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## *Treasury Securities*

In this chapter we will learn about

- Treasury Bills,
- Treasury Notes and Bonds,
- Strips,
- Treasury Inflation Protected Securities,
- and a few other products including Eurodollar deposits.

The primary focus is on the price conventions in these markets. We also will investigate arbitrage restrictions that exist between strips and coupon bonds.

Debt issued by the Treasury with maturities of one year or less are issued as discounted securities called Treasury bills (T-Bills). Securities with maturities greater than a year are coupon securities. Coupon securities with maturities less (greater) than 10 years are called Treasury notes (bonds).

### **2.1 TREASURY BILLS**

A Treasury bill is a zero coupon bond with a maturity of less than one year. Treasury bills are issued in increments of \$5,000 above a minimum amount

of \$10,000. Over \$200 billion are issued per year. T-bills are unusual in that their prices are not directly quoted. The Wall Street Journal, for example, ranks T-bills by maturity. The maturity date is followed by “prices” expressed in bank discount yield form,  $d_y$ , where:

$$d_y = \frac{F - B}{F} \times \frac{360}{m}$$

where  $m$  - number of days to maturity

F - Face value of the Bill

B - Price of the Bill.

Given the discount yield, the invoice price is given by:

$$B = F \times \left[1 - \frac{m \times d_y}{360}\right]$$

For example, if the bank discount yield is 6.78%, and the maturity is 160 days, then the price per \$100 of face value is:

$$B = 100 \times \left[1 - \frac{160 \times 0.0678}{360}\right] = 96.98667$$

The discount yield divides the dollar gain by the face value and so is not a good indicator of return. In addition, the discount is based on a 360 day year. The difference between the bid and asked discount yield is the profit margin of the dealer. The bid ask spread is usually very small, perhaps two basis points.

In order to make yields of T-Bills comparable to yields of other types of coupon bonds, namely T-Notes and T-Bonds, market participants often compute the *Bond Equivalent Yield* (BEY) of the investment. The BEY not only corrects the shortcomings of the bank discount yield but also attempts to set up a yield measure which is comparable to yield measures of coupon bonds with semi annual payments based on a 365 day year. The bond equivalent yield is given by:

$$BEY = \frac{(100 - B)}{B} \times \frac{365}{m}$$

The reported BEY is based on the ask discount quotes.

Actually, the market convention for computing a BEY for a T-Bill with  $m > 182$  days is more complex. Market convention attempts to establish a bond equivalent yield for this T-Bill so that comparisons can be made with other bonds that pay coupons semi-annually. To establish this value we pretend interest of  $y$  is paid after 6 months and that it is possible to reinvest this interest for the remaining time to maturity. Hence, beginning with  $B$  dollars, after six months we would have  $B(1 + y/2)$ . Over the remaining  $n = (m - 365/2)$  days this value is assumed to grow to  $B(1 + y/2)(1 + n \times y/365)$ . Setting

this value to the face value and solving for  $y$  leads to the BEY. Hence:<sup>1</sup>

$$B(1 + \frac{y}{2})[1 + n \times \frac{y}{365}] = F$$

Table 2.1 displays typical T-Bill prices quotations as reported in the financial press.

*Table 2.1* Selected T-Bill Price Quotations on January 4th 1999

Maturity Date	Days to Mat.	Bid	Ask	Ask Yield
Jan 14 99	9	3.92	3.84	3.90
Jan 21 99	16	4.53	4.45	4.52
Jan 28 99	23	4.38	4.30	4.37
April 1 99	86	4.44	4.43	4.54
April 8 99	93	4.42	4.40	4.51
July 1 99	177	4.38	4.37	4.54
Dec 9 99	338	4.39	4.38	4.58
Jan 6 00	366	4.33	4.32	4.53

Consider the April 1 99 T-Bills, with 86 days to maturity. The asked discount yield is 4.43. This implies the cost per \$100 face value is  $100 \times [1 - \frac{86 \times 0.0443}{360}] = \$98.94172$ . The bond equivalent yield is 4.54%, and is indicated in the last column.

