pencil-and-paper strategies. Furthermore, many computations in their daily lives will not require exact answers. (e.g., If three pens each cost \$1.90, can I buy them if I have \$5.00?) Students will also encounter computations in their daily lives for which they can get exact answers quickly in their heads. (e.g., What is the cost of three pens that each cost \$3.00?)

The Implementation of Mental Computational Strategies

General Approach

In general, a strategy should be introduced in isolation from other strategies, a variety of different reinforcement activities should be provided until it is mastered, the strategy should be assessed in a variety of ways, and then it should be combined with other previously learned strategies.

Introducing a Strategy

The approach to highlighting a mental computational strategy is to give the students an example of a computation for which the strategy would be useful to see if any of the students already can apply the strategy. If so, the student(s) can explain the strategy to the class with your help. If not, you could share the strategy yourself. The explanation of a strategy should include anything that will help students see the pattern and logic of the strategy, be that concrete materials, visuals, and/or contexts. The introduction should also include explicit modeling of the mental processes used to carry out the strategy, and explicit discussion of the situations for which the strategy is most appropriate and efficient. The logic of the strategy should be well understood before it is reinforced. (Often it would also be appropriate to show when the strategy would not be appropriate as well as when it would be appropriate.)

Reinforcement

Each strategy for building mental computational skills should be practised in isolation until students can give correct solutions in a reasonable time frame. Students must understand the logic of the strategy, recognize when it is appropriate, and explain the strategy. The amount of time spent on each strategy should be determined by the students' abilities and previous experiences.

The reinforcement activities for a strategy should be varied in type and should focus as much on the discussion of how students obtained their answers as on the answers themselves. The reinforcement activities should be structured to insure maximum participation. Time frames should be generous at first and be narrowed as students internalize the strategy. Student participation should be monitored and their progress assessed in a variety of ways to help determine how long should be spent on a strategy.

After you are confident that most of the students have internalized the strategy, you need to help them integrate it with other strategies they have developed. You can do this by providing activities that includes a mix of number expressions, for which this strategy and others would apply. You should have the students complete the activities and discuss the strategy/strategies that could be used; or you should have students match the number expressions included in the activity to a list of strategies, and discuss the attributes of the number expressions that prompted them to make the matches.

Assessment

Your assessments of mental math and estimation strategies should take a variety of forms. In addition to the traditional quizzes that involve students recording answers to questions that you give one-at-a-time in a certain time frame, you should also record any observations you make during the reinforcements, ask the students for oral responses and explanations, and have them explain strategies in writing. Individual interviews can provide you with many insights into a student's thinking, especially in situations where pencil-and-paper responses are weak.

Assessments, regardless of their form, should shed light on students' abilities to compute efficiently and accurately, to select appropriate strategies, and to explain their thinking.

Response Time

Response time is an effective way for teachers to see if students can use the mental math and estimation strategies efficiently and to determine if students have automaticity of their facts.

For the facts, your goal is to get a response in 3-seconds or less. You would give students more time than this in the initial strategy reinforcement activities, and reduce the time as the students become more proficient applying the strategy until the 3-second goal is reached. In subsequent grades when the facts are extended to 10s, 100s and 1000s, a 3-second response should also be the expectation.

In early grades, the 3-second response goal is a guideline for the teacher and does not need to be shared with the students if it will cause undue anxiety.

With other mental computational strategies, you should allow 5 to 10 seconds, depending upon the complexity of the mental activity required. Again, in the initial application of the strategies, you would allow as much time as needed to insure success, and gradually decrease the wait time until students attain solutions in a reasonable time frame.

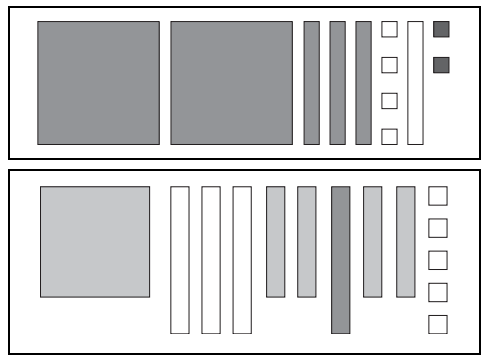
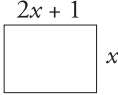
Mental Math: Grade 8 Yearly Plan

In this yearly plan for mental math in grade 8, an attempt has been made to align specific activities with the related topic in the grade 8 text. In some areas, the mental math content is too broad to be covered in the time frame allotted for a single chapter. While it is desirable to match this content to the unit being taught, it is quite acceptable to complete some mental math topics when doing subsequent chapters that do not have obvious mental math connections. For example practice with operations on rational numbers could continue into the data management and geometry chapters.

	Skill	Example
Squares, Square root and, Pythagoras	<p><i>Review</i> multiplication and division facts through</p> <p>a) rearrangement/ decomposition</p> <p>b) multiplying by multiples of 10</p> <p>c) multiplication strategies such as doubles, double/double, double plus one, halve/double etc.</p> <p><i>(Intent is to practice facts through previously learned strategies)</i></p>	<p>a) $8 \times 7 \times 5 = 8 \times 5 \times 7$ $12 \times 25 = 3 \times 4 \times 25 = 3 \times 100$</p> <p>b) $70 \times 80 = 7 \times 8 \times 10 \times 10$ $4 \ 200 \div 6 = 7 \times (600 \div 6)$</p> <p>c) $12.5 \times 4 = 12.5 \times 2 \times 2$ $3 \times 15 = (2 \times 15) + (1 \times 15) = 30 + 15$</p>

	Skill	Example
	<p>Mentally be able to determine perfect squares between 1 and 144 and the corresponding square roots. Daily practice should move to automaticity so students can use these facts in further work.</p> <p>a) Have students find the square root of larger numbers by looking at the factors of the numbers.</p> <p>b) Understanding powers of 10 smaller than 1 that are perfect squares, and how to use them in finding square roots. ($0.01 = 0.1 \times 0.1$, $0.0001 = 0.01 \times 0.01$, etc.)</p> <p>c) Find the square root of a quotient</p>	<p>$8^2 = 64$ so $\sqrt{64} = 8$ $1^2 = 1$ so $\sqrt{1} = 1$ $5^2 = 25$ so $\sqrt{25} = 5$ $11^2 = 121$ so $\sqrt{121} = 11$</p> <p>a) $\sqrt{6400} = \sqrt{64} \times \sqrt{100}$ $= 8 \times 10 = 80$ $\sqrt{900} = \sqrt{100} \times \sqrt{9} = 10 \times 3 = 30$</p> <p>b) $\sqrt{0.09} = \sqrt{0.01} \times \sqrt{9} = 0.1 \times 3 = 0.3$</p> <p>c) $\sqrt{\frac{144}{36}} = \frac{12}{6} = 2$ $\sqrt{\frac{9}{100}} = \frac{3}{10} = 0.3$</p>

	Skill	Example
	<p>Estimation: have students use boundaries to estimate square roots of numbers. (Students should already have the following facts committed to memory and can use them as a toolkit to help them do these problems</p> <p>$2^2 = 4$ $3^2 = 9$. . . $12^2 = 144$)</p> <p>It is important here that students learn to make judgments about what two whole numbers the square root is between and which whole number it is closer to.</p>	<p>a) $\sqrt{60}$ is between 7 and 8 but much closer to 8 b) $\sqrt{92}$ is between 9 and 10 but slightly closer to 10</p>
Algebraic Expressions and Solving Equations	<p>An important topic from grade 7 that can be practiced mentally at the start of this unit is operations on integers. Students will need to see that operations on algebraic expressions require the same approach as operations on integers.</p>	<p>Calculate</p> <p>a) $(+2) + (-3)$ b) $(+2a) + (-3a)$ c) $(+2) - (-3)$ d) $0 - (+4)$ e) $2c - 7c$ f) $2 - 7$ e) $(-1) + (-3) - (5)$ f) $(-2) \times (-3)$ g) $(+4) \times (-8)$ h) $(-2)^2$ i. -2^2 j) $(-72) \div (-8)$ k) $(-64) \div (+8)$ l) $(-3) \times (+4) \times (-2)$ m) $(-32) \div (+8)$</p>

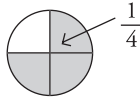
	Skill	Example
<p>Addition and Subtraction of Algebraic Terms</p>	<p>Students should be able to</p> <p>a) evaluate algebraic expressions</p> <p>b) be given a model and ask for the expression in simplest form (The overhead or magnetic white board tiles work well)</p> <p>c) symbolically simplify expressions by combining like terms (show the connection to operations on integers)</p>	<p>a) If $x = +2$ and $y = -5$</p> <p>i. x^2 ii. $3y$ iii. $x - y$ iv. $y - x$ v. $2y - 1$ vi. $2x - 1$ vii. $x + y + 15$ viii. $-2x^2$ ix. $-3y^2$</p> <p>b) State in simplest form the expressions illustrated below:</p>  <p>c) Combine Like Term</p> <p>i. $2x + (-3y) + 6x + (-5y) + 2$ ii. $3x + 4 - 6x - 10$ iii. $3x - 2y - y - 4$ iv. $(3x + 4) + (8 - x)$ v. $(8x + 4) + (18 - 2x)$ vi. Find an expression for the perimeter</p>  <p>vii. $3x - 4 - 2x - 1$ viii. $y - 2x - 3y - x$ ix. $1 - 4x - y + 7x - 3y$ x. $(2x^2 - x - 1) - (x^2 - 2x + 3)$ xi.</p>
	<p>Students should be able to use the distributive property to mentally multiply an expression by a scalar.</p>	<p>Multiply</p> <p>a) $2(x + 2)$ b) $4(3 - 3x)$ c) $3(2x - 1)$ d) $2(x + y) + 3(y)$</p>


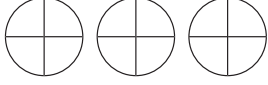
	Skill	Example
Solving Linear Equations:	<p>In grade 7, students are to mainly use concrete materials to solve simple linear equations. Before students can do anything mentally with solving equations, they need a lot of practice moving from the concrete and being confident and competent as they solve equations symbolically. Therefore the type of equations they should be asked to solve mentally at this stage should be fairly simple.</p>	<p>Mentally solve for x:</p> <p>a) $x - 4 = 6$ b) $x + 6 = -1$ c) $3x = -9$ d) $\frac{1}{2}x = -8$ e) $\frac{1}{2}x = 8$ f) $2x - 1 = 5$ g) $\frac{1}{3}x = 4$</p> <div style="border: 1px solid black; border-radius: 50%; padding: 10px; width: fit-content; margin-left: auto; margin-right: auto;"> <p>Think $\frac{1}{3}x = 4$ so $\frac{3}{3}x = 12$</p> </div>
Fraction Operations	<p>a) Learn common fractions and their decimal equivalents for thirds, fourths, fifths, eights, ninths, tenths</p> <p>b) Review</p> <ul style="list-style-type: none"> - equivalent fractions - converting from improper fractions to mixed numbers and vice versa <p>- simplest form of a fraction</p>	<p>a) Express as a decimal</p> <p>i. $\frac{1}{2}$ ii. $\frac{3}{4}$ iii. $\frac{1}{3}$ iv. $\frac{1}{9}$ v. $\frac{1}{8}$ vi. $\frac{7}{8}$ vii. $\frac{3}{4}$ viii. $\frac{3}{4}$</p> <p>- Express as a fraction</p> <p>i. 0.25 ii. $0.\overline{033}$ iii. 0.125 iv. 0.4 v. 0.44</p> <p>b) Find the missing number(s)</p> <p>i. $\frac{3}{5} = \frac{15}{\square}$ ii. $\frac{9}{11} = \frac{\square}{55}$ iii. $\frac{72}{81} = \frac{\square}{\square}$ iv. $3\frac{3}{4} = \frac{\square}{4}$ v. $\frac{21}{5} = \frac{\square}{\square}$</p> <p>- Express in simplest form</p> <p>i. $\frac{21}{14}$ ii. $\frac{60}{36}$ iii. $\frac{125}{100}$</p>

