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Interest Rate Risk Stemming from Liquidity Risk, Gap and Duration Analysis in Banks

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Abstract. In this article, Interest Rate Risk of Liquidity Risk which banks are most exposed and the most commonly used Gap and Duration methods in measuring this risk are studied. To calculate exposure to interest rate risk Gap and Duration Gap methods were thoroughly investigated and real analyzes on banks was led.

Key Words and Phrases: Liquidity, Interest Rate Risk, Liquidity Risk, Value at Interest Rate Risk, Gap, Duration, Duration Gap.

1. The interest rate risk created by liquidity risk

One of the liquidity risks is the risk arising due to the difference between the Assets and Liabilities over the maturity difference (gap). If the difference of the payment duration between the Asset and Liability is positive, then there is a surplus liquidity or a re-investment risk. Otherwise, there is a risk of placing the positive balance in the market at the favorable interest rate. If the difference between the payment duration of Asset and Liability is negative, then there is a risk of re-financing. In other words, there will be a risk of borrowing from market at lower interest rates. Interest rate risk stemming from liquidity risk is a risk created by the difference between interest-earning assets and liabilities in all currencies.

To summarize , If the difference between the amount of sensitive assets and liabilities in each currency is negative, there will be a interest rate risk arising from the risk of liquidity shortage within the interval. That is, the risk of re-financing will create a risk of interest. If the difference in the interval is negative, then the bank will face the loss of interest as the result of an increase in interest rates. Similarly, in each currency, if the difference between the interest rate sensitive assets and liabilities on the interval is positive then there is a interest rate risk created by the excess liquidity risk on that interval. That is, the risk of recurring investment will generate interest rate risk. If the difference in interval is positive, then the loss of interest rates will result in bank loss.

The following two methods are used to measure the interest rate risk created by the liquidity risk:

1. Gap method
2. Duration analyses

2. Gap method

Another name of this method is a method of re-evaluation. Gap analysis is one of the first methods to measure the interest rate risk exposed by the liquidity risk of banks. Through this method, sensitive assets and liabilities are revalued at current market rates (market current) and the loss arising from changes of interest rates are calculated.

First of all, let's see if there is any liquidity risk. For this purpose, term report of assets and liabilities of the bank in each currency is prepared in manat equivalence. This report is the distribution of assets and liabilities falling into same periods. A distribution Schedule captures payment intervals of up to 1 month, 1 to 3 months, 3 to 6 months, 6 to 9 months, 9 months to 1 year, 1 year to 3 years, 3 to 5 years, more than 5 years and non-specified payment time intervals. Non-performing loans are those have past unpaid due and other non-current interest-bearing assets and liabilities are included in the uncertain term. Then, Assets and Liabilities at relevant intervals are deduced from each other. The difference between assets and liabilities relating to the same maturity bucket is called liquidity gap. The gap is calculated as follows:

$$GAP_i = A_i - L_i$$

where, GAP_i, the gap of i-th interval, A_i, assets of i-th interval, L_i, liabilities of i-th interval. Let's look at the following table:

Assets	**up to 1 month**	**1 to 3 months**	**3 to 6 months**	**6 to 9 months**	**9 to 12 months**	**1 to 3 years**	**3 to 5 years**	**more than 5 year**	**uncertain**
Cash resources	11000								
Nostro correspondent accounts	17000								
Securities	7000	3000	2500	2000	3000	5000			
Interbank loans and deposits	9000	5000							
Receipts from other financial institutions		2000		3500					
Investments receipts									
Receipts from customer loans	27000	17000	22000	39000	37000	43000	42000	5000	27000
Other receipts									
Receipts on total assets	**71000**	**27000**	**24500**	**44500**	**40000**	**48000**	**42000**	5000	27000
Liabilities	**up to 1 month**	**1 to 3 months**	**3 to 6 months**	**6 to 9 months**	**9 to 12 months**	**1 to 3 years**	**3 to 5 years**	**more than 5 year**	**uncertain**
Current accounts of customers	65 000								
Vostro correspondent accounts	4 000								
Loans and deposit liabilities to banks		3000		2500	7000	6000	1000		
Obligations to other financial institutions									
Securities obligations									
Liabilities on foreign projects	500	500	500	500	1 500	4500	500	4500	
Commitments on local projects		2000		3000		2500			
Customer deposits	7000	13000	31000	27000	37000	42000	47000		
Other commitments									
Payables on total liabilities	**76500**	**18500**	**31500**	**33000**	**45500**	**55000**	**48500**	4500	
Liquidity gap	**(5500)**	**8500**	**(7000)**	**11500**	**(5500)**	**(7000)**	**(6500)**	500	27000
Cumulative Liquidity gap	**(5500)**	**3000**	**(4000)**	**7500**	**2000**	**(5000)**	**(11500)**	(11000)	16000
Liquidity gap including off-balance sheet liabilities									
Credit lines	2000	3500	1500	4500	2500	3000			
Liquidity gap	**(7500)**	**5000**	**(8500)**	**7000**	**(8000)**	**(10000)**	**(6500)**	500	27000
Cumulative Liquidity gap	**(7500)**	**(2500)**	**(11000)**	**(4000)**	**(12000)**	**(22000)**	**(28500)**	(28000)	(1000)