### Example

Consider a 13-week \$10,000 face value T-bill with an asked discount yield of 8.88%.

- The cost of this bond is

$$B = F[1 - (m/360)d_y] = 10,000[1 - (91/360)0.0888] = \$9,775.53$$

- The bond equivalent yield is

$$\frac{F - B}{B} \times \frac{365}{m} = \frac{10,000 - 9,775.53}{9,775.53} \times \frac{365}{91} = 9.21\%.$$

<sup>1</sup>An alternative formula might require

$$B(1 + y/2)^{1 + \frac{2n}{365}} = 100$$

but this is not the market convention here!

- The effective annual rate of return compounded on a daily basis is

$$\left[\frac{10,000}{9,775.53}\right]^{365/91} - 1 = 0.0953 \text{ or } 9.53\%$$

- The effective annual rate of return compounded on a continuous basis is:

$$\log\left[\frac{10,000}{9,775.53}\right] \times \frac{365}{91} = 0.0911 \text{ or } 9.11\%$$

## 2.2 TREASURY NOTES AND BONDS

T-bills are short term instruments that pay no coupons prior to maturity. We now turn attention to coupon bonds issued by the treasury. All treasury bonds are identified by their coupon and maturity. The 10 1/2 of 1999 means the Treasury bond with a 10 1/2% annual coupon rate that matures in 1999. The dollar amount of interest paid per year is 10 1/2% of the face value. In practice the coupon are paid in two equal installments, six months apart.

Quotations for Treasury notes and Treasury bonds are usually reported together, ordered by maturity. The bid and ask prices are reported in a special form. For example a quotation of 86 – 12 means the price is 86 12/32% of face value. A plus sign following the the number of 32nds means that a 64th is added to the price. To actually purchase a bond, the investor must pay the asked price together with accrued interest since the last coupon payment. Accrued interest is computed by taking the size of the last coupon payment and multiplying by the ratio of the actual number of days since the last coupon payment, relative to the actual number of days in the coupon period.

Table 2.2 illustrates typical price quotations of selected notes and bonds.

As an example, consider the November 2008 Treasury note with coupon 4 3/4. The “n” indicates the contract is a note rather than a bond. The bid quotation price is 100 16/32 and the ask quotation price is 100 17/32. Interest payments are semiannual on the 15th of November and 15th May. The ask invoice price is the asking quotation price plus the accrued interest. Using this price the yield to maturity is computed as 4.68%.

Some T-bonds are callable, with the first-call date coming 5 years before the bond matures. Newspaper quotations typically indicate if the bond has a call feature. For callable bonds, the yield to maturity is reported if the price is below par. If the price is above par, then the yield to the call date is computed. Over the last decade, however, the auctions have not included callable Treasury securities, so the number of such contracts is diminishing.

Table 2.2 Selected Note and Bond Price Quotations for Settlement on January 6th 1999

Rate	Maturity (Mo/Yr)	Bid Price	Ask Price	Ask Yield
6 3/8	Jan 99n	100:00	100:02	3.95
5	Jan 99n	99:31	100:01	4.46
5 7/8	Jan 99	100:01	100:03	4.43
5	Feb 99n	100:00	100:02	4.35
8 7/8	Feb 99n	100:13	100:15	4.49
7 1/8	Feb 00n	102:19	102:21	4.72
4 5/8	Dec 00n	100:02	100:03	4.5
6 1/4	Feb 02n	104:10	104:12	4.74
4 1/4	Nov 03n	98:20	98:21	4.56
4 3/4	Nov 08n	100:16	100:17	4.68
8 1/8	Aug 21	134:12	134:18	5.45
5 1/4	Nov 28	101:16	101:17	5.15

Viewed from a coupon date, and including the date 0 coupon, the yield-to-maturity of a Treasury bond is linked to its market price by the usual bond pricing equation:

$$B_0 = \sum_{j=0}^{m-1} \frac{C/2}{(1+y/2)^j} + \frac{100+C/2}{(1+y/2)^m}$$

where  $100y\%$  is the annual yield to maturity,  $C$  is the annual coupon rate and  $m$  is the number of coupon payouts remaining to maturity.