	Skill	Example
	Compare and order fractions using	
	a) benchmarks	a) Use benchmarks to help you arrange these fractions in order $\frac{11}{12}, \frac{1}{3}, \frac{5}{8}, \frac{1}{20}, \frac{7}{2}, \frac{12}{5}, \frac{9}{4}, \frac{10}{3}$
	b) common denominators	b) Use common denominators to help you arrange these fractions $\frac{2}{3}, \frac{3}{4}, \frac{1}{2}, \frac{5}{8}, \frac{7}{12}$
	c) common numerators	c) Use common numerators to help you arrange these fractions $\frac{1}{3}, \frac{3}{7}, \frac{6}{16}$
	d) conversion to decimals	d) Arrange these fractions in order by changing to decimal form $\frac{3}{5}, \frac{5}{8}$
	Estimation: using friendly fractions to simplify	$\frac{1316}{3824} \rightarrow \frac{1300}{3900} = \frac{13}{39} = \frac{1}{3}$

	Skill	Example
	<p>Addition and Subtraction of Fractions – have students mentally add and subtract fractions using previously learned strategies and properties such as :</p> <p>a) “make one” with like denominators</p> <p>b) “make one” with easy denominators</p> <p>c) “make one” using mixed numbers</p> <p>d) working with whole numbers and a fraction</p> <p>e) constant difference</p> <p>Estimation: sums and differences using</p> <p>a) benchmarks</p> <p>b) compensation</p> <p>c) replacements</p>	<p>a) $\frac{3}{8} + \frac{7}{8}$, $\frac{2}{3} + \frac{2}{3}$, $\frac{1}{6} + \frac{5}{6}$,</p> <p>b) $\frac{3}{4} + \frac{5}{8}$, $\frac{1}{2} + \frac{7}{12}$</p> <p>c) $2\frac{3}{4} + 1\frac{3}{4}$, $4\frac{1}{2} + 3\frac{7}{8}$</p> <p>d) $4 + \frac{2}{3}$, $3 - 1\frac{2}{3}$ (think $3 - 1 = 2$ and $2 - \frac{2}{3} = 1\frac{1}{3}$)</p> <p>e) $4\frac{1}{3} - 2\frac{2}{3}$ (add $\frac{1}{3}$ to each part to get $4\frac{2}{3} - 3 = 1\frac{1}{3}$)</p> <p>a) $\frac{3}{7} + \frac{7}{8} + \frac{1}{10} \rightarrow \frac{1}{2} + 1 + 0 = 1\frac{1}{2}$ $3\frac{1}{3} + 4\frac{6}{7} + 9\frac{1}{8} \rightarrow 3\frac{1}{2} + 5 + 9 = 17\frac{1}{2}$</p> <p>b) $4\frac{1}{10} + 2\frac{1}{6} + 3\frac{1}{4} \rightarrow 4 + 2 + 3\frac{1}{2} = 9\frac{1}{2}$ $4\frac{2}{3} - 1\frac{5}{6} \rightarrow 4\frac{1}{2} - 2 = 2\frac{1}{2}$ $9\frac{1}{7} - 3\frac{1}{8} - 2\frac{1}{4} \rightarrow 9 - 3 - 2\frac{1}{2} = 3\frac{1}{2}$</p> <p>c) $\frac{1}{8} + \frac{1}{9} + \frac{1}{10} \rightarrow \frac{1}{9} + \frac{1}{9} + \frac{1}{9} = \frac{3}{9} = \frac{1}{3}$</p>

	<p>Multiplying Fractions:</p> <p>a) Multiply whole number by a fraction</p> <p>b) Multiply fraction by a whole number</p> <p>c) Using the commutative property to help get an answer to a multiplication problem $(48 \times \frac{7}{8})$ can be thought of as $\frac{7}{8} \times 48$. Students can think that since $\frac{1}{8} \times 48 = 6$, then $\frac{7}{8} \times 48 = 7 \times 6 = 42$)</p> <p>d) Multiplying a mixed number by a whole number using the distributive law</p> $7 \times 3 \frac{1}{5}$ $= (7 \times 3) + (7 \times \frac{1}{5})$ $= 21 + \frac{7}{5}$ $= 21 + 1 \frac{2}{5}$ $= 22 \frac{2}{5}$	<p>a) i. $\frac{2}{3} \times 12$, Think: $\frac{1}{3}$ of 12 = 4, so $\frac{2}{3}$ of 12 = $2(4) = 8$</p> <p>ii. $\frac{5}{6} \times 36$</p> <p>b) i. $8 \times \frac{3}{4} = \frac{24}{4} = 6$</p> <p>ii. $16 \times \frac{1}{8}$</p> <p>c) i. $72 \times \frac{7}{9}$</p> <p>ii. $\frac{4}{5} \times 11$</p> <p>d) i. $8 \times 3 \frac{1}{8}$</p> <p>ii. $9 \times 3 \frac{1}{3}$</p> <p>iii. $32 \times 2 \frac{1}{4}$</p>
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	<p>Using the halve/double strategy to multiply</p> $12 \times 2 \frac{1}{2}$ $= 6 \times 5$ $= 30$ <p>e) Multiplying a fraction by a fraction</p> <p>i) Visualize</p> $\frac{1}{3} \times \frac{3}{4}$  <p>ii) Algorithm using simplification before multiplying</p> $\frac{3}{\cancel{14}} \times \frac{\cancel{14}}{5} = \frac{3}{5}$ <p>f) Estimate the product of fractions:</p> <p>i) using benchmarks</p> $5 \frac{1}{7} \times 6 \frac{9}{10} \rightarrow$ $5 \times 7 = 35$ <p>ii) using friendly fractions and/or commutative property</p> $34 \times \frac{2}{7} \rightarrow$ $35 \times \frac{2}{7} = \frac{2}{7} \times 35 = 10$	<p>e) i. $\frac{1}{5} \times \frac{5}{6}$</p> <p>ii. $\frac{4}{9} \times \frac{9}{10}$</p> <p>f) i. $3 \frac{1}{10} \times 4 \frac{4}{5}$</p> <p>ii. $\frac{7}{8} \times 65$</p>
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	<p>Dividing Fractions</p> <p>a) visualize a unit fraction divided by a whole number</p> <p>b) visualize a whole number divided by a unit fraction</p> <p>c) whole number divided by a fraction using the common denominator method</p> <p>d) Estimate the quotient of 2 fractions</p> <p>Check curriculum guide page 8-31, B 9.2 for additional questions</p>	<p>a) $\frac{1}{4} \div 3 =$  $= \frac{1}{12}$</p> <p>b) $3 \div \frac{1}{4} = 12$ </p> <p>c) $3 \div \frac{3}{4} = \frac{12}{4} \div \frac{3}{4} = 4$</p> <p>d) $\frac{5}{8} \div \frac{1}{8} = 5$</p> <p>Check curriculum guide, page 8-21, B 9.1 for more examples</p>																					
	<p>Order of Operations</p> <p>Create problems for students that encourage the use of previously learned strategies as well as the order of operations. (Some students may prefer to use a “quick calculation” where they record the intermediate steps.)</p>	<p>a) $3 - \frac{3}{5} \times \frac{5}{6}$</p> <p>b) $1\frac{1}{3} \times \frac{3}{4}$</p> <p>c) $3 \times 2\frac{2}{3} \div \frac{1}{2}$</p> <p>d) $3\frac{5}{6} + 1\frac{7}{10} + 1\frac{1}{6} + 2\frac{9}{10}$</p> <p>e) $2\frac{1}{2} \times 8 - 4\frac{1}{2}$</p>																					
<p>Proportions</p>	<p><i>Review</i> fractions, decimals, and percent equivalencies. Students should have automaticity for</p> <ul style="list-style-type: none"> - halves, fourths, eights - thirds, ninths - tenths, fifths <p>The use of flashcards and numberlines will promote automaticity</p>	<p>Complete this chart</p> <table border="1" data-bbox="862 1388 1325 1755"> <thead> <tr> <th>Fraction</th> <th>Decimal</th> <th>Percent</th> </tr> </thead> <tbody> <tr> <td>$\frac{1}{2}$</td> <td></td> <td></td> </tr> <tr> <td></td> <td>0.2</td> <td></td> </tr> <tr> <td></td> <td></td> <td>90%</td> </tr> <tr> <td></td> <td>$0.\bar{3}$</td> <td></td> </tr> <tr> <td>$\frac{2}{5}$</td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td>12.5%</td> </tr> </tbody> </table>	Fraction	Decimal	Percent	$\frac{1}{2}$				0.2				90%		$0.\bar{3}$		$\frac{2}{5}$					12.5%
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	<p>Fractions, Decimals and Percents:</p> <p>a) Estimate what percentage one number is of another using friendly fractions. 13 is what % of 19? $\frac{13}{19} \rightarrow \frac{14}{20} = \frac{7}{10} = 70\%$</p> <p>b) Find mentally what percentage one number is of another, using equivalent fractions. 12 is what % of 15 $\frac{12}{15} \rightarrow \frac{4}{5} = \frac{80}{100} = 80\%$</p> <p>c) Mentally find the percentage of a given number. What is 28% of 1200 20% 1200 + 8% of 1200</p> <p>d) Estimate the percent of a given number by rounding. 28% of 1231, think 30% of 1200</p> <p>e) Mentally find the whole when a percent is given using proportional thinking and/or converting to common fractions.</p> <p>f) Estimate the mixed number equivalent for percents. Round to friendly fractions i) $149\% \rightarrow 150\% = \frac{150}{100} = \frac{15}{10} = \frac{3}{2} = 1\frac{1}{2}$ ii) $119\% \rightarrow 120\% = 1\frac{1}{5}$ iii) $249\% \rightarrow 250\% = 2\frac{1}{2}$</p>	<p>a) i. 12 is what % of 55? ii. 35 is what % of 68?</p> <p>b) i. 9 is what % of 12? ii. 18 is what % of 30? iii. 22 is what % of 66?</p> <p>c) i. 26% of 800 ii. 31% of 90 iii. 50.5% of 80</p> <p>d) i. 24% of 79 ii. 98% of 720 iii. 32% of 69</p> <p>e) i. 5 is 10% of <input type="text"/> ii. 12 is 33.$\bar{3}$% of <input type="text"/> iii. 8 is 25% of <input type="text"/> iv. 30 is 20% of <input type="text"/> v. 9 is 60% of <input type="text"/></p> <p>f) Estimate the mixed number equivalent for: i. 112% ii. 224% iii. 183%</p>
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	<p>g) Estimate percents equivalent for mixed numbers.</p> $3\frac{3}{7} \rightarrow 3\frac{1}{2} = 3.5 = 350\%$ <p>h) Estimate fraction equivalents for percents less than 1</p> <p>i) $0.38\% \rightarrow 0.4\% =$</p> $\frac{0.4}{100} = \frac{4}{1000} = \frac{1}{250}$ <p>ii) $0.072\% \rightarrow 0.075\% =$</p> $\frac{0.075}{100} = \frac{75}{100000} = \frac{3}{4000}$ <p>i) Estimate percents less than 1% for very small fractions</p> <p>i) $\frac{1}{243} \rightarrow \frac{1}{250} =$</p> $\frac{2}{500} = \frac{.04}{100} = 0.4\%$ <p>ii) $\frac{13}{1500} \rightarrow \frac{12}{1500} = \frac{4}{500}$</p> $= \frac{8}{1000} = \frac{.08}{100} = 0.8\%$	<p>g) Estimate the % equivalent for:</p> <p>i. $2\frac{5}{8}$</p> <p>ii. $3\frac{9}{11}$</p> <p>iii. $1\frac{17}{18}$</p> <p>h) i. 0.8%</p> <p>ii. 0.25%</p> <p>iii. 0.71%</p> <p>iv. 0.059%</p> <p>i) i. $\frac{1}{488}$</p> <p>ii. $\frac{57}{1600}$</p> <p>iii. $\frac{18}{2925}$</p>
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	<p>Applying Proportions: There are many applications of proportions. Create problems using numbers that encourage mental calculations.</p> <p>a) Solving Proportions</p> <p>b) Unit Rates</p> <p>c) Comparison Shopping</p> <p>d) Distance, speed, and time</p>	<p>a) i. $7 : 9 = 63 : \square$ ii. $6 : 7 = \square : 63$ iii. $\frac{5}{\square} = \frac{25}{30}$ iv. $\frac{\square}{8} = \frac{24}{64}$</p> <p>b) i. $\frac{64¢}{8 \text{ candy}} = \frac{\square}{\text{candy}}$ ii. $\frac{300 \text{ km}}{4 \text{ h}} = \square / \text{h}$ iii. $\frac{12 \text{ beads}}{15 \text{ sec}} = \square / \text{min}$</p> <p>c) What is the better deal: a dozen oranges for \$2.40 or 3 oranges for 45 cents?</p> <p>d) If you travel 300km in 4 hrs, how long would it take you to travel 450 km?</p>
<p>Data Management</p>	<p>As a <i>brief review</i>, create sets of data and have students mentally calculate the mean using previously learned strategies</p>	<p>Calculate the mean of each.</p> <p>a) $28 + 36 + 22 + 34$ (compatibles)</p> <p>b) $75 + 29 + 46 + 54$ (break up and bridge)</p> <p>c) $4.6 + 3.5 + 8.4 + 1.5 + 2$ (make one)</p> <p>d) $410 + 120 + 330 + 140$ (front end addition or break up and bridge)</p> <p>e) 43, 37, 46, 32, 47 (central value method – choose a central value, such as 40, and then find the mean of the positive and negative differences between the central value and the numbers and add to the central value. $+3 + (-3) + +6 + -8 + +7 = +1$ Add +1 to 40 to get 41 as the mean</p>