The source: *The author's work*

As a result cumulative interest sensitive difference is arriveed at adding differences of each preceeding intervals

As noted above, a negative difference between the amounts of the Assets and Liabilities in each currency is a risk of liquidity shortage within that interval. As it can be seen in the table above, there is a negative balance in the amount of 5 500 000, -7 000 000 and -5 500 000 AZN, in the period up to one [0,1], [3,6] and [9,12] intervals. Likewise, a positive difference between the Assets and Liabilities at intervals in each currency is a risk of excess liquidity on that interval. As seen in the table above, there is a positive gap in the amount of 8 500 000 and 11 500 000 AZN according to [1,3] and [6,9] intervals.

Let us now consider the interest rate risk created by the liquidity risk. For this purpose, the Bank shall prepare a report on the interest rate sensitive assets and interest rate sensitive liabilities in manat equivalence in each currency. The report is the same as for the method of preparing a liquidity statement above, but the difference here is the consideration of not all assets and liabilities but that of sensitive assets and liabilities. This report is a distribution of sensitive assets and liabilities to the same period. Subsequently, differences in interest rate sensitive Assets and Liabilities are deducted from each other at appropriate intervals. The difference between the interest rate sensitive assets and liabilities relating to the payment process is called interest bearing liquidity gap. The gap is calculated as follows:

$$ISGAP_i = ISA_i - ISL_i$$

Where, $ISGAP_i$ is the interest rate sensitive gap on i-th interval, ISA_i is the interest rate sensitive assets on i-th interval, ISL_i is the interest rate sensitive liabilities on i-th interval. Following table is prepared by considering only interest rate sensitive aseets and liabilities from above table:

Interest bearing assets	**up to 1 month**	**1 to 3 months**	**3 to 6 months**	**6 to 9 months**	**9 to 12 months**	**1 to 3 years**	**3 to 5 years**	**more than 5 year**	**uncertain**
Securities	7000	3000	2500	2000	3000	5000			
Interbank loans and deposits	9000	5000							
Receipts from other financial institutions		2000		3500					
Investments receipts									
Receipts from customer loans	27000	17000	22000	39000	37000	43000	42000	5000	27000
Total Interest bearing assets	**43000**	**27000**	**24500**	**44500**	**40000**	**48000**	**42000**	**5000**	**27000**
Interest bearing liabilities	**up to 1 month**	**1 to 3 months**	**3 to 6 months**	**6 to 9 months**	**9 to 12 months**	**1 to 3 years**	**3 to 5 years**	**more than 5 year**	**uncertain**
Loans and deposit liabilities to banks		3000		2500	7000	6000	1000		
Obligations to other financial institutions									
Securities obligations									
Liabilities on foreign projects	500	500	500	500	1 500	500	4500		
Commitments on local projects		2 000		3 000		2500			
Customer deposits	7000	13000	31000	27000	37000	42000	47000		
Total interest bearing liabilities	**7500**	**18500**	**31500**	**33000**	**45500**	**55000**	**48500**	**4500**	**-**
Interest sensitive liquidity gap	**35500**	**8500**	**(7000)**	**11500**	**(5500)**	**(7000)**	**(6500)**	**(500)**	**(27000)**
Cumulative Interest sensitive liquidity gap	**35500**	**44000**	**37000**	**48500**	**43000**	**36000**	**29500**	**30000**	**57000**