If, however, the time to the first coupon date is not exactly 6 months, then the bond pricing equation must be modified. Specifically, the pricing equation is given by

$$B_0 = \frac{1}{(1+y/2)^p} \left[ \sum_{j=0}^{m-1} \frac{C/2}{(1+y/2)^j} + \frac{100}{(1+y/2)^{m-1}} \right]$$

where  $p = t_n/t_b$  and  $t_n$  is the number of days to the next coupon payment, and  $t_b$  is the number of days from the last coupon date to the next coupon date.

The price that an investor pays is the quoted price, together with the accrued interest. The accrued interest,  $AI$  say, equals the proportion of the current coupon period that has elapsed, times the coupon size. That is:

$$AI = [1-p]C/2$$

and

$$B_0 = Q_0 + AI$$

where  $Q_0$  is the quoted bond price.

The total price paid for a bond is often referred to as the *full price* or *dirty price* while the quoted price is often called the *flat price* or *clean price*

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### Example

(i) Consider a T-bond with a 10% coupon which pays out 20 more coupons. The time from the last to the next coupon date,  $t_b$ , is 183 days. Since the last coupon was paid 83 days before the settlement date, the time to the next coupon,  $t_n$ , is 100 days. The quoted price for the bond,  $Q$ , based on \$100 face value is 104 : 08. In decimal form the price is  $104 \frac{8}{32} = \$104.25$ . To obtain the market price,  $B_0$ , the accrued interest must be added. Since the last coupon was paid 83 days ago, the accrued interest per \$100 face value,  $AI$ , is given by

$$AI = [83/183] \times 5 = \$2.26775$$

The actual market price for the bond is its quoted price plus accrued interest. That is

$$B_0 = 104.25 + 2.26775 = \$106.51775.$$

The yield to maturity,  $y$ , is obtained by solving the yield in the bond pricing equation.

(ii) Consider the Feb 2000 T-note that pays a  $7 \frac{1}{8}$  coupon in the table above. The quoted price is  $102 \frac{21}{32} = \$102.656$ . There is 183 days in this coupon interval and there are 39 days remaining till the next coupon is paid. The previous owner is entitled to  $183 - 39 = 144$  days of interest. The accrued interest is

$$AI = 3.5625 \times \frac{144}{183} = \$2.803$$

The market price of the bond is therefore \$105.459. Plugging this value into the left hand side of the bond pricing equation, and solving for the bond equivalent yield provides the answer of 4.72%.

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### Settlement Dates

The *trade date* is the date when parties agree on a price and promise to transact, and the *settlement date* is the date when money and bonds change hands. Treasury bonds usually settle “next day”, or one business day after the trade date. While this is the normal procedure, the actual time to settlement can be negotiated between buyer and seller.

The accrued interest is based from the settlement date, not the trade date. The first day of accrued interest is therefore the day after the settlement date. Hence, if the bond settles on a coupon date, the seller gets the full coupon.

### 2.3 STRIPPED COUPON BONDS

Consider an investor with a one year holding horizon. If the investor purchase a 2 year zero coupon bond, then after one year, the investor has to sell the bond at a price which will depend on interest rates in one years time. The investment has *price* risk. If the investor purchased a one year zero coupon bond, then the strategy is absolutely riskless. The investor will receive the face value in one year, regardless of interest rates. Finally, if the investor purchased a six month T-Bill, with the intent of rolling this investment over into a new 6 month investment, then the return over the year would depend on the reinvestment rate after 6 months. This strategy has *reinvestment* risk.