	<p>Estimation:</p> <p>a) Applying samples to population: – If 20% of sample voted yes, about how many in a population of 769 would vote yes?</p> <p>b) Working with samples for friendly fractions</p> $\frac{27}{164} \rightarrow \frac{28}{164} =$ $\frac{7}{41} \rightarrow \frac{7}{42} = \frac{1}{6}$	
	<p>Adding some number to all members of a set: Find new mean, medium, and mode.</p>	<p>If for the set {9, 9, 9, 14, 15, 17, 18}</p> <p>Mean = 13 Median = 14 Mode = 9</p> <p>Find the new mean median, mode if</p> <ul style="list-style-type: none"> - all numbers of the set are multiplied by 7 - all numbers of the set are divided by 2 - etc.
	<p>Circle Graphs: Find percents of 360° by using proportional thinking</p>	<p>a) 100% of 360° b) 50% of 360° c) 25% of 360° d) 12.5% of 360° e) $33.\bar{3}$ % of 360° f) $66.\bar{6}$ % of 360° g) 10% of 360° h) 20% of 360° i. 5% of 360° j) 15% of 360° k) 27.5% of 360° l) 16% of 360°</p>

<p>Rational Numbers</p>	<p><i>Review</i> the comparing and ordering of</p> <p>a) Integers</p> <p>b) Fractions</p> <p>c) Decimals</p> <p><i>Review</i> previously learned strategies for integer, fractions, and decimal operations</p> <p>d) Integers</p> <p>e) Fractions</p> <p>f) Decimals</p> <p>g) Exponents</p>	<p>Order from smallest to largest</p> <p>a) $-1, 0, -100, +3, -3$</p> <p>b) $\frac{1}{10}, \frac{3}{4}, \frac{16}{29}, \frac{5}{4}, \frac{12}{25}$</p> <p>c) $2.7, 2.71, 2.17, 2.017, 0.2017$</p> <p>Perform the indicated operation</p> <p>d) i) $-3 + (-5) + (+2)$ ii. $-3 - (-5)$ iii. $(+7) \times (-1) \times (-2)$ iv. $(-64) \div (+8)$ v. $-10 - 3 \times (+4)$</p> <p>e) i. $\frac{3}{4} + \frac{7}{8}$ ii. $\frac{9}{10} - \frac{1}{2}$ iii. $1\frac{3}{4} \times 8$ iv. $\frac{2}{3} \div \frac{1}{2}$</p> <p>f) i. $2.9 + 3.4 + 1.1$ ii. $6 - 1.27$ iii. 0.25×80 iv. $9 \div 0.1$ v. $3.5 - 0.5 \times 4$</p> <p>g) Evaluate:</p> <p>i. 2^3 ii. 10^5 iii. $\left(\frac{1}{2}\right)^2$ iv. $(-3)^3$ v. $\left(\frac{2}{3}\right)^2$ vi. 0^{10}</p>
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	<p>Negative Exponents:</p> <p>a) <i>Review</i> established patterns</p> <p>b) Extend these patterns to negative exponents</p> <p>c) After students have learned the patterns for negative and zero powers of 10, include these in daily practice. (Note – Do not use exponent laws. The intent is to link powers to standard form)</p>	<p>a) $10^2 = 100$ $10^1 = 10$ $10^0 = 1$</p> <p>b) $10^{-1} = 0.1$ $10^{-2} = 0.01$ $10^{-3} = 0.001$</p> <p>c) i. $4 \times 10^0 \times (-8)$ ii. $-6 \times (-7) \times 10^{-1}$ iii. $10^{-2} \times 40$ iv. $20 \times 30 \times 10^{-3}$ v. $40 \times 50 \times 10^{-2}$ vi. $10^3 \times 10^{-1}$</p>
	<p>Scientific Notation</p> <p>a) have students convert numbers expressed in standard form to scientific notation and vice versa</p> <p>b) Arrange in order from smallest to largest</p>	<p>a) Express in scientific notation</p> <p>i. 186 000 = ii. 93 000 000 = iii. 0.00016 = iv. 0.0000402 =</p> <p>Express in standard form</p> <p>v. $3.4 \times 10^5 =$ vi. $6.25 \times 10^{-4} =$ v. $9.0 \times 10^{-6} =$</p> <p>b) Arrange from smallest to largest</p> <p>i. 420, 4.2×10^{-3}, 0.42 ii. 2.5×10^{-3}, 0.25×10^{-4}, 25×10^{-2}</p>

	<p>Compare and Order Rational Numbers Students should be encouraged to use the previously learned strategies listed below and apply them to an extended number set.</p> <p>a) a negative is always less than a positive</p> <p>b) use benchmarks (-1, -1/2, 0, +1/2, +1)</p> <p>c) change to common denominators</p> <p>d) change to common numerators</p> <p>e) convert to decimals</p>	<p>Arrange from small to large</p> <p>a) i. -3.1, 2, + 2.42, -1.6, -1.75 ii. $+\frac{1}{3}, -\frac{9}{10}, +\frac{3}{7}, -\frac{4}{9}, -\frac{4}{3}$</p> <p>b) $+\frac{7}{8}, +\frac{3}{4}, +\frac{3}{7}, +\frac{1}{10}, 0.99$</p> <p>c) $-\frac{2}{3}, -\frac{1}{6}, -\frac{1}{2}, -\frac{5}{6}, -\frac{4}{3}$</p> <p>d) $-\frac{3}{4}, -\frac{2}{7}, -\frac{1}{3}, -\frac{6}{11}$</p> <p>e) $+\frac{5}{8}, +\frac{2}{3}, -\frac{3}{5}, -\frac{5}{9}, -0.64$</p>
	<p>Operations with Rational Numbers: <i>Review</i> operations with rational numbers using strategies and properties such as:</p> <ul style="list-style-type: none"> • Front End Addition • Compatible Addends • Front End Multiplication • Compatible Factors • Making Compatible Numbers • Halve/ Double • Compensate: <p>(140 – 69 can be thought of as 140 – 70 then <i>compensate</i> by adding 1.)</p> <p>(12.5 – 4.7 can be thought of as 12.5 – 4.5 then <i>compensate</i> by subtracting 0.2.)</p>	<p>Use the properties of numbers (Associative, Commutative, Distributive, and Identity) to assist mental calculation:</p> <p>a) $2 \times 24 \times 50$ b) $2 \times 3.4 \times 5$ c) $4 \times \frac{3}{10} \times 2.5$ d) 50×14 e) 2.5×16 f) $7 \times \frac{3}{4} \times 12$ g) $\overline{0.33} \times 21 \times 5$ h) $0.25 \times 25 \times 16$ i) 299×15</p> <p>Use your compensation strategies for mentally calculating the following:</p> <p>j) $167 - 38$ k) $3\frac{1}{2} - 2\frac{7}{8}$ l) $6.7 - 7.8$</p>

	<p>a) Associative property</p> <p>b) Identity Property</p> <p>c) Commutative Property</p> <p>d) Distributive Property</p> <p><i>Review</i> the four operations on integers, fractions, and decimals. Students should be able to mentally perform the sum, difference, product, or quotient of two “friendly” rational numbers using strategies from prior grades as well as those learned in grade 8. Bring in such things as the zero principle as well as the product of a number and its reciprocal is one.</p>	<p>a) i. $(3\frac{3}{4} + 3\frac{3}{4}) + 2\frac{1}{4}$ ii. $-4.9 + (-6.3) + (-5.1)$</p> <p>b) i. $+6.1 + (-18) + (-6.1)$ ii. $(\frac{1}{2} \times \frac{3}{4} \times (-\frac{4}{3}))$</p> <p>c) $\frac{3}{4} \times 7 = 7 \times \frac{3}{4}$</p> <p>d) i. $8 \times -3\frac{1}{4} \rightarrow 8 \times -3 + 8 \times (-\frac{1}{4})$ ii. $-4 \times (1.25) - 4 \times (0.25)$ $= -4 \times (1.25 + 0.25)$</p>
	<p>Order of Operations: Create problems using “friendly numbers” that practice using the properties and strategies previously learned.</p>	<p>a) $7 - \frac{2}{3} \times \frac{3}{2}$</p> <p>b) $4 + \frac{4}{5} \div \frac{1}{10}$</p> <p>c) $(-\frac{1}{2})^2 - \frac{1}{4}$</p> <p>d) $-\frac{1^2}{2} - \frac{1}{4}$</p> <p>e) $(\frac{1}{2} \times \frac{2}{5}) + (\frac{1}{2} \times \frac{2}{5})$</p> <p>f) $3 \times 1.3 + 10.1$</p> <p>g) $12\frac{1}{2} \times 8 \div \frac{1}{2}$</p> <p>h) $\sqrt{6.25 \times 16}$</p>