The interest rate risk arising from liquidity shortage risk will be available similarly, if there is a difference between the amounts of interest rate sensitive assets and liabilities in each currency,. As it is seen in the table, there is a negative balance in the amount of 7,000,000 and -5,500,000 manats at intervals of [3,6] and [9,12] within one year. If the difference in interval is negative, the bank will face the loss of interest as the interest rate increases. Similarly, if there is a difference between the amounts in the interval to the interest rate sensitive assets and liabilities in each currency, there will be a interest rate risk created by an exposure to liquidity risk over that interval. There is a positive gap in the amount of 35,000,000, 8,500,000 and 11,500,000 manats in the [0,1], [1,3] and [6,9] intervals as long as it appears in the table above. Likewise, if the difference in intervals is positive, then the bank will face the loss of interest as a result of the decline in interest rates.

3. Gap ratio

Gap ratio is calculated as follows:

$$Gap\ Ratio = \frac{ISA}{ISL}$$

If ***Gap Ratio >1*** there is re-investment, if ***Gap Ratio <1*** then re-financing is existed.

4. Gap ratio limits

In international practice Gap Ratio is kept at the following intervals:

$$0.8 < Gap\ Ratio < 1.2$$

This interval allows banks to accept a more acceptable interest rate risk and manage interest rate risk.

If the bank owes a short-term and makes long-term placement, then the short-term gap in the bank will be negative and the long-term gap will be positive, and if the bank ows long-term and makes short-term placement, then the short-term gap in the bank will be positive, long-term gap will be negative. Both situations expose the bank to interest rate risk. Let's look at the following table:

Interest bearing assets	**Average interest rate**	**up to 1 month**	**1 to 3 months**	**3 to 6 months**	**6 to 9 months**	**9 to 12 months**	**1 to 3 years**	**3 to 5 years**	**more than 5 year**	**uncertain**
Securities	4%	7 000	3 000	2 500	2 000	3 000	5000			
Interbank loans and deposits	7%	9 000	5 000							
Receipts from other financial institutions	9%		2 000		3 500					
Investments receipts	0%									
Receipts from customer loans	17%	27 000	17 000	22 000	39 000	37 000	43000	42000	5000	27000
Total Interest bearing assets		**43000**	**27000**	**24500**	**44500**	**40000**	**48000**	**42000**	**5000**	**27000**
Interest bearing liabilities	**Average interest rate**	**up to 1 month**	**1 to 3 months**	**3 to 6 months**	**6 to 9 months**	**9 to 12 months**	**1 to 3 years**	**3 to 5 years**	**more than 5 year**	**uncertain**
Loans and deposit liabilities to banks	3.0%		3 000		2 500	7 000	6000	1000		
Obligations to other financial institutions	0.0%									
Securities obligations	0.0%									
Liabilities on foreign projects	2.0%	500	500	500	500	1 500	4500	500	4500	
Commitments on local projects	3.0%		2 000		3 000		2500			
Customer deposits	9.0%	7000	13000	31000	27000	37000	42000	47000		
Total interest bearing liabilities		**7500**	**18500**	**31500**	**33000**	**45500**	**55000**	**48500**	**4500**	
Interest sensitive liquidity gap		**35500**	**8500**	**(7000)**	**11500**	**(5500)**	**(7000)**	**(6500)**	**500**	**27000**
Cumulative Interest sensitive liquidity gap		**35500**	**44000**	**37000**	**48500**	**43000**	**36000**	**29500**	**30000**	**57000**
Interest sensitive margin		**4860**	**2210**	**1040**	**4420**	**2840**	**3385**	**2870**	**760**	**4590**
Cumulative interest sensitive margin		**4860**	**7070**	**8110**	**12530**	**15370**	**18755**	**21625**	**22385**	**26975**