Zero coupon bonds are attractive investments for investors such as pension funds and insurance companies who have fixed obligations in the future and want to ensure that these obligations will be met regardless of future interest rates. Zero coupon bonds permit terminal values to be locked in at known rates of return without incurring price and reinvestment risk. Given the likelihood of sizable demand for zeros by pension funds and insurance firms, for example, it is somewhat surprising that such few zeros have been issued. Indeed, apart from short term Treasury Bills, the US Treasury has not issued any zeros.

Financial intermediaries do construct securities which resemble zero coupon debt. Beginning in 1982, the practice of stripping US Treasury securities developed. The first two big strippers were Merrill Lynch, who created TIGRS (Treasury Investment Growth Receipts) and Salomon Bros. who created CATS (Certificates of Accrual on Treasury Receipts). These firms, would purchase Treasury bonds, place them in trust, and then sell claims on the individual cash flows. If a 10 year bond with 20 semi-annual coupons is stripped, the underlying bond is resold as 21 zeros (20 coupons and a principal strip) each of which trades separately in the securities market. In a perfect market, the portfolio of the individual strips has the same value as the whole bond. Differences in values could occur if tax treatments were different. Indeed, the creation of this product by Merrill Lynch and Salomon was primarily motivated by the favorable tax treatment of strips, and the perception that there was an unsatisfied clientele who demanded zeros, and who were prepared to pay a premium for them.

In practice, the cost of stripping and reselling the packages of zeros was quite high. First, trust accounts had to be set up. Second, buyers for the strips had to be found. Third, interest rate risk over the interim period had

to be hedged. In order to reduce these costs and expedite stripping, in 1985, the US Treasury designated some Treasury securities eligible to be stripped through the Federal Reserve book entry system. The resulting zero coupon bonds, called STRIPS (Separate Trading of Registered Interest and Principal of Securities) now dominate this market because it is the least costly form of stripping.

As an example assume the date is January 4th and that a financial institution is holding a 100,000 dollar face value T-note that pays 5% coupons and matures in two years. This bond will pay \$2500 in January 15th and July 15th of this year and the next, as well as the terminal 100,000 dollars. If this bond is delivered to the Treasury to be stripped, then five Strips will be created. The first four strips, each having a face value of 2500 dollars, are called coupon strips or C-strips, while the 100,000 dollar two year strip, with maturity on July 15th of next year, is called a principal strip or P-strip. Each of these strips are assigned a CUSIP number and can trade separately.

Institutions are allowed to unbundle or reconstitute these bonds. For example, an investor could deliver these five strips to the Treasury and request that the original 5% coupon bond be reconstituted. Actually, in reconstituting a bond, any C-strips can be used for the coupon payments, but the principal payments must come from the appropriate P-strip. That is C-strips are completely fungible, but P-strips are not. Specifically, any particular P-strip can only be used to reconstitute a particular bond.

Table 2.3 shows selected prices of STRIPS on January 4th 1999.

There are three types of STRIPS. Stripped coupon interest (ci), treasury bond stripped principal (bp) and treasury note stripped principal (np). Principal strips and interest strips may differ by a small amount. However, arbitrage forces ensure that they do not differ by enough to make purchasing the cheaper and selling the expensive profitable.<sup>2</sup>

A plot of the strip prices, with face values equal to one dollar, against maturity yields a graph that should be a decreasing function, starting from one (at date 0) and declining towards zero. This information is very useful for establishing a discount function, but is not the only set of data that provides information on discount factors. Clearly, T-Bill information adds to the information set. Also, coupon bonds provide information on discount factors. We will explore methods for estimating and constructing a reliable discount function in a later chapter.

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### Example

<sup>2</sup>A distinction is made between STRIPS created from coupons and principal because of tax treatments by non US entities.