	<p>Estimation: Revisit the ideas developed in the unit on fractions. Problems can now be extended to include negative rational numbers.</p>	<p>Further examples can be found in Interactions 8, chapter 4 and chapter 8.</p>
Geometry	<p>You may wish to continue working on mental math topics from the last chapter or from previous chapters as you do the geometry work.</p>	
Measurement	<p>Topics to <i>briefly review</i> at the start of the measurement unit are:</p> <p>a) <i>review</i> the metric units for area and volume</p> <p>b) <i>review</i> the connections between</p> <ul style="list-style-type: none"> – 1 cm^3, 1 ml, and 1 g – 1 dm^3, 1L, and 1kg; – 1 m^3, 1 metric ton and 1 Kl <p><i>For all of the above use models and visuals to reinforce the connections</i></p> <p>c) <i>review</i> all the metric prefixes – post visuals for students to use. Avoid using the metric chart for the memorization of prefixes.</p> <p>d) practice SI conversions</p>	<p>a) i. Draw rectangles with these areas and state the dimensions: 20 cm^2, 0.5 cm^2, 0.25 dm^2, 0.2 dm^2.</p> <p>ii. Estimate these areas: classroom door, your thumbprint, a soccer field, your room, your home.</p> <p>iii. Which of these areas would fit on a scribbler page: $20\,000\text{ mm}^2$, 180 cm^2, 0.8 dm^2, 0.02 m^2?</p> <p>b) Show various shapes and ask for a measurement estimate of the volumes (can extend this later to surface area)</p> <p>d) Convert each of the following:</p> <ul style="list-style-type: none"> i. $250\text{ cm} = \underline{\hspace{2cm}}\text{ m}$ ii. $2.5\text{ km} = \underline{\hspace{2cm}}\text{ m}$ iii. $0.4\text{ km} = \underline{\hspace{2cm}}\text{ m}$ iv. $0.5\text{ m}^2 = \underline{\hspace{2cm}}\text{ cm}^2$ v. $400\text{ mm}^2 = \underline{\hspace{2cm}}\text{ cm}^2$

e) Area and Perimeter of Quadrilaterals

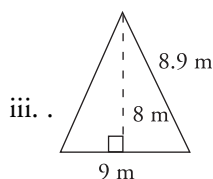
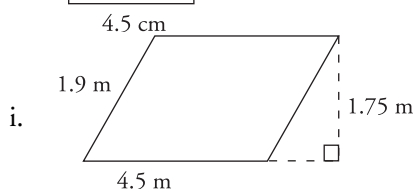
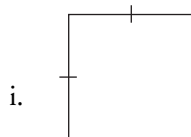
- Practice estimating the area and perimeter of quadrilaterals already studied.

- Present students with “friendly numbers” for dimensions of quadrilaterals and ask them to mentally calculate the area.

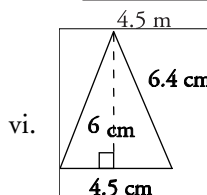
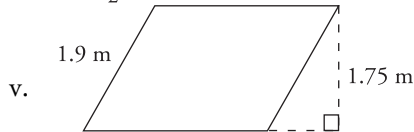
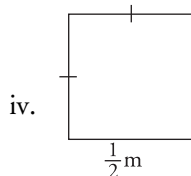
Create problems that will allow students to practice previously learned strategies.

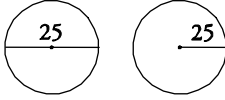
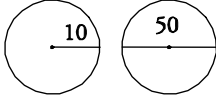
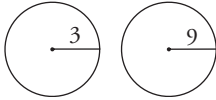
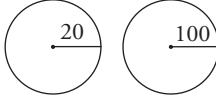
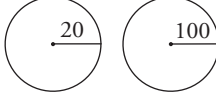
- vi. $3.5 \text{ km}^2 = \underline{\hspace{2cm}}$ ha
- vii. $2 \text{ m}^3 = \underline{\hspace{2cm}}$ cm^3
- viii. $4\,000 \text{ mm}^3 = \underline{\hspace{2cm}}$ cm^3
- ix. $250 \text{ cm}^3 = \underline{\hspace{2cm}}$ L
- x. $500 \text{ L} = \underline{\hspace{2cm}}$ m^3
- xi. $1.5 \text{ dm}^2 = \underline{\hspace{2cm}}$ cm^2

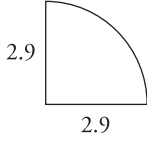

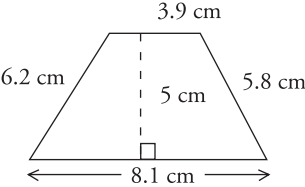
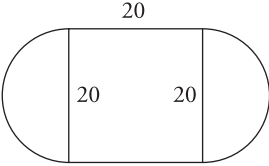
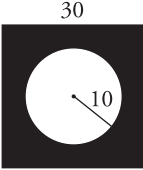
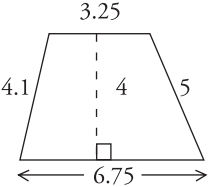
e) Estimate the perimeter and area of each of these figures.

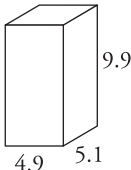
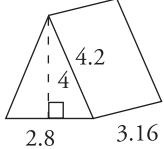
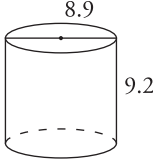
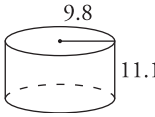
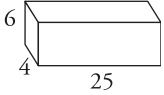
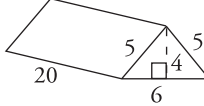
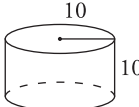
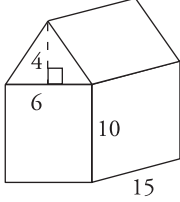


Calculate the area

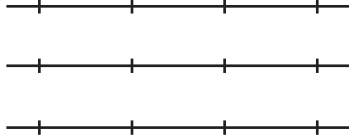


	<p>f) Circumference and Area of Circles:</p> <ul style="list-style-type: none"> - Estimate the circumference of circles using 3 to approximate π - Mentally calculate the circumference using 3.14 for π and having the radius or diameter to be a multiple or power of 10 - Estimate the area of circles by squaring the diameter (rough estimate) by squaring the radius and multiplying by 3 - Mentally calculate the area using: $A = \pi r^2$ 	<p>Estimate the circumference of these circles.</p>  <p>Calculate the circumference of these circles.</p>  <p>Estimate the area of these circles.</p>   <p>Calculate the area of these circles.</p> 
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	<p>Composite Figures</p> <p>a) Estimate the area of these figures (A good chance to practice rounding and compensating. Use 3 for π.)</p> <p>b) Mentally calculate the area of these figures (Create problems with “friendly numbers” allowing students to practice previously learned strategies.)</p>	<p>a) Estimate the area of these figures</p> <p>i. </p> <p>ii.  (Shaded part)</p> <p>iii. </p> <p>b) Calculate the area of each figure</p> <p>i. </p> <p>ii.  (Shaded Part)</p> <p>iii. </p>
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	<p>Have students estimate the solution to problems involving volume and surface area using appropriate estimation strategies. <i>Suitable models and diagrams for estimation of surface area are rectangular and triangular prisms, cylinders and composite 3-D figures. For estimating volume, suitable models and diagrams may include rectangular and triangular prisms, cylinders and composite 3-D figures.</i></p>	<p>Estimate the surface area and volume of each figure</p> <p>i. </p> <p>ii. </p> <p>iii. </p> <p>iv. </p>
	<p>Make decisions on what volume and surface area problems can be solved mentally. Recognize those that can use previously learned strategies (front-end, compatible factors, halve/double etc). Solve volume and surface area problems for prisms, cylinders and composite figures mentally when appropriate. <i>Suitable models and diagrams are rectangular and triangular prisms, cylinders, and composite figures.</i></p>	<p>Calculate the volume and surface area of each figure.</p> <p>i. </p> <p>ii. </p> <p>iii. </p> <p>iv. </p>

<p>Patterns and Relations</p>	<p>You may wish to use some of the mental math time for this chapter to finish some of the suggestions from other chapters</p>																																																													
	<p>Evaluate a single variable expression (start with whole numbers, then fractions and decimals). Progression of the types of expressions is also important (e.g. $2x + 4$, $4 + 2x$, $2x - 4$, $-2x + 4$, $4 - 2x$)</p>	<p>Evaluate the following expressions for the given value : (Do each evaluation separately)</p> <p>a) $3x + 1$ $x = 2$; $x = -6$; $x = \frac{1}{3}$; $x = -\frac{4}{3}$ $x = 0.3$ $x = -0.5$</p> <p>b) $5 + 4x$ $x = 10$; $x = -4$; $x = -\frac{1}{4}$ $x = \frac{3}{2}$; $x = 0.75$; $x = 1.25$</p> <p>c) $6x - 8$ $x = 0$; $x = -2$; $x = \frac{1}{3}$; $x = -\frac{1}{12}$; $x = 1.5$; $x = -2.5$</p> <p>d) $\frac{1}{2}x + 10$ $x = 6$; $x = -8$; $x = \frac{4}{7}$ $x = 4\frac{2}{3}$; $x = -12.6$</p> <p>e) $4 - x^2$ $x = 3$; $x = -2$; $x = \frac{1}{3}$ $x = 0.3$; $x = 0.5$</p> <p>f) $2^x + 3$ $x = 0$; $x = 3$; $x = -2$</p> <p>g) $(4x) \div 3$ $x = 6$; $x = 33$, $x = -15$ $x = \frac{3}{4}$; $x = 1.5$</p>																																																												
	<p>Have students determine the pattern in a table and complete the missing part(s). Include asking students to determine the n^{th} term</p>	<p>Study the table, determine the pattern and use it to complete the missing parts.</p> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td>n</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>10</td><td>20</td><td>n</td> </tr> <tr> <td>term</td><td>4</td><td>7</td><td>10</td><td>13</td><td></td><td></td><td></td><td></td><td>$\frac{7}{6}$</td> </tr> </table> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td>n</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>10</td><td>20</td><td>n</td> </tr> <tr> <td>term</td><td>100</td><td>99</td><td>98</td><td>97</td><td></td><td></td><td></td><td></td><td>$\frac{3}{2}$</td> </tr> </table> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td>n</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>10</td><td>20</td><td>n</td> </tr> <tr> <td>term</td><td>-1</td><td>3</td><td>7</td><td>11</td><td></td><td></td><td></td><td></td><td>$\frac{39}{5}$</td> </tr> </table>	n	1	2	3	4	5	6	10	20	n	term	4	7	10	13					$\frac{7}{6}$	n	1	2	3	4	5	6	10	20	n	term	100	99	98	97					$\frac{3}{2}$	n	1	2	3	4	5	6	10	20	n	term	-1	3	7	11					$\frac{39}{5}$
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<p>Probability</p>	<p>Reinforce equivalency among common fractions, decimals, and percent.</p> <p>Develop automaticity for conversions as well as associations with the words Never, Seldom, About half of the time, Often, and Always.</p> <p><i>Refer to the grade 7 Mental Math Yearly Plan.</i></p>	<p>Use three stacked number lines:</p> <p>Show 0, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ and 1 on the first, the equivalent decimals on the second line and percents on the third line.</p> 
	<p>Experimental Probability:</p> <p>a) use context to set up experimental probability questions</p> <p>b) use “friendly fractions” to estimate experimental probability</p>	<p>a) There are 3600 fish in the pond and 889 are speckled trout. Estimate the probability that you will catch a trout when you go fishing. (Students should see that 889 is close to 900 and so the probability is about $\frac{1}{4}$ or 0.25 or 25%)</p> <p>b) Estimate the experimental probability of:</p> $\frac{124}{506} \Rightarrow \frac{125}{500} \xrightarrow{+25} = \frac{5}{20} = \frac{1}{4}$ $\frac{31}{46} \Rightarrow \frac{30}{45} \xrightarrow{+15} = \frac{2}{3}$
	<p>Complementary Probabilities:</p> <p>Create problems that give students a chance to practice strategies such as subtraction by parts, finding compatibles. Having them use the idea of money when working with decimals is also beneficial.</p>	<p>Find the complementary probability</p> <p>If P(drawing a red ball) = 0.47</p> <p>Then P(not drawing a red ball) = 1-0.47 = 0.53</p> <p>If P(drawing a blue ball) = 0.112</p> <p>P(not drawing a blue ball) = 1-0.112 = 0.888</p>



