Solving Exponential Equations:

There are two strategies used for solving an exponential equation. The **first strategy**, if possible, is to write each side of the equation using the same base.

Ex 1: Solve: $4^{x-1} = 32^{2x+3}$

Both bases, 4 and 32, can be written as powers of base 2.

$$(2^2)^{x-1} = (2^5)^{2x+3}$$

Use the exponent rule for a power to a power (multiply exponents).

$$2^{2x-2} = 2^{10x+15}$$

Since the bases are the same, the exponents must be the same.

$$2x - 2 = 10x + 15$$

$$-17 = 8x$$

$$\boxed{-\frac{17}{8} = x}$$

Use the procedure demonstrated in example 1 to solve the following exponential equations.

Ex 2: (a) $25^{5x} = 625^{x-3}$ (b) $81^{\frac{1}{2}x-1} = \left(\frac{1}{3}\right)^{x+2}$

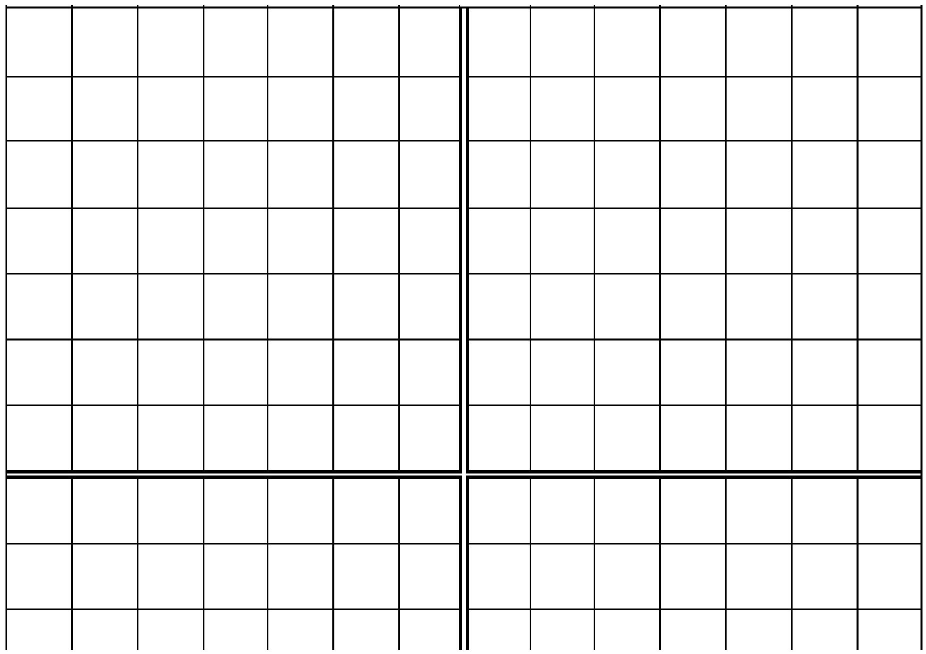
The **second strategy** to solve an exponential equation will be discussed in a future lesson.

Exponential Functions

<u>Exponential Functions:</u> <u>A basic exponential function</u> has the form $f(x) = b^x$ or $y = a \cdot b^x$, where the base b is any positive real number other than 1 , x the exponent is any real number, and the constant a is any real number.