Table 2.3 Selected Prices of Strips

Maturity	Type	Bid	Ask	Ask Yield
Feb 99	ci	99:16	99:16	4.62
May 99	ci	98:12	98:12	4.60
May 99	np	98:12	98:12	4.62
Aug 99	ci	97:08	97:08	4.62
Aug 99	np	97:08	97:08	4.63
Nov 99	ci	96:05	96:05	4.62
Nov 99	np	96:04	96:04	4.66
Feb 00	ci	95:01	95:01	4.64
Feb 02	ci	86:17	86:19	4.68
Feb 03	ci	82:15	82:18	4.72
Feb 03	np	82:16	82:19	4.71
Feb 06	ci	71:02	71:07	4.84
Feb 06	np	71:21	71:26	4.72
Feb 06	bp	71:13	71:18	4.76

Consider the strip maturing on February 15th 2000. Its price is  $B_0 = 95 \frac{1}{32}$  and its yield is 4.64%. The current trade date is January 4th with the settlement date January 6th, 1999. Take the February 2000 strip as an example. The ask price is 95:01 or  $95 \frac{1}{32}$ . The yield is semiannual based on coupon dates on Feb 15th and Aug 15th. There are 183 days between the last coupon date and the next February coupon date. There are 40 days until Feb. 15th. Hence, the number of semiannual periods to the next “coupon” date is  $p = 40/183$ . Thereafter, there are exactly 2 “coupon dates” to maturity.

The price of the strip is connected to its yield by:

$$B_0 = \frac{100}{(1 + \frac{y}{2})^{2 + \frac{40}{183}}}$$

Using the bond pricing equation should lead us to the asking yield of 4.64%.

When the strip has a maturity less than the current coupon period, the market convention is to quote a simple interest rate.

Table 2.4 Prices of Selected Strips

Maturity	Type	Bid	Ask	Discount Factor
Feb 99	ci	99:16	99:16	0.9950000
Aug 99	ci	97:08	97:08	0.9725000
Feb 00	ci	95:01	95:01	0.9503125
Aug 00	ci	92:28	92:29	0.9290625
Feb 01	ci	90:24	90:25	0.9578125
Aug 01	ci	88:20	88:21	0.8865625
Feb 02	ci	86:17	86:19	0.8659375
Aug 02	ci	84:15	84:17	0.8453125
Feb 03	ci	82:15	82:18	0.8256250

## 2.4 ARBITRAGE RESTRICTIONS BETWEEN STRIPS AND COUPON BONDS

Table 2.4 shows the prices of selected strips, separated by six months intervals, that existed for settlement on January 6th 1999. The discount factor is based on the ask price. For example the discount factor for cash flows in February 15th 2000, is based on the price 95 1/32 or \$95.03125. One can view the value 0.9503125 as the present value, as of January 6th 1999, of \$1 which is obtained on February 15th 2000.

The plot of these discount factors against their maturity provides one possible estimate of the discount function.

Now consider the February 2000 T-note that pays 7 1/8. It has a quoted price of 102 21/32 or \$102.656. There are 183 days in the coupon interval and 40 days remain until the next coupon. Hence the accrued interest is

$$AI = 3.5625 \times \frac{144}{183} = \$2.7838$$

The market price of the bond is therefore \$105.440.

Using the above discount factors, the theoretical price of a one hundred dollar face value bond can easily be computed. Cash flows of \$3.5625 are due in Feb 99, Aug 99 and \$103.5625 are due at maturity. Consider a portfolio of 3.5625 strips with a face value of \$1 that matures in Feb 99, 3.5625 strips that mature in August 99 and 103.5625 strips that mature in Feb 2000.

Clearly, this portfolio of strips provides the exact same cash flows as the coupon bond. Hence, to avoid riskless arbitrage, it should follow that the price of the portfolio of strips equals the price of the coupon bond. Using the discount factors we compute the price of the strip portfolio as:

$$(0.995 + 0.9725) \times 3.5625 + (0.9503125) \times 103.5625 = 105.426.$$

As can be seen the portfolio of strips that replicates the T-note has almost identical value. Arbitrageurs will ensure that the market for T-notes and T-bonds is closely connected with the STRIP market.