**Mental Math
Yearly Plan
Grade 9**

Draft — September 2006

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Introduction

Definitions

It is important to clarify the definitions used around mental math. Mental math in Nova Scotia refers to the entire program of mental math and estimation across all strands. It is important to incorporate some aspect of mental math into your mathematics planning everyday, although the time spent each day may vary. While the *Time to Learn* document requires 5 minutes per day, there will be days, especially when introducing strategies, when more time will be needed. Other times, such as when reinforcing a strategy, it may not take 5 minutes to do the practice exercises and discuss the strategies and answers.

For the purpose of this booklet, fact learning will refer to the acquisition of the 100 number facts relating the single digits 0 to 9 for each of the four operations. When students know these facts, they can quickly retrieve them from memory (usually in 3 seconds or less). Ideally, through practice over time, students will achieve automaticity; that is, they will abandon the use of strategies and give instant recall. Computational estimation refers to using strategies to get approximate answers by doing calculations in one's head, while mental calculations refer to using strategies to get exact answers by doing all the calculations in one's head.

While we have defined each term separately, this does not suggest that the three terms are totally separable. Initially, students develop and use strategies to get quick recall of the facts. These strategies and the facts themselves are the foundations for the development of other mental calculation strategies. When the facts are automatic, students are no longer employing strategies to retrieve them from memory. In turn, the facts and mental calculation strategies are the foundations for estimation. Attempts at computational estimation are often thwarted by the lack of knowledge of the related facts and mental calculation strategies.

Rationale

In modern society, the development of mental computation skills needs to be a major goal of any mathematical program for two major reasons. First of all, in their day-to-day activities, most people's calculation needs can be met by having well developed mental computational processes. Secondly, while technology has replaced paper-and-pencil as the major tool for complex computations, people need to have well developed mental strategies to be alert to the reasonableness of answers generated by technology.

Besides being the foundation of the development of number and operation sense, fact learning itself is critical to the overall development of mathematics. Mathematics is about patterns and relationships and many of these patterns and relationships are numerical. Without a command of the basic relationships among numbers (facts), it is very difficult to detect these patterns and relationships. As well, nothing empowers students with confidence and flexibility of thinking more than a command of the number facts.

It is important to establish a rationale for mental math. While it is true that many computations that require exact answers are now done on calculators, it is important that students have the necessary skills to judge the reasonableness of those answers. This is also true for computations students will do using pencil-and-paper strategies. Furthermore, many computations in their daily lives will not require exact answers. (e.g., If three pens each cost \$1.90, can I buy them if I have \$5.00?) Students will also encounter computations in their daily lives for which they can get exact answers quickly in their heads. (e.g., What is the cost of three pens that each cost \$3.00?)

The Implementation of Mental Computational Strategies

General Approach

In general, a strategy should be introduced in isolation from other strategies, a variety of different reinforcement activities should be provided until it is mastered, the strategy should be assessed in a variety of ways, and then it should be combined with other previously learned strategies.

Introducing a Strategy

The approach to highlighting a mental computational strategy is to give the students an example of a computation for which the strategy would be useful to see if any of the students already can apply the strategy. If so, the student(s) can explain the strategy to the class with your help. If not, you could share the strategy yourself. The explanation of a strategy should include anything that will help students see the pattern and logic of the strategy, be that concrete materials, visuals, and/or contexts. The introduction should also include explicit modeling of the mental processes used to carry out the strategy, and explicit discussion of the situations for which the strategy is most appropriate and efficient. The logic of the strategy should be well understood before it is reinforced. (Often it would also be appropriate to show when the strategy would not be appropriate as well as when it would be appropriate.)

Reinforcement

Each strategy for building mental computational skills should be practised in isolation until students can give correct solutions in a reasonable time frame. Students must understand the logic of the strategy, recognize when it is appropriate, and explain the strategy. The amount of time spent on each strategy should be determined by the students' abilities and previous experiences.

The reinforcement activities for a strategy should be varied in type and should focus as much on the discussion of how students obtained their answers as on the answers themselves. The reinforcement activities should be structured to insure maximum participation. Time frames should be generous at first and be narrowed as students internalize the strategy. Student participation should be monitored and their progress assessed in a variety of ways to help determine how long should be spent on a strategy.

After you are confident that most of the students have internalized the strategy, you need to help them integrate it with other strategies they have developed. You can do this by providing activities that includes a mix of number expressions, for which this strategy and others would apply. You should have the students complete the activities and discuss the strategy/strategies that could be used; or you should have students match the number expressions included in the activity to a list of strategies, and discuss the attributes of the number expressions that prompted them to make the matches.

Assessment

Your assessments of mental math and estimation strategies should take a variety of forms. In addition to the traditional quizzes that involve students recording answers to questions that you give one-at-a-time in a certain time frame, you should also record any observations you make during the reinforcements, ask the students for oral responses and explanations, and have them explain strategies in writing. Individual interviews can provide you with many insights into a student's thinking, especially in situations where pencil-and-paper responses are weak.

Assessments, regardless of their form, should shed light on students' abilities to compute efficiently and accurately, to select appropriate strategies, and to explain their thinking.

Response Time

Response time is an effective way for teachers to see if students can use the mental math and estimation strategies efficiently and to determine if students have automaticity of their facts.

For the facts, your goal is to get a response in 3-seconds or less. You would give students more time than this in the initial strategy reinforcement activities, and reduce the time as the students become more proficient applying the strategy until the 3-second goal is reached. In subsequent grades when the facts are extended to 10s, 100s and 1000s, a 3-second response should also be the expectation.

In early grades, the 3-second response goal is a guideline for the teacher and does not need to be shared with the students if it will cause undue anxiety.

With other mental computational strategies, you should allow 5 to 10 seconds, depending upon the complexity of the mental activity required. Again, in the initial application of the strategies, you would allow as much time as needed to insure success, and gradually decrease the wait time until students attain solutions in a reasonable time frame.

Mental Math: Grade 9 Yearly Plan

In this yearly plan for mental math in grade 9, an attempt has been made to align specific activities with the related chapter in the new grade 9 text, *Mathematics 9: Focus on Understanding*. In some areas, the mental math content is too broad to be covered in the time frame allotted for a single chapter. While it is desirable to match this content to the unit being taught, it is quite acceptable to complete some mental math topics when doing subsequent chapters that do not have obvious mental math connections. In the grade 9 mathematics program, students need to make decisions on whether an exact answer or approximate answer is required. When an exact answer is required, they need to decide whether to use mental math, pencil and paper computation, or a calculator. Keeping this in mind it is appropriate to allow practice in mental math to happen at the start or within a lesson.

The following are strategies from previous grades that students should be familiar with and can be briefly reviewed at the start of the year and reinforced through the development of appropriate questions and practice throughout the year.

Mental Math (M) (Grade 7–B2)

- Front-end (addition & multiplication)
- Compatible numbers
- Compatible factors
- Halve / Double
- Compensate, break up and bridge (Grade 4)

Estimation (E) (Grade 7 – B1)

- Rounding
- Front-end
- Special numbers
- Clustering of near compatibles
- Compatibles
- Using referents

Properties to be reviewed and used throughout the year, where appropriate

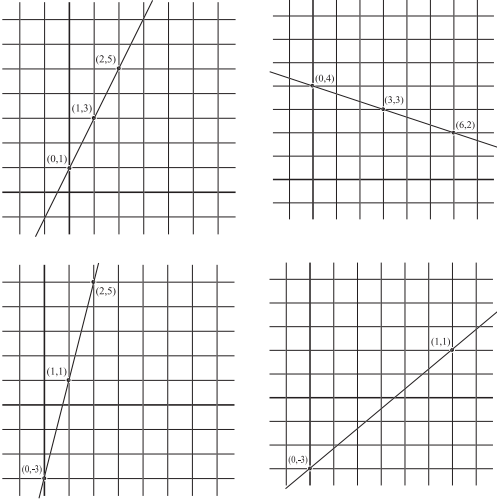
- Distributive
- Associative
- Commutative
- Identity

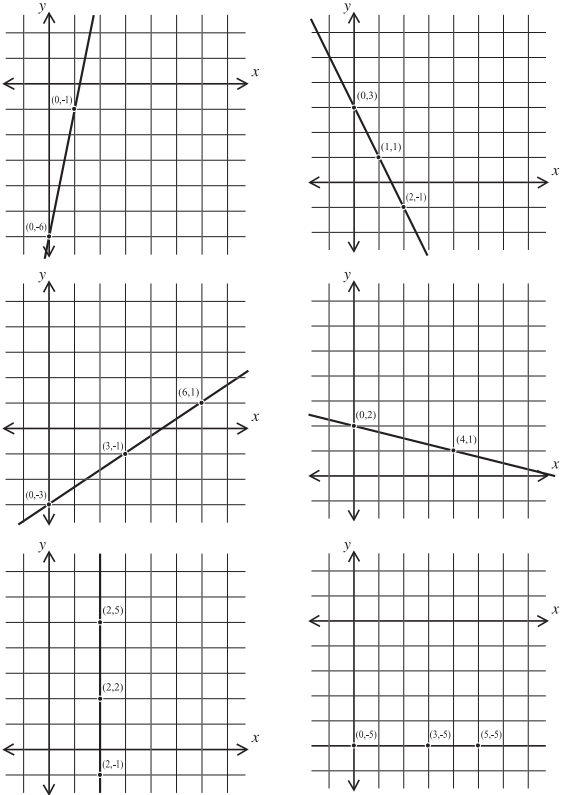
Chapter	Skill	Example
Number Sense	Compare Real Numbers: <i>reviewing</i>	<p style="text-align: right;">>, <, or =) between the</p> <p>two fractions.</p> <p>a) $\frac{5}{7}, \frac{3}{7}$</p> <p>b) $\frac{5}{6}, \frac{5}{8}$</p> <p>c) $\frac{3}{7}, \frac{5}{8}$</p> <p>d) $\frac{9}{10}, \frac{5}{8}$</p> <p>Decimals:</p> <p>Place the appropriate symbol (>, <, or =) between the two decimals.</p> <p>a) 0.2, 0.20</p> <p>b) 0.123, 0.1234</p> <p>c) 2.002, 2.020</p> <p>d) 2.5, 2.49</p> <p>e) -1.3, -1.5</p> <p>Write a decimal number between each pair:</p> <p>a) -2.3, 2.31</p> <p>b) 0.54, -0.541</p> <p>Calculate the square root of the following numbers:</p> <p>a) 36</p> <p>b) 144</p> <p>c) 400</p> <p>d) 4900</p> <p>e) $\frac{9}{100}$</p> <p>f) 0.25</p> <p>Approximate the square root of the following numbers:</p> <p>a) 29</p> <p>b) 99</p> <p>c) 63</p> <p>d) 150</p>
	:	<p>Identify the larger value:</p> <p>a) $\frac{1}{3}, 0.4$</p> <p>d) $\sqrt{49}, -7$</p>