The loss on interest rate risk created by the liquidity risk is calculated by the following formula:

$$\Delta NII_i = GAP_i \times \Delta R_i$$

Where, ΔNII_i, net interest income on i-th interval, GAP_i, gap on i-th interval, ΔR_i – the change of the interest rates of assets or liabilities on i-th interval. If the gap is positive then the change of interest rates of assests should be considered, otherwise interest rate change of liabilities should be. As the table shows, the sensitive margin on interest rates on the interval is as follows:

Intervals	**up to 1 month**	**1 to 3 months**	**3 to 6 months**	**6 to 9 months**	**9 to 12 months**	**1 to 3 years**	**3 to 5 years**	**more than 5 year**	**uncertain**
Interest sensitive margin	**4 860**	**2 210**	**1 040**	**4 420**	**2 840**	**3 385**	**2 870**	**760**	**4 590**

5. Value at Interest Rate Risk

If any gap on any interval is negative, then there is a risk of liquidity shortage as it is mentioned. The current interest rate and actual interest rate difference should be looked at for this interval. If the actual interest rate exceeds the current interest rate then the gap is calculated by multiplying to the change of interest rate. Likewise, if the gap for any interval is positive, then there is a risk of excess liquidity on that interval. The current interest rate and actual interest rate difference should be looked at for this interval. If the actual interest rate is lower than the current interest rate then the loss gap is calculated by multiplying to the change of interest rate.
In our example, if the interest rates fall below 1% then the loss will be as follows:

Intervals	**up to 1 month**	**1 to 3 months**	**3 to 6 months**	**6 to 9 months**	**9 to 12 months**	**1 to 3 years**	**3 to 5 years**	**more than 5 year**	**uncertain**
Interest sensitive margin	**4 860**	**2 210**	**1 040**	**4 420**	**2 840**	**3 385**	**2 870**	**760**	**4 590**
Loss in case of 1% change of interest rate	**49**	**22**	**10**	**44**	**28**	**34**	**29**	**8**	**46**

As can be seen from the table, if interest rate drops below 1% in the [6,9] month interval, then the loss will be 44,000 AZN.

6. Duration method

The shortcoming of the gap model is that the interest-bearing assets and liabilities in this model are dealt with at their fair values. For certain periods of time, for example, customers' credit interest payments and deposit interest payments to customers are not taken into account in this calculation. In this case, a more accurate and effective method of duration analysis can be applied.

Duration is the weighted average of all the cash flows that can be derived from a financial asset until the end of the asset. In other words, the duration is the average of the cash flows, that is, duration is the average term of the financial asset. The duration is an effective period of a financial asset. This type of duration is called Macaulay duration.

Calculation formula for the duration is as follows:

$$Duration = \frac{\sum_{t=1}^{n} PV_t \times t}{\sum_{t=1}^{n} PV_t}$$

Where, PV_t – current value, t – cash flow flood numbers.

7. Current and Future values

$$FV = PV \times (1 + r)^n$$

$$PV = \frac{FV}{(1 + r)^n}$$

Where, FV – future value , PV – present value, r – interest rate and t – cash flow numbers.

8. Duration formula

Given the above, another calculation formula for the duration will be as follows:

$$Duration = \frac{\sum_{t=1}^{n} \frac{CF_t}{(1+r)^t} \times t}{\sum_{t=1}^{n} \frac{CF_t}{(1+r)^t}}$$

Where, CF_t – cash flows, n – the number of cash flows, r – interest rate.

9. Calculation of duration

Let's assume that the bank gives 10,000 AZN loan at 20% for 12 months. The form of payment should be quarterly and equal to the principal amount in the form of accrued interest. The payment schedule will be as follows:

Months	**Payment on main amount**	**Payment on interest amount**	**Total payment**
1	833	167	1,000
2	833	153	986
3	833	139	972
4	833	125	958
5	833	111	944
6	833	97	931
7	833	83	917
8	833	69	903
9	833	56	889
10	833	42	875
11	833	28	861
12	833	14	847

The table above includes 10,000 manat loan, with 12 months maturity at 20% interest rate which pays it's principal amount with 833 manat installments with interest payments for the balance of the loan. Here, the total payment will be the cash flow (CF) for us. Taking this into consideration

$$PV = \frac{CF_t}{(1+r)^t}$$

the formula for the present value (PV) calculate:

CF	**PV**	**PV*t**
1,000	984	984
986	954	1,908
972	925	2,776
958	897	3,588
944	870	4,348
931	843	5,056
917	817	5,716
903	791	6,328
889	766	6,894
875	742	7,417
861	718	7,897
847	695	8,337

The duration of the calculation, taking into account the values in this table and the formula for duration

$$Duration = \frac{\sum_{t=1}^{n} PV_t \times t}{\sum_{t=1}^{n} PV_t}$$

$= 6.12$

will be obtained. Therefore, a loan with an amount of 10 000 manat at 20% interest rate with 12 months maturity, paying back its principal amount in equal installments plus interest amount to the residual balance will have an average duration of 6.12 months.

Another example again, this time with 20% of 10 000 credit to be given for a period of 12 months. This time the form of payment is in annuity form. The payment schedule will be as follows:

Months	**Payment on main amount**	**Payment on interest amount**	**Total payment**
1	760	167	926
2	772	154	926
3	785	141	926
4	798	128	926
5	812	115	926
6	825	101	926
7	839	87	926
8	853	73	926
9	867	59	926
10	882	45	926
11	896	30	926
12	911	15	926

The total payment will be cash flow (CF) for us. Considering this, the present value (PV) calculate:

CF	**PV**	**PV*t**
926	911	911
926	896	1,792
926	882	2,645
926	867	3,468
926	853	4,264
926	839	5,033
926	825	5,776
926	812	6,493
926	798	7,185
926	785	7,852
926	772	8,496
926	760	9,116

Taking into account the values in this table in the formula of duration,

$$Duration = 6.30$$

will be obtained as above. If the loan amount is 10,000 manat, 20 percent interest rate and 12 months maturity, the loan term will be 6.30 months.

If the loan is given for a period of 1 year with principal amount and interest rate payable at maturity, then the duration of the loan will be 1 year. Because, in this case there will be only 1 time cash flow.

To calculate the duration of the loan portfolio, the duration of each loan is calculated and the duration of the portflio is arrived at with weighted average method. A similar calculation can be made for a deposit portfolio. Therefore, each interest bearing asset and liability duration can easily be calculated.

If our deposit portfolio consists of deposits maturing at the end of the deposit term in full amount , then the duration of our deposit portfolio will be equal to the entire portfolio duration. Here are some examples of deposits.

1. Let's look at the deposit of 5,000 AZN, invested for 12 months at a rate of 10% and monthly interest payments:

Months	**Payment on main amount**	**Payment on interest amount**	**Total payment**	**CF**	**PV**	PV*t
1	-	42	42	42	41	41
2	-	42	42	42	41	82
3	-	42	42	42	41	122
4	-	42	42	42	40	161
5	-	42	42	42	40	200
6	-	42	42	42	40	238
7	-	42	42	42	39	275
8	-	42	42	42	39	312
9	-	42	42	42	39	348
10	-	42	42	42	38	383
11	-	42	42	42	38	418
12	5,000	42	5,042	5,042	4,564	54,765

   The duration of this deposit will be 11.47 months.

2. Let's look at the deposit of 5,000 AZN, placed for 12 months at a rate of 10% and quarterly interest rate repayment:

Months	Payment on main amount	Payment on interest amount	Total payment	CF	PV	PV*t
1	-	-	-	-	-	-
2	-	-	-	-	-	-
3	-	125	125	125	122	366
4	-		-	-	-	-
5	-		-	-	-	-
6	-	125	125	125	119	714
7	-		-	-	-	-
8	-		-	-	-	-
9	-	125	125	125	116	1,044
10	-		-	-	-	-
11	-		-	-	-	-
12	5,000	125	5,125	5,125	4,639	55,671

In this case, the deposit will be 11.57 months.

10. Relationship between interest rate risk and duration

To prevent a financial asset (credit, securities, etc.) from being exposed to interest rate risk, the effect of changes in interest rates on the fair value of the financial asset can be met the re-investment of cash flows from the financial asset (re-investment risk) . This can be done by applying a concept of duration.