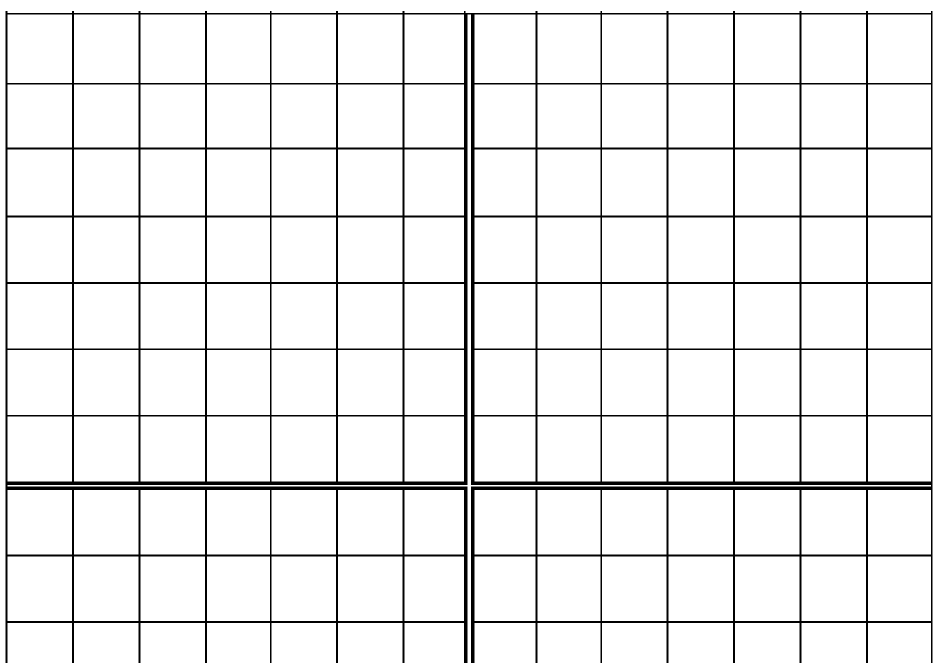
<u>Ex 3:</u> Complete the table for the exponential function $g(x) = \left(\frac{3}{2}\right)^x$ and sketch its graph.

x	y
-3	
-2	
-1	
0	
1	
2	
3	



<u>Ex 4:</u> Complete the table for the exponential function $h(x) = \left(\frac{1}{2}\right)^{x+1}$ and sketch its graph.

x	y
-3	
-2	
-1	
0	
1	
2	
3	



The graphs of the exponential functions on the previous page and others that could be sketched, lead to the following characteristics of a basic exponential function $y = f(x) = b^x$ $(a\ is\ 1)$.

(1) The x-axis is a horizontal asymptote and the graph will approach the x-axis (line with equation $y = 0$) either as the graph goes toward positive infinity or toward negative infinity.

(2) The function values (y-values) of all basic exponential functions are positive. Therefore the graphs always lie above the x-axis.

(3) Basic exponential functions have a y-intercept of (0, 1).

(4) If the base b is between 0 and 1 ($0 < b < 1$), the graph will be decreasing and the function represents exponential decay. If the base b is greater than 1 ($b > 1$), the graph will be increasing and the function represents exponential growth.

Exponential functions can model real life situations.

Ex 8: For example, suppose a mall found that the following function could be used to find the average amount a random person spent at the mall after a time of x hours.

$$f(x) = 42.2(1.56)^x$$

Find the average amount spent by a person who is at the mall for 3 hours. Round to the nearest dollar.

$$f(3) = 42.2\,(1.56)^3$$

Ex 9: Suppose a culture of bacteria begins with 1000 bacteria at time 0. The number of bacteria is doubling every day. The function $f(t) = 1000(2)^t$ represents the number of bacteria present in the culture after t days. Approximate the number present after the following number of days.

(a) 3.5 days

(b) $11\frac{1}{3}$ days

<u>Ex 10:</u> The half-life of an isotope is the time it takes for one-half of the original amount in the given sample to decay. Suppose the polonium isotope ^{210}Po has a half-life of approximately 140 days. If 20 grams is present initially, then this function will determine the amount remaining after *t* days.

$$A(t) = 20\left(\frac{1}{2}\right)^{\frac{1}{140}t}$$

Find the number of grams present after (a) 100 days, (b) 300 days, and (c) 1000 days. Round to the nearest whole number.

One of the most common exponential functions used in real life are the compound interest formulas used by the financial institutions.

If interest is compounded periodically *n* times per year, then the following determines the amount in an account *A* that begins with *P* dollars. (*r* is the annual interest rate <u>as a decimal</u>, *t* is time in years).