## 2.5 INFLATION INDEXED TREASURY SECURITIES

In 1997 the US Treasury began to issue Treasury securities that adjust for inflation. These securities are referred to as Treasury Inflation Protection Securities or TIPS. Issues with maturities of 2, 5, 10, and 30 years have been made. Each contract has a fixed coupon rate, called the real rate. Let  $c$  represent this annual rate. Cash flows occur every 6 months. The adjustment for inflation is made by altering the principal every 6 months according to the inflation. For example, assume the initial principal was \$100, and the coupon (real) rate was  $c = 3.5\%$  per year. Now assume the inflation rate came in at  $i = 4\%$  per year. Then the inflation adjusted principal, IAP, say, would be increased from  $IAP = 100$  to  $IAP = 100(1 + \frac{1}{2}i) = 100(1.02) = \$102$ , and the cash flow would be  $\frac{c}{2} \times IAP$ , or  $\frac{0.03}{2} \times 102 = \$1.53$ . At the maturity date the final inflation adjusted principal is returned to the investor. Because of possible disinflation, the inflation adjusted principal may turn out to be less than the initial par value. The Treasury has structured their inflation protected bonds in such a way that the final principal payment is the greater of the final inflation adjusted principal or the initial par value.

The inflation index used for the adjustments is the nonseasonally adjusted Consumer Price Index for all Urban Consumers, CPI-U), with a three month lag. This index measures the average change in prices paid by urban consumers for a fixed basket of more than 360 categories of goods and services.

Table 2.5 shows prices of selected TIPS on December 27th 1999:

The April 2029 TIPS has a real rate coupon of 3.875%. The ask price is 93-15/32 or 93.46875% of principal. The accrued or inflation protected principal is 1.023 times greater than the original principal. The yield to maturity, is the bond equivalent yield based on the current principal.<sup>3</sup>

TIPS provide investors with a risk free opportunity to efficiently protect themselves against a common measure of inflation risk.

<sup>3</sup>This is the market convention used for obtaining yields in this market.

Table 2.5 Selected TIPS Prices

Rate	Maturity	Bid/Asked	Yield	Accrued Principal
3.625	07/02	98-30/31	4.053	1051
3.375	01/07	94-05/06	4.34	1062
3.625	01/08	95-06/07	4.334	1042
3.875	01/09	96-18/19	4.333	1026
3.625	04/28	89-11/12	4.276	1040
3.875	04/29	93-14/15	4.267	1023

## 2.6 OTHER SHORT TERM MONEY MARKET INSTRUMENTS

Below is a very brief description of the more important short term instruments.

### Demand Deposits

Interest bearing bank deposits are offered by commercial banks and savings and loan institutions. Typically, these accounts are guaranteed by agencies of the government. The rate of interest varies with market conditions. Investors can withdraw funds at any time.

### Time Deposit and Certificates of Deposits

In a time deposit, the interest rate is guaranteed over the time period. If the investor withdraws the funds prior to maturity a penalty is incurred. A Certificate of Deposit, or CD, is similar. CDs are issued in standard denominations. CDs can be sold in a secondary market.

### Eurodollar Deposits

Eurodollar deposits (ED) are dollar deposits that are maintained out of the United States. Eurodollar deposits in foreign banks and in foreign branches of US banks are not subject to the Federal Reserve regulations that apply to domestic deposit markets. Eurodollar deposits are direct obligations of the commercial banks and are not guaranteed by any government. Due to the credit risk of the issuing bank, and due to political risk, Eurodollar deposits typically provide a higher rate of return than T-bills of equal maturity. The rates that applies to Eurodollar deposits in interbank transactions are known as the London Interbank Offered Rates or LIBOR rates. The LIBOR spot

market is active in maturities ranging from a few days to 10 years. The market is especially liquid in the 3 month and six month maturities. Interest on ED deposits is simple based on an “Actual/360” day count convention. The interest payment on an  $m$ -day ED deposit of  $P_0$  is:

$$\text{Interest} = P_0 \times \text{LIBOR} \times \frac{m}{360}$$

where  $L \times 100$  is the LIBOR rate in percentage terms.