11. Duration gap

The duration gap is the difference between the duration of the Assets and the duration of the liability. Duration gap method used to measure the risks arising from fluctuations in interest rates. In other words, this method is way of measuring the effect of changes of interest rate sensitive assets and liabilities on interest rates when market interest rates change. The duration gap is calculated as follows:

$$DUR_{GAP_i} = DUR_{ASSET_i} - DUR_{LIAB_i} \times \frac{LIAB_i}{ASSET_i}$$

Where, DUR_{GAP_i}, the duration gap on i-th interval, DUR_{ASSET_i}, duration of vulnerable assets against interest rate on i – th interval, DUR_{LIABi}, duration of vulnerable liabilities against interest rate on i – th interval, $ASSET_i$, vulnerable assets against interest rate on i – th interval, $LIAB_i$, vulnerable liabilities against interest rate on i – th interval. The duration of both the assets and the liabilities can be calculated easily based on the examples given above.

We can say the following hypotheses about the duration gap:

1. If the gap is positive then the bank is sensitive to interest rate changes . In this case, when the interest rates rise, the assets will lose more value than liabilities, and if the interest rates will fall, the assets are more valuable than liabilities.

2. If the gap is negative, then the bank is sensitive to the interest rate changes. Similarly, when the interest rates rise, liabilities will lose more value to assets and interest rates will fall, whereas liabilities will be more valuable than assets.

There are some difficulties with duration gap mentioned as follow:

1. Early repayments of customers may disrupt the planned cash flows

2. Customers' delayed credit repayment can disrupt planned cash flows

3. Early withdrawal can disrupt planned cash flows

4. Cash flows for some assets and liabilities may not be clearly defined

12. Duration gap analysis

Let's take into account their assets and liabilities as well:

Assets	**Amount**	**Duration**
Cash resources	11 000	0
Nostro correspondent accounts	17 000	0
Securities	22 500	0.7
Interbank loans and deposits	14 000	0.1
Receipts from other financial institutions	5 500	0.5
Investments receipts	0	0
Receipts from customer loans	276 000	1.5
Other receipts	0	0
Receipts on total assets	**346 000**	
Liabilities	**Amount**	**Duration**
Current accounts of customers	65 000	0
Vostro correspondent accounts	4 000	0
Loans and deposit liabilities to banks	19 500	1.2
Obligations to other financial institutions	0	0
Securities obligations	0	0
Liabilities on foreign projects	13 000	2.7
Commitments on local projects	7 500	1.0
Customer deposits	204 000	1.6
Other commitments	0	0
Payables on total liabilities	**313 000**	

We can use the formula

$$DUR_{asset} = \sum_i A_i \times D_i \quad and \quad DUR_{liab} = \sum_i L_i \times D_i$$

to calculate the average duration of both assets and liabilities. Where, DUR_{asset} and DUR_{liab} the average duration of the assets and liabilities, respectively. A_i and L_i assets and liabilities respectively, D_i D_i is their duration. By calculating the average positions of both assets and liabilities using these formulas,

$$DUR_{asset} = 1.25 \quad and \quad DUR_{liab} = 1.28$$

are obtained. Taking these values into account in terms of the formula of gap duration,

$$DUR_{GAP} = DUR_{ASSET} - DUR_{LIAB} \times \frac{LIAB}{ASSET} = 1.25 - 1.28 \times \frac{313\ 000}{346\ 000} = 0.092$$

is obtained.
Net income / expense resulting from changes in interest rates is calculated as follows:

$$\Delta NW = -DUR_{GAP} \times \frac{\Delta i}{i+1}$$

Where, ΔNW – net incomes / expence, Δi – change of interest rate, i – interest rate.
As mentioned above, if the duration gap is positive, then the bank is sensitive to interest rate fluctuations. Due to this case, when the interest rates rise, the assets will lose more value than liabilities, and if the interest rates will fall, the assets are more valuable than liabilities.
Let's assume that the average interest rate on assets is 10% and the average interest rate is 1%. Then,

$$\Delta NW = -DUR_{GAP} \times \frac{\Delta i}{i+1} = -0.092 \times \frac{0.01}{1+0.1} \approx -0.00084 = -0.084\%$$

Loss of bank as a result of 1% interest rate is

$$\Delta NW = \%\Delta NW \times AKT = -0.084\% \times 346\ 000\ 000 = \ -290\ 640$$

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