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

<u>Ex 11:</u> Find the amount of money in an account whose initial investment was \$2500 with an annual interest rate of $1\frac{3}{4}\%$ annual interest compounded quarterly for 3 years.

<u>Ex 12:</u> To the nearest dollar, how much money should be invested <u>now</u> in an account that earns 4% annual interest compounded monthly to amount to $2000 in 10 years.

Some banks or financial institutions offer continuously compounded interest. The formula for this type of account includes the number e. The number e, like the number π, is an irrational number that never terminates nor repeats. Your TI-30XA calculator has e^x above the 'ln' key. To determine the value of e (e^1): enter a 1 in the calculator, press the 2^{nd} key, then the 'ln' key. You can see that the number e is approximately 2.71828 to 5 decimal places. If you need to find e^5: Enter 5 in the calculator, press the 2^{nd} key, then the 'ln' key.

If interest is compounded continuously, then the following determines the amount in an account *A* that begins with *P* dollars (*r* is the annual interest rate as a decimal and *t* is time in years).

$$A = Pe^{rt}$$

<u>Ex 13:</u> Jon invests $5000 in an account that earns 3.7% annual interest compounded continuously. How much does Jon have in his account in 4 ½ years?

The number e is just like any other number. So the rules for exponents and other rules in algebra apply. Simplify these expressions.

$e^5 e^{12} e^{-7} =$ $\qquad$ $\dfrac{ee^2}{e^4} =$

$e^{15x} e^{25x} =$ $\qquad$ $e^{2x} e^6 =$

$(e^x)^{11} =$ $\qquad$ $\dfrac{e^4}{e^4} =$

$(e^{2x} + e^{-2x})^2 =$

$(e^{5x} + e^{-5x})(e^{5x} - e^{-5x}) =$

$(e^{12x} - e^{-12x})^2 =$

<u>Using your TI-30XA calculator to approximate power of *e*:</u>
Enter the exponent first. Enter the 2nd key and then the LN key. (Notice that e^x if over the LN key, so using the 2nd function key yields powers of e.)

$e^3 =$ $\qquad$ $e^{-1.5} =$ $\qquad$ $e^0 =$

<u>Ex 14:</u> Evaluate the following function at the given values.

$$f(x) = -3e^{0.6x} + 2.45$$

(a) $f(0) =$

(b) $f(2.8) =$

(c) $f(-4.2) =$

Exponential functions can sometimes be determined given certain information.
Ex 14: For example, suppose there is a culture of a radioactive material that begins at 0 hours with 5 dekagrams. After 10 hours, there are 2 dekagrams. If the radioactive material is decreasing exponentially, find a function to represent this situation. Hint: Begin with the general format $y = a \cdot b^t$ and solve a system of two equations. (You will have to find the constants, a and b.)

Ex 15: In 1938, a federal law established a minimum wage. It was initially set as $0.25 per hour; the minimum wage had risen to $5.15 per hour by 1997. (a) Find a simple exponential function of the form $y = ab^t$ that models the minimum wage for 1938 through 1997. (Approximate b to 4 decimal places.)
(b) Use your function to find the minimum wage in 2016 in Indiana, if this function was still relevant. (Indiana's minimum wage in 2016 is $7.25.)

<u>Ex 16:</u> A sample of plutonium-240 decays according to the model $N(t) = 34.52e^{-0.000106t}$ where N is the amount remaining in grams and t is time in days.

(a) How much is in the initial sample?

(b) What amount will remain after 68 days?

(c) What is the amount after 182.4 hours?

(d) What is the **percent** that remains t after 4 years? Round to 2 decimal places.

<u>Ex 17:</u> The population of a certain country starting in 1980 can be modeled by $P(t) = 4.03e^{0.011t}$ where P is in millions of people and t is the time in years. Round the answers below to the nearest hundredth of a million.

(a) What was the population in 1980?

(b) What was the population in 2010? 2016?