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**Example**

Consider a 3 month \$1m Eurodollar time deposit. Assume the 3 month LIBOR rate is 10%. Then, after 3 months (90 days say) the interest will be  $1,000,000 \times (10/100) \times (90/360) = \$2500$ . If LIBOR was 10.01% then the interest would be an additional \$25. Hence each basis point corresponds to \$25.

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The market for ED deposits is one of the largest and LIBOR rates have become the benchmark from which many other rates are derived. Swaps, and other derivative contracts that are sensitive to LIBOR rates are very actively traded and will be considered later.

**Commercial Paper**

These are unsecured short term loans to corporations. They are discounted securities like Treasury Bills. Corporations with strong credit reputations are able to borrow directly from the public at rates that are lower than rates that they can get by borrowing from banks. Highly rated companies are able to borrow at sub LIBOR rates.

**2.7 EXERCISES**

1. Consider the February 2002 Treasury note with a coupon of 6 1/4 in Table 2.2. Compute its accrued interest as of the January 6th settlement date. Also, confirm its yield to maturity.
2. Use Table 2.4 of coupon interest STRIP prices in the chapter, to come up with the value of a STRIP portfolio that replicates the payouts of the Treasury note in Question (1). What is the difference in value between the strip portfolio and the T-note? Do you think there are arbitrage opportunities here? Discuss.
3. Refer to Table 2.1 of T-Bill quotations in this chapter. Compute the bond equivalent yields for the 93 day bill, the 177 day bill and the 338 day bill.
4. In this problem we will learn how to compute T-bond prices using excel. In particular, we will write a program that will compute the price of a coupon bond given the settlement date, maturity date, coupon, and yield to maturity. Proceed as follows.

- Open up excel. In the first four rows enter a settlement date, a maturity date, a coupon rate and a yield to maturity. These are the basic inputs. Make sure that the format for the first two dates are in DATE format.

If you want to know the number of days between these two dates, just subtract the two cells. Make sure that you format the cell, where the answer is stored as, general. Now, we can use this date feature to compute accrued interest for a coupon bond. All we need is the last coupon date and the next coupon date. Since coupons are paid semiannually, given the maturity date, we can easily work out when the last coupon was paid out. It is always on the 15th day of the appropriate month.

- In the next row of your spreadsheet, enter the last coupon date. Underneath this enter the next coupon date. Now, by subtracting the appropriate entries, you can compute the number of days in the coupon period, the number of days since the last coupon, and the number of days till the next coupon. With this information you can compute the accrued interest, and the ratio,

$$p = \frac{\text{days to next coupon}}{\text{days between coupon payments}}.$$

- Now, using the settlement and maturity dates compute the number of coupon payments remaining.

- Now, pretend we are at the next coupon date. Use the analytical form for the coupon bond price to obtain its price. (Recall the present value of the coupons can be computed as an annuity). Make sure you take into consideration the “current” coupon. Then, discount this value back using the  $p$  value computed earlier. This gives you the market price of the bond.
  - To establish the clean price subtract out the accrued interest. This gives you a clean price in decimal form. To compute the quoted price in 32nds you can use the built in function DOLLARFR( ). This function has two arguments. The first, is the dollar price in decimal form. The second argument is the value 32.
  - Use some of the price information given in Table 2.2 to validate that your routines work.
5. Excel has a built in function that automates this entire procedure. The function name is PRICE(). It has several arguments: The settlement date, the maturity date, the coupon, the yield, the face value, the number of coupons per year and the time convention. For our purposes, the face value can be set at 100, the number of coupons per year is 2, and the market convention is actual/actual, which in excel is coded as 1. Using a few of the bonds from Table 2.2 to make sure that your results are correct. The excel function PRICE just automates what you have done in the previous problem.
6. In Excel, there is a built in function called YIELD. Its arguments are the same as PRICE, except it takes in the quoted price, and spits out the yield to maturity, rather than the reverse. Use YIELD on a few of the bonds from Table 2.2 to obtain the yields to maturity.
7. There are a number of other functions built into excel that are useful for T-bond calculations. These include:
- DOLLARDE converts quotes to decimals
  - DOLLARFR converts decimal quotes to prices in 32nds.
  - COUPNCD computes the next coupon date.
  - COUPPCD computes the previous coupon date.
  - COUPDAYSBS computes the number of days since the last coupon
  - COUPDAYS computes the number of days between the last and next coupon.