		<p>e) $\sqrt{1.21}$, 0.1</p> <p>f) $\sqrt{70}$, 8.8</p> <p>g) 0.40404..., 0.404004...</p>
	<p>Order Real Numbers: Students will use their comparing strategies to order a group of real numbers.</p>	<p>Order from least to greatest:</p> <p>a) $\frac{5}{6}$, $1\frac{3}{4}$, $\sqrt{2}$</p> <p>b) -0.25, $\sqrt{0.25}$, $\frac{1}{4}$</p> <p>c) π, $\frac{-10}{-3}$, 3.04</p>
	<p>Operations with Rationals: <i>Review</i> operations with rational numbers using strategies such as:</p> <ul style="list-style-type: none"> ▪ Front End Addition (<i>Mental Math in the Junior High</i>, page 41) ▪ Compatible Addends (<i>Mental Math in the Junior High</i>, pages 51-56, 65, 99-102) ▪ Front End Multiplication (<i>Mental Math in the Junior High</i>, page 73) ▪ Compatible Factors (<i>Mental Math in the Junior High</i> page 123) ▪ Making Compatible Numbers (<i>Mental Math in the Junior High</i> page 125) ▪ Halve/ Double (<i>Mental Math in the Junior High</i> pages 117-120) ▪ Compensate: 140 – 69 can be thought of as 140 – 70 then <i>compensate</i> by adding 1. <p>12.5 – 4.7 can be thought of as 12.5 – 4.5 then <i>compensate</i> by subtracting 0.2.</p> <ul style="list-style-type: none"> ▪ Review the four 	<p>Use the properties of numbers (Associative, Commutative, and Distributive) to calculate mentally:</p> <p>a) $2 \times 24 \times 50$</p> <p>b) $2 \times 3.4 \times 5$</p> <p>c) $4 \times \frac{3}{10} \times 2.5$</p> <p>d) 50×14</p> <p>e) 2.5×16</p> <p>f) $7 \times \frac{3}{4} \times 12$</p> <p>d) $0.3333... \times 21 \times 5$</p> <p>e) $0.25 \times 25 \times 16$</p> <p>f) 299×15</p> <p>Use your compensation strategies for mentally calculating the following:</p> <p>a) $167 - 38$</p> <p>b) $3\frac{1}{2} - 2\frac{7}{8}$</p> <p>c) $6.7 - 7.8$</p>

Chapter	Skill	Example
	<p>operations on integers. Students should be able to mentally perform the sum, difference, product, or quotient of two “friendly” integers using strategies from prior grades. Include discussion of the zero principle as well.</p>	
	<p>Mentally calculate the squares of numbers</p>	<p>Mentally calculate the following:</p> <p>a) 20^2 b) 1.2^2 c) $\left(\frac{3}{4}\right)^2$ d) 99^2 e) 5.5^2 f) $(\sqrt{4.3})^2$</p>
	<p>Laws of exponents: Mentally calculate problems using the laws of exponents:</p> <ul style="list-style-type: none"> ▪ Students should be able to simplify appropriate expressions using their laws of exponents. Including problems with “friendly” or reasonable coefficients that will also allow the use of other strategies. 	<p>Express each in standard form:</p> <p>a) $\frac{7^2}{7^4}$ b) $(5^2 \times 3^4)^0$ c) $8^{-2} \times 8^4$ d) $\frac{2^4}{8^2}$ e) $\left(\frac{1}{3}\right)^{-3} \div 9^2$</p> <p>Simplify:</p> <p>a) $(2x^3)(-3x^5)$ b) $\left(\frac{1}{2}x\right)^2(4x^4)$ c) $(0.2x)^{-4}(40x^5)$</p>

Chapter	Skill	Example
	<p>Scientific Notation: Estimate: Students should be able to use estimation strategies with scientific notation.</p> <p>Students should first be able to identify the conditions necessary for easy mental calculations. They can then proceed to mentally calculate the answers. The starred questions are examples of those that can be done mentally.</p>	<p>Estimate the answer for each of the following:</p> <p>a) $1.78 \times 10^5 + 5.34 \times 10^5$ b) $7.01 \times 10^3 + 3.97 \times 10^{-3}$ c) $(2.46 \times 10^2)(3.2 \times 10^4)$ d) $9.86 \times 10^6 \div 2.21 \times 10^2$</p> <p>Determine which of the following can be found mentally and calculate:</p> <p>a) $2.5 \times 10^4 + 3.5 \times 10^7$ b) $*3.4 \times 10^5 + 4.8 \times 10^5$ c) $1.8 \times 10^{-4} + 1.8 \times 10^4$ d) $*5.79 \times 10^{-5} - 4.01 \times 10^{-5}$ e) $6.3 \times 10^{-9} - 6.2 \times 10^{-5}$ f) $*4.4 \times 10^3 - 6.4 \times 10^3$ g) $*5.5 \times 10^{-3} (2 \times 10^{-5})$ h) $(3.48 \times 10^6) 2.7 \times 10^6$ i) $*4.5 \times 10^6 \div 1.5 \times 10^{-5}$ j) $3.67 \times 10^4 \div 2.81 \times 10^{-2}$</p>
<p>Patterns and Relations</p>	<p>Evaluate a single variable expression (start with whole numbers, then fractions and decimals). Progression of the types of expressions is also important (e.g. $2x + 4$, $4 + 2x$, $2x - 4$, $-2x + 4$, $4 - 2x$)</p>	<p>Evaluate the following expressions for the given value : (Do each evaluation separately)</p> <p>a) $3x + 1$ $x = 2$; $x = -6$; $x = \frac{1}{3}$; $x = -\frac{4}{3}$ $x = 0.3$ $x = -0.5$</p> <p>b) $5 + 4x$ $x = 10$; $x = -4$; $x = -\frac{1}{4}$ $x = \frac{3}{2}$; $x = 0.75$; $x = 1.25$</p> <p>c) $6x - 8$ $x = 0$; $x = -2$; $x = \frac{1}{3}$; $x = -\frac{1}{12}$; $x = 1.5$; $x = -2.5$</p> <p>d) $\frac{1}{2}x + 10$ $x = 6$; $x = -8$; $x = \frac{4}{7}$ $x = 4\frac{2}{3}$; $x = -12.6$</p> <p>e) $4 - x^2$ $x = 3$; $x = -2$; $x = \frac{1}{3}$ $x = 0.3$; $x = 0.5$</p> <p>f) $2^x + 3$ $x = 0$; $x = 3$; $x = -2$</p> <p>g) $(4x) \div 3$ $x = 6$; $x = 33$; $x = -15$ $x = \frac{3}{4}$; $x = 1.5$</p>

Chapter	Skill	Example																																						
	<p>Determine from a table of values if the slope of the graph will be negative or positive.</p> <p>Determine from a table of values if the graph of the relationship will be linear or non-linear.</p>	<table border="1" style="display: inline-table; margin-right: 20px;"> <tr><th>x</th><th>y</th></tr> <tr><td>-3</td><td>0.125</td></tr> <tr><td>-2</td><td>0.25</td></tr> <tr><td>-1</td><td>0.5</td></tr> <tr><td>0</td><td>1</td></tr> <tr><td>1</td><td>2</td></tr> </table> <table border="1" style="display: inline-table; margin-right: 20px;"> <tr><th>x</th><th>y</th></tr> <tr><td>-3</td><td>9</td></tr> <tr><td>-2</td><td>7</td></tr> <tr><td>-1</td><td>5</td></tr> <tr><td>0</td><td>3</td></tr> <tr><td>1</td><td>-1</td></tr> <tr><td>2</td><td>-3</td></tr> </table> <table border="1" style="display: inline-table;"> <tr><th>x</th><th>y</th></tr> <tr><td>-8</td><td>-11</td></tr> <tr><td>-4</td><td>-5</td></tr> <tr><td>0</td><td>1</td></tr> <tr><td>4</td><td>7</td></tr> <tr><td>8</td><td>13</td></tr> </table>	x	y	-3	0.125	-2	0.25	-1	0.5	0	1	1	2	x	y	-3	9	-2	7	-1	5	0	3	1	-1	2	-3	x	y	-8	-11	-4	-5	0	1	4	7	8	13
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	<p>Mentally calculate the slope of a line from a graph</p>																																							
	<p>Give a linear equation and ask to calculate the slope</p>	<p>a) $y = 2x + 4$, b) $2y = 4x + 10$, c) $5y = -15x$, d) $\frac{1}{2}y = 4x + 3$</p>																																						
	<p>Give a linear equation and ask to calculate the y-intercept</p> <p>Give a linear equation and ask to graph the equation of the line:</p> <p>Hand out a photocopied coordinate system and a transparency sheet. The transparency should have a diagonal line drawn from corner to corner. The students are shown an equation on the board and then are asked to</p>	<p>a) $y = 2x + 4$, b) $2y = x - 10$, c) $\frac{1}{3}y = x + 1$</p>																																						

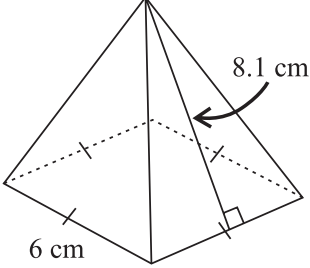
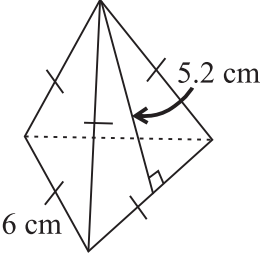
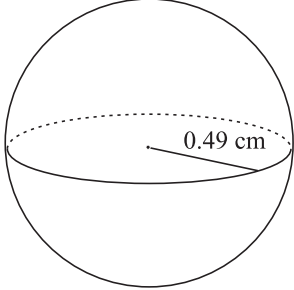
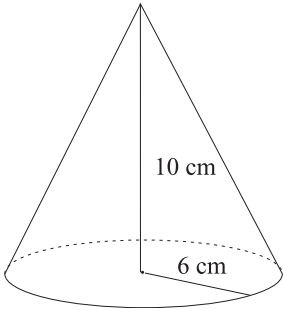
Chapter	Skill	Example
	<p>place the “line” on the coordinate system to graph the equation. Teachers can easily look at the graph on the students’ desks to see if the graphing was completed correctly.</p>	
	<p>Show graphs of various “friendly” lines including horizontal and vertical lines and ask for the equation of the line</p>	<p><i>Determine the equations of these lines mentally.</i></p>  <p>The example section contains six coordinate planes, each with a line and several points labeled:</p> <ul style="list-style-type: none"> Graph 1: A line passing through (0, -1) and (0, -6). Graph 2: A line passing through (0, 3), (1, 1), and (2, -1). Graph 3: A line passing through (0, -3), (3, -1), and (6, 1). Graph 4: A line passing through (0, 2) and (4, 1). Graph 5: A vertical line passing through (2, 5), (2, 2), and (2, -1). Graph 6: A horizontal line passing through (0, -5), (3, -5), and (5, -5).

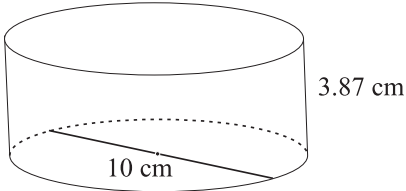
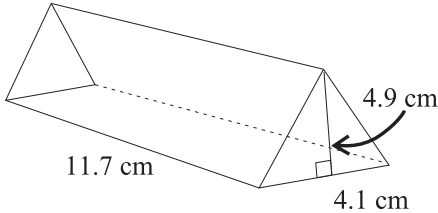
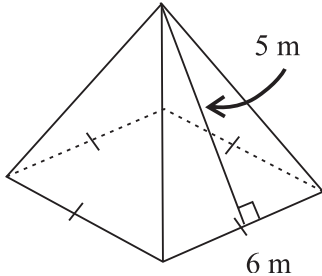
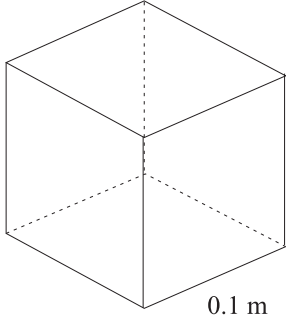
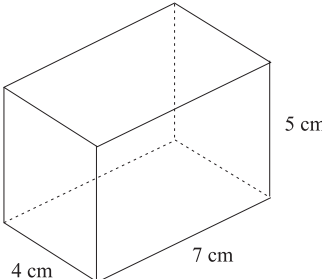
Chapter	Skill	Example
	<p>Make decisions on what single variable equations and inequations could be solved mentally</p>	<p>Determine which of these equations could be solved mentally :</p> <p>a) $2x + 1 = 9$ b) $3x + 13 = -1.8x$ c) $\frac{x}{4} = -3$ d) $2(x + 6) = 5(2x - 1)$ e) $5(x - 4) = -10$ f) $3x - 1 = -7$ g) $\frac{2}{3}x + 6 = \frac{4}{5}x - 2$ h) $4x + 5 = 3x + 10$ i) $x^2 + 1 = 101$ j) $11 = \frac{1}{3}x - 1$ k) $4.6x + 3.4x = -24$ l) $\frac{3}{4}x - 16 = -\frac{1}{4}x$ m) $1 - \frac{7}{8}x + 2x + \frac{1}{8}x = 32$ n) $3x + 2.3 = x - 5.7$ o) $5x - 2.25 - \frac{3}{4} > 4x - 1$ p) $2x - + 10.7 < -5x + 4.23$</p>
	<p>Solve appropriate single variable equations and inequations by inspection.</p> <p>Create equations that reinforce the strategies identified in chapter 1. Again use the real number progression from whole numbers to fractions to decimals and then give a mixture.</p>	<p>Use the equations and inequations in a, c, e, f, h, i, j, k, l, m, n, and o as examples of ones that can be solved mentally.</p> <p>Work through these with students.</p> <p>Many of the equations above use the ‘make one’ strategy developed in early grades. Discuss the many ways ‘make one’ can be disguised in an equation.</p>
<p>Probability</p>	<p>Benchmarks: Students have had a lot of exposure to benchmarks. They have had to determine which benchmark a number is closer to, if it is greater than that benchmark, or if it is less than that benchmark.</p>	<p>Determine which benchmark ($0, \frac{1}{2}, 1$) the following numbers are closer to.</p> <p>a) $\frac{2}{7}, \frac{11}{36}, \frac{36}{40}, \frac{27}{52}, \frac{1}{15}$ b) $0.9, 0.09, 0.125, 0.65, 0.659$</p>

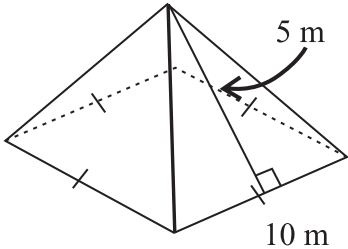
Chapter	Skill	Example
	For probability, the most common benchmarks are 0, $\frac{1}{2}$, and 1.	Complete the fraction so that it is close to $\frac{1}{2}$ but a bit larger: a) $\frac{x}{33}$ b) $\frac{42}{x}$ c) $\frac{x}{98}$ d) $\frac{60}{x}$
	Estimate probabilities	There are 3600 fish in the pond and 889 are speckled trout. Estimate the probability that you will catch a trout when you go fishing. (Students should see that 889 is close to 900 and so the probability is about $\frac{1}{4}$ or 0.25 or 25%)
	Compare and order numbers	Can reinforce work found in the Compare and Order Real Numbers sections in chapter 1
	Translating between the various forms of fractions, decimals, and percents Progression of this may be: <ul style="list-style-type: none"> ▪ familiar fractions to decimal form and/or percent form(eg: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{10}$) ‘Familiar fractions’ may vary according to the experience of the students. ▪ less familiar fractions/decimals to percent form($\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{8}$, $\frac{1}{6}$, $\frac{5}{9}$) ▪ fraction/decimal percents to decimal form. 	When given the following fraction, decimal or percent, supply the other two equivalent forms. <ul style="list-style-type: none"> a) $\frac{1}{4}$ b) $\frac{2}{3}$ c) $\frac{3}{8}$ d) 0.75 e) 0.125 f) 32% g) 9% h) $\frac{7}{100}$ i) $\frac{12}{20}$ (take the opportunity to discuss strategies for these translations; i.e. $\frac{12}{20}$ may be translated easily by expressing it in equivalent forms such as $\frac{60}{100}$, $\frac{6}{10}$, $\frac{3}{5}$)


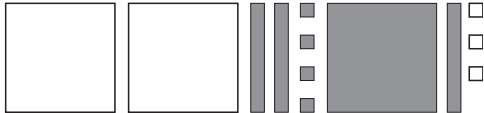
Chapter	Skill	Example
	<p>Review multiplying fractions and decimals, by whole numbers as well as fractions by fractions symbolically.</p> <p>The following are concepts students should recognize to shorten their computation time:</p> <ul style="list-style-type: none"> ▪ Discussion of multiplying reciprocals ▪ ‘Dividing out ones’ Example: In question c) students should notice that the denominator 3 and the numerator 6 have a common factor of 3. This common factor should be divided out. ▪ Whole number by a fraction, where the demoninator is a factor of the whole number. 	<p>Perform the indicated operations mentally:</p> <p>a) $\frac{3}{4} \times 12$</p> <p>b) $-10 \times \frac{2}{5}$</p> <p>c) $\frac{2}{3} \times \frac{6}{7}$</p> <p>d) $\frac{3}{5} \times \frac{4}{5}$</p> <p>e) $\frac{4}{9} \times \frac{9}{4}$</p> <p>f) 0.25×16</p> <p>g) 20×0.3</p>
	<p>Review calculating percents of a whole number using various strategies.</p>	<p>Mentally calculate each:</p> <p>a) $25\% \times 44$ (think $\frac{1}{4}$ of 44)</p> <p>b) $33 \frac{1}{3}\%$ of 93 (think $\frac{1}{3}$ of 93)</p> <p>c) 50% of 248 (think $\frac{1}{2}$ of 248 or halve/double strategy)</p> <p>d) 75% of 16 (think $\frac{3}{4}$ of 16)</p> <p>e) 12% of 300 (think $10\% \times 300 + 2\% \times 300$) or $(1\% \times 300) \times 12$</p> <p>f) 54% of 600 (think $\frac{1}{2} \times 600 + 4\% \times 600$)</p> <p>g) $\frac{1}{2}\%$ of 84 (think halve/double)</p> <p>h) $\frac{1}{4}\%$ of 200 (think halve/double twice)</p> <p>i) 9% of 500 (think $10\% \times 500 - 1\%$ of 500)</p>

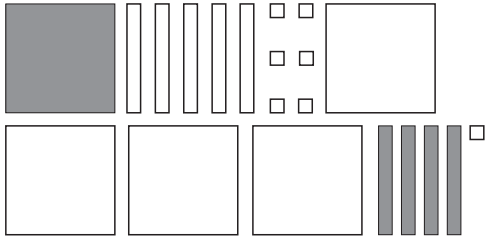
Chapter	Skill	Example
	<p>Review multiplying fractions, decimals, and/or percents in context Finding approx. percents in context.</p>	<p>$\frac{3}{4}$, (75%), or 0.75 of a class of 28 students are girls. How many are girls?</p> <p>20% of the grade nine students at your school like rap music. If 30 students like rap music, how many students are in your school?</p> <p>17 out of 24 students in your class have brown hair. Approximately, what percent of the students in your class have brown hair?</p>
<p>Measurement</p>	<p>Conversions can become memorizations of meaningless information for the students if they do not have practice in estimation of the measurements. Give students opportunities to practice their estimation skills. Example: Have the students work in partners, one with the metre stick and the other to do the estimating. Have the student put their hands together in a ‘prayer-like’ fashion and they will pull them apart according to the length given by the teacher. The student with the metre stick will measure the distance between the hands and record the amount the estimate was over or under the required distance. Variation: Have the students draw lines on their paper that are approximately the distance given by the teacher. They will then measure the distance and record the number of units over estimated or under estimated. This activity can be adapted to area, volume, and capacity by using familiar benchmarks of textbook covers, bottles of water, etc. Simple SI conversions (length, area, volume, volume to capacity)</p>	<p>Show the approximate length of</p> <p>a) 15 cm b) 70 mm c) 100 mm d) 100cm e) 50 cm f) 50mm</p> <p>Convert each of the following:</p> <p>g) 120 cm = _____m h) 1.2 km = _____m i) 0.3km = _____cm j) 2 m² = _____cm² k) 500 mm² = _____cm² l) 3.5 km² = _____m² m) 2 m³ = _____cm³ n) 5 000 mm³ = _____cm³ o) 500 cm³ = _____L p) 300 L = _____m³</p>

Chapter	Skill	Example
	Review square numbers and square roots.	Can reinforce work found in chapter one.
	<p>Have students estimate the solution to problems involving volume and surface area using appropriate estimation strategies. <i>Suitable diagrams for estimation of surface area are rectangular and triangular prisms, square and triangle based pyramids, cones, and spheres. For estimating volume, suitable diagrams may include rectangular and triangular prisms, square and triangle based pyramids, cylinders, and cones.</i></p>	<p>1. Estimate the surface area of each figure:</p> <p>a) square based pyramid</p>  <p>b) tetrahedron (4 equal triangles)</p>  <p>c) sphere with a radius of 0.49 cm</p>  <p>2. Estimate the Volume of these figures</p> <p>a) cone</p> 