Use some of these functions to compute accrued interest on some of the bonds in Table 2.2.

8. In this problem we will learn how to compute T-bill prices and yields using excel. In particular, we will write a program that will compute

the price (or yield) of a T-bill given the settlement date, maturity date, and yield to maturity (or price). Proceed as follows.

- Enter the settlement date, maturity date, and bid discount yield.
  - Use TBILLPRICE function to compute the price of the T-Bill
  - Compute the number of days to maturity.
  - Use TBILLEQ function to compute the bond equivalent yield.
  - Use Table 2.1 to validate some of your results.
9. In this problem we will learn how to compute prices and yields of T-bonds and notes that have only one payment to go. These bonds are referred to as short governments.
- Enter the settlement, maturity and issue date of the bond, as well as the coupon.
  - Enter the yield.
  - Use the function PRICEMAT() to price the security.
  - If the price is given, use the function YIELDMAT() to establish the yield.
  - Check your results using information from a financial newspaper such as the Wall Street Journal.



## Appendix

### 2.8 DETAILS OF CALENDAR CONVENTIONS

#### (1) Actual/Actual Calendar

Suppose the coupon bond pays coupons every 6 months, and that the settlement date is between coupons.

- Price the bond as if you were at the next coupon date. Make sure to add in the coupon that is received on that date.
- Present value this figure using the bond pricing equation. The fractional period is determined based on the number of days.

$$p = \frac{t_n}{t_b}$$

The discount factor is

$$\frac{1}{(1 + y/2)^p}$$

#### (2) 30/360 Calendar

Let date 1 be the early date and date 2 the late date. Let  $Y_i$ ,  $M_i$  and  $D_i$  be the year, month, day date for  $i = 1, 2$ .

Then

- $d_1 = 30$  if  $D_1 = 31$
- $d_1 = 30$  if  $D_1 \geq 28$  and  $M_1 = 2$  and date 1 is a coupon date.
- $d_1 = D_1$  otherwise. Let
- $d_2 = 30$  if  $D_2 = 31$  and  $D_1 \geq 30$
- $d_2 = D_2$  otherwise.
- Then, the number of days between date 1 and 2 is:

$$n_{days} = 360(Y_2 - Y_1) + 30(M_2 - M_1) + (d_2 - d_1)$$

Now:

- Compute the days in 30/360 form between settlement to next coupon,  $t_n$ .
- Compute the days in 30/360 form between coupons, (last coupon to next),  $t_b$ .
- Compute  $p = \frac{t_n}{t_b}$ .
- Use the bond pricing equation.

Corporate, Municipal and Federal Agency bonds use this method.

### (3) Actual/360 Calendar

- Compute the actual days between settlement to maturity,  $t_{s,m}$ .
- Compute the actual days between issue to settlement,  $t_{i,s}$ .
- Compute the actual days between issue to maturity,  $t_{i,m}$ .
- Redefine these numbers by dividing them by 360.

For a CD with coupon  $c$ , quoted in annual terms, the interest payment is:

$$100 \times c \times \frac{t_{i,m}}{360}$$

Given a simple interest quote of  $y$ , the full dollar price of a \$100 par amount of a CD with coupon rate  $c$  is

$$B = 100 \times \frac{1 + c(t_{i,m}/360)}{1 + y(t_{s,m}/360)}$$

The accrued interest is

$$100 \times c \times \frac{t_{i,s}}{360}$$

and the flat price is obtained by subtracting the accrued interest from the full price.