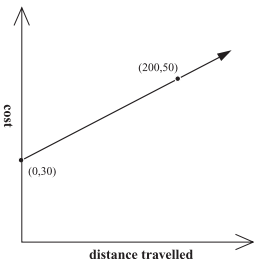
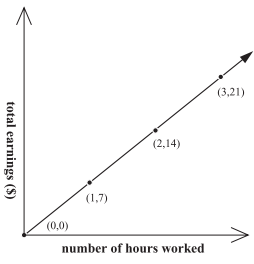
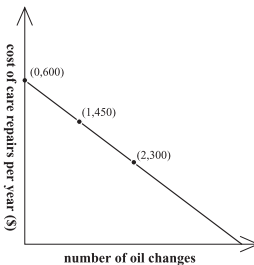
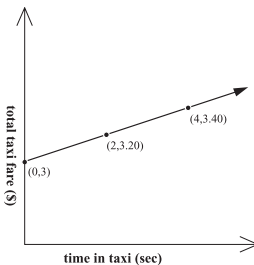
Chapter	Skill	Example
		<p>b) cylinder</p>  <p>c) prism</p> 
	<p>Make decisions on what problems about volume and surface area can be solved mentally. Recognize those that can use previously learned strategies (front-end, compatible factors, halve/double etc) Solve volume and surface area problems for prisms, pyramids, cylinders and cones mentally when appropriate. <i>Suitable diagrams are rectangular and triangular prisms, and pyramids.</i></p>	<p>1. a) Find the volume of a square based pyramid with base of 11m and height of 6 m</p>  <p>b) Find the Surface Area of this cube</p>  <p>c) Find the Volume of</p> 

Chapter	Skill	Example
		<p>d) Find the Surface Area of a square based pyramid with a base of 10 cm and a slant height of 5 cm</p> 
	<p>Have students create problems that can be estimated and/or solved mentally. These can then be presented to the class followed by a discussion of the use of efficient strategies.</p>	
<p>Geometry</p>	<p>You may wish to continue working on mental math topics from previous chapters.</p>	
	<p>Reinforce equivalent fractions.</p> <p>b) Determine equivalent fractions. Students are given a fraction that is not expressed in simplest form and asked to identify equivalent fractions from a list.</p>	<p>a) State three fractions equivalent to each of the following:</p> <p>i) $\frac{2}{3}$</p> <p>ii) $\frac{3}{5}$</p> <p>iii) $\frac{7}{8}$</p> <p>iv) $\frac{11}{12}$</p> <p>v) $\frac{13}{15}$</p> <p>b) In the list identify equivalent fractions for each given fraction (or you can also ask to identify ones that are not equivalent) and discuss why.</p> <p>i) $\frac{9}{12}$ ($\frac{3}{4}, \frac{8}{9}, \frac{15}{20}, \frac{2}{3}$)</p> <p>ii) $\frac{10}{15}$ ($\frac{15}{20}, \frac{24}{36}, \frac{6}{9}, \frac{1}{5}$)</p>

Chapter	Skill	Example															
	<p>Determine the missing value in a proportion. Use “friendly” numbers, whole at first, so the students can easily work between or within the ratios.</p> <p>This provides an opportunity for the students to focus on the multiplicative relationship of the ratios in the proportion.</p> <p>Progress then to other rational numbers.</p>	<p>Calculate the missing value :</p> $\frac{x}{6} = \frac{8}{12}$ $\frac{10}{x} = \frac{6}{18}$ $\frac{0.5}{2} = \frac{2}{x}$ $\frac{2}{6} = \frac{x}{27}$ $\frac{4.5}{x} = \frac{1}{4}$ $\frac{x}{4} = \frac{9}{x}$															
	<p>Students will be able to determine and use mapping rules. Given two of the three pieces of data(pre-image coordinates, image coordinates, and mapping rule) determine the third.</p>	<p>Complete the table : (present each row of the table as an individual problem)</p> <table border="1" data-bbox="824 867 1398 1157"> <thead> <tr> <th>Pre-image</th> <th>Mapping rule</th> <th>Image</th> </tr> </thead> <tbody> <tr> <td>(-4,7)</td> <td>$(x,y) \rightarrow (x+3,y-8)$</td> <td></td> </tr> <tr> <td>(-3,2)</td> <td></td> <td>(-3,-2)</td> </tr> <tr> <td></td> <td>$(x,y) \rightarrow (2x, 2y)$ $(x,y) \rightarrow (x- 1,y+6)$</td> <td>$(\frac{6}{8}, \frac{10}{12})$ (5,0)</td> </tr> <tr> <td>(3,4)</td> <td></td> <td>(-4,-3)</td> </tr> </tbody> </table>	Pre-image	Mapping rule	Image	(-4,7)	$(x,y) \rightarrow (x+3,y-8)$		(-3,2)		(-3,-2)		$(x,y) \rightarrow (2x, 2y)$ $(x,y) \rightarrow (x- 1,y+6)$	$(\frac{6}{8}, \frac{10}{12})$ (5,0)	(3,4)		(-4,-3)
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(3,4)		(-4,-3)															
<p>Polynomials</p>	<p>Use Alge-Tiles to model or represent expressions and have the students simplify these expressions. Overhead or magnetic tiles work well for this.</p>	<p>This is reinforcing the work done in chapter 3.</p>															
	<p>Show several different arrangements of Alge-tiles (A, B, ...E)</p> <p>a) Ask students to simplify and record the simplified expressions.</p> <p>b) Have them determine the resultant expression for such things as $A + B$, $A - B$, $2C$.</p>	<p>a) </p> <p>b) </p>															

Chapter	Skill	Example
	<p>c) Give students a resultant expression and an operation, and ask students which expression(s) underwent the operation to produce the resultant.</p> <p>Progression for this may begin with monomials, to binomials to trinomials and then to a mixture of polynomials. The progression depends on the experience of the students.</p>	<p>c) </p> <p>a) answers : A. $2x^2 + 3x - 2$ B. $-x^2 + 3x + 1$ C. $-3x^2 - x - 7$</p> <p>b) Simplify A + C B - C 2A</p> <p>c) Which two expressions will give the sum of $-4x^2 + 2x - 6$? The product of 2 and what expression will yield $-2x^2 + 6x + 2$?</p>
	<p>Review laws of exponents</p>	<p>See Chapter 1 for examples</p>
	<p>Identify the GCF of a binomial mentally. The progression may be from a numerical factor, to a variable and then both as GCF's.</p>	<p>Identify the GCF of each pair:</p> <p>a) 24, 36 b) 44, -88 c) -56, -63 d) x, 2x e) $3x^2, x^2$ f) $-4x^2, 6x$ g) $45y^3, 36y^2$</p> <p>Identify the GCF in each expression: $6x + 9$ $-4y - 10$ $12k - 30$ $3x^2 + 4x$ $13y^4 - 4y^2$ $x^2y^6 - 7y^2$ $10x + 15x^3$ $2m^6 - 4m^3$</p>

Chapter	Skill	Example								
	Students will be able to multiply a monomial by polynomials using “friendly” numbers with previously learned strategies.	Determine the product of each: $(3x)(4x^5)$ $(4x^2)(13x^3)(25x^{-1})$ $(\frac{1}{4}x^3)(12x^{-2})$ $(0.5x^9)(86x^2)$ $(22x)(15x)$ $(4x^6)(99x^{-2})$ $(25y^{-5})(36y^{-7})$ think $25 \times (4 \times 9) = (25 \times 4) \times 9$								
	Have students solve problems such as “find two numbers that multiply to give _____ and add to give _____” and visa versa	Find two numbers that multiply to give _____ and add to give _____ <table border="1" data-bbox="824 724 1396 861"> <thead> <tr> <th>Multiply to give</th> <th>Add to give</th> </tr> </thead> <tbody> <tr> <td>6</td> <td>5</td> </tr> <tr> <td>-20</td> <td>8</td> </tr> <tr> <td>30</td> <td>-11</td> </tr> </tbody> </table>	Multiply to give	Add to give	6	5	-20	8	30	-11
Multiply to give	Add to give									
6	5									
-20	8									
30	-11									
	Multiply two binomials, and factor expressions into two binomials using “friendly” numbers.	Determine the product of each: $(x + 3)(x + 8)$ $(y - 4)(y + 5)$ $(z - 12)(z - 3)$ $(b + 7)(b - 7)$ Factor each expression into two binomials $x^2 + 4x + 3$ $x^2 + 7x - 18$ $y^2 - 10y + 24$ $x^2 + x + 0.25$ $x^2 - 36$ $x^2 - 1.44$ $x^2 + \frac{9}{2}x + 2$ $x^2 + \frac{37}{3}x + 4$								
	Make decisions on which polynomials can be factored into two binomials.	Determine which expressions can be factored and discuss why. a) $x^2 + 16$ b) $x^2 - 16$ c) $x^2 + 7x + 10$ d) $x^2 - 9x + 18$ e) $x^2 + 4x - 5$ f) $x^2 + 8x - 7$ g) $x^2 + x$								

Chapter	Skill	Example
	Divide a polynomial by a monomial.	Simplify a) $\frac{36x^6}{4x^4}$ b) $\frac{2.4x^3}{1.2x^2}$ c) $\frac{(12x^2 + 4x + 20)}{4}$ d) $\frac{15x^6}{0.5x}$
	Have students create expressions that can be added, subtracted, multiplied or divided mentally.	
Data Management	Create sets of data and have students mentally calculate the mean using prior strategies.	Calculate the mean of each. 28 + 36 + 22 + 34 (compatibles) 75 + 29 + 46 + 54 (break up and bridge) 4.6 + 3.5 + 8.4 + 1.5 + 2 (make one) 410 + 120 + 330 + 140 (front end addition or break up and bridge)
	Review work on slope. This can be extended to students calculating and describing the meaning of the slope and y-intercept in context when presented with labeled graphs.	Reinforcement of work done in chapter two where students determined the slope from a graph and from an equation. <div style="display: flex; flex-wrap: wrap; justify-content: space-around;"> <div style="text-align: center; margin: 10px;">  <p>cost</p> <p>distance travelled</p> <p>Points: (0,30), (200,50)</p> </div> <div style="text-align: center; margin: 10px;">  <p>total earnings (\$)</p> <p>number of hours worked</p> <p>Points: (0,0), (1,7), (2,14), (3,21)</p> </div> <div style="text-align: center; margin: 10px;">  <p>total cost of oil changes (\$)</p> <p>number of oil changes</p> <p>Points: (0,600), (1,450), (2,300)</p> </div> <div style="text-align: center; margin: 10px;">  <p>total taxi fare (\$)</p> <p>time in taxi (sec)</p> <p>Points: (0,3), (2,3.20), (4,3.40)</p> </div> </div>

Chapter	Skill	Example
	Present scenarios and ask students to determine if the data would represent continuous or discrete data.	Determine if the data collected would be an example of continuous or discrete data: a) Filling up the gas tank of a car. Cost vs number of litres bought b) buying a quantity of newspapers. Cost vs number of papers purchased.
	Review work on evaluating expressions	Reinforcement of work done in chapter two where students evaluated various expressions using whole numbers, fractions, and decimals.

